Two-loop corrections to $t\bar{t}$ and VV production

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PRISMA

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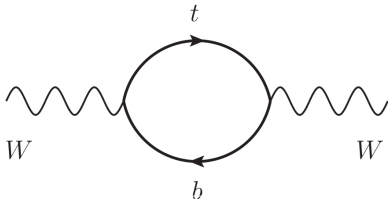
Top pair production at the LHC

LHC is top factory:

- large rates
- precise mass determination, crucial for EW precision observables



- background to many processes, notably: missing energy (new physics)

theory prediction for $pp \to t\bar{t}$:

- LHC precision below NLO accuracy
- resummations: see talks by B. Pecjak, A. Broggio
- **total σ at NNLO** (semi-numerical methods) Czakon, Bärnreuther, Fiedler, Mitov '08-'14

Analytic NNLO calculation

ingredients:

- **VV**: two-loop ME for $q\bar{q} \to t\bar{t}$ and $gg \to t\bar{t}$
  leading N_c, (light) fermionic: Bonciani, Ferroglia, Gehrmann, Maitre, AvM, Studerus '08-'13
  poles: Ferroglia, Neubert, Pecjak, Yang '09
  small mass: Czakon, Mitov, Moch '06
  one-loop^2: Kniehl, Körner, Merebashvili, Rogal '05-'08, Anastasiou, Aybat '08
- **RV**: one-loop ME for $t\bar{t}$ + 1 parton

  Dittmaier, Uwer, Weinzierl '07, '09; Bevilacqua, Czakon, Papadopoulos, Worek '10, '11; Melnikov, Schulze '10
- **RR**: tree level ME for $t\bar{t}$ + 2 partons
- **subtraction terms**: up to 2 unresolved partons needed, see talk by G. Abelof

two-loop corrections:

- 253 master integrals (w/products, wo/crossings)
- advanced mathematical functions

this talk:

- $gg \to t\bar{t}$: light fermionic (recently)
- $q\bar{q} \to t\bar{t}$: all contributions (in preparation)

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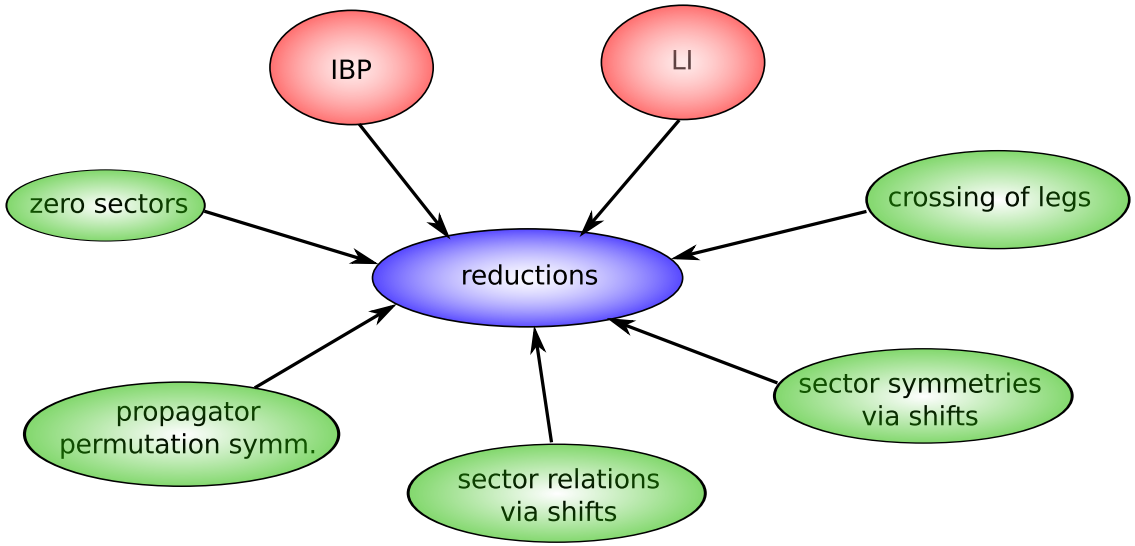
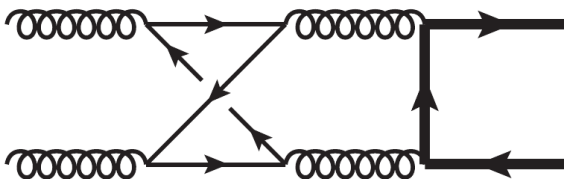
Reduze 2 (AvM, C. Studerus)
arXiv:1201.4330, HepForge

uses GiNaC (Bauer, Frink, Kreckel)
and Fermat (Lewis)



- distributed **Feynman integral reduction**
- advanced **shift finders**
- upcoming version features:
  - **bilinear propagators**
    3-loop heavy flavour Wilson coefficients in DIS, see talk by A. Hasselhuhn
  - **phase space integrals**
    RRV threshold contributions to N^3LO Higgs and DY, Li, AvM, Schabinger, Zhu '14
  - **family finder**, . . .

