Colour, Flavour and multi-particle QCD

Tom Melia, CERN Theory

Loopfest 2014, CUNY

Based on: TM, Phys. Rev. D. 88, 014020 (2013); TM, Phys. Rev. D. 89, 074012 (2014)
& on work with Stefano Frixione, Robbert Rietkerk to appear

Outline

Colour plays an important role in modern unitarity pQCD calculations.

Recently discovered properties of QCD amplitudes

- Dyck words and multi-quark amplitudes

  T Melia, Phys. Rev. D. 88, 014020 (2013)

- Getting more flavour out of one-flavour QCD

  T Melia, Phys. Rev. D. 89, 074012 (2014)

Role in modern pQCD calculations

- Colourful FKS subtraction

  S. Frixione, JHEP 1109 , 091 (2011)

- 1/Nc expansions and multi-jet LHC physics

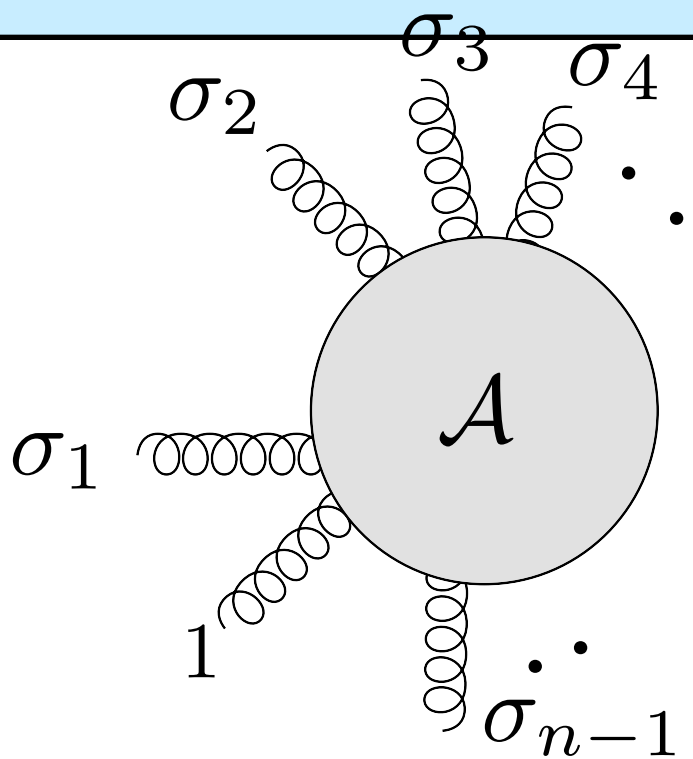
Colour

Colour decomposition for tree-level gluons:

$$A^{\text{full}} = \sum_{\sigma \in S_{n-1}} \mathrm{tr}(\lambda^1 \lambda^{\sigma_1} \lambda^{\sigma_2} \ldots \lambda^{\sigma_{n-1}})\, \mathcal{A}(1, \sigma_1, \sigma_2, \ldots, \sigma_{n-1})$$

