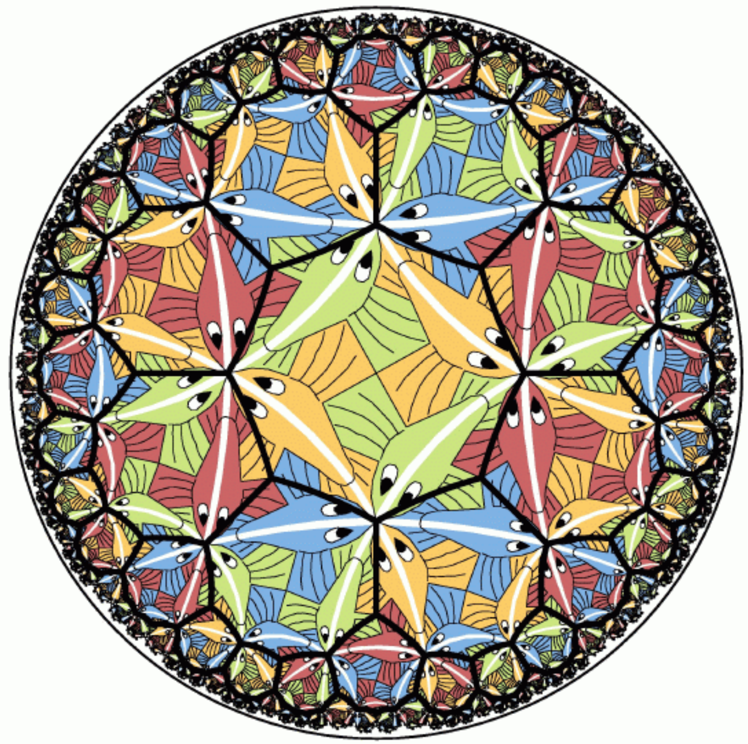
Calculating non-global logarithms to higher order in perturbation theory



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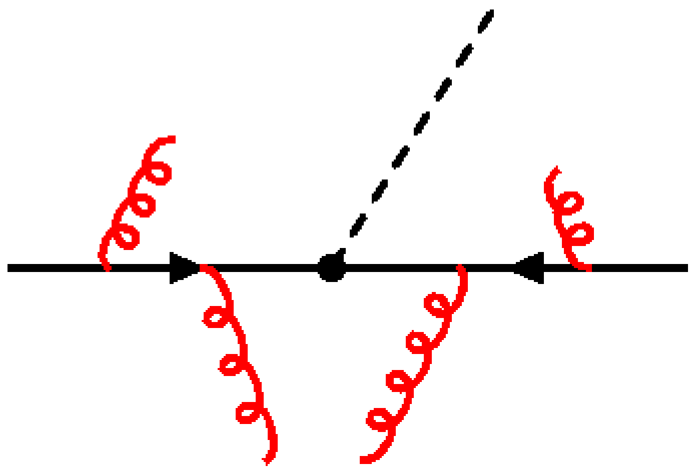
What is NGLs and why it's interesting:I

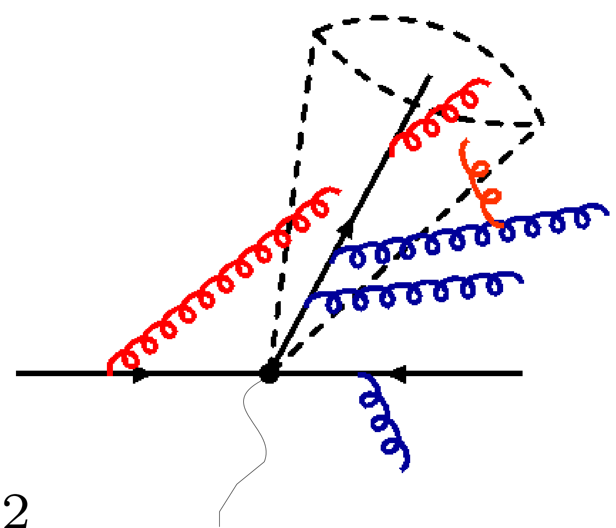
- Non-global observable [Dasgupta, Salam, 2001]
  - Observable only sensitive to emissions in a restricted angular region. Examples:
    - Invariant mass of individual jets
    - Distributions of interjet energy flow
    - Original Sterman-Weinberg jet cross section
    - ...