Non-planar master integrals for light fermionic $gg \to t\bar{t}$



new master integrals [AvM, C. Studerus '13]:

- method of differential equations
- constants & checks: regularity, symmetry, Mellin-Barnes
  - `MB.m` [Czakon '05], planar: `AMBRE` [Gluza, Kajda, Riemann '07 + Yundin '10]
  - `SecDec 2.1` [Borowka, Heinrich '13]

kinematics:

- diff. eq. contain $\sqrt{-s(4m^2 - s)}$, rationalize with Landau var. x:

$$s = -m^2(1-x)^2/x, \qquad t = -m^2 y, \qquad u = -m^2 z$$

- non-linear relations for crossed kinematics

$$(1-x)^2/x + y + z = -2$$

- involved functional identities, explicite imaginary parts

Multiple polylogarithms

Remiddi, Gehrmann; Goncharov

$$G(a_1, a_2, \ldots, a_n; x) = \int_0^x dt \; \frac{dt}{t - a_1} G(a_2, \ldots, a_n; t),$$

with $G(; x) = 1$, complex weights a_i and complex argument x.

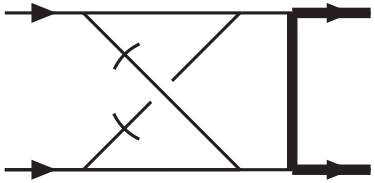
we employ also **generalised weights** $[f(o)]$:

$$G([f(o)], w_2, \ldots, w_n; x) = \int_0^x \mathrm{d}t \frac{f'(t)}{f(t)} G(w_2, \ldots, w_n; t)$$

example:

$$G([o^2 + 1]; x) = \int_0^x \mathrm{d}t \frac{2t}{t^2 + 1} = \int_0^x \mathrm{d}t \frac{1}{t - i} + \int_0^x \mathrm{d}t \frac{1}{t + i} = G(i; x) + G(-i; x)$$

see AvM, Schabinger, Zhu '13, related: Ablinger, Blümlein, Schneider '11 (cyclotomic polylogs)



$$= \frac{x^2}{m^4(1-x)^2(1-x+x^2+xy)} \sum_{i=-4}^{0} I_i \epsilon^i + \mathcal{O}(\epsilon)\,,$$

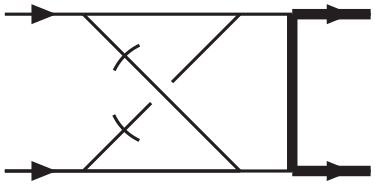
$$I_{-4} = \frac{7}{384}$$

$$\begin{aligned}
I_{-3} = &-\frac{5}{192} G(-(1-x+x^2)/x;y) + \frac{1}{64} G(-1;y) - \frac{5}{192} G([1-o+o^2];x) - \frac{1}{16} G(1;x) \\
&+ \frac{11}{192} G(0;x) - \frac{5}{192} i\pi
\end{aligned}$$

$$\begin{aligned}
I_{-2} = &+\frac{1}{192} G(-(1-x+x^2)/x;y)^2 - \frac{1}{32} G(-(1-x+x^2)/x;y) G(-1;y) \\
&+ \frac{1}{96} G(-(1-x+x^2)/x;y) G([1-o+o^2];x) + \frac{1}{8} G(-(1-x+x^2)/x;y) G(1;x) \\
&- \frac{7}{96} G(-(1-x+x^2)/x;y) G(0;x) - \frac{1}{64} G(-1;y)^2 - \frac{1}{32} G(-1;y) G([1-o+o^2];x) \\
&+ \frac{1}{32} G(-1;y) G(0;x) + \frac{1}{192} G([1-o+o^2];x)^2 + \frac{1}{8} G([1-o+o^2];x) G(1;x) \\
&- \frac{7}{96} G([1-o+o^2];x) G(0;x) + \frac{1}{16} G(1;x)^2 - \frac{3}{16} G(0;x) G(1;x) + \frac{1}{12} G(0;x)^2 \\
&+ \frac{1}{96} i\pi G(-(1-x+x^2)/x;y) - \frac{1}{32} i\pi G(-1;y) + \frac{1}{96} i\pi G([1-o+o^2];x) \\
&+ \frac{1}{8} i\pi G(1;x) - \frac{7}{96} i\pi G(0;x) - \frac{35}{1152} \pi^2
\end{aligned}$$

$I_{-1} = \ldots$ (very lengthy)

where $x = \frac{\sqrt{1-4m^2/s}-1}{\sqrt{1-4m^2/s}+1}$, $y = -t/m^2$



$$= \frac{x^2}{m^4(1-x)^2(1-x+x^2+xy)} \sum_{i=-4}^{0} I_i \epsilon^i + \mathcal{O}(\epsilon)\,,$$

simplified representation:

$$I_{-4} = \frac{7}{384}$$

$$I_{-3} = -\frac{1}{32}\ln(y_1+z_1) + \frac{1}{64}\ln y_1 - \frac{5}{192}\ln z_1 + \frac{1}{32} i\pi$$

$$I_{-2} = \frac{1}{64}\ln^2(y_1+z_1) - \frac{1}{64}\ln^2 y_1 + \frac{1}{192}\ln^2 z_1 - \frac{1}{32}\ln y_1 \ln z_1 + \frac{1}{16}\ln z_1 \ln(y_1+z_1)$$
$$- \frac{1}{32} i\pi \ln(y_1+z_1) - \frac{1}{16} i\pi \ln z_1 - \frac{47}{1152}\pi^2$$

