COMPARING DIRECT AND EFFECTIVE METHODS OF QCD RESUMMATION

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OUTLINE

Based on: Leandro Almeida, Steve Ellis, CL, George Sterman, Ilmo Sung, and Jonathan Walsh, JHEP 1404 (2014), 174 [arXiv:1401.4460]

- Resummation in Soft Collinear Effective Theory
- Correspondence between QCD and SCET Resummation
- Improved Accuracy in Differential Momentum-Space Distributions

LARGE LOGARITHMS IN THRUST

- Take as a concrete example e^+e^- thrust distribution. Computing in fixed-order perturbation theory, we encounter large logs in the two-jet kinematic region:

$$\sigma(\tau) \equiv \int_0^\tau d\tau' \frac{1}{\sigma_0}\frac{d\sigma}{d\tau'} = 1 + \frac{\alpha_s}{4\pi}\Big(F_{12}\ln^2\tau + F_{11}\ln\tau + F_{10}\Big)$$
$$+ \left(\frac{\alpha_s}{2\pi}\right)^2\Big(F_{24}\ln^4\tau + F_{23}\ln^2\tau + F_{22}\ln^2\tau + F_{21}\ln\tau + F_{20}\Big)$$

- Need to reorganize perturbative series to regain convergence:

$$\begin{aligned}\ln\sigma(\tau) \sim \quad & \alpha_s(\ln^2\tau + \ln\tau) \\ & + \alpha_s^2(\ln^3\tau + \ln^2\tau + \ln\tau) \\ & + \alpha_s^3(\ln^4\tau + \ln^3\tau + \ln^2\tau + \ln\tau) \\ & + \quad \vdots \qquad\qquad \vdots \qquad\qquad \vdots \qquad\qquad \vdots\end{aligned}$$

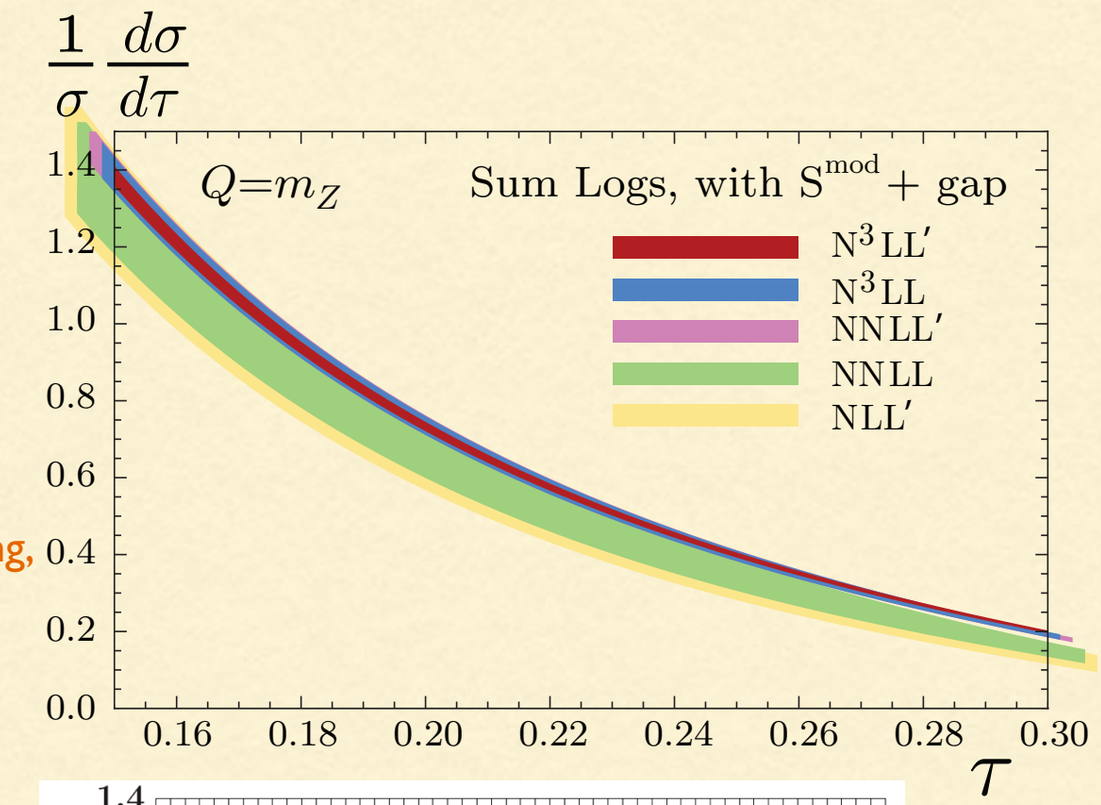
Leading Log (LL) | Next-to-Leading Log (NLL) | NNLL

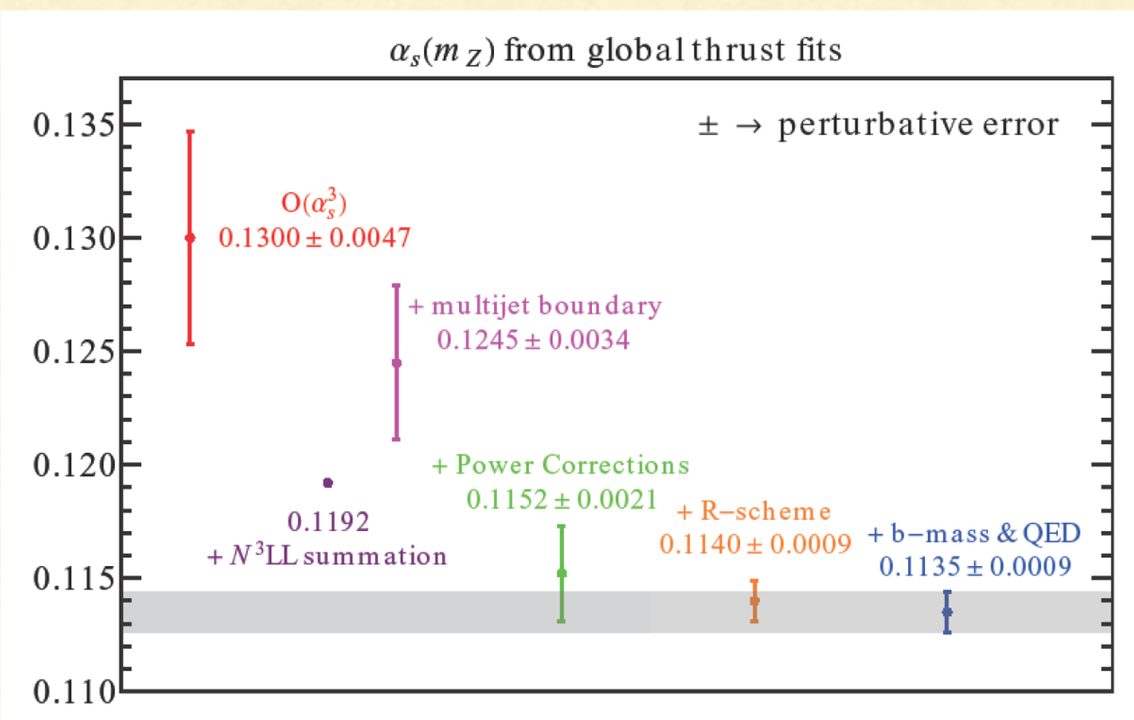
STATE-OF-THE-ART RESUMMATION

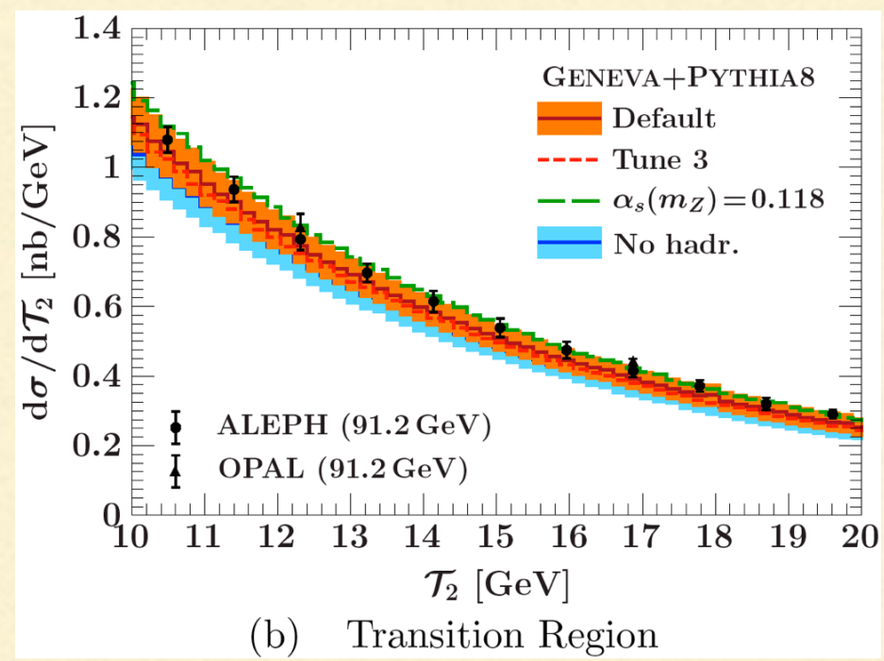
Thrust computed to NNNLL accuracy using Soft Collinear Effective Theory Becher, Schwartz (2008)

NNNLL perturbative results + consistent treatment of nonperturbative power corrections + other corrections used to achieve most precise and reliable extraction of strong coupling from event shapes Abbate, Fickinger, Hoang, Mateu, Stewart (2010)

$$\alpha_s(M_Z) = 0.1135 \pm 0.0011$$







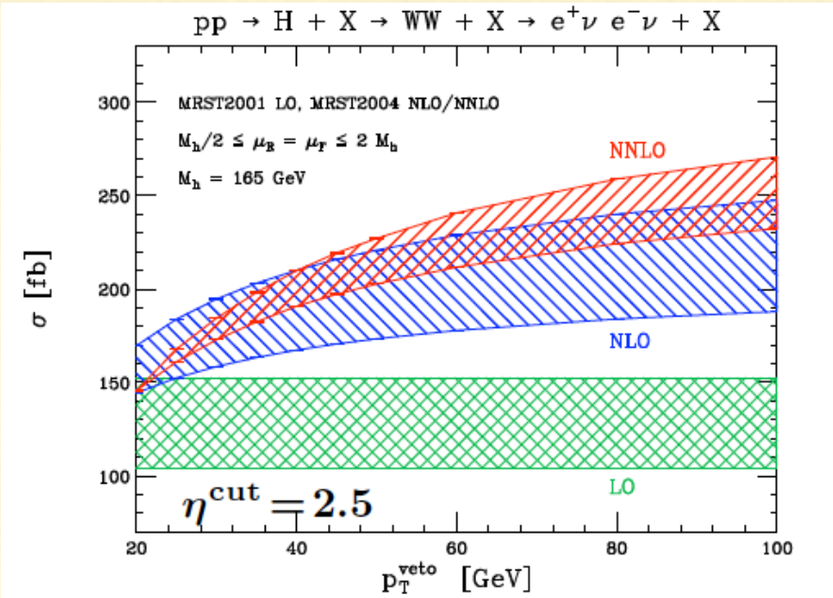
GenEvA Collaboration (2012) fit to LEP data gives consistent value