Global observable: Higgs qT



qT = - (transverse momentum of all QCD radiations)

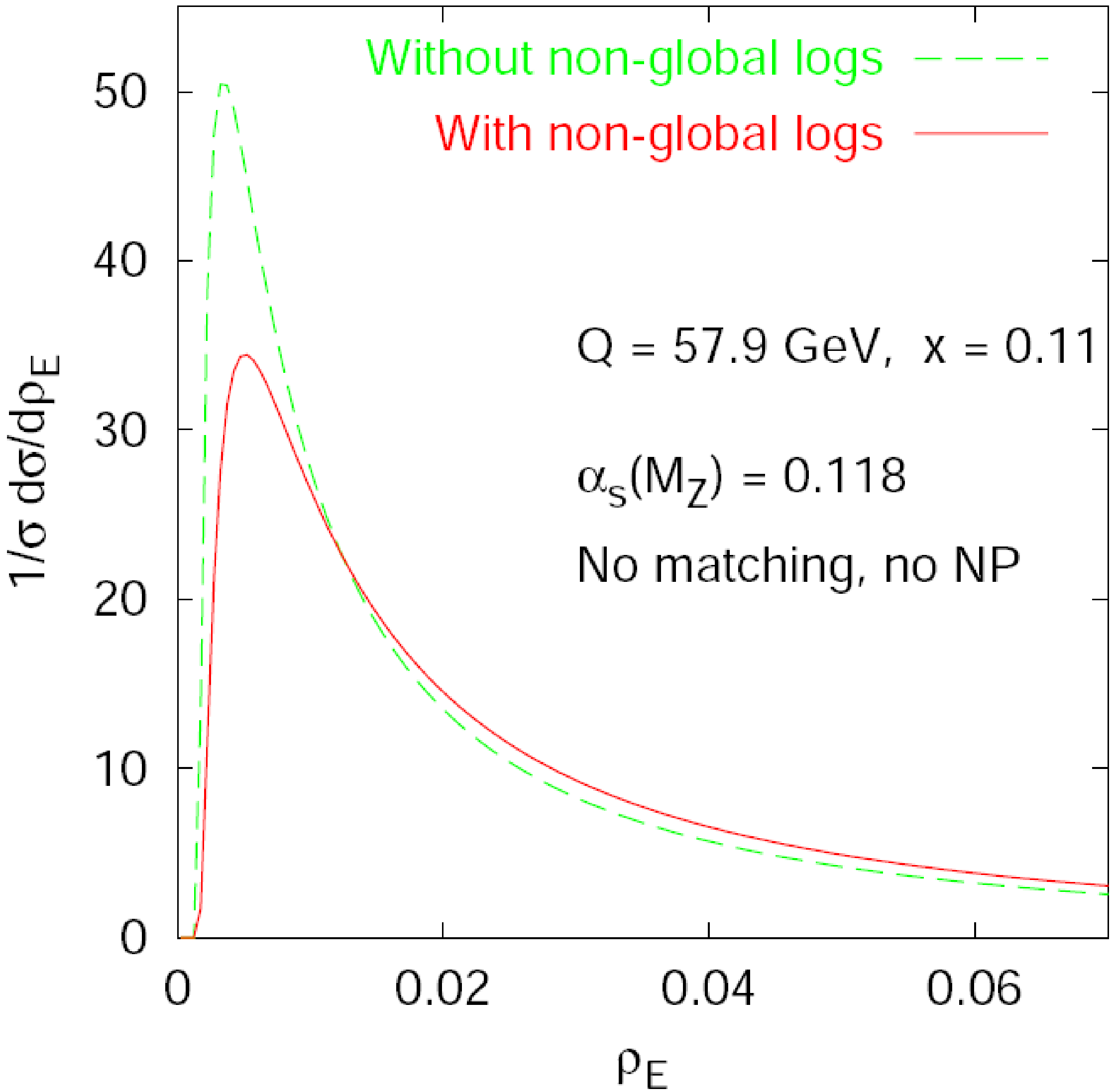
non-global observable: individual jet mass



m_J^2 (momentum of parton inside the jet)^2

What is NGLs and why it's interesting:II

- Impact of non-global logarithms: reduction in the peak height by ~ 30%



Invariant mass sum in DIS current hemisphere

$$\rho_E = \frac{\left(\sum_{\mathcal{H}_C} P_i\right)^2}{4\left(\sum_{\mathcal{H}_C} |\vec{P}_i|\right)^2}$$