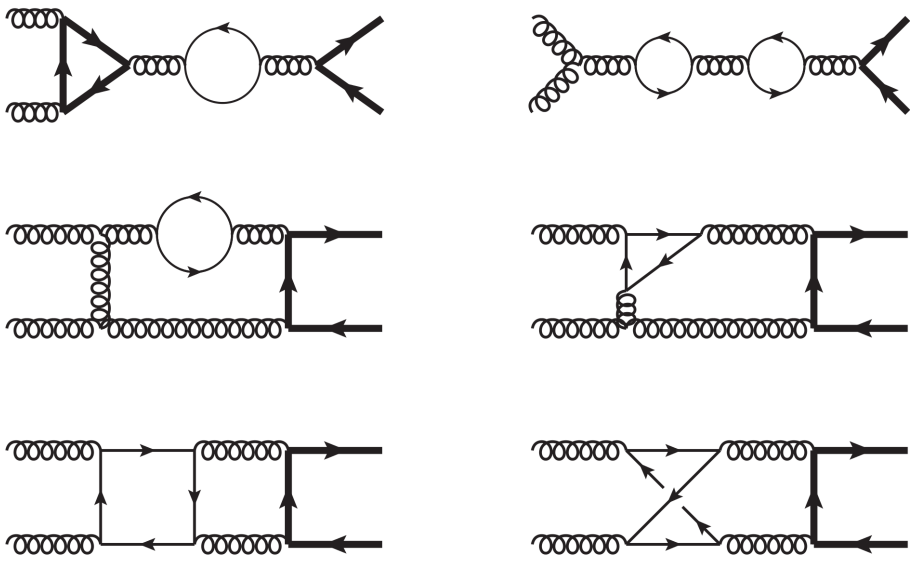
new functional basis for all poles: $\mathrm{Li}_3\left(\frac{y_1 z_1}{y_1+z_1}\right)$, $\mathrm{Li}_2\left(\frac{y_1 z_1}{y_1+z_1}\right)$, $\ln(y_1+z_1)$, $\log y_1$, $\ln z_1$

$(y_1 = y+1,\ z_1 = z+1)$ instead of 28 (65 expanded) GPLs

coproduct based and other algorithms for the multiple polylogarithms

- partly from Brown '11, Duhr '12, Duhr, Gangl, Rhodes '11
- numerical routines from Vollinga, Weinzierl '04

Light fermionic two-loop corrections to $gg \to t\bar{t}$



+ more

[Bonciani, Ferroglia, Gehrmann, AvM, Studerus '13]

gg **channel**: 789 two-loop diagrams (+ ghost init.)

$$\mathbf{2\,Re}\left\langle \mathcal{M}^{(0)} | \mathcal{M}^{(2)} \right\rangle = 2 C_F N_c \Big(N_c^3 \mathbf{A} + N_c \mathbf{B} + \frac{1}{N_c} \mathbf{C} + \frac{1}{N_c^3} \mathbf{D}$$
$$+ N_c^2 n_l \mathbf{E_l} + n_l \mathbf{F_l} + \frac{n_l}{N_c^2} \mathbf{G_l} + N_c n_l^2 \mathbf{H_l} + \frac{n_l^2}{N_c} \mathbf{I_l}$$
$$+ N_c^2 n_h \mathbf{E_h} + n_h \mathbf{F_h} + \frac{n_h}{N_c^2} \mathbf{G_h} + N_c n_h^2 \mathbf{H_h} + \frac{n_h^2}{N_c} \mathbf{I_h}$$
$$+ N_c n_l n_h \mathbf{H_{lh}} + \frac{n_l n_h}{N_c} \mathbf{I_{lh}} \Big)$$

PRIMARY RESULT

$$G(\ldots;y), \quad \text{weights} \in \left\{-1, 0, -\frac{1}{x}, -x, -(1+x^2)/x, -(1-x+x^2)/x\right\}$$

$$G(\ldots;x), \quad \text{weights} \in \left\{-1, 0, 1, [1+o^2], [1-o+o^2]\right\}$$

OPTIMISED FUNCTIONAL BASIS

choose real valued $\ln$, $\mathrm{Li}_n(R_1)$, $\mathrm{Li}_{2,2}(R_1, R_2)$ with