STATE-OF-THE-ART RESUMMATION

- "Beam thrust" in *pp* collisions as jet veto

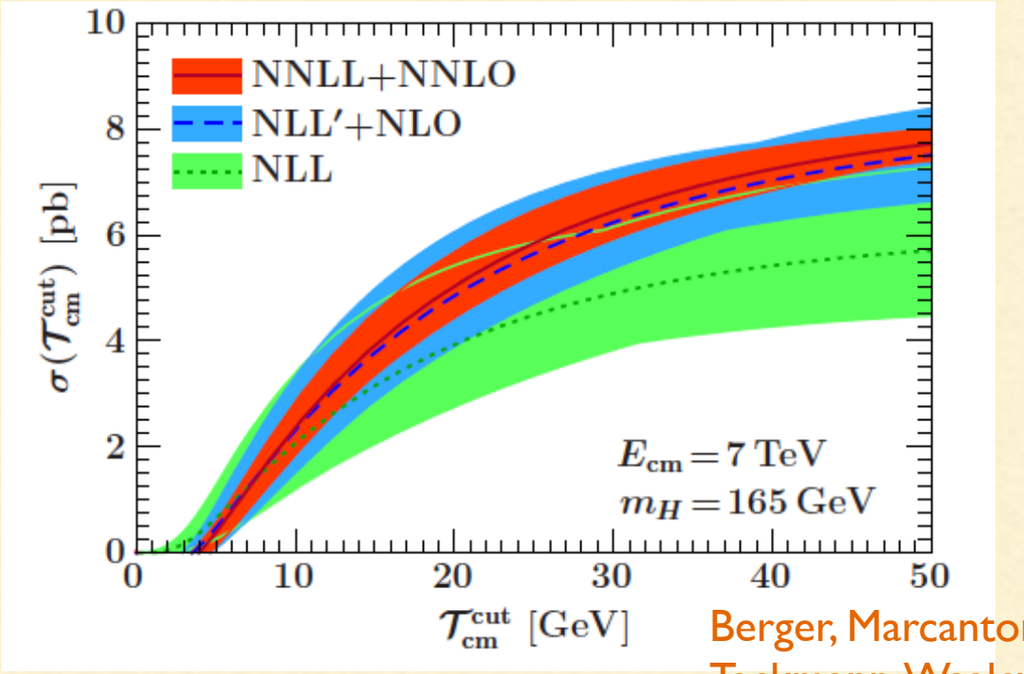
$$\mathcal{T}_{\rm cm} = \sum_k |\vec{p}_{kT}| e^{-|\eta_k|} = \sum_k (E_k - |p_k^z|)$$

Fixed-order:



[Anastasiou, Dissertori, Stöckli (arXiv:0707.2373)]

Resummed in SCET:



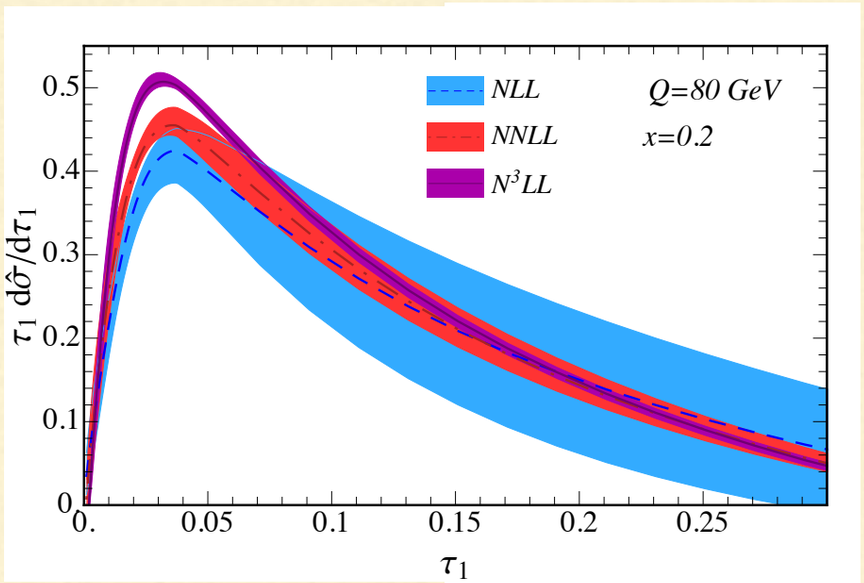
Berger, Marcantonini, Stewart, Tackmann, Waalewijn (2010)

- "1-jettiness" / thrust in DIS:

$$\tau_1 = \frac{2}{Q^2} \sum_{i \in X} \min\{q_B \cdot p_i, q_J \cdot p_i\}$$

Resummed to NLL — Antonelli, Dasgupta, Salam (2000)

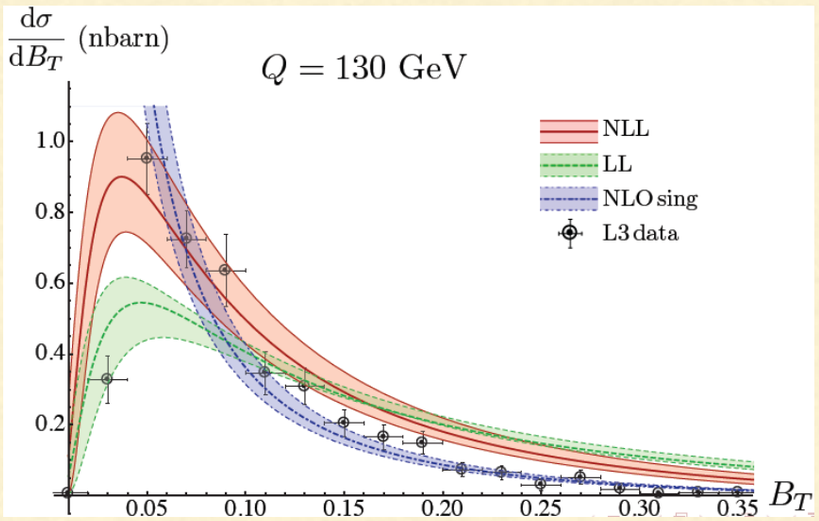
Resummed to NNLL — Z. Kang, X. Liu, S. Mantry, J.-W. Qiu (2013); D. Kang, CL, Stewart (2013)



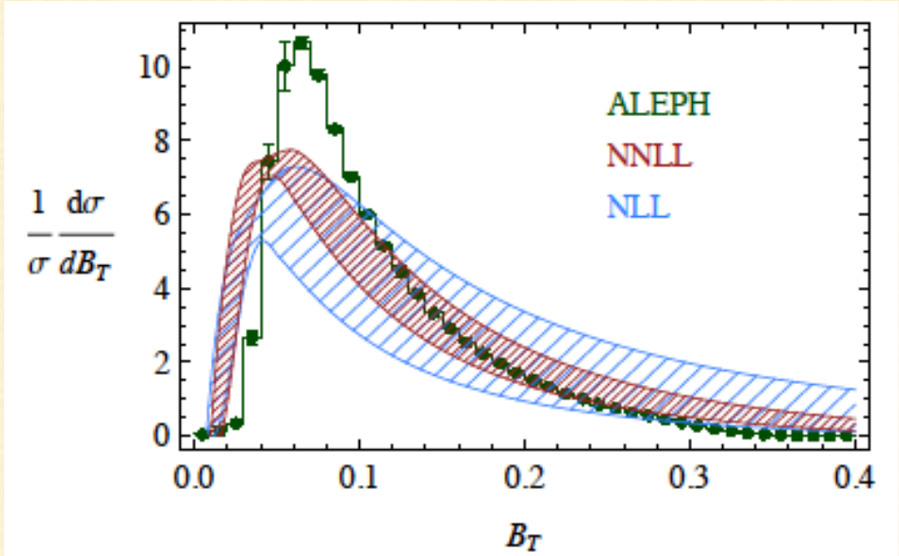
N³LL
D. Kang, CL, Stewart (2014)
preliminary

STATE-OF-THE-ART RESUMMATION

- Other event shapes
  - Jet Broadening at NLL and NNLL



Chiu, Jain, Neill, Rothstein (2012)

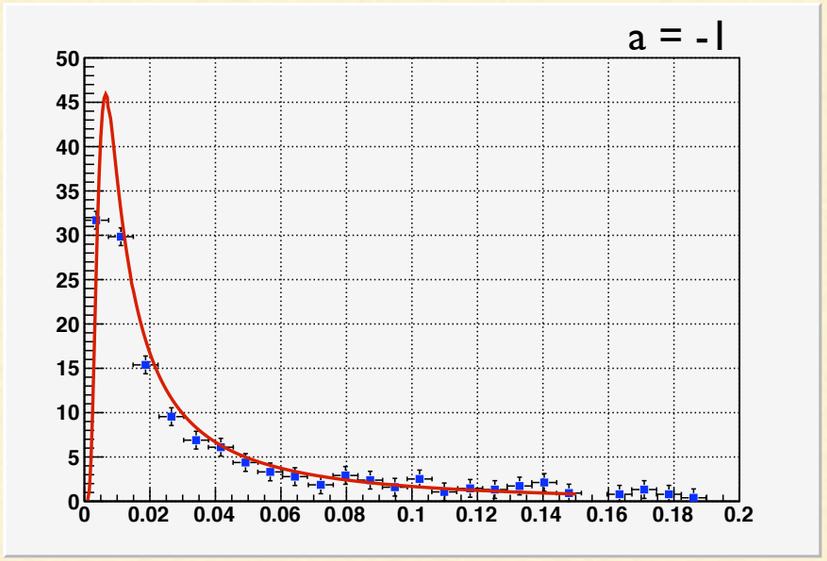


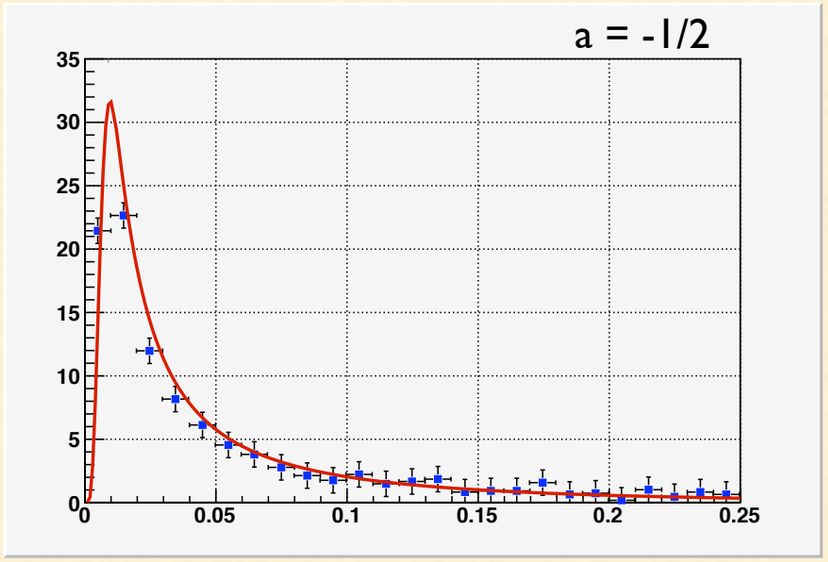
Becher and Bell (2012)

