## NNLOsing and $N^{3} L L$ for WIZ and Higgs

## production at large $p_{T}$



Loopfest XIII, NY City College of Technology June 18th to 20th, 2014

Inclusive production of single, electroweak boson is the most basic hard-scattering process at hadron colliders.

- QCD benchmark
- PDF determination, Mw determination
- Jet-energy calibration
- Search for New Physics
see Michael Schmitt's Talk


## New measurements for $Z$ production




CMS-PAS-SMP-13-013
$0.5 \%$ Precision at low $р_{т}$, few \% at higher $\boldsymbol{p}_{\tau}$. Among the most precise measurements at LHC!

## Talk based on

- Factorization theorem, NNLL resummation for photon production TB, Schwartz '09
- W/Z to N3'LLpartial+NLO TB, Lorentzen, Schwartz '12
- Electroweak Sudakov effects TB, Garcia Tormo '13
- Two-loop jet functions TB Neubert '06, TB, Bell '11 twoloop soft function TB, Bell, Marti '12
- New: Two-loop hard function, full $\mathrm{N}^{3}$ LL+NLO for W/Z and Higgs TB, Bell, Lorentzen, Marti ' 13
- First resummation of " 3 jet obs." at this accuracy


## Fixed order at $q_{T} \neq 0$

- LO

- NLO known since the 80's. (Elis, Matinelli, Petronzici'83; Amodd Fenn 'g8; Gonsaves, Pamowski, wai 'r9) Implemented in numerical codes.
- NNLO computations in progress.
- 2-loop virtual corrections known (Garann, Germman, Glover, Koukoursakis and Remiddi '01 '02, Gehrmann and Tancredi '11 + Weihs '13; Gehrmann, Jaquier, Glover and Koukoutsakis '11)
- Difficulty: singularity structure of real emissions!
- First results for $g g \rightarrow H+X$ Boughezal et al. '13
update: talks by Fabrizio Caola and Matthieu Jaquier


## Simplification near threshold



Real emission simplify drastically near the partonic threshold $m_{X}^{2} \rightarrow 0$ :

- single, low-mass jet, recoiling against V ,
- accompanied by soft radiation

Use SCET to compute real emissions.

## Factorization theorem

Cross section at large $p_{\text {т }}$ near the partonic threshold $m x^{2} \rightarrow 0$
TB, Schwartz '09


$$
\hat{s} \frac{\mathrm{~d} \hat{\sigma}}{\mathrm{~d} \hat{u} \mathrm{~d} \hat{t}}=H_{a b}(\hat{u}, \hat{t})\left(J_{c} \otimes S_{a b}\right)\left(m_{X}^{2}\right)
$$

## Threshold cross section

$$
\begin{aligned}
& +\left(\frac{\alpha_{S}}{4 \pi}\right)^{2}\left[\delta\left(m_{X}^{2}\right)\left(h^{(2)}+h^{(1)} \cdot p_{0}^{(1)}+p_{0}^{(2)}\right)+\left[\frac{1}{m_{X}^{2}}\right]_{*}\left(h^{(1)} \cdot p_{1}^{(1)}+p_{1}^{(2)}\right)\right. \\
& \left.\left.+\left[\frac{\ln \frac{m_{x}^{2}}{k_{2}}}{m_{X}^{2}}\right]_{*}\left(h^{(1)} \cdot p_{2}^{(1)}+p_{2}^{(2)}\right)+\left[\frac{\ln ^{2} \frac{m_{x}^{2}}{k^{2}}}{m_{x}^{2}}\right]_{*} p_{3}^{(2)}+\left[\frac{\ln { }^{3} \frac{m_{x}^{2}}{k^{2}}}{m_{X}^{2}}\right]_{*} p_{4}^{(2)}\right]\right\}
\end{aligned}
$$

- Theorem gives leading terms for $m x^{2} \rightarrow 0$ : Singular distributions in $m x^{2}$.
- Numerically dominant.
- In addition, there are regular contributions. Known at NLO.