$$|R_1| < 1, \qquad |R_1 R_2| < 1$$

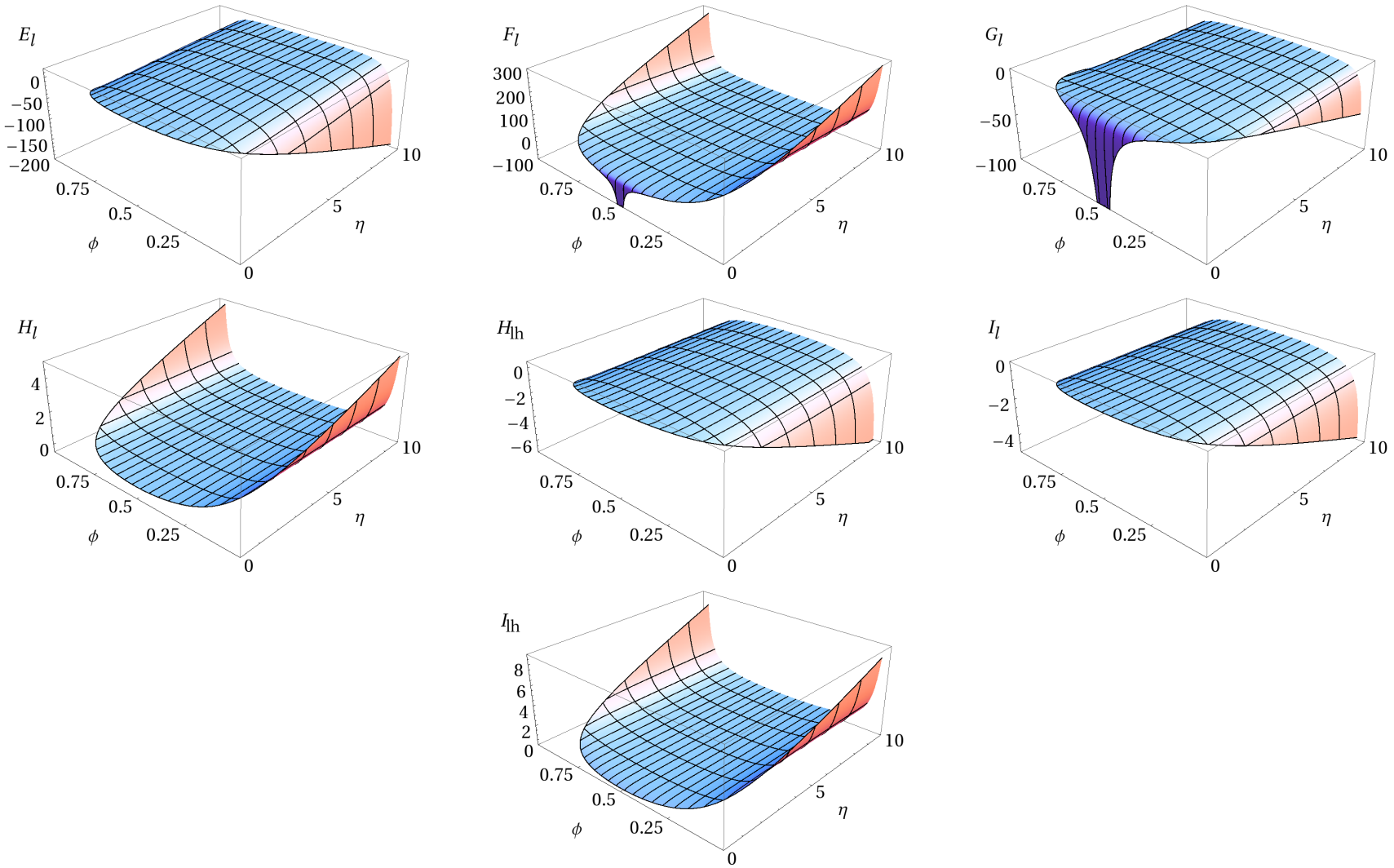
such that Li functions have convergent power series

$$\mathrm{Li}_n(R_1) = -\sum_{j_1=1}^{\infty} \frac{R_1^{j_1}}{j_1^n}, \qquad \mathrm{Li}_{2,2}(R_1, R_2) = \sum_{j_1=1}^{\infty}\sum_{j_2=1}^{\infty} \frac{R_1^{j_1}}{(j_1+j_2)^2} \frac{(R_1 R_2)^{j_2}}{j_2^2}$$

features:

- $R_i \in \left\{\pm x,\ x^2,\ \frac{1}{y+1},\ -xy,\ \frac{(1-x)y}{y+1},\ \frac{(1-x)z(x,y)}{z(x,y)+1}, \ldots\right\}$
- **very fast and stable** numerical evaluation

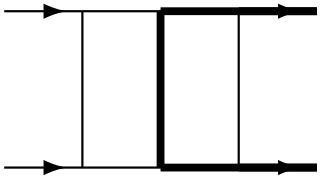
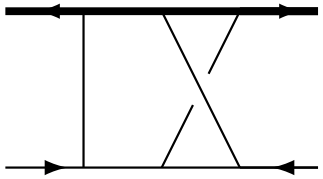
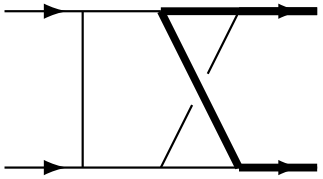
Results for $gg \to t\bar{t}$ light N_f



$$\eta = s/(4m^2) - 1, \quad \phi = -(t - m^2)/s$$

Next: complete $q\bar{q} \to t\bar{t}$

105 master integrals (w/ products, w/o crossings)