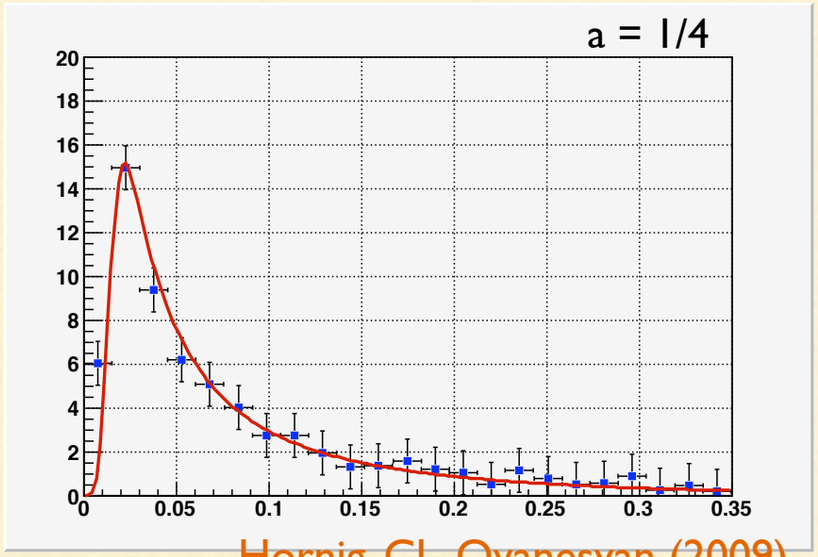
  - Angularities to NLL'

$$\tau_a = \frac{1}{Q}\sum_{i\in X}|\mathbf{p}_T^i|e^{-|\eta_i|(1-a)}$$

Berger, Kucs, Sterman (2003)





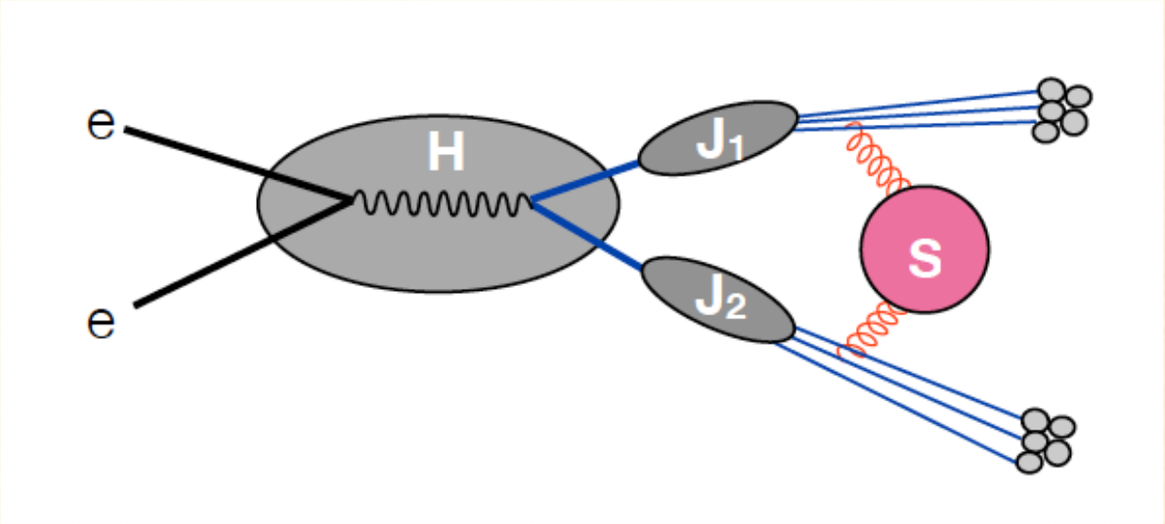


Hornig, CL, Ovanesyan (2009)

FACTORIZATION FOR EVENT SHAPES

Collins, Soper, Sterman

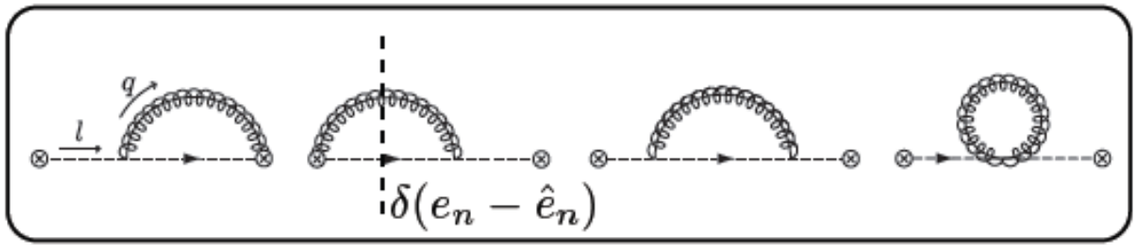
- Two-jet event shapes in e^+e^-, e.g. thrust:



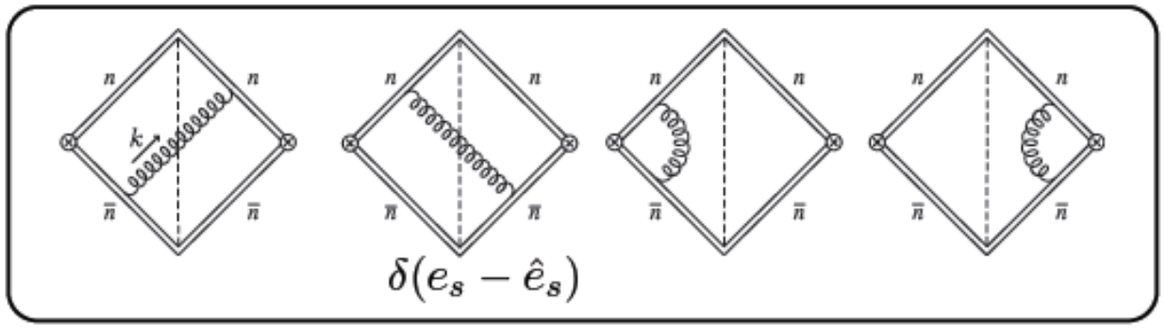
$$\frac{1}{\sigma_0}\frac{d\sigma}{d\tau} = H_2(Q^2,\mu)\int dt_n dt_{\bar{n}} dk_S\, \delta\Big(\tau - \frac{t_n + t_{\bar{n}}}{Q^2} - \frac{k_S}{Q}\Big) J_n(t_n,\mu) J_{\bar{n}}(t_{\bar{n}},\mu) S_2(k_S,\mu)$$

Fleming, Hoang, Mantry, Stewart (2007)
Bauer, Fleming, CL, Sterman (2008)

- Jet Function

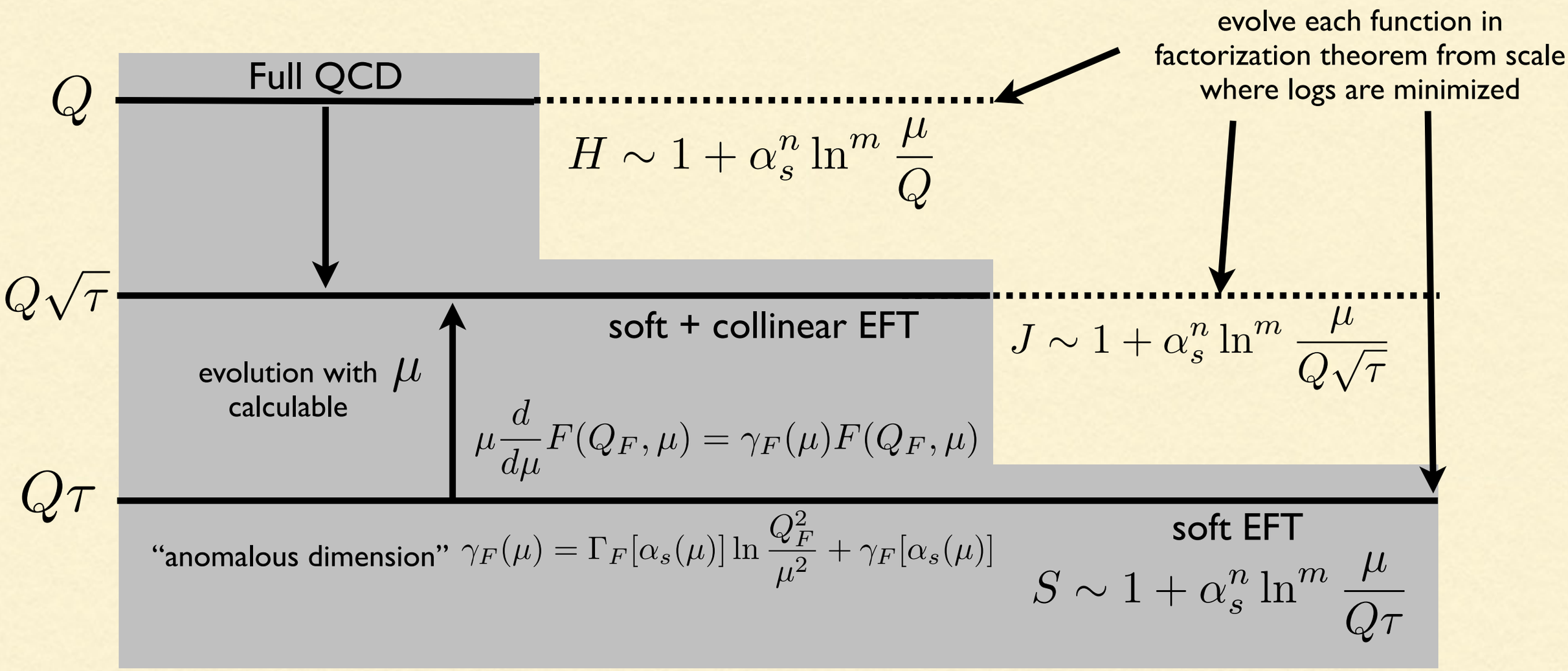


- Soft Function



RESUMMATION FROM RG EVOLUTION

- SCET RG gives equations for evolution of hard, jet, and soft functions in factorization theorem with energy scale μ.



- **Solutions of evolution equations contain logs resummed to all orders in α_s**

RESUMMATION IN DIRECT QCD VS. SCET

- In CTTW (1993) we find the formulae

$$\boxed{\sigma(\tau) = C(\alpha_s)\Sigma(\tau,\alpha_s) + D(\tau,\alpha_s)}$$

$$C(\alpha_s) = 1 + \sum_{n=1}^{\infty}\left(\frac{\alpha_s}{2\pi}\right)^n C_n \qquad \ln\Sigma(\tau,\alpha_s) = \sum_{n=1}^{\infty}\sum_{m=1}^{n+1}\left(\frac{\alpha_s}{2\pi}\right)^n G_{nm}\ln^m\frac{1}{\tau}$$

  - Achieving resummation requires computing the coefficients G_{nm}

- In SCET, we find the formulae

$$\sigma(\tau) = e^{K_H+2K_J+K_S}\left(\frac{\mu_H}{Q}\right)^{\omega_H}\left(\frac{\mu_J}{Q\tau^{1/2}}\right)^{\omega_J}\left(\frac{\mu_S}{Q\tau}\right)^{\omega_S} H_2(Q^2,\mu_H)$$
$$\times\,\widetilde{J}\left(\partial_\Omega + \ln\frac{\mu^2}{Q^2\tau},\mu_J\right)^2\widetilde{S}\left(\partial_\Omega + \ln\frac{\mu_S}{Q\tau},\mu_S\right)\frac{e^{\gamma_E\Omega}}{\Gamma(1-\Omega)}$$

e.g. Hornig, CL, Ovanesyan (2009)
cf. Becher, Neubert (2006)

$$K_F(\mu,\mu_F) = \int_{\mu_F}^{\mu}\frac{d\mu'}{\mu'}\left\{\Gamma_F[\alpha_s(\mu')]\ln\frac{\mu'}{\mu_F} + \gamma_F[\alpha_s(\mu')]\right\} \qquad \omega_F(\mu,\mu_F) = \int_{\mu_F}^{\mu}\frac{d\mu'}{\mu'}\Gamma_F[\alpha_s(\mu')] \qquad \Omega = 2\omega_J + \omega_S$$

  - Goal is to compute the hard, jet, and soft functions and their anomalous dimensions Γ_F, γ_F

