REcursive Computation of One-Loop Amplitudes

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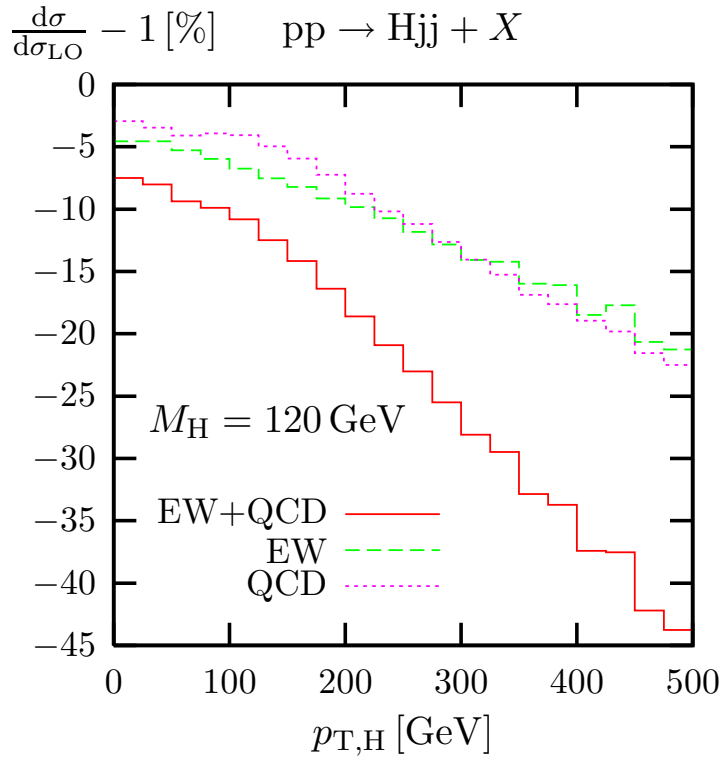
After the discovery of the Higgs boson:

- Precise investigation of the Standard Model and beyond
- Need to have under control potential large corrections for several processes

After the discovery of the Higgs boson:

- Precise investigation of the Standard Model and beyond
- Need to have under control potential large corrections for several processes
- QCD corrections are known to be large
- EW corrections can be enhanced:
  - in high energy regions (Sudakov log's)
  - in Higgs physics
  - by photon emission (mass-singular log's)



Let's concentrate on one loop corrections

Les Houches wishlist 2013 at one loop

- **QCD:**

  $pp \to t\bar{t}H, \qquad pp \to t\bar{t} + j$ (top decays)

- **EW:**

  $pp \to 3j,$

  $pp \to t\bar{t}, \qquad pp \to t\bar{t}H, \qquad pp \to t\bar{t} + j$ (top decays)

  $pp \to V + 2j, \qquad pp \to VV', \qquad pp \to VV + j,$
  $pp \to VV + 2j \qquad pp \to VV'\gamma, \qquad pp \to VV'V'',$

  $(V, V', V'' = W, Z$ decay leptonically)

- Many issues at hadronic level:

  Multi-channel MCs, Real emission, PDFs, Parton Shower, ...

- At least the partonic processes should be **automatized**

Many codes have been produced:

MCFM	Campbell, Ellis
FormCalc	Agrawal, Hahn, Mirabella
BlackHat	Berger, Bern, Dixon, Febres Cordero, Forde, Ita, Kosower, Maître
VBFNLO	Arnold, Bähr, Bozzi, Campanario, Englert, Figy, Greiner, Hackstein, Hankele, Jäger, Klämke, Kubocz, Oleari, Plätzer, Prestel, Worek, Zeppenfeld
HELAC-NLO	Bevilacqua, Czakon, Garzelli, van Hameren, Kardos, Papadopoulos, Pittau, Worek
GoSam	Cullen, Greiner, Heinrich, Luisoni, Mastrolia, Ossola, Reiter, Tramontano
SANC	Sadykov, Arbuzov, Bardin, Bondarenko, Christova, Kalinovskaya, Kolesnikov, Sapronov, Uglov
NJet	Badger, Biedermann, Uwer, Yundin
AMC@NLO	Hirschi, Frederix, Frixione, Garzelli, Maltoni, Pittau
OpenLoops	Cascioli, Maierhöfer, Pozzorini

Most of them are efficient codes for **QCD**

R E C O L A

REcursive Computation of One Loop Amplitudes
(in the **full Standard Model**)

Based on **recursive relations** for **off-shell currents**

Off-shell tree currents

Given a process with L external legs:

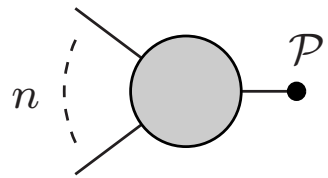
$$\underbrace{\mathcal{P}_1 + \ldots + \mathcal{P}_{L-1}}_{\text{primary}} + \underbrace{\mathcal{P}_L}_{\text{last}} \to 0$$

Off-shell current of a particle $\mathcal{P}$ with n primary legs:

Def: Amplitude made of n primary on-sheel particles and the off-sheel particle $\mathcal{P}$

colour

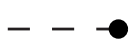
$$w(\mathcal{P}, \mathcal{C}, \{l_1, \ldots, l_n\}) =$$



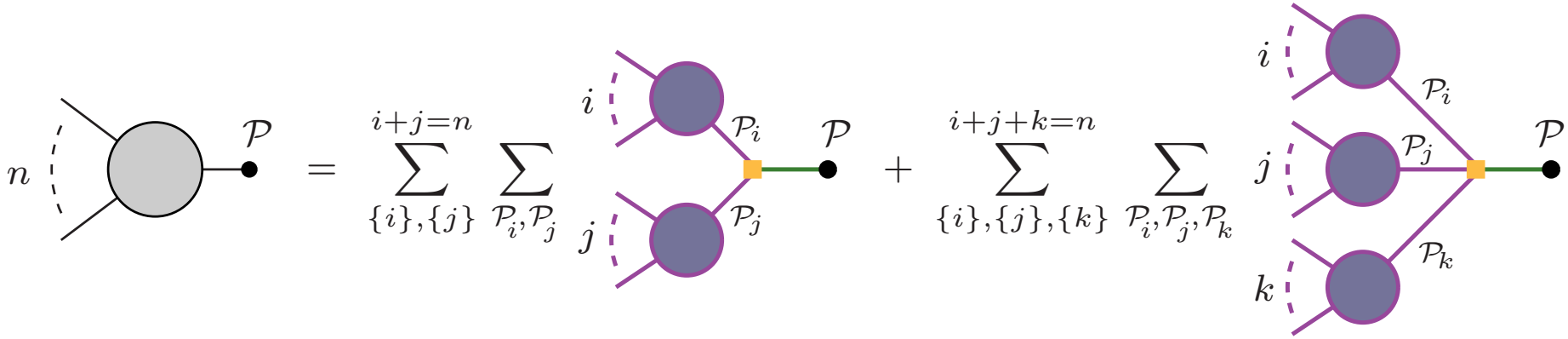
particle: e^-, e^+, γ, ...