## Resummation at $\mathrm{N}^{3} \mathrm{LL}$



- Two-loop H, J, S
- Three-loop anomalous dimensions $\gamma_{H}, \gamma_{J}, \gamma_{s}$. All known.
- $\gamma_{H}$ follows from two-jet result using factorization constraints TB, Neubert '09.
- Four-loop $\gamma_{\text {cusp }}$
- Not available, but numerically insignificant, use Padé


## Soft and jet functions

Jet functions (imaginary part of two-point functions)


## Soft function

3 Wilson lines


## Hard function



Contains 2-loop virtual corrections. Computed quite some time ago. Garland, Gehrmann, Glover et al. '02, ...

- Helicity amplitudes, given in electronic form.

Assemble two-loop hard function from two-loop helicity amps. TB, Bell, Lorentzen, Marti '13

- Use $\boldsymbol{C}$ and $\boldsymbol{P}$ and analytic continuation to obtain all helicity amplitudes from minimal set.
- Convert from Catani IR subtraction to $\overline{\mathrm{MS}} \mathrm{IR}$ subtraction.


## Conversion Catani to $\overline{\mathrm{MS}}$

$\overline{\mathrm{MS}}$ subtraction

H in SCET


$$
\left|\mathcal{M}^{\mathrm{ren}}(\{p\}, \mu)\right\rangle=\lim _{\epsilon \rightarrow 0} Z^{-1}(\epsilon,\{p\}, \mu)|\mathcal{M}(\epsilon,\{p\})\rangle
$$

Catani subtraction

$$
\uparrow_{\text {given by }}^{\left|\mathcal{M}^{\mathrm{fin}}(\{p\}, \mu)\right\rangle=\left[1-\frac{\alpha_{s}}{2 \pi} \boldsymbol{I}^{(1)}(\epsilon)-\left(\frac{\alpha_{s}}{2 \pi}\right)^{2} \boldsymbol{I}^{(2)}(\epsilon)+\ldots\right]|\mathcal{M}(\epsilon,\{p\})\rangle}
$$

Gehrmann et al.

## Result for conversion

$$
\begin{aligned}
\mathcal{M}^{(2), \text { ren }}= & \mathcal{M}^{(2), \text { fin }}+\mathcal{C}_{0} \mathcal{M}^{(1), \text { fin }}+\left\{\frac{1}{2} \mathcal{C}_{0}^{2}+\frac{\gamma_{1}^{\text {cusp }}}{8}\left(\mathcal{C}_{0}+\frac{\pi^{2}}{128} \Gamma_{0}^{\prime}\right)\right. \\
& \left.+\frac{\beta_{0}}{2}\left(\mathcal{C}_{1}+\frac{\pi^{2}}{32} \Gamma_{0}+\frac{7 \zeta_{3}}{96} \Gamma_{0}^{\prime}\right)-\frac{1}{8}\left[\Gamma_{0}, \mathcal{C}_{1}\right]\right\} \mathcal{M}^{(0)},
\end{aligned}
$$

$C_{0}$ and $C_{1}$ from $\varepsilon$ expansion of $\mathrm{I}^{(1)}(\varepsilon)$

$$
\begin{aligned}
\mathcal{C}_{\mathbf{0}} & =\sum_{(i, j)} \frac{\boldsymbol{T}_{i} \cdot \boldsymbol{T}_{j}}{16}\left[\gamma_{0}^{\text {cusp }} \ln ^{2} \frac{\mu^{2}}{-s_{i j}}-\frac{4 \gamma_{0}^{i}}{C_{i}} \ln \frac{\mu^{2}}{-s_{i j}}\right]-\frac{\pi^{2}}{96} \Gamma_{0}^{\prime} \\
\boldsymbol{\mathcal { C }}_{\mathbf{1}} & =\sum_{(i, j)} \frac{\boldsymbol{T}_{i} \cdot \boldsymbol{T}_{j}}{48}\left[\gamma_{0}^{\text {cusp }} \ln ^{3} \frac{\mu^{2}}{-s_{i j}}-\frac{6 \gamma_{0}^{i}}{C_{i}} \ln ^{2} \frac{\mu^{2}}{-s_{i j}}\right]-\frac{\pi^{2}}{48} \boldsymbol{\Gamma}_{0}-\frac{\zeta_{3}}{24} \Gamma_{0}^{\prime}
\end{aligned}
$$

See 1309.3245 for details.

hep-ph/recent SPIRES-HEP Search Blogs $\pi$ inspire Readability arXiv hep-ph/new

## PeTeR

## Resummation for electroweak bosons at large transverse momentum

## Christian Lorentzen

This is the development page of the PeTeR project. PeTeR is a C++ code for the computation of the transverse momentum spectrum of the production of an electroweak boson (photon, W, Z, Higgs).