F. A. Berends and W. Giele, Nucl.Phys. B294 , 700 (1987).

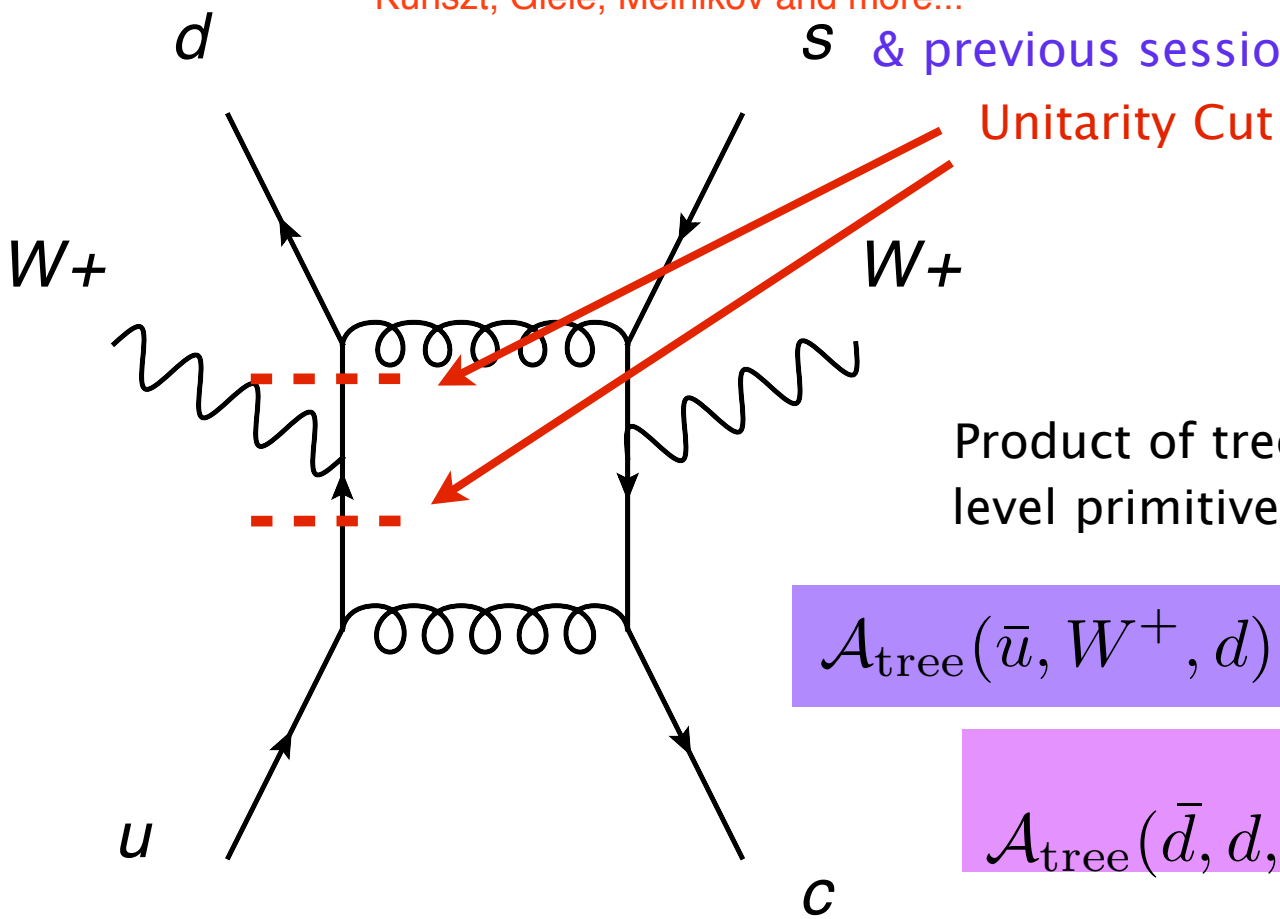
M. L. Mangano, S. J. Parke, and Z. Xu, Nucl.Phys. B298 , 653 (1988).



Colour

Plays an important role in modern unitarity pQCD calculations Bern, Dixon, Kosower; Cachazo, Britto, Feng; Ellis, Ossola, Papadopoulos, Pittau; Kunszt, Giele, Melnikov and more...

e.g.: & previous sessions talks Yang & Pierpaolo



Product of tree level primitives

$\mathcal{A}_{\text{tree}}(\bar{u}, W^+, d)$ x

$\mathcal{A}_{\text{tree}}(\bar{d}, d, \bar{s}, W^+, c, \bar{u}, u)$

No Feynman diagrams: use recursion relations.

e.g.: R. Britto, F. Cachazo, B. Feng, and E. Witten, Phys.Rev.Lett. 94 (2005).
F. A. Berends and W. Giele, Nucl.Phys. B294 , 700 (1987).

Colour

Plays an important role in modern unitarity pQCD calculations

Colour decomposition/ putting colour back

Tree level with quarks

M. L. Mangano, S. J. Parke, and Z. Xu, Nucl.Phys. B298 , 653 (1988).

M. L. Mangano, Nucl.Phys. B309 , 461 (1988).

One-loop QCD decompositions & quark primitive amplitudes

Z. Bern, L. J. Dixon, and D. A. Kosower, Nucl.Phys. B437 , 259 (1995)
V. Del Duca, L. J. Dixon, and F. Maltoni, Nucl.Phys. B571 , 51 (2000)

For a spontaneously broken gauge group

L. Dai, K. Melnikov, and F. Caola, JHEP 1204 , 095 (2012), arXiv:1201.1523 [hep-ph].

All tree and one-loop QCD primitives -> full amplitudes

R. K. Ellis, Z. Kunszt, K. Melnikov, and G. Zanderighi, Phys.Rept. 518 , 141 (2012)

C. Reuschle and S. Weinzierl, (2013),

T. Schuster, (2013), arXiv:1311.6296 [hep-ph].

Colour

And in more formal amplitude studies

Parke-Taylor Formula

S. J. Parke and T. Taylor, Phys.Lett. B157 , 81 (1985).

All N=4 SYM tree-level colour stripped amplitudes

J. Drummond and J. Henn, JHEP 0904 , 018 (2009).

Deep duality between colour and kinematics

Z. Bern, J. Carrasco, and H. Johansson, Phys.Rev. D78 , 085011 (2008),

From N=4 trees -> QCD trees for up to four quark lines

L. J. Dixon, J. M. Henn, J. Plefka, and T. Schuster, JHEP 1101 , 035 (2011)

Effective supersymmetry: Z. Kunszt, Nucl.Phys. B271 , 333 (1986).