List of primary legs

- w is a scalar, spinor or vector
- The off-shell currents for external legs are the wave functions:

 $= u_\lambda(p)$  $= \bar{u}_\lambda(p)$  $= \epsilon_\lambda(p)$  $= 1$

Recursion relation for tree amplitudes



(**incoming currents**) $\times$ (**coupling**) $\times$ (**propagator**)

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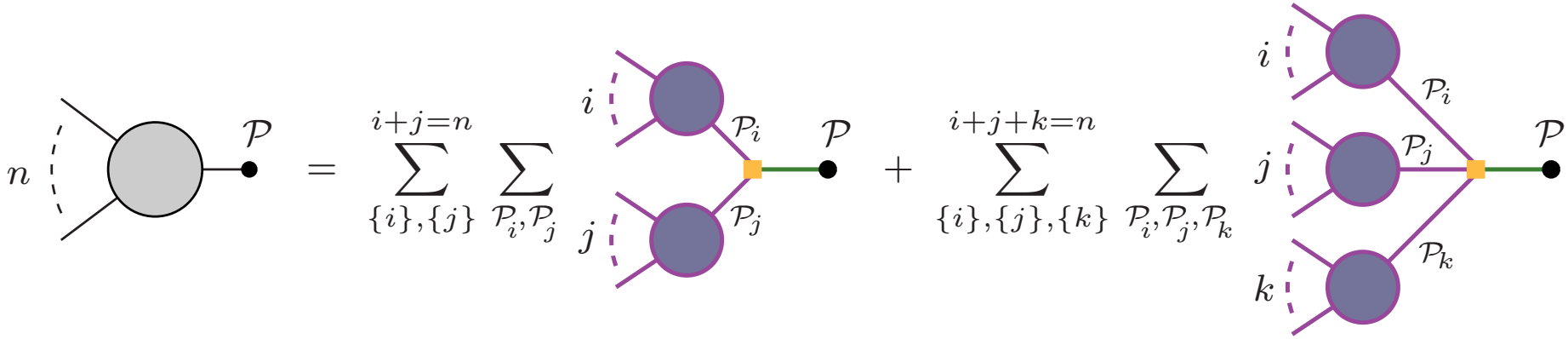
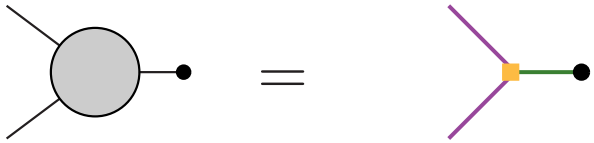


(**incoming currents**) $\times$ (**coupling**) $\times$ (**propagator**)

- Recursive procedure:

2-leg currents:



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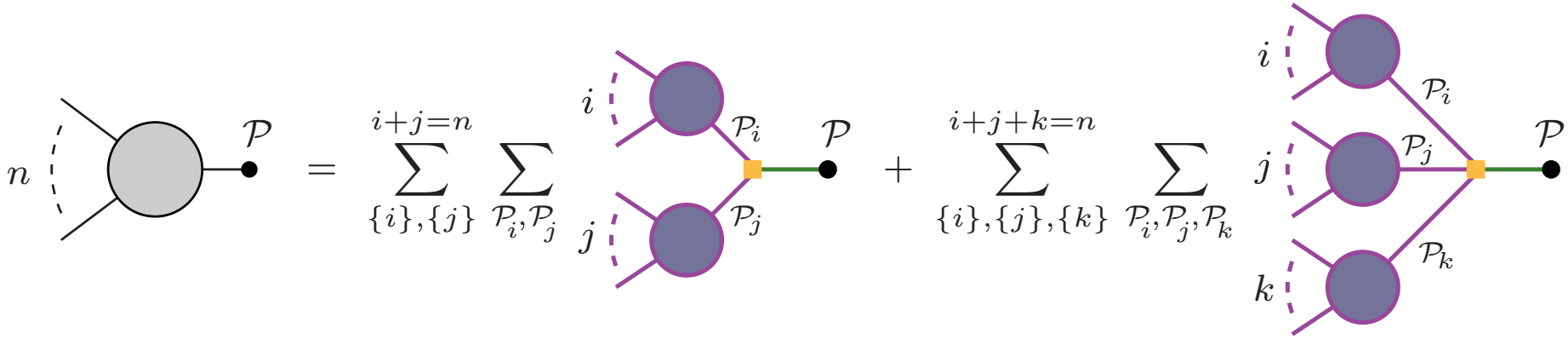

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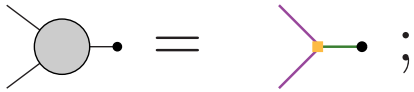
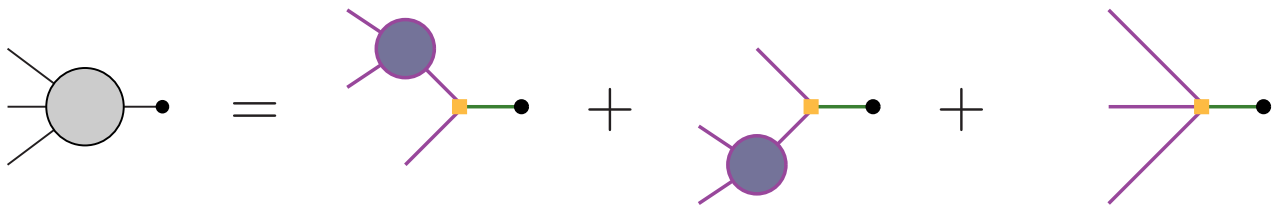


3-leg currents:



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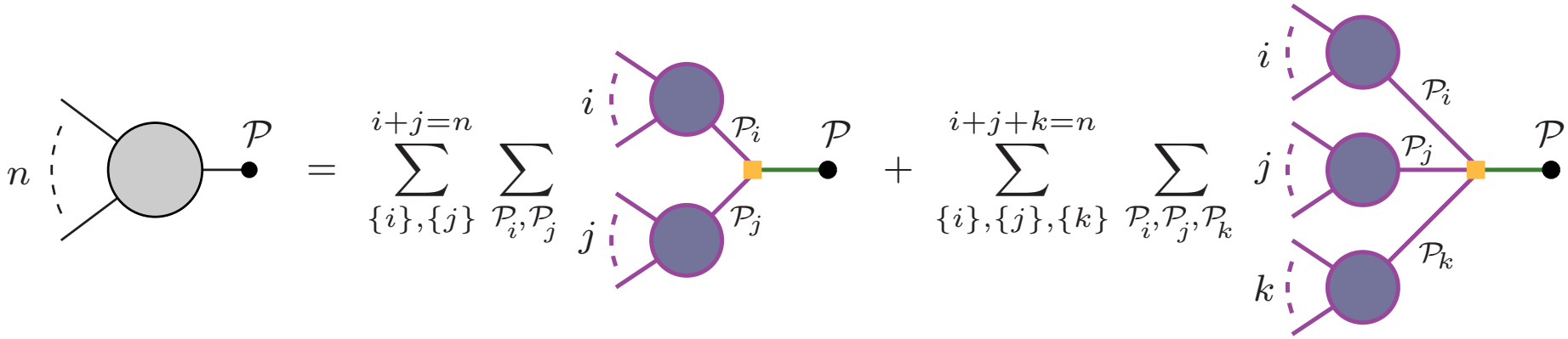

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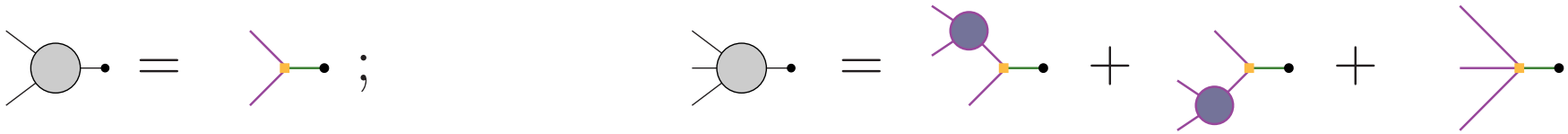
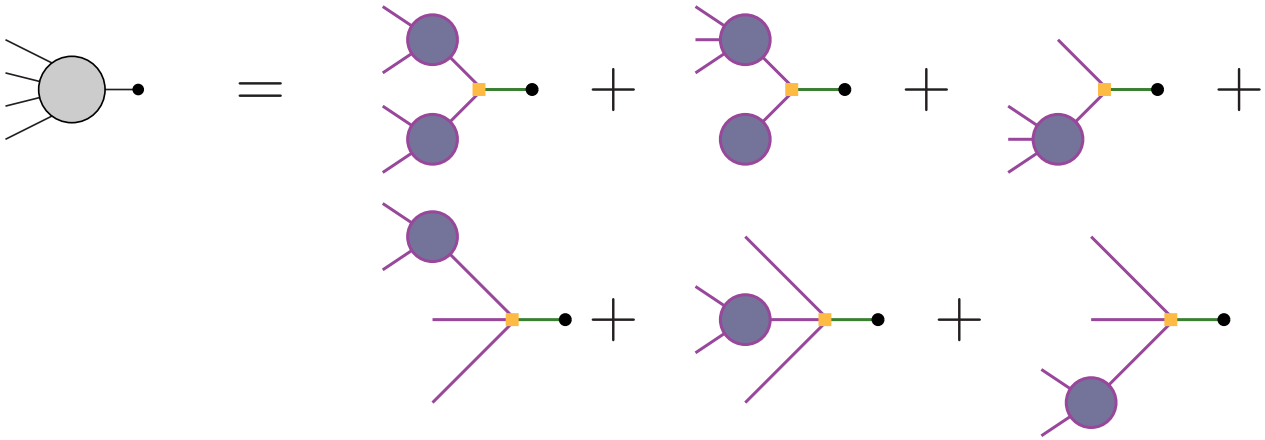