[Dasgupta, Salam, hep-ph/0208073]

Known facts about NGLs

- Large logarithms L=ln(m) in jet mass due to soft and collinear singularities of QCD
- When $\alpha_S L \sim 1$ perturbation expansion in strong coupling break down. Need resummation!
- A typical non-global observable receive both global and non-global contributions

$$\Sigma_J(M_J) = \frac{1}{\sigma}\int_0^{M_J} dm_J \frac{d\sigma}{dm_J}$$

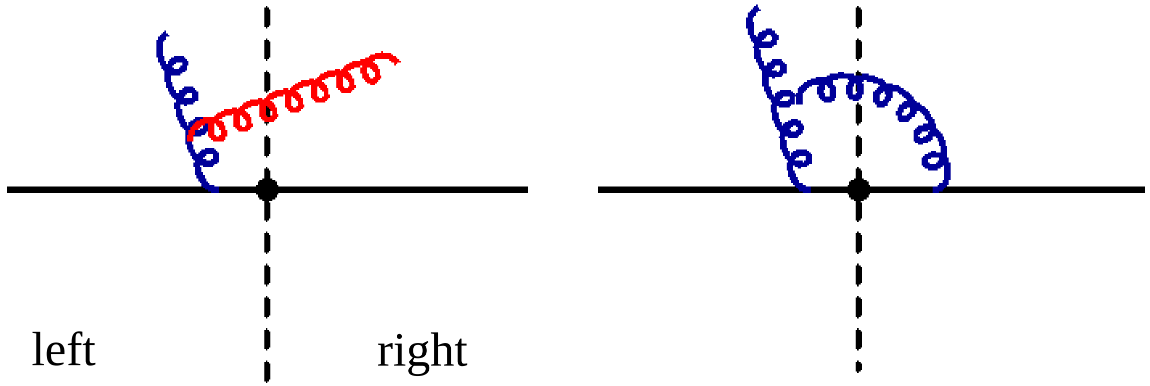
$$\Sigma_J(M_J) = S(\ln M_J) \times \Sigma_P(\ln M_J)$$

- $\Sigma_P(\ln M_J)$ Global contribution, can be resumed using standard resummation tech.
- $S_J(\ln M_J)$ Non-global contribution, difficult to resum analytically
- The leading logarithmic contribution to $S_J(\ln M_J)$ originates from soft radiations
- Is single log at each order in perturbation expansion

$$S_J(\ln M_J) = 1 + \alpha_S^2 L^2 + \alpha_S^3 L^3 + \alpha_S^4 L^4 + \ldots$$

LO calculation of NGLs

- The LO contribution (two loops) first calculated for hemisphere mass distribution (Dasgupta, Salam, 2001)



- LO contribution known exactly: $S^{(2)} = -\frac{\zeta(2)}{4}\overline{\alpha}_S^2 N_c^2 \ln^2 \rho$
- Get contribution only from strong-gluon-energy-ordered configuration
- Analytical treatment at all orders is difficult. Method of [Dasgupta and Salam 2001]:
  - Complicated color structure: large Nc approximation
  - Complicated geometry structure: use Monte Carlo numerical branching algorithm

$$P_{\mathcal{C}'}(L') = \bar{\alpha}_{\mathrm{s}}(L')\, \Delta_{\mathcal{C}}(L, L')\, F_{\mathcal{C}}(\theta', \phi') P_{\mathcal{C}}(L)$$

$$\mathcal{S}(\alpha_{\mathrm{s}} L) = \frac{1}{\sqrt{\Delta_{ab}(L)}} \sum_{\mathcal{C}|\mathcal{H}_{\mathrm{R}}\ \mathrm{empty}} P_{\mathcal{C}}(L)$$

$$F_{\mathcal{C}}(\theta_k, \phi_k) = \sum_{\mathrm{dipoles}-ij} \frac{2C_A}{(1-\cos\theta_{ik})(1-\cos\theta_{kj})}$$

  - Can we solve the geometry structure analytically, at least in the first few orders?