. . .

Properties of QCD amplitudes

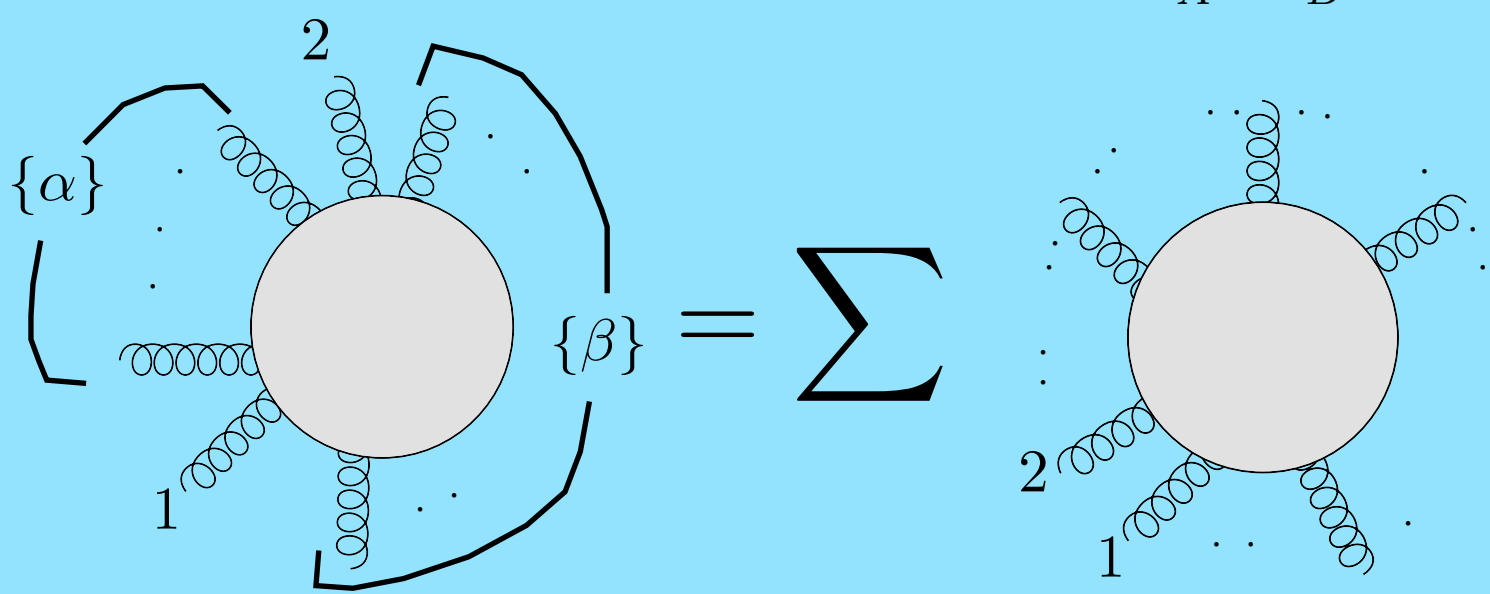
Group theory relations

Kleiss-Kuijf relations

R. Kleiss and H. Kuijf, Nucl.Phys. B312 , 616 (1989)

All gluon: # Independent = $(n-2)!$

$$\mathcal{A}(1,\{\alpha\},2,\{\beta\}) = \sum_{\mathrm{OP}\{A\}\{B\}} \mathcal{A}(1,2,\underbrace{\{\alpha^T\}}_{A}\underbrace{\{\beta\}}_{B})$$



$\{\alpha\} = 3\ 4\ 5$

$\{\alpha^T\} = 5\ 4\ 3$

$\{\beta\} = 6\ 7$

OP $\{\alpha^T\}\{\beta\}$

5 4 3 6 7 5 6 4 3 7 5 4 6 7 3 6 5 4 7 3 6 5 7 4 3

5 4 6 3 7 6 5 4 3 7 5 6 4 7 3 5 6 7 4 3 6 7 5 4 3

Group theory relations

Dyck words for quark amplitudes (k flavours)