4-leg currents:



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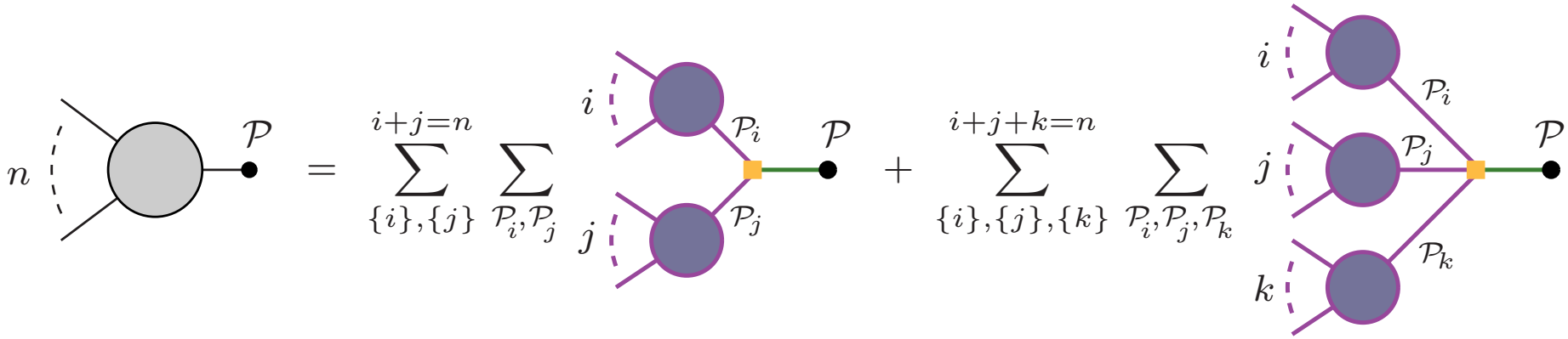

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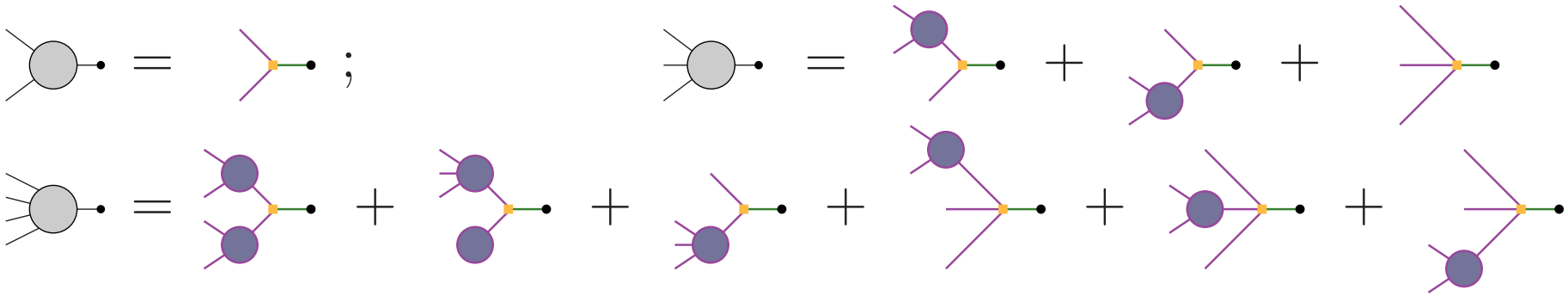

etc. . . .

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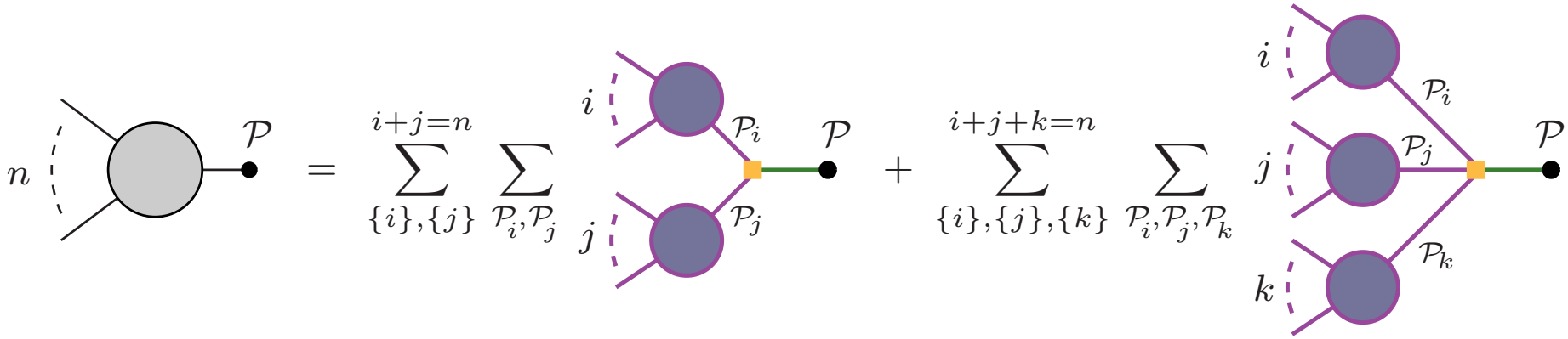

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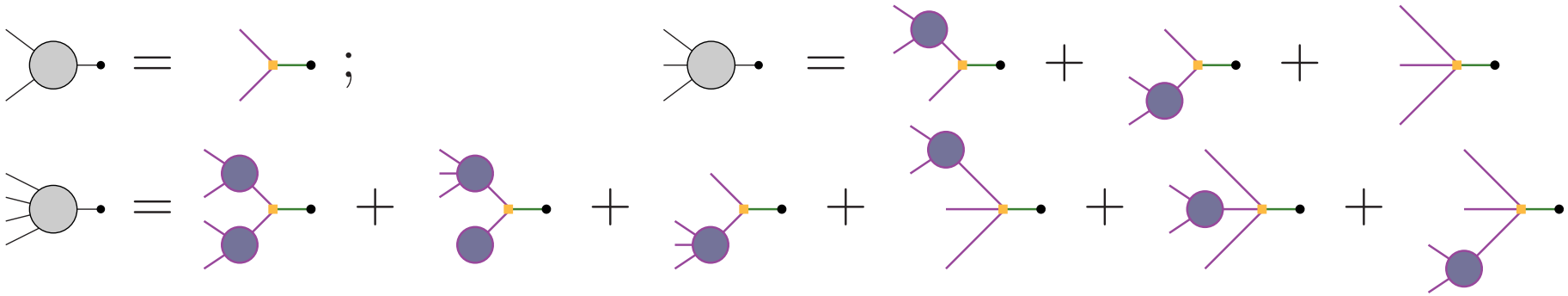

etc. . . .

- Amplitude: $\mathcal{A} = w(\overline{\mathcal{P}}_L, 2^{L-1}-1) \times$ (**propagator**)$^{-1} \times w(\mathcal{P}_L, 2^{L-1})$

Recursion relation for loop amplitudes

General form of the amplitude:

Tensor Coefficients (TCs)

$$\mathcal{A} = \sum_t c^{(t)}_{\mu_1 \ldots \mu_{r_t}} T^{\mu_1 \ldots \mu_{r_t}}_{(t)}$$

Tensor Integrals (TIs)

$$T^{\mu_1 \cdots \mu_{r_t}}_{(t)} = \int \frac{d^n q \, q^{\mu_1} \cdots q^{\mu_{r_t}}}{D^{(t)}_0 \cdots D^{(t)}_{k_t}} \qquad D^{(t)}_{k_t} = (q + p^{(t)}_{k_t})^2 - (m^{(t)}_{k_t})^2$$

Indices $\mu_1, \ldots, \mu_{r_t}$ are computed numerically in D=4 dimensions.