Diagrammatic construction of integrand beyond LO: I

- Approximation used in the simplification of integrand:
  - Strong energy ordering limit (Bassetto, Ciafaloni, Marchesini, Pyhy.Rept. 100, 201)
  - Large Nc approximation: only planar corrections
- At N^kLO
  - Tree-level matrix element for k+2 soft gluon emission $W_{RR\ldots R}$
  - One-loop matrix element for k+1 soft gluon emission $W_{R\ldots V\ldots R}$
  - Two-loop matrix element for k soft gluon emission $W_{R\ldots V\ldots V\ldots R}$
  - ….
  - K-loop matrix element for 2 soft gluon emission $W_{V\ldots R\ldots R\ldots V}$
- At leading logarithmic approximation, loop matrix element can be related to real matrix element by unitarity

$$\int W_{\ldots R} + W_{\ldots V} = 0$$

Diagrammatic construction of integrand beyond LO: II

- Matrix elements for m real gluon emission

$$\left|\mathcal{M}_{ab}^{1\cdots m}\right|^2 = N_c^m g^{2m} \frac{1}{\omega_1^2 \cdots \omega_m^2} \mathcal{P}_{ab}^{1\cdots m}$$

$$\mathcal{P}_{ab}^{1\ldots m} = \sum_{\text{perms. of } 1\ldots m} \frac{(ab)}{(a1)(12)\ldots(mb)} \qquad (ij) \equiv \frac{p_i \cdot p_j}{\omega_i \omega_j} = 1 - \cos\theta_{ij}$$

- Real-virtual and virtual amplitude from unitarity

$$\mathcal{W}_R = \mathcal{P}_{ab}^1, \quad \mathcal{W}_V = -\mathcal{W}_R$$

$$\mathcal{W}_{RR} = \mathcal{P}_{ab}^{12} \qquad \mathcal{W}_{RV} = -\mathcal{W}_{RR} \qquad \mathcal{W}_{VR} = -\mathcal{P}_{ab}^1 \mathcal{P}_{ab}^2 \qquad \mathcal{W}_{VV} = -\mathcal{W}_{VR}$$

$$C_1 = \mathcal{P}_{ab}^{123} = \mathcal{W}_{RRR} = -\mathcal{W}_{RRV}$$

$$C_2 = \mathcal{P}_{ab}^{12}\left(\mathcal{P}_{a1}^3 + \mathcal{P}_{b1}^3\right) = \mathcal{W}_{RVV} = -\mathcal{W}_{RVR}$$

$$C_3 = \mathcal{P}_{ab}^1 \mathcal{P}_{ab}^{23} = -\mathcal{W}_{VRR} = \mathcal{W}_{VRV}$$

$$C_4 = \mathcal{P}_{ab}^1 \mathcal{P}_{ab}^2 \mathcal{P}_{ab}^3 = \mathcal{W}_{VVR} = -\mathcal{W}_{VVV}$$

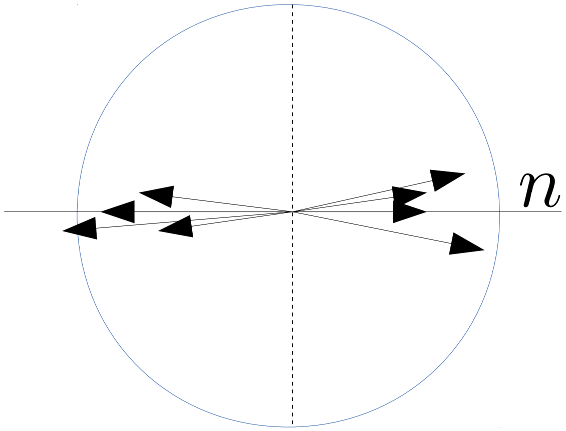
Diagrammatic construction of integrand beyond LO: III