named after Walter von Dyck

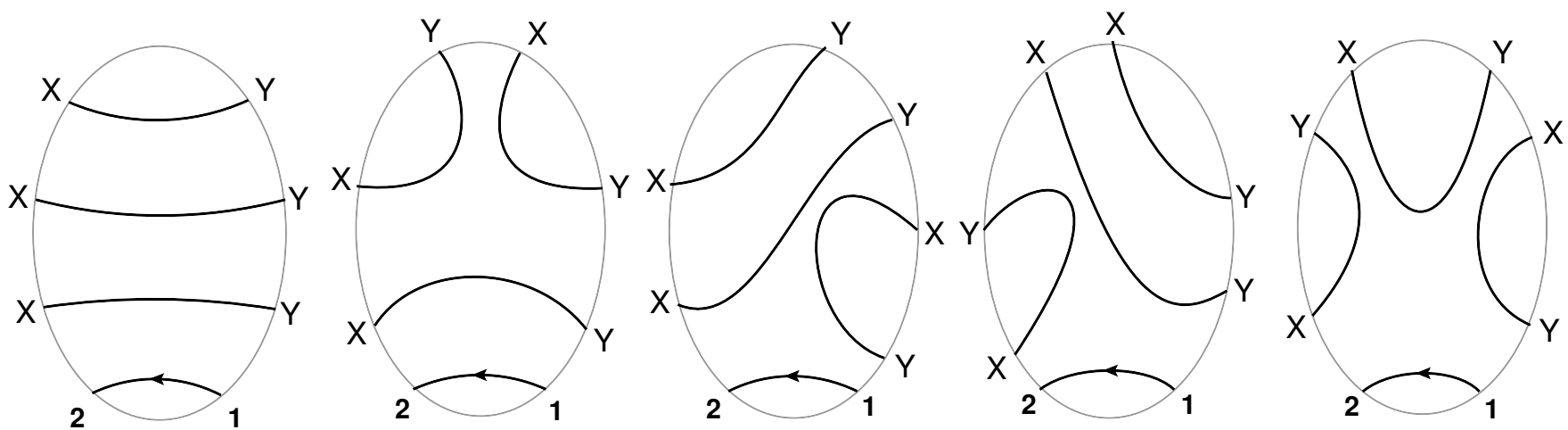
T Melia, Phys. Rev. D. 88, 014020 (2013)

String of r Xs and r Ys such that the number of Xs is always greater than or equal to the number of Ys in any initial segment of the string.

r=1 XY

r=2 XXYY XYXY

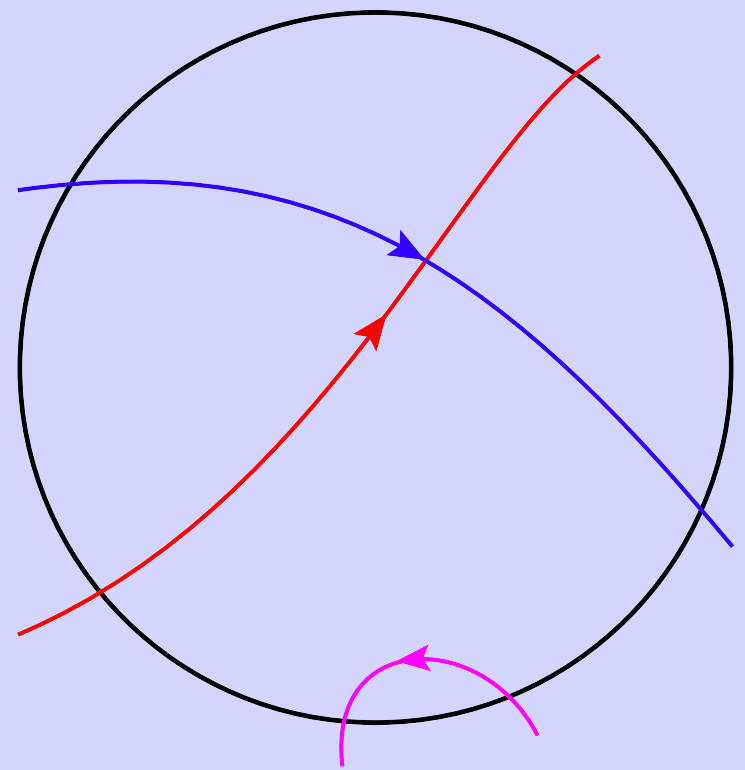
r=3 XXXYYY XXYXYY XXYYXY XYXXYY XYXYXY

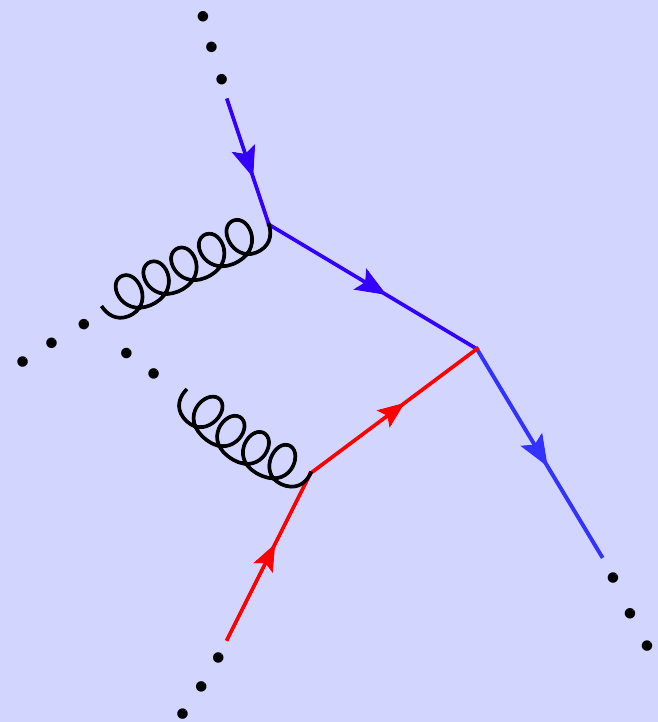


Group theory relations

Dyck words for quark amplitudes (k flavours)

nam





No crossing quark lines.