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$\rightsquigarrow$ Add the rational part $\mathcal{A}_{\mathrm{R2}}$

- Effective Feynman rules
  [Draggiotis, Garzelli, Malamos, Papadopoulos, Pittau '09-'10]

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General form of the amplitude:

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⇝ Add the rational part $\mathcal{A}_{\mathrm{R2}}$ → **tree-like amplitudes**

- Effective Feynman rules
  [Draggiotis, Garzelli, Malamos, Papadopoulos, Pittau '09-'10]

⇝ Add the counterterms contribution $\mathcal{A}_{\mathrm{CT}}$

Basic idea: Cut the loop line and consider tree diagrams with two more legs.

[A. van Hameren, JHEP 0907 (2009) 088]

Given the loop process

$$\mathcal{P}_1 + \ldots + \mathcal{P}_L \to 0$$

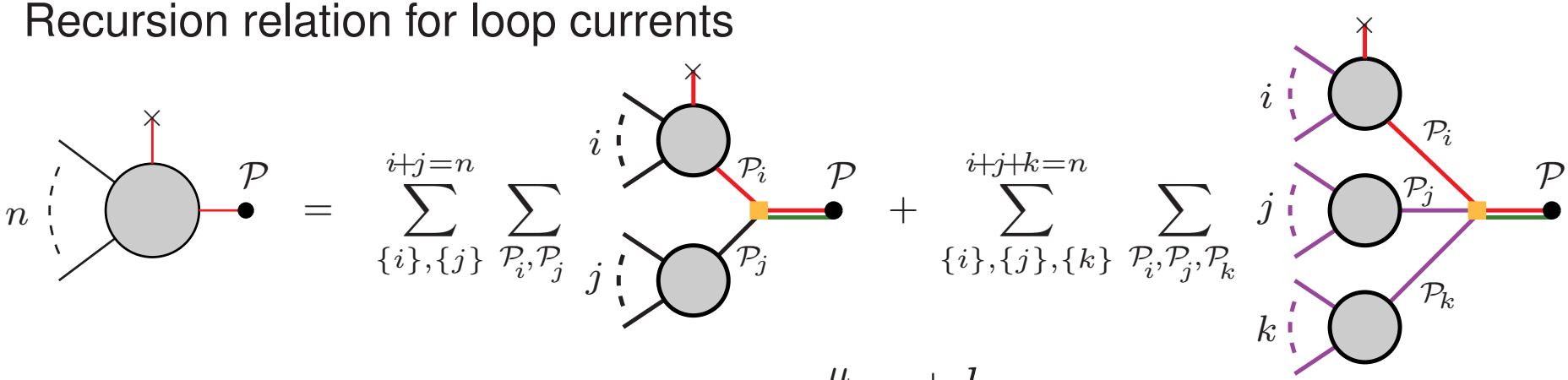
we consider the tree processes

$$\underbrace{\mathcal{P}_1 + \ldots + \mathcal{P}_L + \mathcal{P}}_{\text{primary}} + \underbrace{\overline{\mathcal{P}}}_{\text{last}} \to 0 \qquad \forall\, \mathcal{P} \in \{\text{Particle of the SM}\}$$

Problem: Associated tree diagrams are more than the original loop diagrams

$\rightsquigarrow$ **Selection rules to avoid double counting of the associated trees**

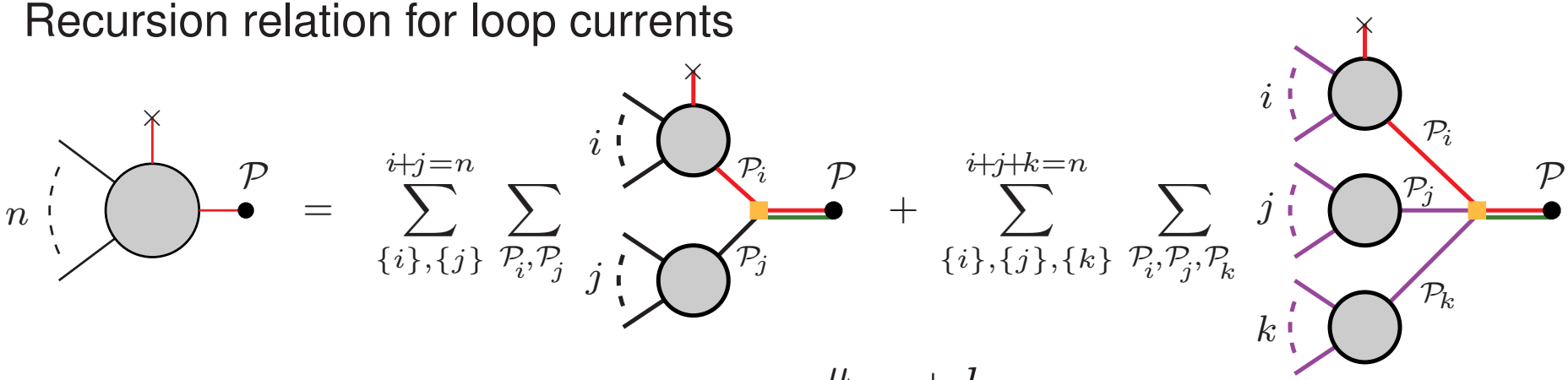
- Recursion relation for loop currents



$$(\textbf{coupling}) \times (\textbf{propagator}) = \frac{a^\mu q_\mu + b}{(q+p)^2 - m^2}$$

$q =$ loop momentum

- Recursion relation for loop currents



$$(\textbf{coupling}) \times (\textbf{propagator}) = \frac{a^\mu q_\mu + b}{(q+p)^2 - m^2} \qquad q = \text{loop momentum}$$

loop current (q) $= \sum_{r=0}^{k} a_{k,r}^{\mu_1 \cdots \mu_r} \dfrac{q_{\mu_1} \cdots q_{\mu_r}}{\prod_{h=0}^{k}[(q+p_h)^2 - m_h^2]}$

number of propagators

rank

computed in the recursion relation

goes in the TIs

Remark: Indices $\mu_1, \ldots, \mu_r$ are symmetrized at each step

- The coefficients $a_{k,r}^{\mu_1 \cdots \mu_r}$ of the last current give the TCs $c^{(t)}_{\mu_1 \ldots \mu_{r_t}}$

Colour-flow representation

[Kanaki, Papadopoulos '00; Maltoni, Paul, Stelzer, Willenbrock '02]

Structure of amplitude:

$$\mathcal{A}^{i_1\cdots i_n}_{j_1\cdots j_n} = \sum_{P(j_1,\ldots,j_n)} \delta^{i_1}_{j_1}\cdots\delta^{i_n}_{j_n}\,\mathcal{A}_P$$

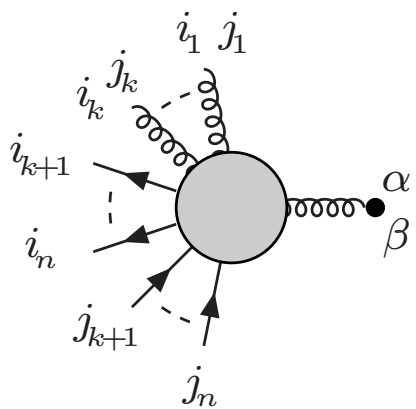
Structure-dressed (or colour-ordered) amplitudes:

$\rightarrow$ Compute $\mathcal{A}_P$ for all possible P $(n!)$

Squared amplitude:

$$\overline{\mathcal{M}^2} = \sum_{P,P'} \mathcal{A}^*_P\, C_{PP'}\, \mathcal{A}_{P'}$$

It requires structure-dressed currents:



$\rightarrow$

saturated part open part

$$\delta^{i_1}_{j_1}\cdots\delta^{i_{n-1}}_{j_{n-1}}\;\delta^{\alpha}_{j_n}\;\delta^{i_n}_{\beta}$$

$$w(\,\mathcal{P},\;\mathcal{C},\;\mathcal{B}_n\,)$$

with all possible permutations of $j_1,\ldots j_n,\beta$

Optimization for colour

- U(1)-gluons are unphysical:

$$\text{Gluon field} \;:\quad \sqrt{2}\; A^a_\mu\, (\lambda^a)^i_j \;=\; (\mathcal{A}_\mu)^i_j$$