ORDERS OF LOGARITHMIC ACCURACY

- Resummed logarithmic accuracy determined by order in α_s to which anomalous dimensions and fixed-order matrix elements are known:

	Γ_F	γ_F	$\beta[\alpha_s]$	H, J, S
LL	α_s	1	α_s	1
NLL	α_s^2	α_s	α_s^2	1
NNLL	α_s^3	α_s^2	α_s^3	α_s
NNNLL	α_s^4	α_s^3	α_s^4	α_s^2

	H, J, S
LL	1
NLL'	α_s
NNLL'	α_s^2
NNNLL'	α_s^3

"Primed" counting: better correspondence in accuracy amongst Laplace transform, cumulative, and (especially) differential distributions

see Almeida, Ellis, CL, Sterman, Sung, Walsh (2014)

RESUMMATION IN DIRECT QCD VS. SCET

- CTTW form generalized to a more systematic all-orders form, closer to SCET:

$$\sigma(\tau) = \mathcal{N}(Q)\exp\Big[\sum_{n=2}^{\infty}\frac{1}{n!}\bar{E}^{(n)}\partial^n_{\bar{E}'}\Big]\frac{e^{\bar{E}(\ln 1/\tau)}}{\Gamma(1-\bar{E}'(\ln 1/\tau))}$$

Almeida, Ellis, CL, Sterman, Sung, Walsh (2014); cf. Contopanagos, Sterman (1993)

$$E^{(n)} = \frac{d^n E}{d[\ln(1/\tau)]^n}$$

$$E(\ln\nu) = 2\int_{Q(e^{\gamma_E}\nu)^{-1/2}}^{Q}\frac{d\mu'}{\mu'}\Big\{A[\alpha_s]\ln\Big(\frac{\mu'}{Q}\Big)^2 + B_J[\alpha_s]\Big\} - 2\int_{Q(e^{\gamma_E}\nu)^{-1}}^{Q(e^{\gamma_E}\nu)^{-1/2}}\frac{d\mu'}{\mu'}\Big\{A[\alpha_s]\ln\Big(\frac{\mu'}{Q(e^{\gamma_E}\nu)}\Big)^2 + B_S[\alpha_s]\Big\}$$

- derivatives come from inversion of Laplace transform, where logs naturally exponentiate algebraically

$$\widetilde{\sigma}(\nu) = \mathcal{N}(Q)e^{\bar{E}(\ln\nu)} \qquad \sigma(\tau) = \frac{1}{2\pi i}\int_{\mathcal{C}}\frac{d\nu}{\nu}\widetilde{\sigma}(\nu)$$

- Repeating the form derived in SCET:

$$\sigma(\tau) = e^{K_H+2K_J+K_S}\Big(\frac{\mu_H}{Q}\Big)^{\omega_H}\Big(\frac{\mu_J}{Q\tau^{1/2}}\Big)^{\omega_J}\Big(\frac{\mu_S}{Q\tau}\Big)^{\omega_S}H_2(Q^2,\mu_H) \times \widetilde{J}\Big(\partial_\Omega + \ln\frac{\mu^2}{Q^2\tau},\mu_J\Big)^2\widetilde{S}\Big(\partial_\Omega + \ln\frac{\mu_S}{Q\tau},\mu_S\Big)\frac{e^{\gamma_E\Omega}}{\Gamma(1-\Omega)}$$

SCET & DIRECT QCD CORRESPONDENCE

Almeida, Ellis, CL, Sterman, Sung, Walsh (2014)

- Showed that two forms are exactly equal with the dictionary:

$$A[\alpha_s] = \Gamma_{\rm cusp}[\alpha_s] \qquad B_F[\alpha_s] = \gamma_F[\alpha_s] - \frac{d\ln \tilde{F}(0,\mu_F)}{d\ln \mu_F}$$

$$\mathcal{N}(Q) = H(Q)\tilde{J}(0,Q)^2\tilde{S}(0,Q)$$

$$E = 2E_J + E_S \qquad E_F(\mu,\mu_F) = K_F(\mu,\mu_F) - \int_{\mu_F}^{\mu} \frac{d\mu'}{\mu'}\frac{d\ln \tilde{F}(0,\mu')}{d\ln \mu'}$$

- In addition, the dQCD formula has the hard, jet, and soft scales already fixed to:

$$\mu = \mu_H = Q\,, \qquad \mu_J = Q(\tau e^{-\gamma_E})^{1/2}\,, \qquad \mu_S = Q\tau e^{-\gamma_E}$$

- In SCET the scales are typically varied about the central values:

$$\mu = \mu_H = Q\,, \qquad \mu_J = Q\tau^{1/2}\,, \qquad \mu_S = Q\tau$$

  - by factors of 2, or more sophisticated "profile scales" for reliable uncertainty estimation and smooth matching onto both fixed-order and nonperturbative regimes.

SCET & DIRECT QCD CORRESPONDENCE

Almeida, Ellis, CL, Sterman, Sung, Walsh (2014)

- In principle, these scales can (should) also be kept free and then varied to estimate uncertainties in dQCD formalism

- Our proof of equivalence with SCET leads to unified formula, generalizing dQCD to variable jet/soft scales, and SCET to more explicitly exponentiated form:

$$\sigma(\tau) = \sigma_0 H(\mu_H)\left(\frac{\mu_H}{Q}\right)^{\omega_H} e^{K_H}\,\tilde{J}(0,\mu)^2\,\tilde{S}(0,\mu)e^{2E_J(\mu,\mu_J)+E_S(\mu,\mu_S)}$$
$$\times \exp\Big[\sum_{n=2}^{\infty}\frac{1}{n!}\Big(2E_J^{(n)}(\mu_J)\partial^n_{2E'_J} + E_S^{(n)}(\mu_S)\partial^n_{E'_S}\Big)\Big]$$
$$\times\left(\frac{Q(e^{-\gamma_E}\tau)^{1/2}}{\mu}\right)^{2E'_J(\mu,\mu_J)}\left(\frac{Q\tau}{\mu e^{\gamma_E}}\right)^{E'_S(\mu,\mu_S)}\frac{1}{\Gamma(1+2E'_J+E'_S)}$$

GOING BETWEEN LAPLACE AND MOMENTUM SPACE

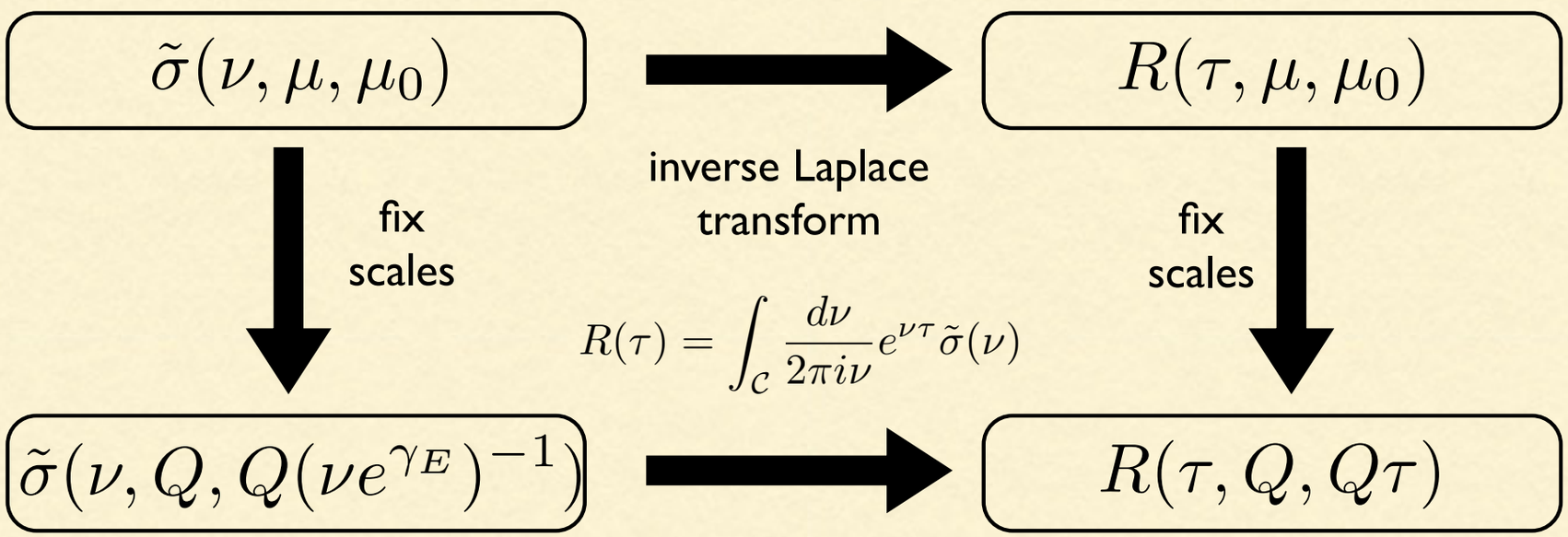
Almeida, Ellis, CL, Sterman, Sung, Walsh (2014)

- Both dQCD and SCET approaches typically begin by resumming in Laplace space:

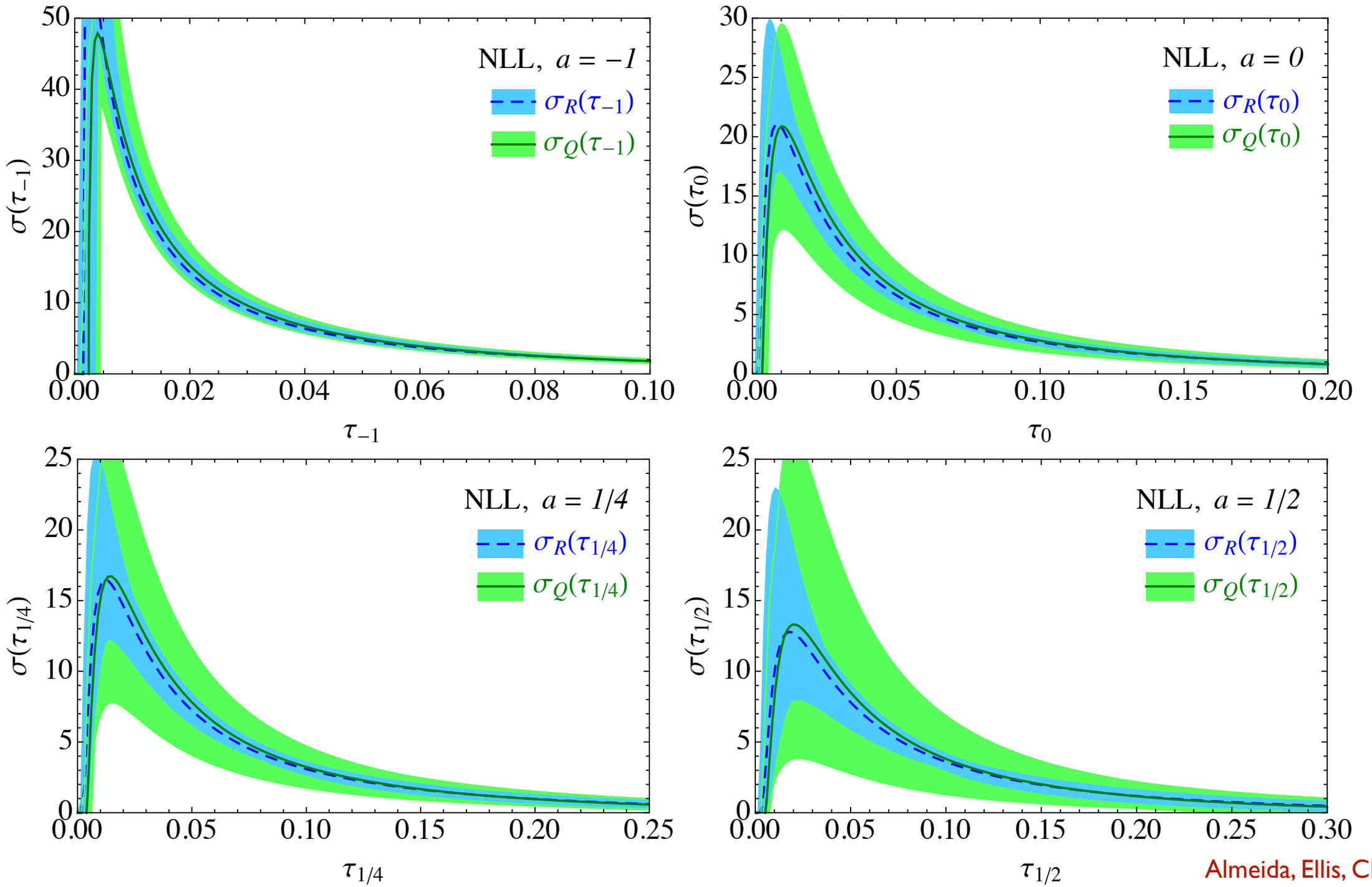
$$\tilde{\sigma}(\nu) = \sigma_0 e^K H(\mu_H)\left(\frac{\mu_H}{Q}\right)^{\omega_H(\mu,\mu_H)} \tilde{J}(L_J,\mu_J)^2 \tilde{S}(L_S,\mu_S)\left(\frac{\mu_J \nu^{1/2}}{Q e^{-\gamma_E}}\right)^{\omega_J(\mu,\mu_J)}\left(\frac{\mu_S \nu}{Q e^{-\gamma_E}}\right)^{\omega_S(\mu,\mu_S)}$$