Group theory relations

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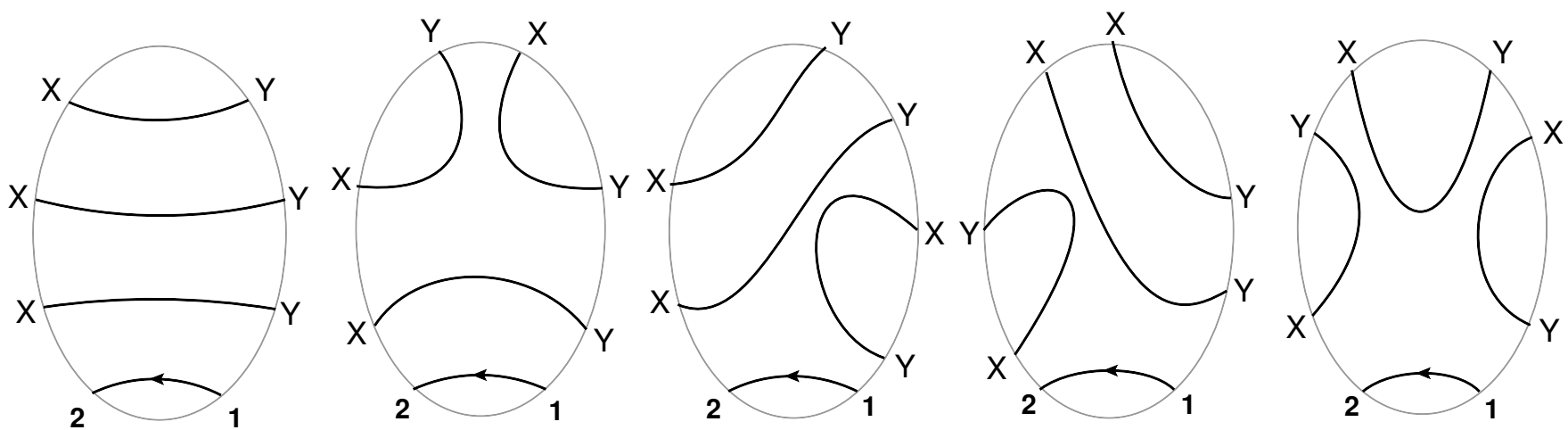

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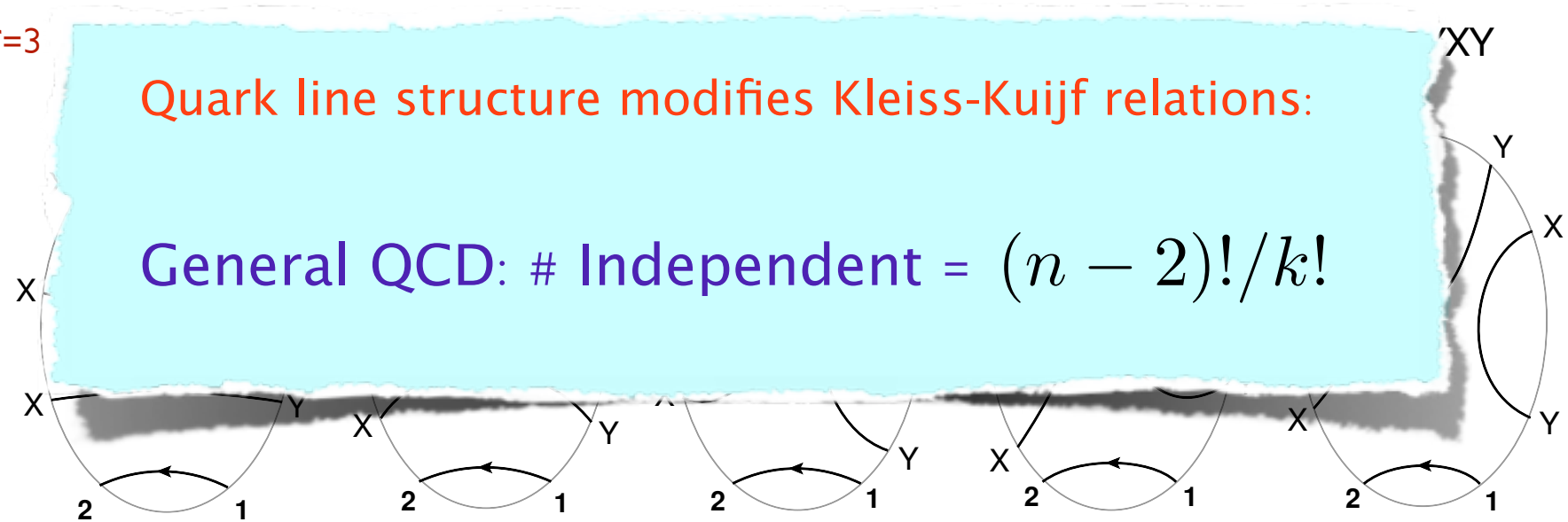
Quark line structure modifies Kleiss-Kuijf relations:

General QCD: # Independent = $(n-2)!/k!$



Interplay between colour & flavour

All of massless QCD with nf flavours of quark can be obtained from ONE-flavour QCD

Shown explicitly at tree-level in T Melia, Phys. Rev. D. 89, 074012 (2014)

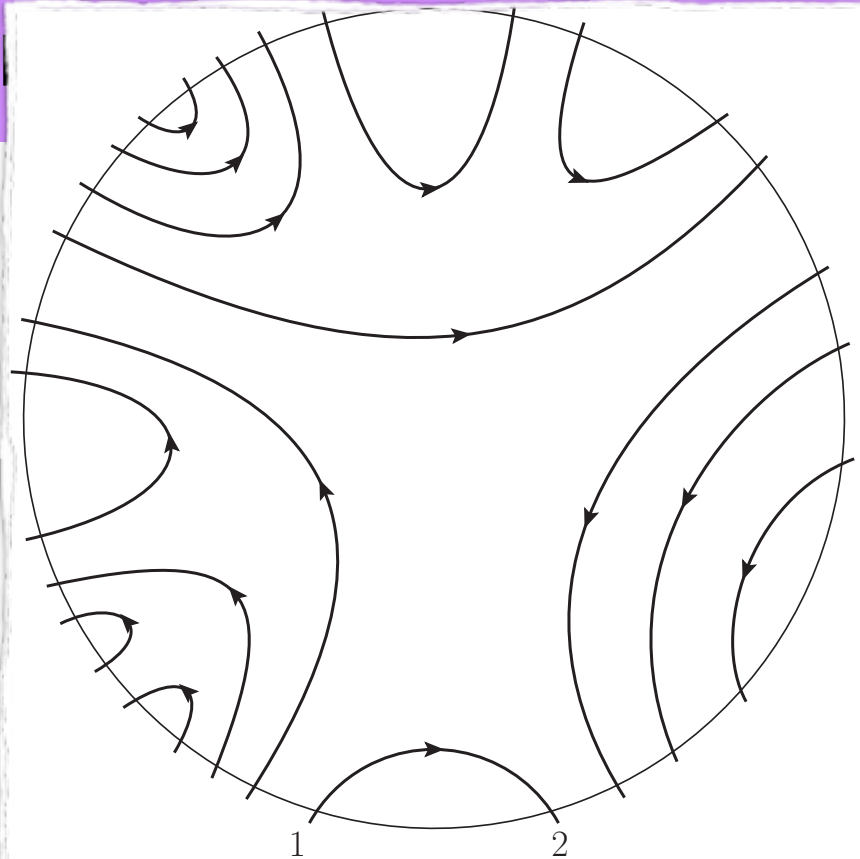
Usually

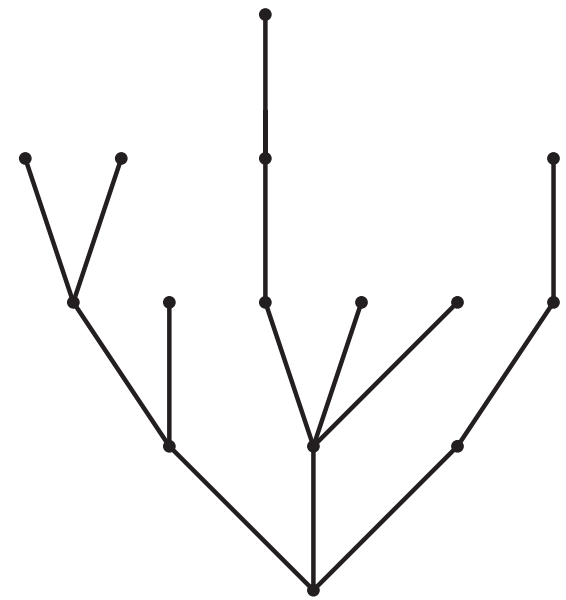
$$\mathcal{M}_{u,u,u} = \sum_{\mathcal{P}(u,d,s)} \mathcal{M}_{u,d,s} \, (-1)^{\mathrm{sgn}\mathcal{P}}$$