"usual" gluon with colour index $a = 1, \ldots, 8$

gluon with colour-flow ^i_j
$i, j = 1, 2, 3$
$\sum_i (A_\mu)^i_i = 0$

Each external gluon with indices i, j has to be contracted with:

$$P^{ii'}_{jj'} = \delta^i_{j'}\delta^{i'}_j - \frac{1}{N_C}\,\delta^i_j\delta^{i'}_{j'}$$

$\rightsquigarrow$ All colour structures with $\ldots\,\delta^i_j\,\ldots$ does not contribute

Compute just colour structures without δ^i_j for each ^i_j-gluon

Optimization for colour

- **Compute just colour structures without δ^i_j for each ^i_j-gluon**

  $\rightsquigarrow$ $g + g + g \to 0$:

$$\delta^{i_1}_{j_1}\delta^{i_2}_{j_2}\delta^{i_3}_{j_3}\ ,\quad \delta^{i_1}_{j_1}\delta^{i_2}_{j_3}\delta^{i_3}_{j_2}\ ,\quad \delta^{i_1}_{j_2}\delta^{i_2}_{j_1}\delta^{i_3}_{j_3}\ ,\quad \delta^{i_1}_{j_2}\delta^{i_2}_{j_3}\delta^{i_3}_{j_1}\ ,\quad \delta^{i_1}_{j_3}\delta^{i_2}_{j_1}\delta^{i_3}_{j_2}\ ,\quad \delta^{i_1}_{j_3}\delta^{i_2}_{j_2}\delta^{i_3}_{j_1}$$

Optimization for colour

- **Compute just colour structures without δ^i_j for each ^i_j-gluon**

$\rightsquigarrow$ $g + g + g \to 0$: 2 colour structures instead of 6

$$\cancel{\delta^{i_1}_{j_1}\delta^{i_2}_{j_2}\delta^{i_3}_{j_3}}\,,\quad \cancel{\delta^{i_1}_{j_1}\delta^{i_2}_{j_3}\delta^{i_3}_{j_2}}\,,\quad \cancel{\delta^{i_1}_{j_2}\delta^{i_2}_{j_1}\delta^{i_3}_{j_3}}\,,\quad \delta^{i_1}_{j_2}\delta^{i_2}_{j_3}\delta^{i_3}_{j_1}\,,\quad \delta^{i_1}_{j_3}\delta^{i_2}_{j_1}\delta^{i_3}_{j_2}\,,\quad \cancel{\delta^{i_1}_{j_3}\delta^{i_2}_{j_2}\delta^{i_3}_{j_1}}$$

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$\rightsquigarrow$ $g + g + g + g \to 0$: 9 colour structures instead of 24

$\rightsquigarrow$ $g + g + g + g + g \to 0$: 44 colour structures instead of 120

$\rightsquigarrow$ $g + g + g + q + \bar{q} \to 0$: 11 colour structures instead of 24

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Optimization for colour

- **Compute just colour structures without δ^i_j for each ^i_j-gluon**

  - $g + g + g \to 0$: 2 colour structures instead of 6

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- **Compute once currents differing just by the colour structure**

Example:

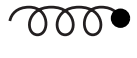
$\bar{u}$	$+$	u	$+$	g	$+$	g	$+$	g	$\rightarrow$	0
1		2		4		8		16		

Example:

$$\underset{1}{\bar{u}} + \underset{2}{u} + \underset{4}{g} + \underset{8}{g} + \underset{16}{g} \rightarrow 0$$

2  $\beta = w(u, \delta^{i_2}_{\beta}, 2)$

4  α β $= w(g, \delta^{i_4}_{\beta}\delta^{\alpha}_{j_4}, 4)$

8  α β $= w(g, \delta^{i_8}_{\beta}\delta^{\alpha}_{j_8}, 8)$

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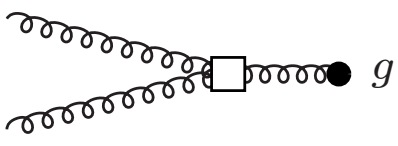
4  $\begin{matrix}\alpha\\ \beta\end{matrix} = w(g, \delta^{i_4}_{\beta}\delta^{\alpha}_{j_4}, 4)$

8  $\begin{matrix}\alpha\\ \beta\end{matrix} = w(g, \delta^{i_8}_{\beta}\delta^{\alpha}_{j_8}, 8)$

8 g

4 g



g $\rightarrow$ $w(g, \delta^{i_8}_{j_4}\delta^{i_4}_{\beta}\delta^{\alpha}_{j_8}, 12)$ $w(g, \delta^{i_4}_{j_8}\delta^{i_8}_{\beta}\delta^{\alpha}_{j_4}, 12)$

Example:

$$\bar{u} + u + g + g + g \rightarrow 0$$

1 2 4 8 16

2  $\beta = w(u, \delta^{i_2}_{\beta}, 2)$

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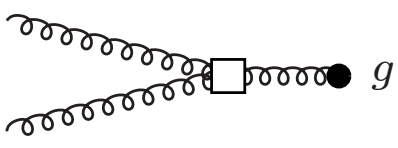

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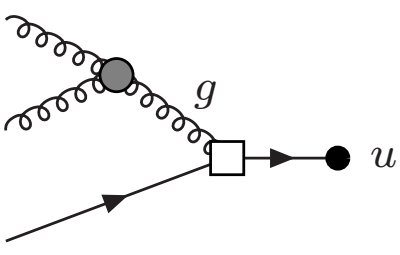
4 g

8 g

2 u



g u

Example:

$$\underset{1}{\bar{u}} + \underset{2}{u} + \underset{4}{g} + \underset{8}{g} + \underset{16}{g} \to 0$$

2  $\beta = w(u, \delta^{i_2}_{\beta}, 2)$

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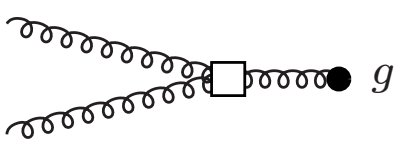
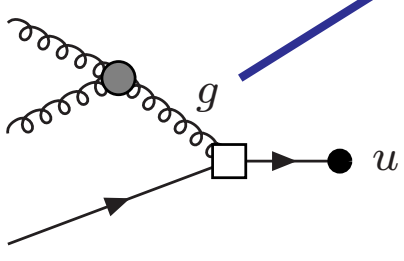


$\to$ $w(g, \delta^{i_8}_{j_4}\delta^{i_4}_{\beta}\delta^{\alpha}_{j_8}, 12)$ $\quad$ $w(g, \delta^{i_4}_{j_8}\delta^{i_8}_{\beta}\delta^{\alpha}_{j_4}, 12)$

4 g
8 g
2 u



$\to$

$$\boxed{\begin{array}{l} w(u, \delta^{i_2}_{j_8}\delta^{i_8}_{j_4}\delta^{i_4}_{\beta}, 14) \\ w(u, \delta^{i_4}_{j_8}\delta^{i_8}_{j_4}\delta^{i_2}_{\beta}, 14) \end{array}}$$

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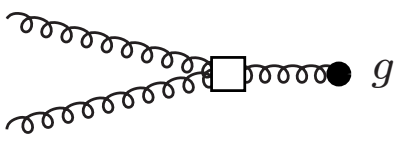

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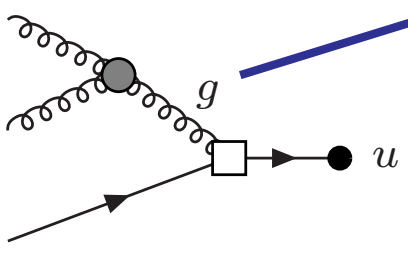

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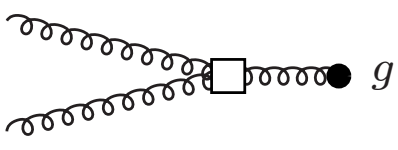

g $\rightarrow$ $\underbrace{w(g, \delta^{i_8}_{j_4}\delta^{i_4}_{\beta}\delta^{\alpha}_{j_8}, 12)}_{A}$ $\underbrace{w(g, \delta^{i_4}_{j_8}\delta^{i_8}_{\beta}\delta^{\alpha}_{j_4}, 12)}_{-A}$