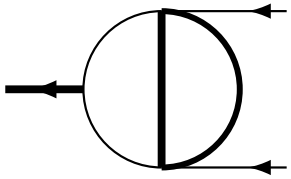




+ more

[J. Henn, AvM, V. Smirnov (in preparation)]

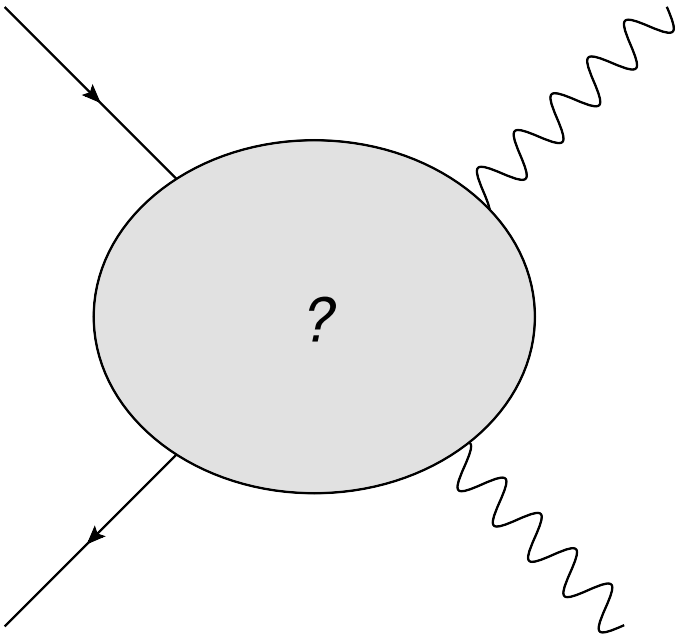
alphabet: similar as before

- contains root $\sqrt{-s(4m^2 - s)}$ (rationalized by x)
- heavy fermionic piece



contains $\sqrt{-s(4m^2+s)}$, known: Bonciani, Mastrolia, Remiddi '04

Vector boson pair production at LHC



- background to Higgs production
- precision tests of electroweak interactions

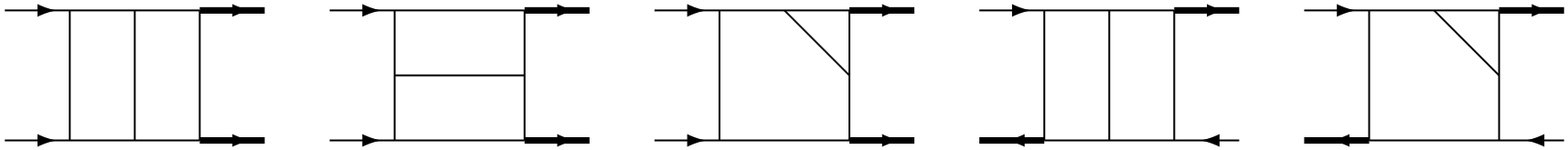
here:

- $q\bar{q} \to ZZ,\ W^+W^-$
- last missing NNLO ingredient: two-loop contributions

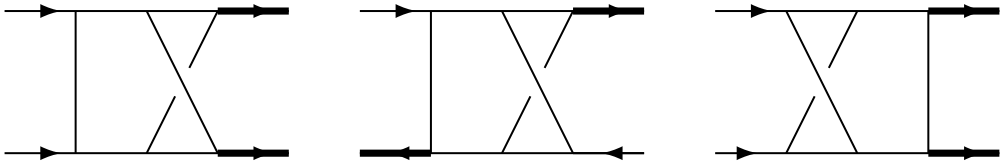
Master integrals for $q\bar{q} \to VV$

61 master integrals (w/ products, w/o crossings)

planar two-loop master integrals [Gehrmann, Tancredi, Weihs '13]



here: non-planar master integrals [Gehrmann, AvM, Tancredi, Weihs '14]



for VV', see [Henn, Melnikov, Smirnov '14], [Caola, Henn, Melnikov, Smirnov '14]

Differential equations

- general form, $\epsilon = (4-d)/2$:

$$\frac{\partial}{\partial x_i} f_j(x_i, \varepsilon) = A^{(i)}_{jk}(x_i, \varepsilon) f_k(x_i, \varepsilon)$$

- in certain cases proper choice of basis achieves:

$$A^{(i)}_{jk}(x_i, \varepsilon) = \varepsilon\, \bar{A}^{(i)}_{jk}(x_i)$$

[Henn '13]; [Henn, A. Smirnov, V. Smirnov '13], see also talk by J. Henn

Canonical form

$$\mathrm{d}f(x_i, \varepsilon) = \varepsilon\, \mathrm{d}A(x_i)\, f(x_i, \varepsilon)$$