Reverse Logic (using colour ordered primitives)

$$A_{d,s,u} = A_{u,u,u} - \sum_{\mathcal{P}(u,d,s)} (1 - \delta_{\mathcal{P},(d,s,u)}) A_{u,d,s} \, (-1)^{\mathrm{sgn}\mathcal{P}}$$

Have to apply colour group theory relations to change cyclic ordering in a way specified by a 'Dyck tree'. And then recurse.





an

2 (2014)

$$A_{d,s,u} = A_{u,u,u} - \sum_{\mathcal{P}(u,d,s)} (1 - \delta_{\mathcal{P},(d,s,u)}) A_{u,d,s} (-1)^{\mathrm{sgn}\mathcal{P}}$$

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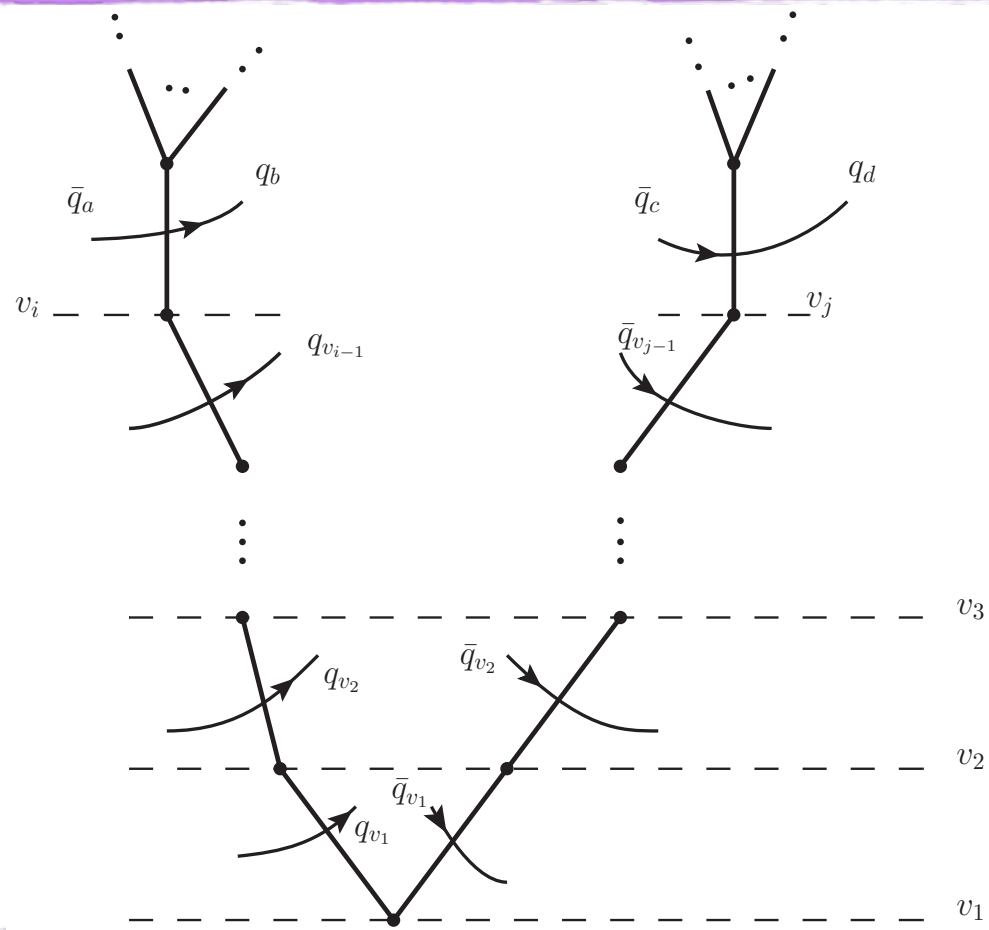

an

2 (2014)