4 g
8 g
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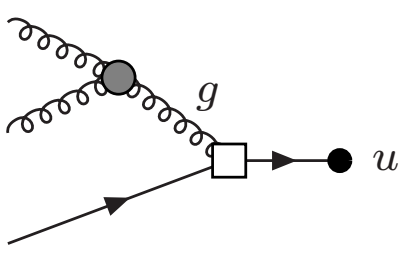

g u $\rightarrow$

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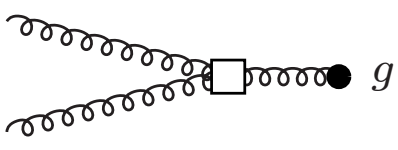

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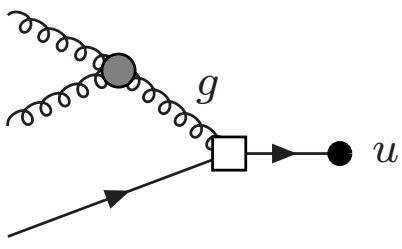

g u $\rightarrow$

$$\overbrace{w(u, \delta^{i_2}_{j_8}\delta^{i_8}_{j_4}\delta^{i_4}_{\beta}, 14)}^{B} \qquad \overbrace{w(u, \delta^{i_2}_{j_4}\delta^{i_4}_{j_8}\delta^{i_8}_{\beta}, 14)}^{-B}$$

$$\underbrace{w(u, \delta^{i_4}_{j_8}\delta^{i_8}_{j_4}\delta^{i_2}_{\beta}, 14)}_{-\frac{B}{N_c}} \qquad \underbrace{w(u, \delta^{i_4}_{j_8}\delta^{i_8}_{j_4}\delta^{i_2}_{\beta}, 14)}_{\frac{B}{N_c}}$$

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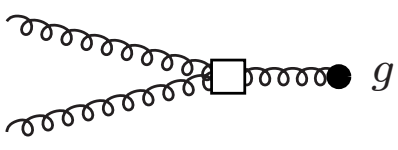

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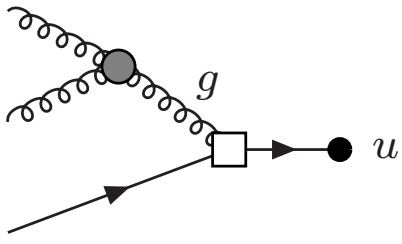

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Features of RECOLA (fortran 95)

- Full Standard Model:
  - Complex mass scheme
  - Feynman rules for rational parts and on-shell Counterterms
  - Select/unselect powers of α_s in the amplitude
  - Selection of resonant contributions

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- Need external libraries for TIs $\rightsquigarrow$ link to the COLLIER library

Hofer, Denner, Dittmaier TO BE PUBLISHED

COLLIER

Complex One Loop Library In Extended Regularizations

- Compute tensor integrals for:
  - arbitrary number of external momenta N
  - arbitrary rank
- Expansion methods for exceptional phase-space points (e.g. small Gram determinant) to arbitrary order
- Mass and dimensional regularization supported for IR-singularities
- Complex masses supported (unstable particles)
- Cache-system to avoid recalculation of identical integrals
- Output: coefficients $T^N_{0\cdots 0 i_1 \cdots i_k}$ or tensors $(T^N)^{\mu_1 \cdots \mu_P}$
- Two independent implementations: COLI and DD

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⇝ QCD corrections ($W^+ \to l^+ \nu_l, W^- \to l^- \bar{\nu}_l$, colour and helicity summed)

$u\,\bar{d} \to W^+\,g\,g$	$t_{\rm gen}$:	2.4 s	$t_{\rm TIs}$:	4.0 ms	$t_{\rm TCs}$:	1.1 ms
$u\,\bar{d} \to W^+\,g\,g\,g$	$t_{\rm gen}$:	15 s	$t_{\rm TIs}$:	67 ms	$t_{\rm TCs}$:	45 ms
$u\,\bar{u} \to W^+\,W^-\,g\,g$	$t_{\rm gen}$:	76 s	$t_{\rm TIs}$:	83 ms	$t_{\rm TCs}$:	16 ms

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⇝ EW + QCD corrections (colour and helicity summed)

$u\,\bar{u} \to l^+l^-\,g\,g$	$t_{\rm gen}$:	3.2 s	$t_{\rm TIs}$:	27 ms	$t_{\rm TCs}$:	25 ms
$u\,\bar{u} \to l^+l^-\,u\,\bar{u}$	$t_{\rm gen}$:	5 s	$t_{\rm TIs}$:	68 ms	$t_{\rm TCs}$:	35 ms
$u\,\bar{u} \to l^+l^-\,g\,g\,g$	$t_{\rm gen}$:	44 s	$t_{\rm TIs}$:	331 ms	$t_{\rm TCs}$:	684 ms
$u\,\bar{u} \to l^+l^-\,u\,\bar{u}\,g$	$t_{\rm gen}$:	50 s	$t_{\rm TIs}$:	835 ms	$t_{\rm TCs}$:	632 ms

EW corrections to $pp \to l^+ l^- jj$

$pp \to V + \text{jets}$:

- tests of QCD and EW Standard Model and backgrounds for Higgs studies and new physics searches
- QCD corrections for $\mathrm{Z} + \leq 4\mathrm{j}$, $\mathrm{W} + \leq 5\mathrm{j}$ [Blackhat collaboration]
- EW corrections for $\mathrm{Z/W} + \mathrm{j}$ [Denner, Dittmaier, Kasprzik, Mück '09, '11, '12]

$pp \to Z + 2$ jets:

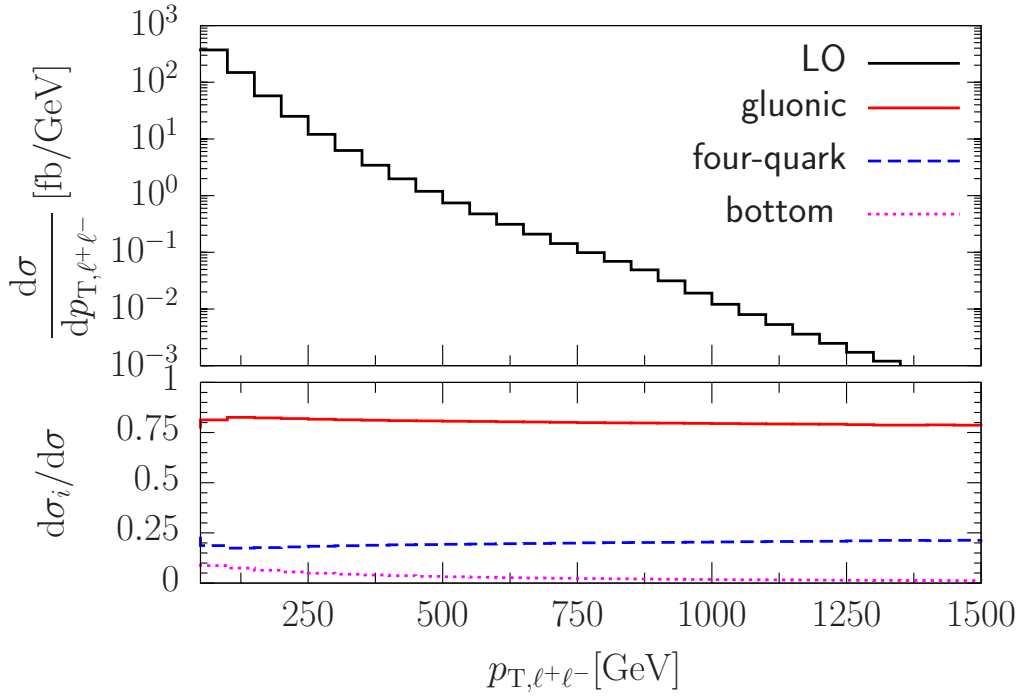
- $\mathcal{O}(\alpha_s^3 \alpha^2)$:
  QCD corrections to QCD diagrams [Campbell, Ellis, Rainwater '02, '03]
- $\mathcal{O}(\alpha_s^2 \alpha^3)$:
  EW corrections to QCD diagrams
  QCD corrections to EW–QCD interferences