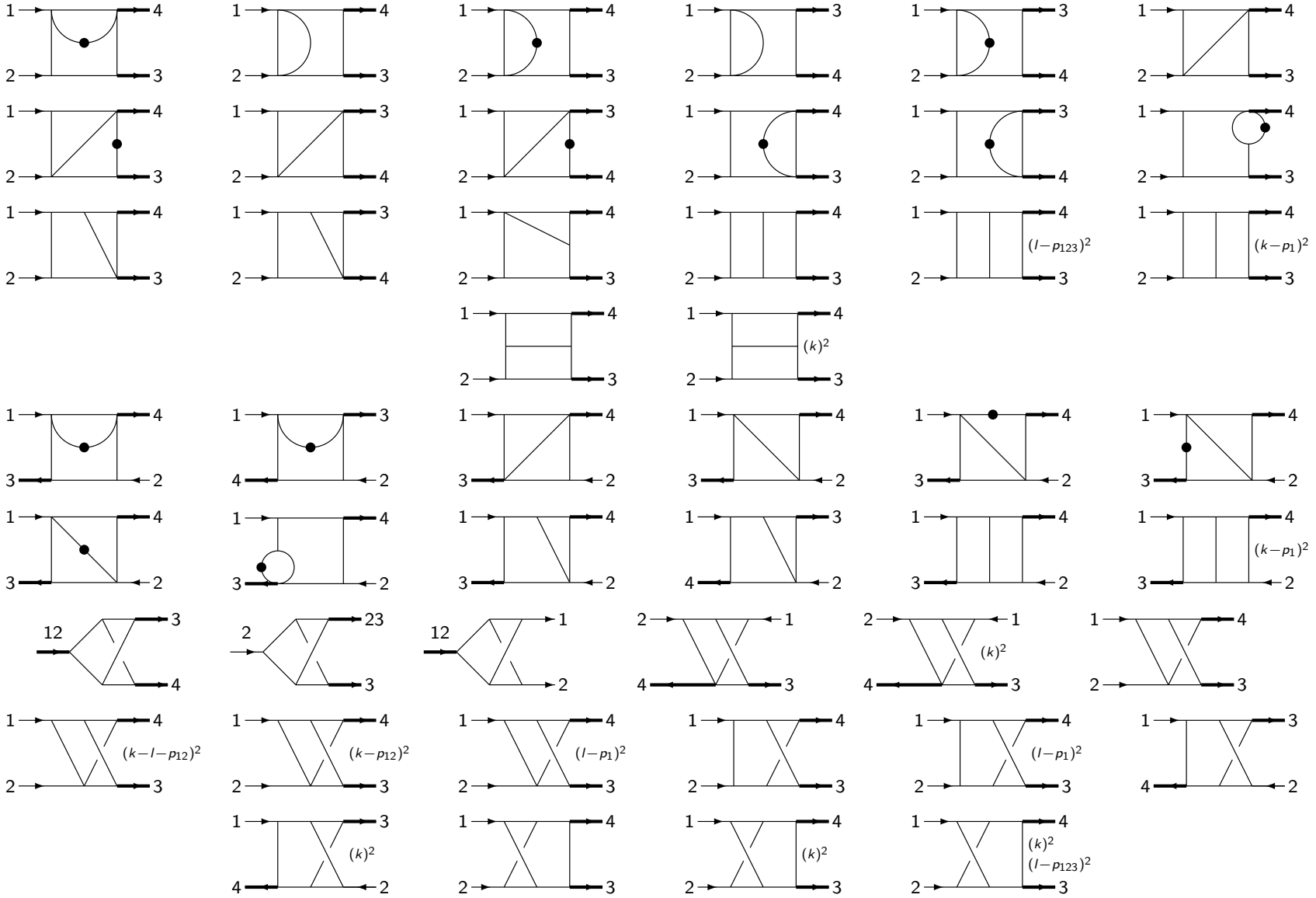
with

$$A(x_i) = A^{(l)} \ln R_l(x_i)$$

- decoupling, clear structure, pure functions for each order in ε
- works for f expandable in terms of multiple polylogarithms, beyond: [Tancredi, Remiddi '13]

Master integrals for $q\bar{q} \to VV$

(planar bubbles, triangles and one-loop products not shown)



[Gehrmann, AvM, Tancredi, Weihs '14]

how to find **canonical basis** ?

- general hints: [Henn '13]; [Smirnov, Smirnov, Henn '13]
- basis changes for special cases: [Argeri, Di Vita, Mastrolia, Mirabella, Schlenk, Schubert, Tancredi '14]; [Caron-Huot, Henn '14]

here:

PROCEDURE

construct canonical basis starting from rough first guess

1. **bottom up:** subtopos canonical
2. for given topo, guess basis: **triangular for** $\epsilon = 0$, diff. eq. **linear in** ϵ (top level only)
3. integrate out **homogeneous part for** $\epsilon = 0$ (top level only)
4. **remove unwanted terms** $1/(1+v\epsilon)^n$**,** 1**,** ϵ^n iteratively
   simplifying assumption: minimal shifts

RESULT

vector of 75 master integrals in canonical basis with alphabet:

$$x,\ 1-x,\ 1+x,\ z,\ 1+z,\ x-z,\ 1-xz,\ 1+x^2-xz,\ 1+x+x^2-xz,\ z(1+x+x^2)-x$$

where $s = m^2(1+x)^2/x,\ \ u = -m^2 z$

integration and boundary terms:

- integrate all masters in uniform setup, "physical" variables
- input some simple bubbles and triangles
- remaining boundary functions fixed by regularity (unphys. region):