- Differential cross section at strong-energy-ordered limit

$$\frac{1}{\sigma_0} d\sigma_m = \bar{\alpha}\frac{d\omega_1}{\omega_1}\frac{d\Omega_1}{4\pi}\left(\mathcal{W}_R + \mathcal{W}_V\right) + \frac{\bar{\alpha}^2}{2!}\frac{d\omega_1}{\omega_1}\frac{d\Omega_1}{4\pi}\frac{d\omega_2}{\omega_2}\frac{d\Omega_2}{4\pi}\left(\mathcal{W}_{RR} + \mathcal{W}_{RV} + \mathcal{W}_{VR} + \mathcal{W}_{VV}\right) + \frac{\bar{\alpha}^3}{3!}\frac{d\omega_1}{\omega_1}\frac{d\Omega_1}{4\pi}\frac{d\omega_2}{\omega_2}\frac{d\Omega_2}{4\pi}\frac{d\omega_3}{\omega_3}\frac{d\Omega_3}{4\pi}\left(\mathcal{W}_{RRR} + \mathcal{W}_{RRV} + \cdots\right)$$

- Right hemisphere mass distribution

$$S(\rho) = \frac{1}{\sigma_0 \Sigma_P(\rho)} \int d\sigma_m \Theta\left(\rho Q - \sum_i 2(p_i \cdot n)\theta_R(p_i)\right)$$



- Additional simplification in the measurement function

$$\Theta\left(\rho Q - \sum_i 2(p_i \cdot n)\theta_R(p_i)\right) \to \Theta\left(\rho Q - \sum \omega_i\right)$$

- Integrand beyond LO can be algorithmically worked out

Integrand bootstrap: I

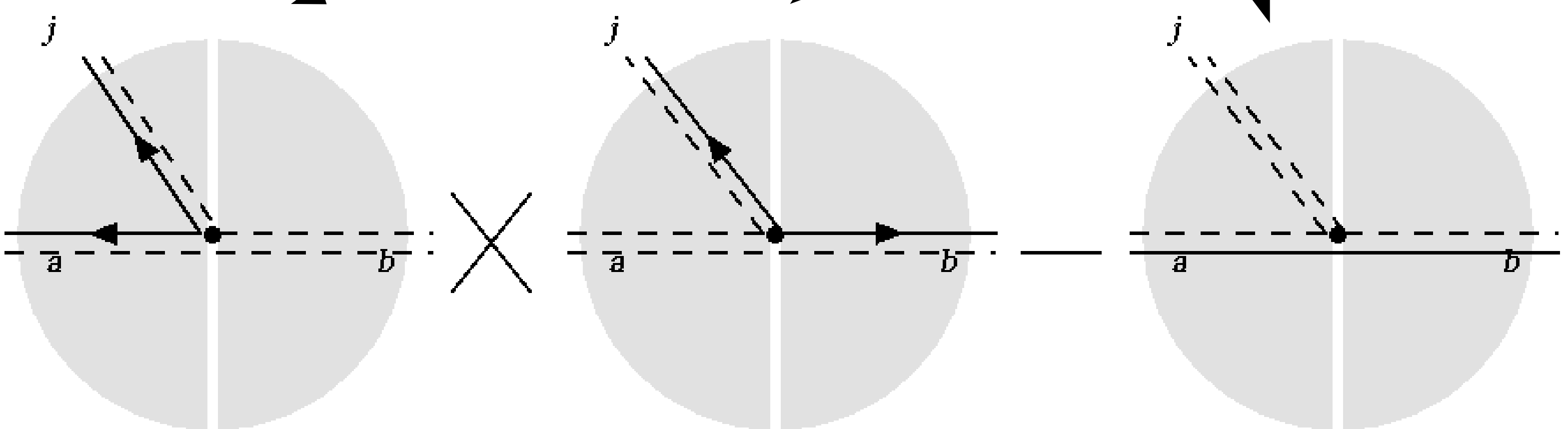
- Non-global logarithms obey an integro-differential equation (Banfi-Marchesini-Syme, 2002). First formulated for interjet energy flow