LHC13, basic cuts PRELIMINARY

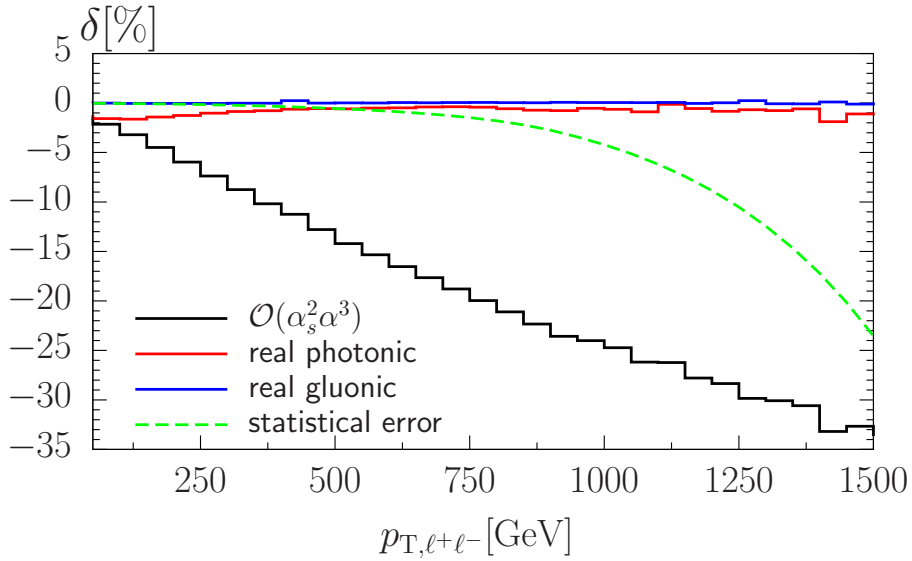
process class	$\sigma^{\rm LO}$ [fb]	$\sigma^{\rm LO}/\sigma^{\rm LO}_{\rm tot}$ [%]	$\sigma^{\rm NLO}_{\rm EW}$ [fb]	$\frac{\sigma^{\rm NLO}_{\rm EW}}{\sigma^{\rm LO}} - 1$ [%]
$qg \to qgl^-l^+$ $\bar{q}g \to \bar{q}gl^-l^+$	34584(8)	67.5	33751(9)	-2.41
$q\bar{q} \to ggl^-l^+$	2713(1)	5.3	2626(1)	-3.21
$gg \to q\bar{q}l^-l^+$	3612(1)	7.1	3556(1)	-1.55
gluonic	40910(8)	79.9	39932(9)	-2.39
four-quark	10299(1)	20.1	10033(1)	-2.58
sum	51209(8)	100	49965(9)	-2.43
bottom quarks	4376(3)	8.54		

- $\rm qg$ channels dominate
- EW corrections small for total cross section: $\sim -2.4\%$

LHC13: Distribution in p_{T} of l^+l^-



PRELIMINARY



statistical error based on $300\,\mathrm{fb}^{-1}$

- gluon channels dominate for all $p_{\mathrm{T},ll}$
- EW corrections -25% for $p_{\mathrm{T},ll} = 1\,\mathrm{TeV}$

  dominated by virtual corrections (Sudakov logarithms)
- subtracted real corrections small ($\lesssim 2\%$)

Summary

- Efficient automatization for elementary EW and QCD processes at NLO
- Recursion relations $\longrightarrow$ good tool also in the EW sector
- **used for EW corrections to** $pp \to l^+ l^- jj$

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Outlook

- Publication of the code
- Allow extensions to other Models
- Let's compute other LHC processes

▪

- Binary notation for $\{l_1, \ldots, l_n\}$ (HELAC):

  Binary numbers $1, 2, 4, 8, ..., 2^{L-1}$ for the primary legs

  $\{l_1, \ldots, l_n\}$ can be expressed by $\mathcal{B}_n =$ sum of the n binaries

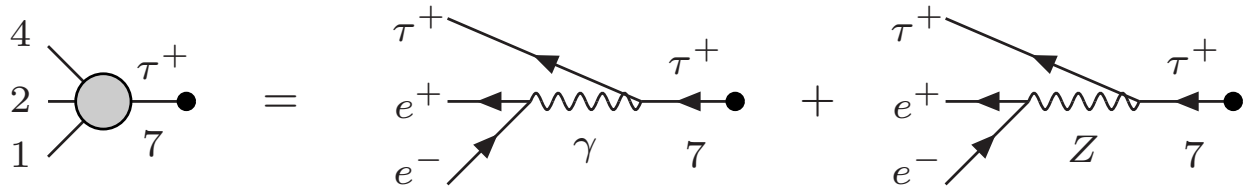
Example: $\{1, 2, 8\} \quad \rightarrow \quad \mathcal{B}_3 = 1 + 2 + 8 = 11$

Note: The off-shell currents just keep trace of the primary legs used to build them, not the way it has been done.

Example: Process $e^- + e^+ + \tau^+ + \tau^- \rightarrow 0$

1 2 4

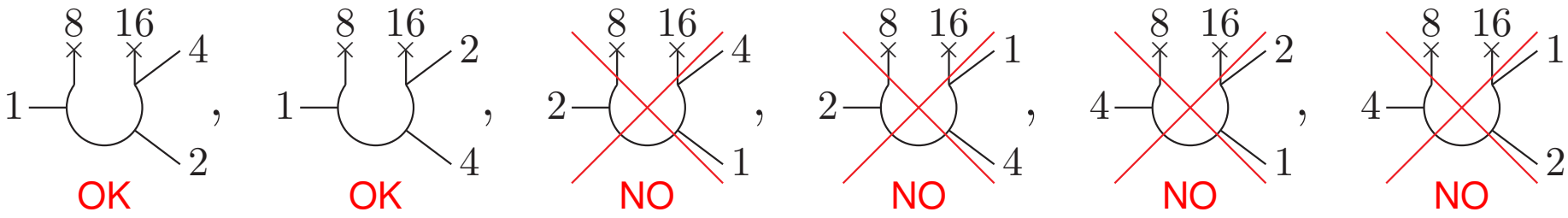
$w(\tau^+, 7) =$



Rules to avoid double counting of the associated trees:

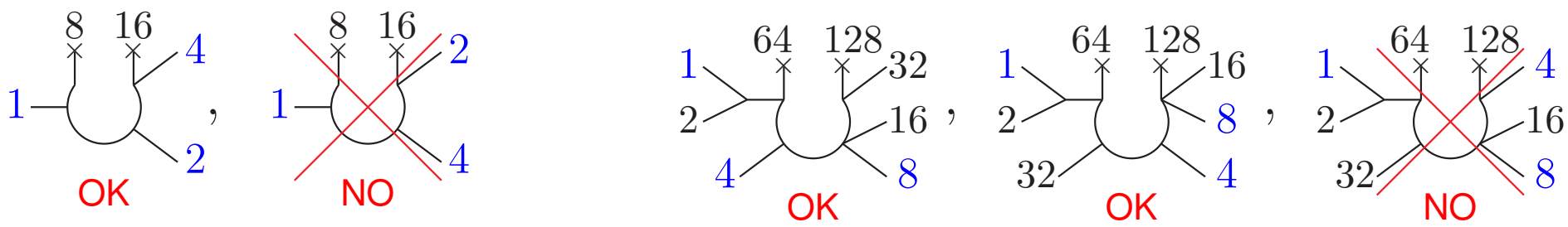
Rule 1: $\longrightarrow$ Fix starting point of loop flow

The current containing the first external line enters the loop flow first



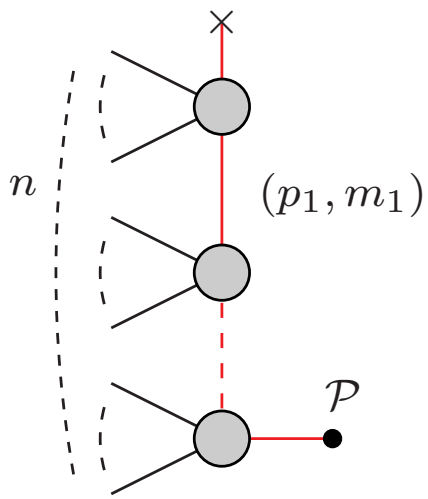
Rule 2: $\longrightarrow$ Fix direction of loop flow