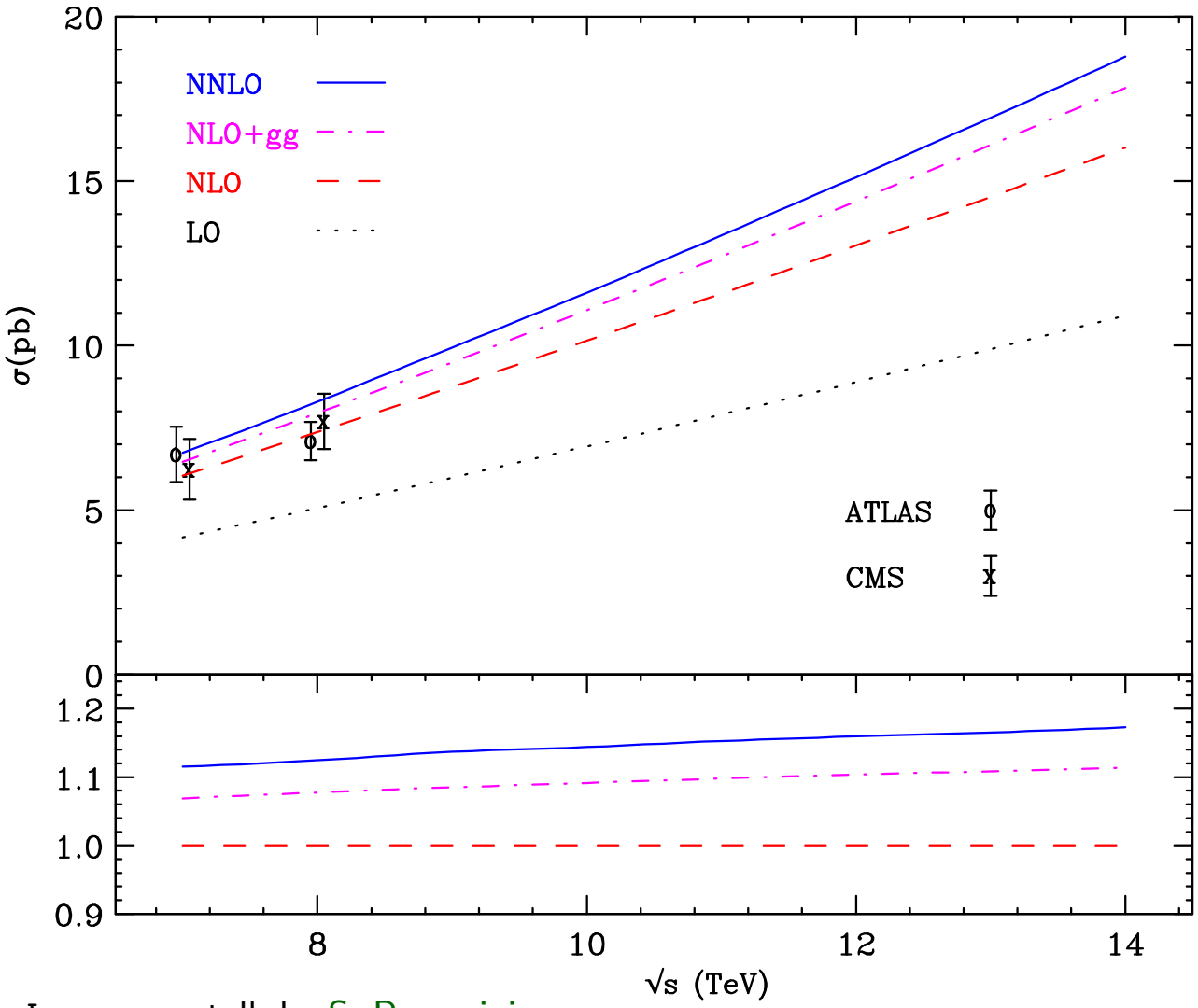
$$z \to x, \qquad z \to 1/x, \qquad z \to -1, \qquad z \to (1 + x + x^2)/x, \qquad x \to 1$$

- checks 1: consistency of bound., diff. eq., known planar masters, real in Eulicidean region
- checks 2: SecDec 2.1

optimisation for numerical evaluation:

- use real valued $\mathrm{Li}_{2,2}$, Li_n, ln, optimize for power numerical evaluation
- evaluation time: $O(30ms)$ for generic point

Result: ZZ production at NNLO



- 1-loop by `OpenLoops`, see talk by S. Pozzorini
- see also talk by D. Rathlev

[Cascioli, Gehrmann, Grazzini, Kallweit, Maierhöfer, AvM, Pozzorini, Rathlev, Tancredi, Weihs '14]

Conclusions & outlook

- technical progress
  - Reduze 2: reductions & shift finding for broad class of integrals
  - algorithms for multiple polylogarithms
  - procedure to put DEQ into normal form, works for typical cases
- $t\bar{t}$: analytic 2-loop
  - leading N_c + (light) fermionic available
  - full $q\bar{q} \to t\bar{t}$ in preparation
- ZZ, W^+W^- : analytic 2-loop
  - complete set of two-loop master integrals
  - first NNLO prediction for ZZ total cross section at LHC
  - outlook: W^+W^-, differential observables