- BMS equation for hemisphere mass distribution

$$U_{abj}(L) = \exp\left[L\int_{\text{right}}\frac{d\Omega_1}{4\pi}\mathcal{P}^1_{ab} - \mathcal{P}^1_{aj} - \mathcal{P}^1_{jb})\right]$$

$$\partial_L g_{ab}(L) = \int_{\text{left}}\frac{d\Omega_j}{4\pi}\mathcal{P}^j_{ab}\left[U_{abj}(L)g_{aj}(L)g_{jb}(L) - g_{ab}(L)\right]$$

$$L = \frac{\alpha_S}{\pi}N_c\ln\frac{1}{\rho} \qquad \mathcal{P}^j_{ab} = \frac{(ab)}{(aj)(jb)}$$



- Difficult to solve analytically

- Expansion in perturbation theory; first non-trivial corrections starting at two loops

$$g_{ab}(L) = 1 + \cancel{g^{(1)}_{ab}(L)} + g^{(2)}_{ab}(L) + g^{(3)}_{ab}(L) + g^{(4)}_{ab}(L) + \ldots \qquad g^{(n)}_{ab}(L) \propto L^n$$

Integrand bootstrap: II

- Expand to two loops

$$\partial_L g_{ab}^{(2)}(L) = \int_{\text{left}} \frac{d\Omega_1}{4\pi} \frac{(ab)}{(a1)(1b)} \left[U_{ab1}^{(1)}(L) + g_{a1}^{(1)}(L) + g_{1b}^{(1)}(L) - g_{ab}^{(1)}(L) \right]$$
$$= \int_{\text{left}} \frac{d\Omega_1}{4\pi} \frac{(ab)}{(a1)(1b)} \left[L \int_{\text{right}} \frac{d\Omega_2}{4\pi} (\mathcal{P}_{ab}^2 - \mathcal{P}_{a1}^2 - \mathcal{P}_{1b}^2) \right]$$
$$= \frac{2}{L} \int \frac{d^3p_1}{2E_1(2\pi)^3} \frac{d^3p_2}{2E_2(2\pi)^3} (\mathcal{W}_{RR} + \mathcal{W}_{RV} + \mathcal{W}_{VR} + \mathcal{W}_{VV}) \Theta(\rho Q > (\text{right hemi. energy}))$$

- Expand to three loops and four loops

$$\partial_L g_{ab}^{(3)}(L) = \int_{\text{left}} \frac{d\Omega_1}{4\pi} \frac{(ab)}{(a1)(1b)} \left[U_{ab1}^{(2)}(L) + g_{a1}^{(2)}(L) + g_{1b}^{(2)}(L) - g_{ab}^{(2)}(L) \right]$$
$$= \frac{3}{L} \int \frac{d^3p_1}{2E_1(2\pi)^3} \frac{d^3p_2}{2E_2(2\pi)^3} \frac{d^3p_3}{2E_3(2\pi)^3} (\mathcal{W}_{RRR} + \cdots + \mathcal{W}_{VVV}) \Theta(\rho Q > (\text{right hemi. energy}))$$

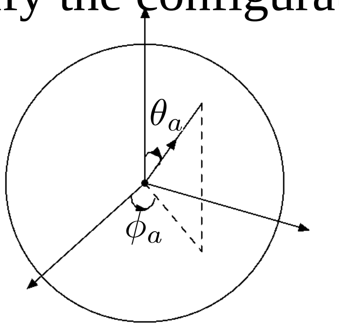
$$\partial_L g_{ab}^{(4)}(L) = \int_{\text{left}} \frac{d\Omega_1}{4\pi} \frac{(ab)}{(a1)(1b)} \left[U_{ab1}^{(3)}(L) + U_{ab1}^{(1)}(L)(g_{a1}^{(2)}(L) + g_{1b}^{(2)}(L)) + g_{a1}^{(3)}(L) + g_{1b}^{(3)}(L) - g_{ab}^{(3)}(L) \right]$$

Integrating the integrand

$$\partial_L g_{\bar{n}n}(L) = \int_{\text{left}} \frac{d\Omega_j}{4\pi} \mathcal{P}^{j}_{\bar{n}n} \left[U_{\bar{n}nj}(L) g_{\bar{n}j}(L) g_{jn}(L) - g_{\bar{n}n}(L) \right]$$