The 3 currents with the 3 smallest binaries enter the loop flow in fixed order



Loop off-shell currents

$$w_{i_k}(\mathcal{P}, \mathcal{C}, \mathcal{B}_n, \{p_1, \ldots, p_k\}, \{m_1, \ldots, m_k\}) =$$



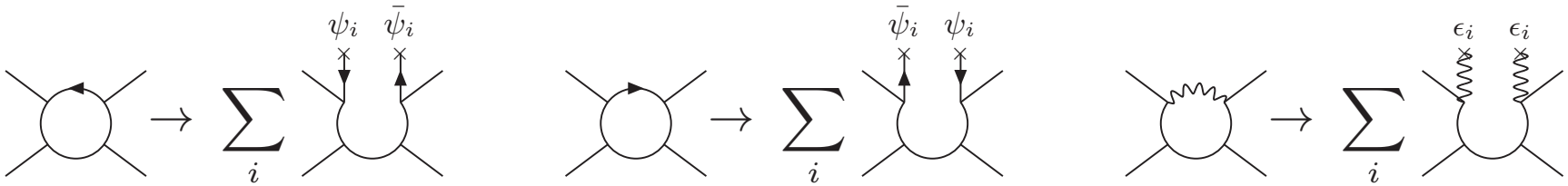
Sequence of momenta in loop propagators

Sequence of masses in loop propagators

- i_k is the tensorial index:

$$\begin{aligned} i_k &= 0 & \rightarrow \quad w_{i_k} &= a_{k,0} \\ i_k &= 1, \ldots, 4 & \rightarrow \quad w_{i_k} &= a_{k,1}^{\mu_1} \\ i_k &= 5, \ldots, 14 & \rightarrow \quad w_{i_k} &= a_{k,2}^{\mu_1 \mu_2} \\ &\ldots \end{aligned}$$

- Special wave functions for the cutted line:



where the components are $(\psi_i)_j = (\bar{\psi}_i)_j = \delta_{ij}$, $\epsilon_i^\mu = \delta_{i\mu}$.

Treatment of the colour

Colour-flow representation [Kanaki, Papadopoulos 2000; Maltoni, Paul, Stelzer, Willenbrock 2002]

$$\text{Gluon field} \;:\quad \sqrt{2}\, A^a_\mu\, (\lambda^a)^i_j = (\mathcal{A}_\mu)^i_j$$

gluon with colour-flow ^i_j

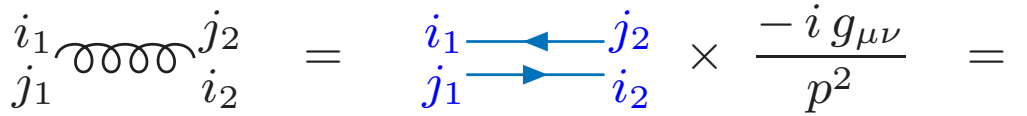
$i, j = 1, 2, 3$

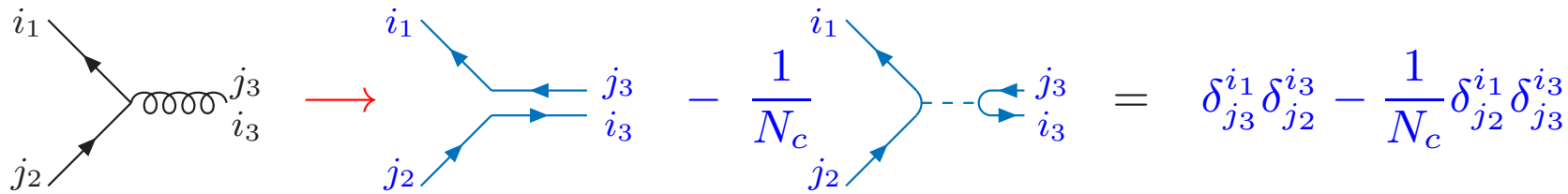
$\sum_i (A_\mu)^i_i = 0$

"usual" gluon with colour index $a = 1, \ldots, 8$

Feynman rules:

- Multiply gluon fields A^a_μ by $(\lambda^a)^i_j/\sqrt{2}$ and use properties of $(\lambda^a)^i_j$
- The colour part of the Feynman rules is just product of deltas:





Colour-flow representation

[Kanaki, Papadopoulos 2000; Maltoni, Paul, Stelzer, Willenbrock 2002]

Structure of amplitude: $$\mathcal{A}^{i_1\cdots i_n}_{j_1\cdots j_n} = \sum_{P(j_1,\ldots,j_n)} \delta^{i_1}_{j_1}\cdots\delta^{i_n}_{j_n}\,\mathcal{A}_P$$

- Colour-dressed amplitudes:

  $\rightarrow$ Compute $\mathcal{A}^{i_1\cdots i_n}_{j_1\cdots j_n}$ for all possible colours $\left(N_c^{2\,n}\right)$

  Squared amplitude: $$\overline{\mathcal{M}^2} = \sum_{i_1\ldots i_n, j_1,\ldots,j_n} (\mathcal{A}^{i_1\cdots i_n}_{j_1\cdots j_n})^* \mathcal{A}^{i_1\cdots i_n}_{j_1\cdots j_n}$$

  It requires colour-dressed currents

- Structure-dressed (or colour-ordered) amplitudes:

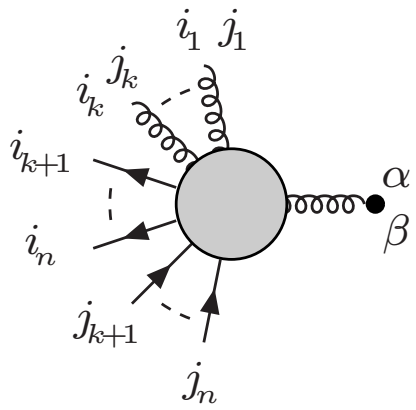
  $\rightarrow$ Compute $\mathcal{A}_P$ for all possible P $(n!)$

  Squared amplitude: $$\overline{\mathcal{M}^2} = \sum_{P,P'} \mathcal{A}^*_P\, C_{PP'}\, \mathcal{A}_{P'}$$

  It requires structure-dressed currents

Structure-dressed off-shell currents

Colour structure of off-shell current:



$\rightarrow$

saturated part open part

$$\delta^{i_1}_{j_1} \cdots \delta^{i_{n-1}}_{j_{n-1}} \quad \delta^{\alpha}_{j_n} \; \delta^{i_n}_{\beta}$$

$$w(\,\mathcal{P},\;\; \mathcal{C},\;\; \mathcal{B}_n\,)$$

with all possible permutations of $j_1, \ldots j_n, \beta$

In the recursion procedure:

- Saturated parts of incoming currents multiply
- Open parts of incoming currents are contracted

Optimization: Compute once currents differing just by the colour structure

$\rightsquigarrow$ Overcome lack of colour factorization

Structure of the code

- Definition of the processes

```
call define_process_rcl(1,'u g -> u g e+ e-','NLO')
call define_process_rcl(2,'u g -> u g e+[+] e-[-]','NLO')
call define_process_rcl(3,'u g -> u g Z(e+ e-)','NLO')
```

- Generation phase

```
call generate_processes_rcl
```

- Computation of the amplitudes

```
call compute_process_rcl(1,p,A2lo(1),A2nlo(1))
call compute_process_rcl(2,p,A2lo(2),A2nlo(2))
call compute_process_rcl(3,p,A2lo(3),A2nlo(3))
```

(the momenta `p(1:legs,0:3)` come from MC)

- PDFs: MSTW2008LO [Martin et al. '09]
- scales: $\mu_R = \mu_F = M_Z$
- jet clustering: anti-k_{T} algorithm with $\Delta R = 0.4$, also for photons [Cacciari, Salam, Soyez '08]
- basic cuts: motivated by [ATLAS '13]

$$p_{\mathrm{T,j}} > 30\ \mathrm{GeV}, \quad |\eta_j| < 4.5$$

$$p_{\mathrm{T,l}} > 20\ \mathrm{GeV}, \quad |\eta_l| < 2.5$$

$$\Delta R_{jl^-} > 0.5, \quad \Delta R_{jl^+} > 0.5$$

$$\Delta R_{l^+l^-} > 0.2, \quad 66\ \mathrm{GeV} < M_{l^+l^-} < 116\ \mathrm{GeV}$$

photon energy fraction in jet $z_\gamma < 0.7$