Using the SCET approach, PeTeR is able to perform a threshold resummation for large p_T up to $N^{3}$ LL matched to NLO. It is based on the following following papers:

- Resummation for $W$ and $Z$ production at large $p T$ Thomas Becher, Christian Lorentzen and Matthew D. Schwartz, Phys.Rev.Lett. 108 (2012) 012001, arXiv:1106.4310 [hep-ph]
- Precision Direct Photon and W-Boson Spectra at High p_T and Comparison to LHC Data Thomas Becher, Christian Lorentzen and Matthew D. Schwartz, Phys.Rev. D86 (2012) 054026, arXiv:1206.6115 [hep-ph]
- Transverse-momentum spectra of electroweak bosons near threshold at NNLO Thomas Becher, Guido Bell, Christian Lorentzen and Stefanie Marti, (2013), arXiv:1309.3245 [hep-ph]

If you use PeTeR, please cite one of the papers above.
For resummation at small transverse momentum see CuTe.



## $Z$ production with PeTeR



- Moderate NNLO corrections of $O(10 \%)$. Vary $\mu=\mu_{\mathrm{h}}=\mu_{\mathrm{j}}=\mu_{\mathrm{s}}$ and $\mu_{\mathrm{f}}$ independently by a factor of two around $\mu_{\mathrm{f}}=\mu=p_{T}$.
- will use the same prescription for Higgs


## NNLOsing versus $\mathrm{N}^{3}$ LL resummation




Resummation reduces scale uncertainty but has small effect on central value. No large logarithms: $m_{X}$ not dramatically lower than $p_{T}$.

## EW corrections

## Kühn Kulesza, Pozzorini, Schulze 05; TB, Garcia Tormo '13



- Resummed simultaneously with QCD corr'ns.
- QCD and EW factorize to good accuracy

- Much larger corrections for Higgs production!
- Real emissions in both cases are described by the same jet and soft functions.
- Large corrections in hard function $H_{g g \rightarrow H g}$ !


## Hard function for total rate



- Hard function for total rate is scalar gluon form factor.
- Convergence much better for $\mu^{2}=-m_{H^{2}}$. Avoids imaginary parts, corresponds to expansion of space-like form factor.

Ahrens, TB, Neubert, Yang '09

- Same procedure cannot immediately be applied to $H+j$. (Imaginary parts from different channels.)


## Hard function for $\mathrm{H}+\mathrm{jet}$



- Large corrections, irrespective of $\arg (\mu)$
- Logs from different channels:

$$
\alpha_{s}^{n}(\mu) \ln ^{2 n} \frac{-\hat{s}}{\mu^{2}} \quad \alpha_{s}^{n}(\mu) \ln ^{2 n} \frac{-\hat{t}}{\mu^{2}} \quad \alpha_{s}^{n}(\mu) \ln ^{2 n} \frac{-\hat{u}}{\mu^{2}}
$$

## RG-equations

$$
H_{g g \rightarrow H g}(\hat{u}, \hat{t}, \mu)=\sum\left|\mathcal{M}_{g g \rightarrow H g}(\hat{u}, \hat{t}, \mu)\right|^{2}
$$

## Renormalized amplitude

$$
\frac{\mathrm{d}}{\mathrm{~d} \ln \mu} \mathcal{M}_{g g \rightarrow H g}(\hat{u}, \hat{t}, \mu)=\left[\frac{C_{A}}{2} \gamma_{\mathrm{cusp}}\left(\alpha_{s}\right)\left(\ln \frac{-\hat{s}}{\mu^{2}}+\ln \frac{-\hat{t}}{\mu^{2}}+\ln \frac{-\hat{u}}{\mu^{2}}\right)+3 \gamma_{g}\left(\alpha_{s}\right)\right] \mathcal{M}_{g g \rightarrow H g}(\hat{u}, \hat{t}, \mu)
$$