$$L_J = \ln(\mu_J^2 e^{\gamma_E}\nu/Q^2) \qquad L_S = \ln(\mu_S e^{\gamma_E}\nu/Q)$$

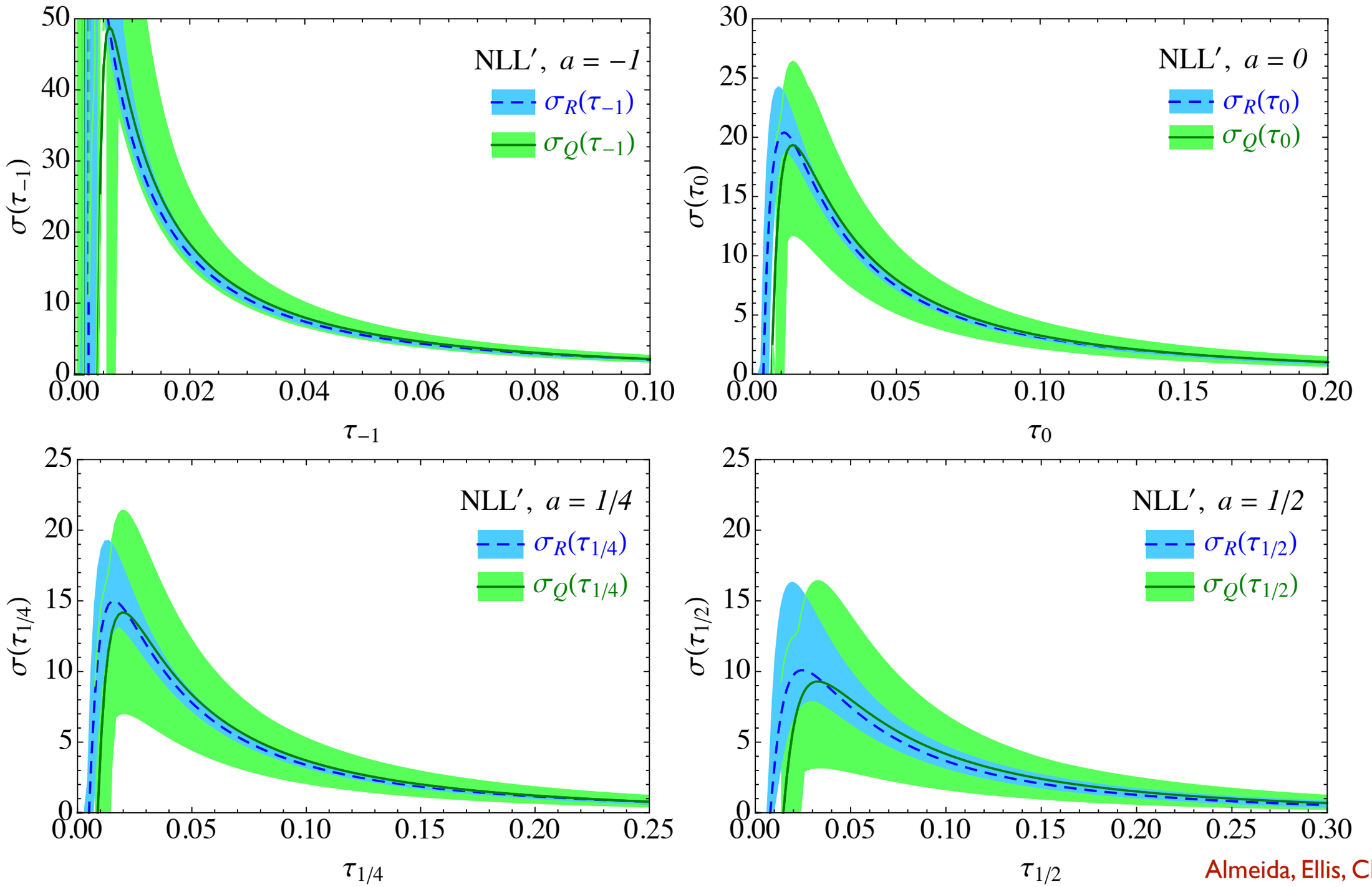
- The difference is the order in which the fixing of scales and the inverse Laplace transform are performed:



- To all orders, the two routes are equivalent. Truncated at a finite accuracy, the two routes may differ in subleading terms.



Almeida, Ellis, CL, Sterman, Sung, Walsh [1401.4460]



Almeida, Ellis, CL, Sterman, Sung, Walsh [1401.4460]

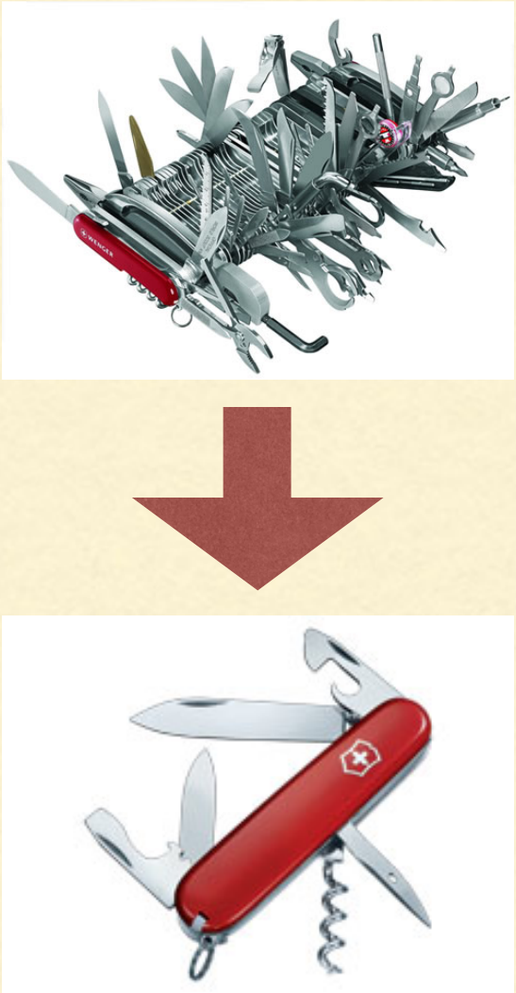
CONCLUSIONS

- We have provided a case study comparing SCET and dQCD resummation for e^+e^- event shapes in exquisite detail.
- We derived correspondence relations that put results from the two formalisms into precisely equivalent all-orders forms.
  - The unified form endows dQCD formulae with variable jet and soft scales, and puts SCET formulae into a more explicitly exponentiated form.
- In practice, different organizations of terms differing at subleading accuracy account for remaining numerical differences.
- These correspondences and improvements have direct parallels in DIS and *pp* event shapes as well
  - Provides common language to compare dQCD and SCET resummed predictions for cross sections at LHC and other collider environments.

BACKUP MATERIAL

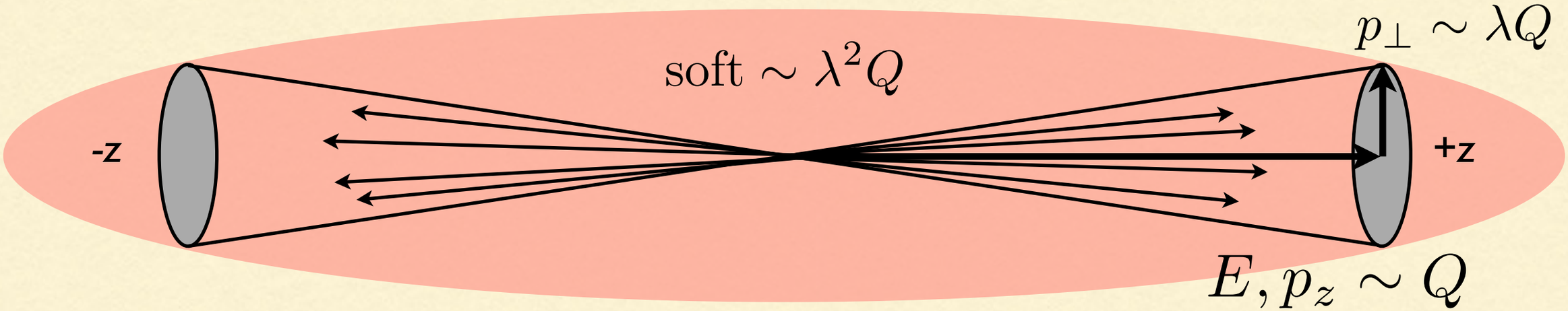
EFFECTIVE FIELD THEORIES

- Simplify "full theory" by integrating out heavy or UV degrees of freedom. Replace with "effective theory" of the accessible, low-energy degrees of freedom.
- e.g. effective EW theory (integrate out W,Z bosons), heavy quark effective theory (integrate out momentum modes ~ m_Q)
- EFT must reproduce same IR, low-energy behavior of full theory. Mismatch in UV encoded in Wilson or "matching" coefficients.
- EFT may possess additional symmetries or computational simplifications beyond full theory (e.g. heavy quark spin-flavor symmetry in HQET, soft-collinear decoupling in SCET.)
- EFT is an approximation to full theory to order-by-order in a power counting expansion in a small parameter λ



SOFT-COLLINEAR EFFECTIVE THEORY: MODES

- SCET modes describe small fluctuations around *lightlike* momentum trajectories:

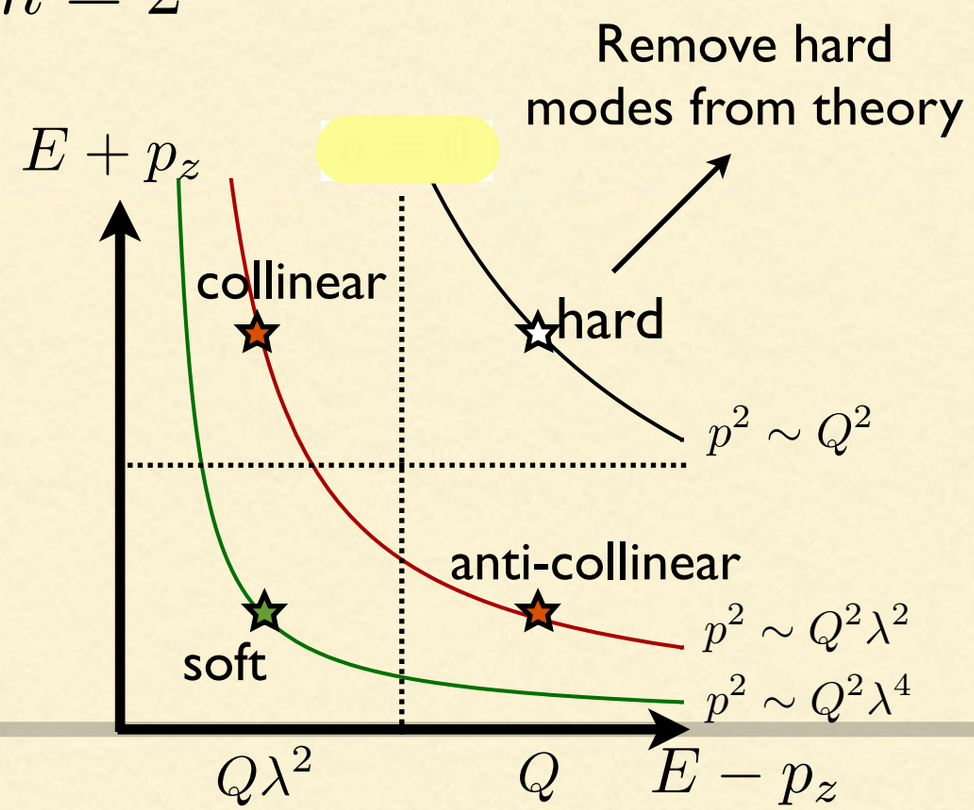


$$n = (1, \vec{z}), \quad \bar{n} = (1, \vec{z})$$
$$n^2 = \bar{n}^2 = 0, \quad n \cdot \bar{n} = 2$$

$$p_n = \tilde{p}_n + k$$
$$k \sim Q\lambda^2$$
$$\tilde{p}_n \equiv \bar{n} \cdot p \frac{n}{2} + \tilde{p}_\perp$$
$$\sim Q \qquad \sim Q\lambda$$

"label" momentum

- In 2-jet events, three EFT modes:



Bauer, Fleming, Luke, Pirjol, Rothstein, Stewart (2000, 2001)

SCET LAGRANGIAN

Bauer, Fleming, Pirjol, Stewart (2000, 2001)

- Factor out large momentum phase and project out "heavy" and "light" components of fermion fields:

$$\psi(x) = \sum_{\tilde{p}} e^{-i\tilde{p}\cdot x}\psi_{n,p}(x)$$

$$\xi_{n,p} = \frac{\not{n}\not{\bar{n}}}{4}\psi_{n,p}$$ massless

$$\Xi_{n,p} = \frac{\not{\bar{n}}\not{n}}{4}\psi_{n,p}$$ effective mass Q, integrate out

- Similarly for gluon fields. Resulting leading-power Lagrangian:

$$\mathcal{L}_{\rm SCET} = \sum_n (\mathcal{L}_{qn} + \mathcal{L}_{gn}) + \mathcal{L}_s$$

$$\mathcal{L}_{qn} = \bar{\xi}_n(x)\left[in\cdot D + i\not{D}^c_\perp W_n(x)\frac{1}{i\bar{n}\cdot\mathcal{P}}W_n^\dagger(x) i\not{D}^c_\perp\right]\frac{\not{n}}{2}\xi_n(x)$$

$$D^c_\mu = \mathcal{P}^\mu - igA^c_\mu$$

$$\mathcal{L}_{gn} = \frac{1}{2g^2}{\rm Tr}\left[i\mathcal{D}^\mu + gA_n, i\mathcal{D}^\nu + gA^\nu_n\right]^2$$

$$\mathcal{D}^\mu = \bar{n}\cdot\mathcal{P}\frac{n^\mu}{2} + \mathcal{P}^\mu_\perp + in\cdot D\frac{\bar{n}^\mu}{2}$$

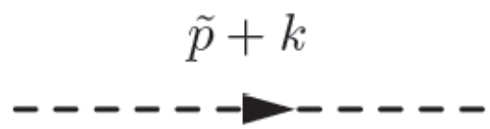
$$\mathcal{L}_s = \mathcal{L}_{\rm QCD}[q_s, A_s]$$

$\mathcal{P}^\mu$ "label momentum operator"

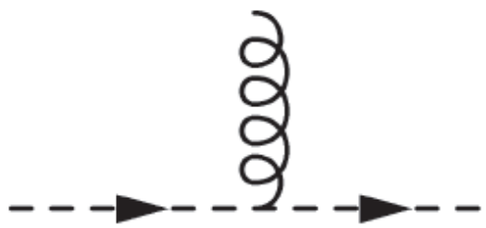
$$\xi_n(x) = \sum_{\tilde{p}} e^{i\tilde{p}\cdot x}\xi_{n,\tilde{p}}(x)$$ similarly for collinear gluons

SCET FEYNMAN RULES

- Collinear quarks:



$$= i\frac{\not{n}}{2}\frac{\bar{n}\cdot\tilde{p}}{n\cdot k\,\bar{n}\cdot\tilde{p}+\tilde{p}_\perp^2+i\epsilon}$$



$$= igT^A n_\mu \frac{\not{\bar{n}}}{2}$$

$$= igT^A\left(n_\mu+\frac{\gamma_\mu^\perp\not{\tilde{p}}_\perp}{\bar{n}\cdot\tilde{p}}+\frac{\not{\tilde{p}}'_\perp\gamma_\mu^\perp}{\bar{n}\cdot\tilde{p}'}-\frac{\not{\tilde{p}}'_\perp\not{\tilde{p}}_\perp}{\bar{n}\cdot\tilde{p}\;\bar{n}\cdot\tilde{p}'}\bar{n}_\mu\right)\frac{\not{\bar{n}}}{2}$$

- Similarly for collinear gluons.

SCET OPERATOR MATCHING

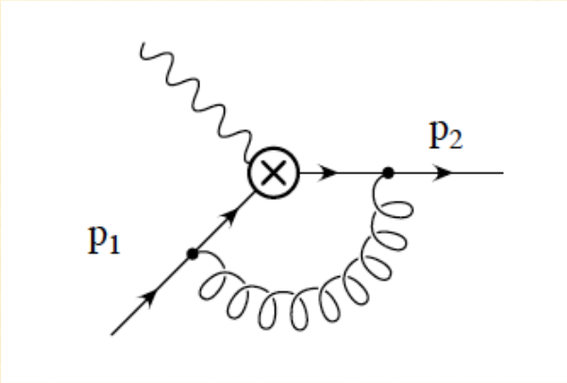
- Besides Lagrangian, also need to match QCD currents onto SCET operators.

- e.g. for e^+e^- to 2 jets, DIS, or Drell-Yan:

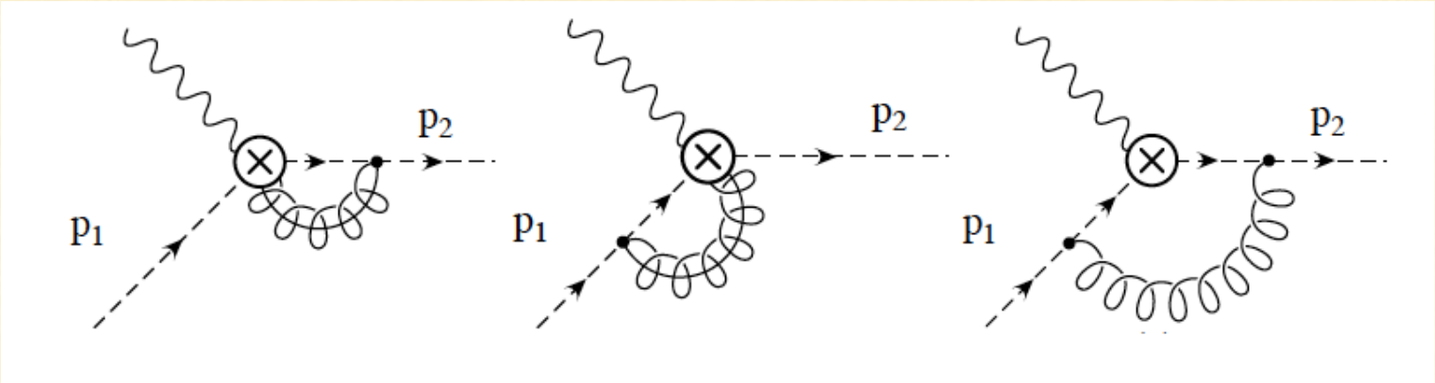
$$\bar{\psi}\gamma^\mu\psi \longrightarrow C_2(\mu)[\bar{\xi}_{n_1}W_{n_1}]_{\tilde{p}_1}\gamma^\mu[W^\dagger_{n_2}\xi_{n_2}]_{p_2}$$

- One-loop matching, difference of QCD and SCET loop graphs. IR divergences match, leaving over UV-dependent Wilson coefficient:

QCD:



SCET:



$$C_2(\mu) = 1 + \frac{\alpha_s(\mu)C_F}{4\pi}\left[-\ln^2\frac{\mu^2}{-\tilde{p}_1\cdot\tilde{p}_2} - 3\ln\frac{\mu^2}{-\tilde{p}_1\cdot\tilde{p}_2} - 8 + \frac{\pi^2}{6}\right]$$

Manohar (2003); Bauer, CL, Manohar, Wise (2003)

SOFT-COLLINEAR DECOUPLING

Bauer, Pirjol, Stewart (2001)

- At leading power, soft-collinear interactions are eikonal:

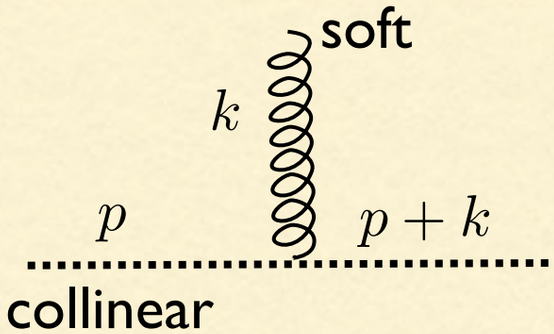
$$\bar{\xi}_n(in\cdot D_s)\xi_n = \bar{\xi}_n(in\cdot\partial + gn\cdot A_s)\xi_n$$

- Soft-collinear interactions resummed into Wilson lines:

$$Y_n(x) = P\exp\left[ig\int_{-\infty}^{0} ds\, n\cdot A_s(ns+x)\right]$$

- Perform field redefinition:

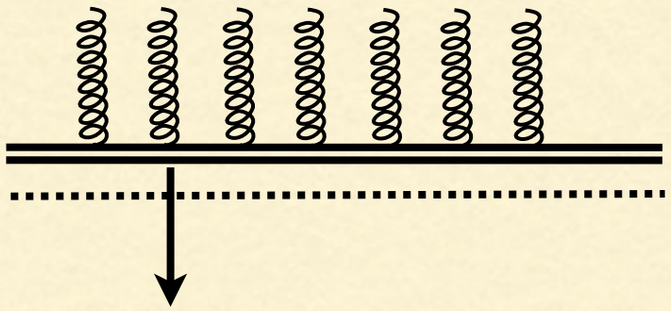
$$\xi_n(x) = Y_n(x)\xi_n^{(0)}(x) \qquad \bar{\xi}_n(in\cdot D_s)\xi_n \longrightarrow \bar{\xi}_n^{(0)}(in\cdot\partial)\xi_n^{(0)}$$



$$\sim \not{p}\,\frac{ig\gamma^\mu t^a}{(p+k)^2}\not{p}$$

keep leading order part of diagram

soft gluons "see" only direction and color of jet



$$\frac{n^\mu t^a}{n\cdot k}\frac{\not{n}}{2}$$

QCD form

- Consider the Laplace transform of the cross section: $\tilde{\sigma}(\nu) = \int_0^\infty d\tau\, e^{-\nu\tau} \frac{d\sigma}{d\tau}$