Supplementary Slides

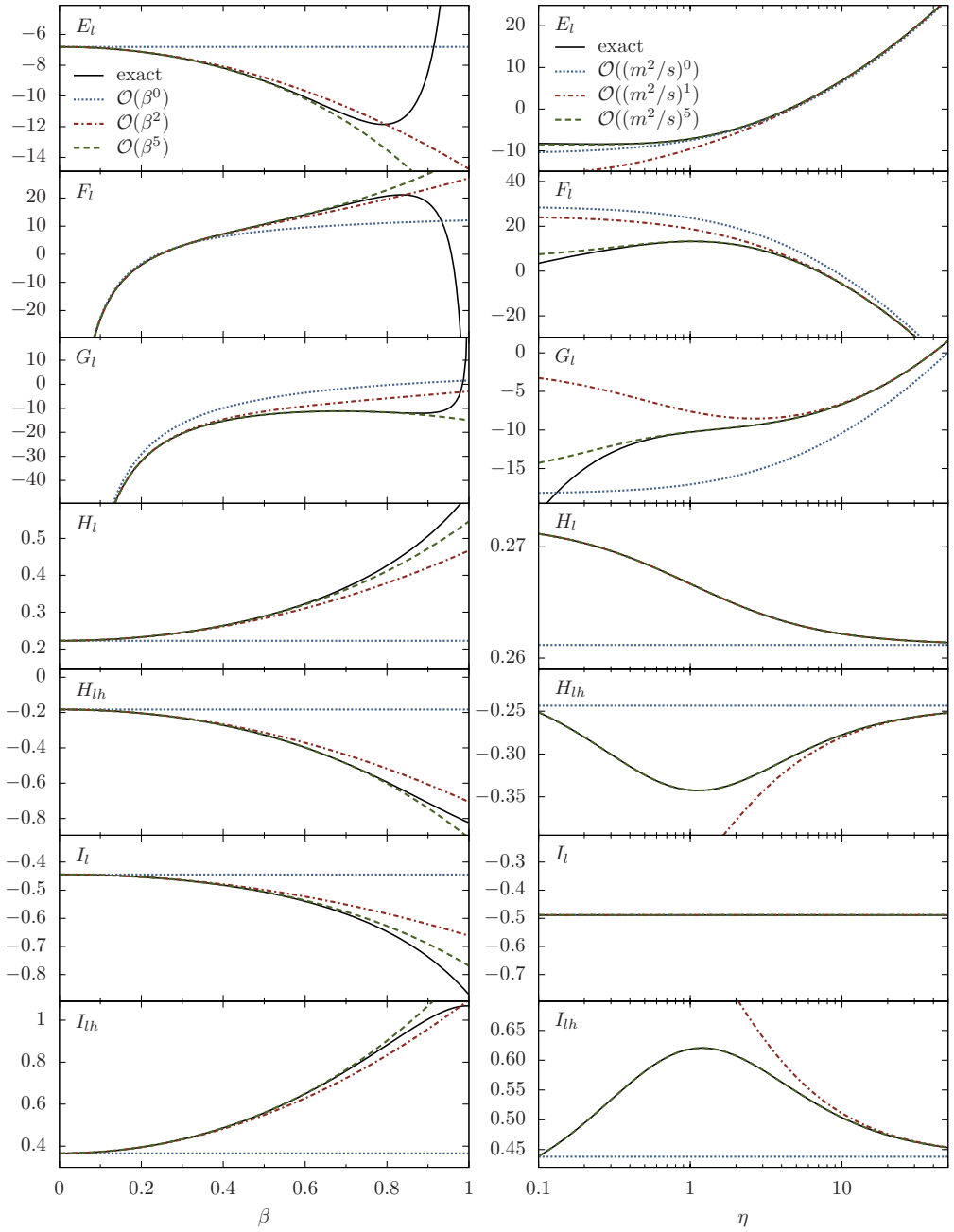
3. Algorithms for multiple polylogarithms

4. Expansions for light N_f corrections to $gg \to t\bar{t}$

Algorithms for Multiple Polylogarithms

main algorithms:

1. **normal form for specific arguments**
   - independent of symbol calculus
   - uses Vollinga, Weinzierl '04 for numerical evaluation, fits constants
2. **coproduct based normal form** for general choice of basis
   - based on Goncharov '02, Brown '11, Duhr '12, Duhr, Gangl, Rhodes '11
   - handles generalised weights
   - identifies products (e.g. $G(0,1;x) + G(1,0;x) \to G(0;x)G(1;x)$)
   - matches irreducible factors *at symbol level*
   - uses Vollinga, Weinzierl '04 for numerical evaluation, fits constants
3. **construct new basis** with desired properties
   - based on Duhr, Gangl, Rhodes '11, apply to generalised weights



left: threshold expansion, $\beta = \sqrt{1 - 4m^2/s}$, $\cos\theta = 0.7$

right: "high energy" expansion, $\eta = s/(4m^2) - 1$, $\phi = -(t - m^2)/s = 0.35$