Scalar $g g \rightarrow H$ form factor $F_{s}$

$$
\frac{\mathrm{d}}{\mathrm{~d} \ln \mu} F_{S}(\hat{s}, \mu)=\left[C_{A} \gamma_{\mathrm{cusp}}\left(\alpha_{s}\right) \ln \frac{-\hat{s}}{\mu^{2}}+2 \gamma_{g}\right] F_{S}(\hat{s}, \mu)
$$

## Simple improvement scheme

Define reduced amplitude

$$
\widetilde{\mathcal{M}}_{g g \rightarrow H g}(\hat{u}, \hat{t})=\frac{\mathcal{M}_{g g \rightarrow H g}(\hat{u}, \hat{t}, \mu)}{\sqrt{F_{S}(\hat{s}, \mu) F_{S}(\hat{t}, \mu) F_{S}(\hat{u}, \mu)}}
$$

Then use RG-method to improve formfactor $F_{s}$. Improved cross section

$$
\left(\frac{d \sigma}{d p_{T}}\right)^{\text {impr. }}=\left|\frac{F_{S}\left(\hat{s}, \mu_{h}\right) U_{S}\left(\mu_{h}, \mu\right)}{F_{S}(\hat{s}, \mu)}\right| \frac{d \sigma}{d p_{T}}
$$

with evolution $U_{S}$ from $\mu_{h}^{2}=-\hat{s}$ to $\mu^{2}=+\hat{s}$.


Divide out form factor pieces, use RG to improve them.


Divide out form factor pieces, use RG to improve them.

## Reduced amplitude




- Reduced hard function (dashed lines) has better perturbative behavior.
- However, at the end of the day, only moderate improvement of cross section.


## Improved cross section




- Moderate improvement: corrections larger than those to the form factor.
- Reduced scale unc. at NNLO.


## Heavy top approximation



- Finite- $m_{t}$ result only known at LO. (Approximate NLO: Harlander et al. '12.)
- Small difference below $p_{T}=200 \mathrm{GeV}$


## Summary and outlook

- $\mathrm{NNLO}_{\text {sing }} / \mathrm{N}^{3} \mathrm{LL}$ result for $\mathrm{W}, \mathrm{Z}$ and Higgs production at large $p_{\text {т }}$.
- Public code PeTeR
- Small NNLO corrections for W/Z, large for $H$.
- Among first hadron collider physics results from 2-loop fourpoint functions. Same hard function will arise in all resummations for single-jet observables.
- Nontrivial check on the full NNLO results once they become available, estimate of $\mathrm{N}^{3} \mathrm{LO}$.
- Will do phenomenological analysis with QCD and EW, comparison to new W/Z measurements by ATLAS and CMS.
- New results at low $p_{T}$ : Two-loop collinear functions. Gehrmann, Lübbert, Yang ' 14 Better understanding of NP effects. TB, Bell 14


## Additional slides

## Individual scale variations





- Dominant uncertainties are from $\mu_{\mathrm{h}}$ and $\mu_{\mathrm{f}}$ variation. $\rightarrow$ Vary those separately.


## Resummation for Z-production



Moderate effect: Difference $\mathrm{NNLO} \mathrm{S}_{\text {sing }}$ vs $\mathrm{N}^{3} \mathrm{LL}$ is $\mathrm{O}\left(\alpha_{s}{ }^{3}\right)$ and logarithms are not very large.


- Numerically $N^{3} L L_{p}$ and $N^{3} L L$ are quite close

NLO correction to hard function:


Correction is of the form

$$
\alpha(\mu)\left(c_{2} L^{2}+c_{1} L+c_{0}\right) \quad L=\ln \left(\mu / p_{T}\right)
$$

Plot shows that $\mu \approx p_{\text {T }}$ is reasonable choice.

## $Z$ production



## Higgs production

Hard NLO


Hard NNLO


Jet NLO


Jet NNLO


Soft NLO


Soft NNLO