- In the QCD literature, a simple exponentiated form has been given for the resummed Laplace transform:

Sterman et al.

$$\tilde{\sigma}(\nu) = \sigma_0 \mathcal{N}(Q) \exp[E(\ln \nu)]$$

$$\mathcal{N}(Q) = 1 + \frac{\alpha_s(Q)}{4\pi} C_1 + \left(\frac{\alpha_s(Q)}{4\pi}\right)^2 C_2 + \cdots$$

$$\begin{aligned} E(\ln \nu) = {} & 4 \int_{Q(e^{\gamma_E}\nu)^{-1/2}}^{Q} \frac{d\mu'}{\mu'} A[\alpha_s] \ln \frac{\mu'}{Q} \\ & - 4 \int_{Q(e^{\gamma_E}\nu)^{-1}}^{Q(e^{\gamma_E}\nu)^{-1/2}} \frac{d\mu'}{\mu'} A[\alpha_s] \ln \frac{\mu'}{Q(e^{\gamma_E}\nu)^{-1}} \\ & + 2 \int_{Q(e^{\gamma_E}\mu)^{-1/2}}^{Q} \frac{d\mu'}{\mu'} B_J[\alpha_s] + 2 \int_{Q(e^{\gamma_E}\nu)^{-1}}^{Q} \frac{d\mu'}{\mu'} B_S[\alpha_s] \end{aligned}$$

E has an expansion in logs of the form on the previous slide.

A, B_J, B_S can be calculated order by order in α_s

At each higher order, another entire colored box on the previous slide is fully predicted.

- The dependence on variable hard, jet, and soft scales is ***not*** fully predicted by this formula. Theoretical uncertainty will be ***underestimated***.

SCET form

- In the SCET literature, the typical form for the resummed cross section is:

$$\tilde{\sigma}(\nu) = \sigma_0 e^K H(\mu_H)\Big(\frac{\mu_H}{Q}\Big)^{\omega_H(\mu,\mu_H)} \tilde{J}(L_J,\mu_J)^2 \tilde{S}(L_S,\mu_S)\Big(\frac{\mu_J \nu^{1/2}}{Qe^{-\gamma_E}}\Big)^{\omega_J(\mu,\mu_J)}\Big(\frac{\mu_S \nu}{Qe^{-\gamma_E}}\Big)^{\omega_S(\mu,\mu_S)}$$

$$L_J = \ln(\mu_J^2 e^{\gamma_E}\nu/Q^2) \qquad L_S = \ln(\mu_S e^{\gamma_E}\nu/Q)$$

- Exponents given by integrals over anomalous dimensions of hard, jet, soft functions:

$$K_F(\mu,\mu_F) = \int_{\mu_F}^{\mu} \frac{d\mu'}{\mu'}\Big\{\Gamma_F[\alpha_s(\mu')]\ln\frac{\mu'}{\mu_F} + \gamma_F[\alpha_s(\mu')]\Big\}$$

$$\omega_F(\mu,\mu_F) = \int_{\mu_F}^{\mu} \frac{d\mu'}{\mu'}\Gamma_F[\alpha_s(\mu')]$$

They can be calculated order by order in α_s like *A,B*

- This formula looks more complicated, but has the correct dependence on variable hard, jet, and soft scales. Allows ***more reliable*** theoretical uncertainty estimates.

- It becomes nearly equivalent to the QCD form for the scale choices:

$$\bar{\mu}_H = Q\,, \quad \bar{\mu}_J = Q(e^{\gamma_E}\nu)^{-1/2}\,, \quad \bar{\mu}_S = Q(e^{\gamma_E}\nu)^{-1}$$

$$\Rightarrow \quad \tilde{\sigma}(\nu) = \sigma_0 H(Q)\tilde{J}(0,\bar{\mu}_J)^2\tilde{S}(0,\bar{\mu}_S)e^K$$

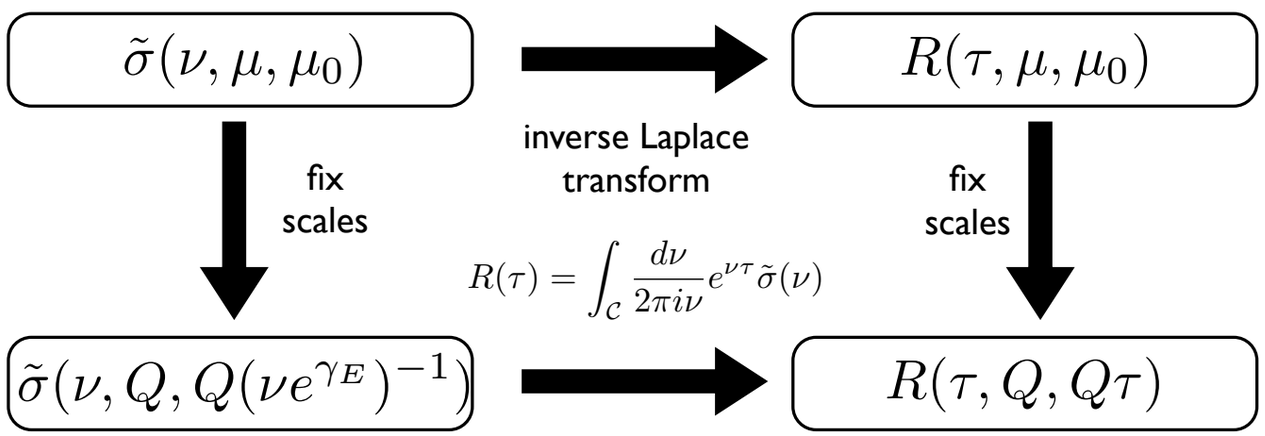
still need to evolve to *Q*

Going back to event shape space

- To transform back to the event shape τ

$$R(\tau) = \frac{1}{2\pi i}\int_{\gamma-i\infty}^{\gamma+i\infty}\frac{d\nu}{\nu}e^{\nu\tau}\tilde{\sigma}(\nu)$$

- We *should* get the same result before or after fixing the hard, jet, and soft scales:



- However, care must be exercised in evaluating the formulas so that this is true at a given order of resummed accuracy.

- Going the top route, one obtains:

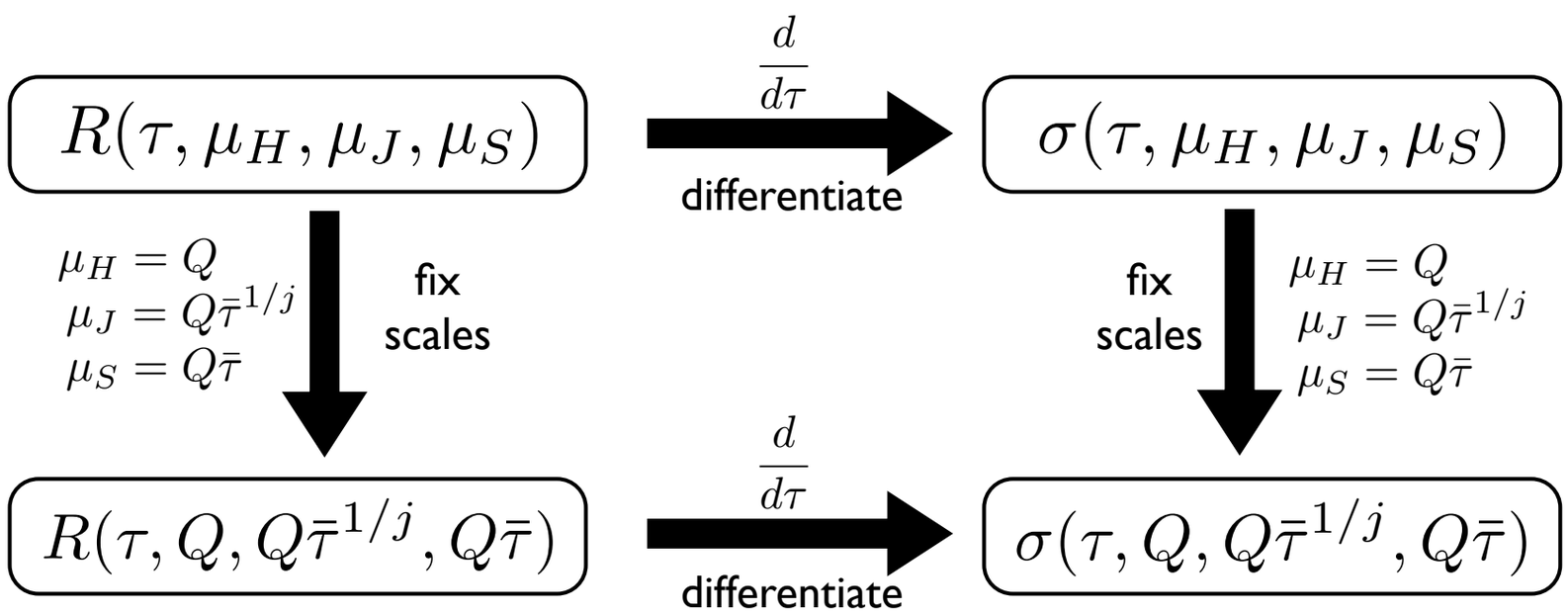
$$R(\tau) = \sigma_0 e^{K_H+2K_J+K_S}\left(\frac{\mu_H}{Q}\right)^{\omega_H}\left(\frac{\mu_J}{Q\tau^{1/2}}\right)^{2\omega_J}\left(\frac{\mu_S}{Q\tau}\right)^{\omega_S}$$
$$\times H(\mu_H)\tilde{J}\left(\partial_\Omega + \ln\frac{\mu_J}{Q\tau^{1/2}},\mu_J\right)^2\tilde{S}\left(\partial_\Omega + \ln\frac{\mu_S}{Q\tau},\mu_S\right)\frac{e^{\gamma_E\Omega}}{\Gamma(1-\Omega)}$$