- To calculate hemisphere NGLs to m-loop, need to know NGLs for a and b point to same hemisphere, or a and b point to opposite hemisphere at m-1 loop
- For arbitrary a and b, need four degrees of freedom to specify the configuration
- Some picture

$$\theta_a \qquad \phi_a \qquad \theta_b \qquad \phi_b$$



- There are two more angles to specify the direction of Ω_j
- Can use an obvious azimuthal symmetry around the hemisphere axis to fix one azimuthal angle
- Still has an integrand with 5 variables, and two integral to go, quite difficult

$$\int_0^{\pi/2} \sin\theta_j d\theta_j \int_0^{2\pi} d\phi_j$$

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(Log[(2 Cos[θj]^2 (1 - Cos[θa] Cos[θb] - Cos[ϕb] Sin[θa] Sin[θb])) /
      ((1 - Cos[θa] Cos[θj] - Cos[ϕj] Sin[θa] Sin[θj])
        (1 + Cos[θb] Cos[θj] - Cos[ϕb - ϕj] Sin[θb] Sin[θj]))]
    (1 - Cos[θa] Cos[θb] - Cos[ϕb] Sin[θa] Sin[θb]) Sin[θj]) /
  (16 π (1 - Cos[θa] Cos[θj] - Cos[ϕj] Sin[θa] Sin[θj])
    (1 - Cos[θb] Cos[θj] - Cos[ϕb - ϕj] Sin[θb] Sin[θj]))
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- Any other simplification?

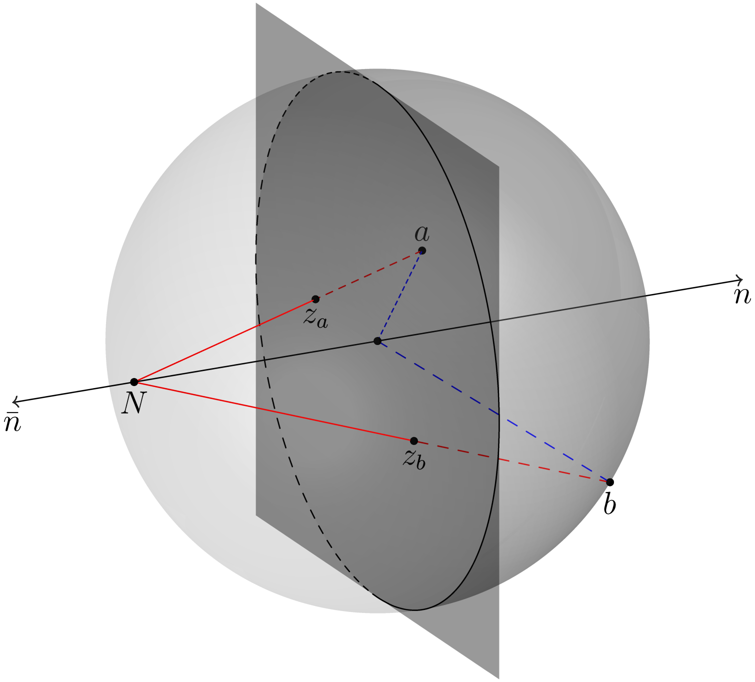
Symmetry of the BMS equation: I

- BMS equation formally similar to Balitsky-Kovchegov equation. There is a well-known SL(2,C) symmetry in the BK equation. Same should be true in BMS equation (Hatta, Ueda, 0909.0056)
- The direction of the integrated momenta j can be parametrized by a conformal 2 sphere. Mapping the 2 sphere to complex plane by stereographic projection

$$z = e^{i\phi}\tan\frac{\theta}{2}$$

$$d\Omega = d\cos\theta d\phi = \frac{4dz\,d\bar{z}}{(1+|z|^2)^2}$$

$$d\Omega_j \frac{(ab)}{(aj)(jb)} = d^2 z \frac{|z_a - z_b|^2}{|z_a - z_j|^2 |z_j - z_b|^2}$$