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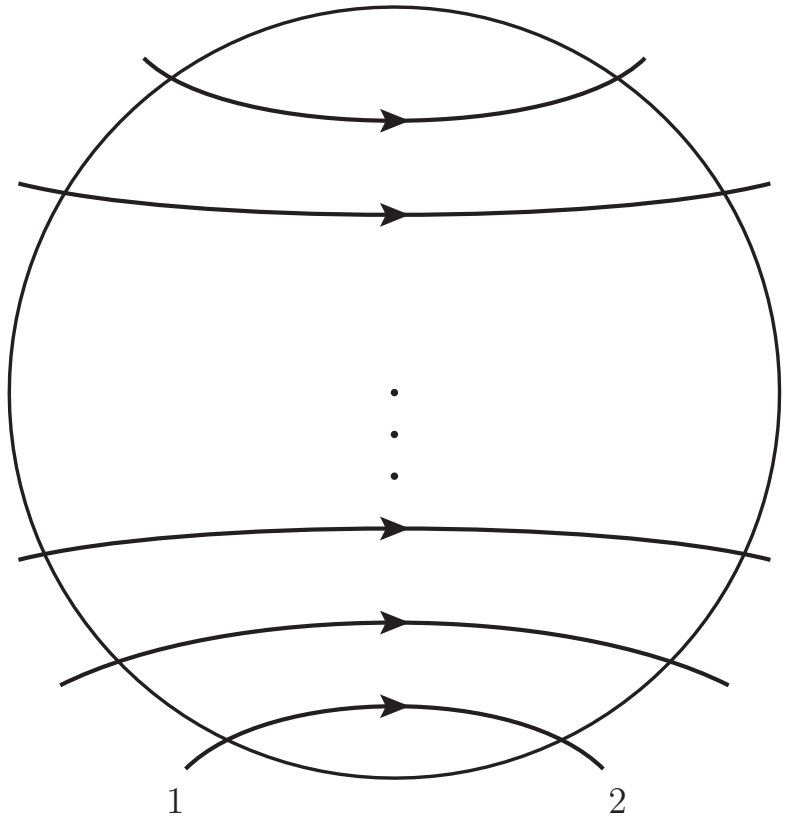

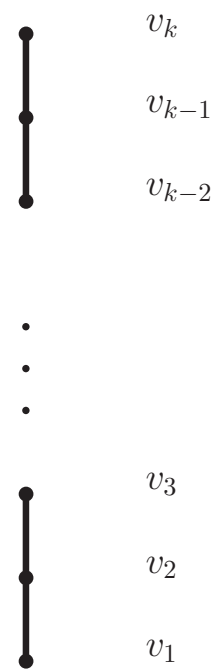

$XX\ldots XXYY\ldots YY$

an

2 (2014)

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One-flavour tree amplitudes are effectively N=1 supersymmetric

Closed subgroup of N=4; there exists closed N=4 formula projected onto one flavour of quark and gluons, as in L. J. Dixon, J. M. Henn, J. Plefka, and T. Schuster, (2011)

Different, multi-flavour approach to obtaining all QCD from N=4: T. Schuster, (2013), arXiv:1311.6296 [hep-ph].

Role in modern pQCD calculations

Colour and FKS subtraction

Tree level amplitudes: simple! but irony is that in the sense of computational time they usually carry the heaviest weight.

Certainly amplitude 'formal' advances important

Faster to evaluate More numerically stable

Subtraction procedures (C-S dipoles, FKS...) usually formulated using <u>full</u> amplitudes

Colourful FKS subtraction at the level of primitives

S. Frixione, JHEP 1109 , 091 (2011)

Colour and FKS subtraction

Pairs of colour ordered amplitudes

$$A^{(n+1)}(\ldots a,\, n+1,\, b\ldots)^\star\, A^{(n+1)}(\ldots c,\, n+1,\, d\ldots)$$

$$\rightarrow_{(n+1)\,\mathrm{soft}}$$

$$\Big([a,c]-[a,d]-[b,c]+[b,d]\Big)\times A^{(n)}(\ldots a,\;\, b\ldots)^\star\, A^{(n)}(\ldots c,\;\, d\ldots)$$

where $\quad [k,l]=\dfrac{p_k\cdot p_l}{p_k\cdot p_{n+1}\;\, p_l\cdot p_{n+1}}$

Colour and FKS subtraction

Pairs of colour ordered amplitudes

$$A'^{(n+1)}(\ldots a,\, n+1,\, b\ldots)^\star\, A^{(n+1)}(\ldots c,\, n+1,\, d\ldots)$$

$$\rightarrow_{n+1\,||n}$$

$$\Big(\delta(a,n)\delta(c,n) - \delta(a,n)\delta(d,n) - \delta(b,n)\delta(c,n) + \delta(b,n)\delta(d,n)\Big)\times$$

$$\left(P(z)A'^{(n)\star}A^{(n)} + \frac{1}{2}Q(z)\left[\frac{\langle k_n k_{n+1}\rangle}{[k_n k_{n+1}]}A_-'^{(n)\star}A_+^{(n)} + \frac{[k_n k_{n+1}]}{\langle k_n k_{n+1}\rangle}A_+'^{(n)\star}A_-^{(n)}\right)$$

P(z) and Q(z) are Altarelli-Parisi (usual and azimuthal) splitting functions

Colour and FKS subtraction

Opens the path to systematic 1/Nc expansion of the full cross section, both real and virtual.

More interesting enumerative mathematics than Dyck words even . . . much use of group theory relations

Many multi-jet already available e.g. in this LoopFest alone, 5 jets from Daniel, Valery,

NLO automated techniques

Public and very useful translate to experimentalists, interfacing with parton showers..

Within well developed framework of MG5_aMC@NLO

how to 'add a few extra jets'

Systematic 1/Nc, real integration bottleneck, plug-in virtuals as Valentin outlined in his talk