$$\Omega = 2\omega_J(\mu,\mu_J) + \omega_S(\mu,\mu_S)$$

Differential cross sections

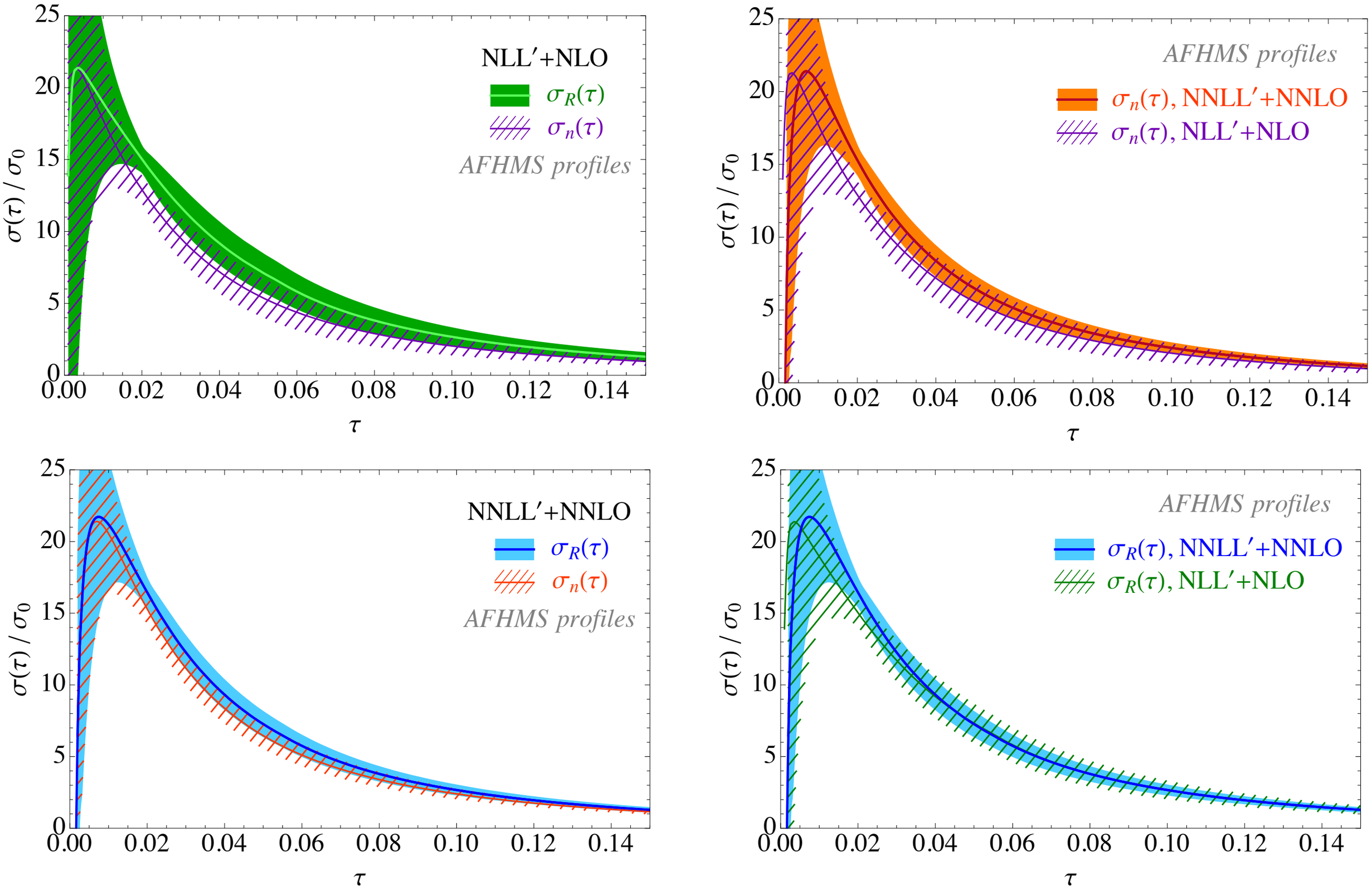
- We made similar observations and gave similar prescriptions for computing differential cross sections to better accuracy at a given resummed order than in past literature.

$$\frac{d\sigma}{d\tau} = \frac{d}{d\tau} R(\tau)$$



- **Our results are the most accurate resummed forms in event shape space to date that also display correct dependence on hard, jet, and soft scales and thus can be used to estimate theoretical uncertainties most reliably.**

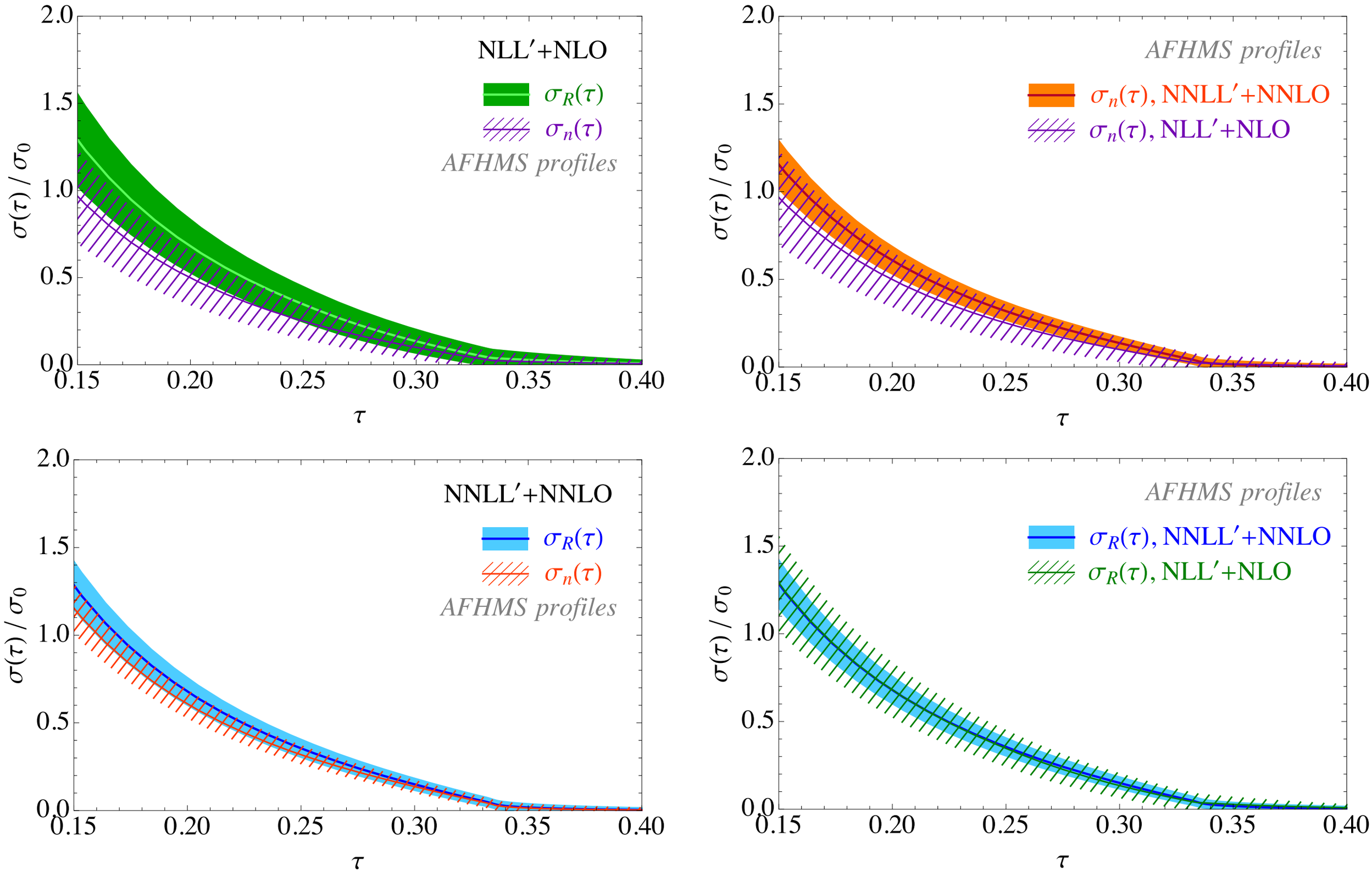
from J. Walsh [preliminary]



Using scale profiles from Abbate, Fickinger, Hoang, Mateu, Stewart (2010)

σ_R exhibits better convergence than σ_n order to order

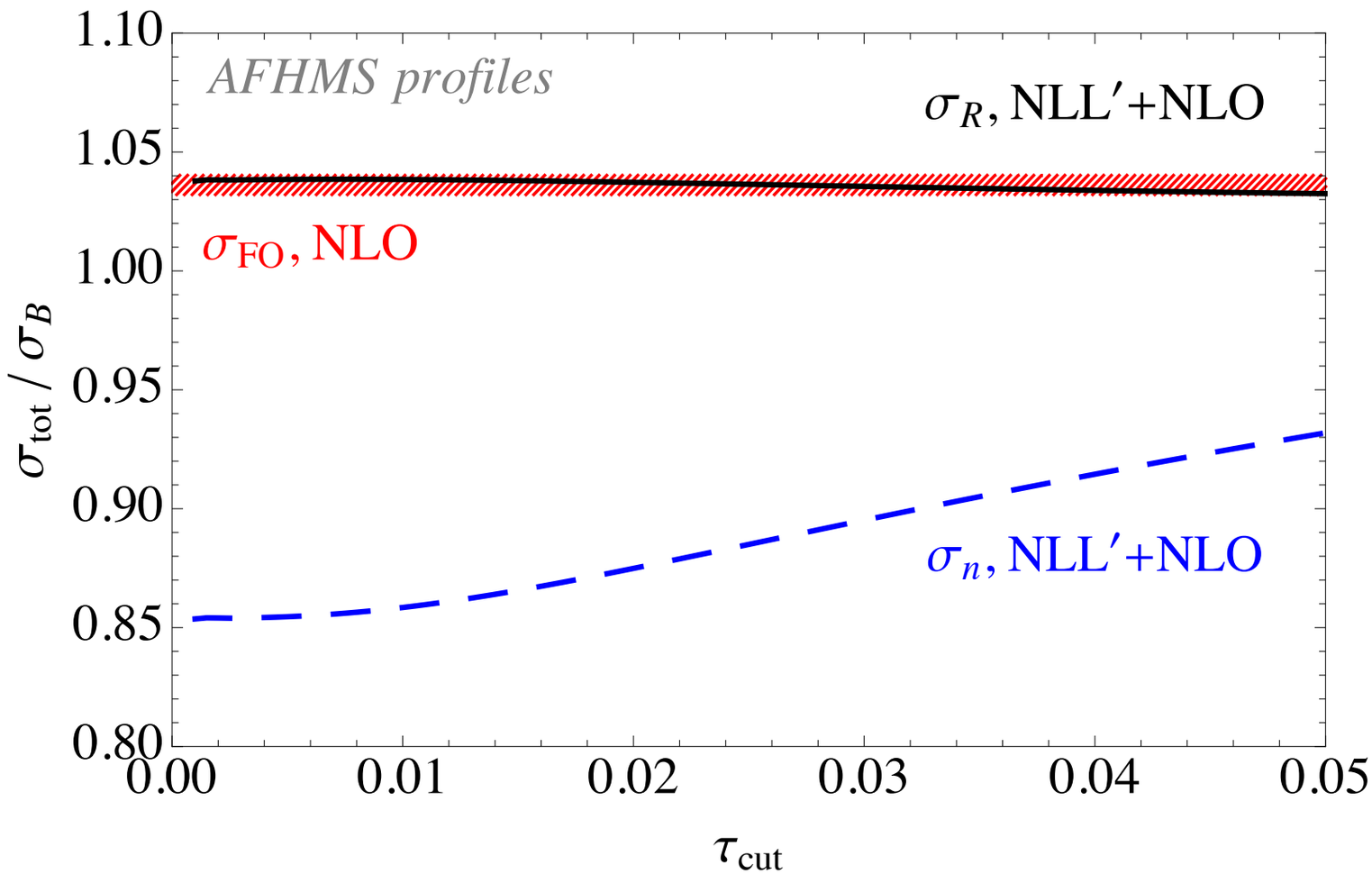
from J. Walsh [preliminary]



Using scale profiles from Abbate, Fickinger, Hoang, Mateu, Stewart (2010)

σ_R exhibits better convergence than σ_n order to order

CL, Tackmann, Walsh [preliminary]



$$\sigma_{\rm tot} = R(\tau_{\rm cut}) + \int_0^{\tau_{\rm cut}} d\tau' \sigma(\tau')$$

σ_R integrates properly to the total cross section, more stable under variations of $\tau_{\rm cut}$