Combination of $d\Omega$ and soft factor is invariant under Möbius transformation

$$z \to \frac{\alpha z + \beta}{\gamma z + \delta}, \qquad \alpha, \beta, \gamma, \delta \in \mathbb{C}$$

$$\begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} \simeq SL(2, \mathbb{C})$$

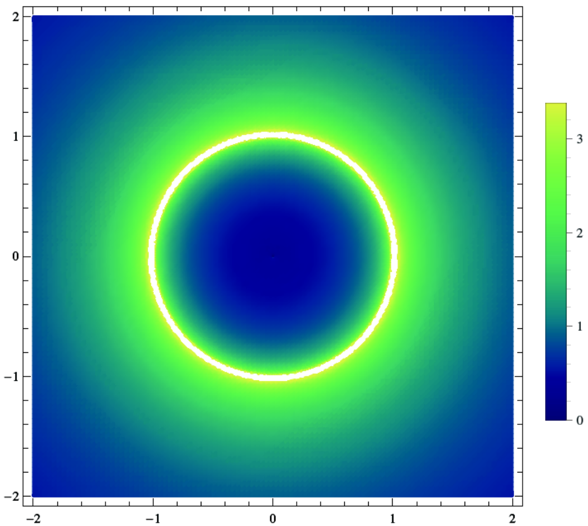
Symmetry of the BMS equation: II

- The actual symmetry group of BMS equation is smaller

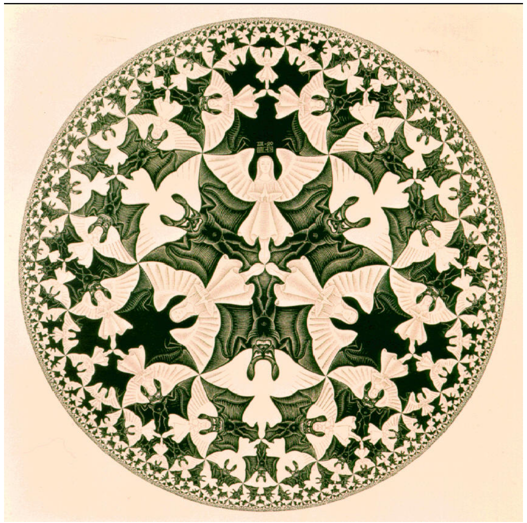
$$\partial_L g_{ab}(L) = \int_{\text{left}} \frac{d\Omega_j}{4\pi} \mathcal{P}^j_{ab} \left[U_{abj}(L) g_{aj}(L) g_{jb}(L) - g_{ab}(L) \right]$$

Integral limited to left hemisphere

- On the projection complex plane, symmetry preserved for a subset of Möbius transformation which map unit disc to unit disc, the (P)SL(2,R) group
- The unit disc with the isometry group PSL(2,R) called Poincaré Disc



Density plot of the two-loop NGL integrand on the complex plane



[Escher, Heaven and Hell]

Symmetry of the BMS equation: III

- Can use isometry group PSL(2,C) to eliminate three degrees of freedom.

$g_{ab}(L)$ only depends on a single isometry invariant

$$\langle ij\rangle = \frac{|z_i - z_j|^2}{(1-|z_i|^2)(1-|z_j|^2)} = \frac{(ij)}{2\cos\theta_i\cos\theta_j}$$

- We can then map, e.g., $z_a = x_a + iy_a$ to the origin of complex plane. Two-loop integrand now simplified to

$$\int_0^{\pi/2} \sin\theta_j d\theta_j \int_0^{2\pi} d\phi_j \, \frac{(1-\cos\theta_b)\ln\frac{2(1-\cos\theta_b)\cos^2\theta_j}{(1-\cos\theta_j)(1+\cos\theta_b\cos\theta_j-\cos\phi_j\sin\theta_b\sin\theta_j)}}{16\pi(1-\cos\theta_j)(1-\cos\theta_b\cos\theta_j-\cos\phi_j\sin\theta_b\sin\theta_j)}$$

- Finite integral, can easily be evaluated numerically

- Can we do the integral analytically, with the help of recent developed integral technique from scattering amplitudes?

Integrating the azimuthal angle: I

- The azimuthal angle integral can be cast into a contour integral

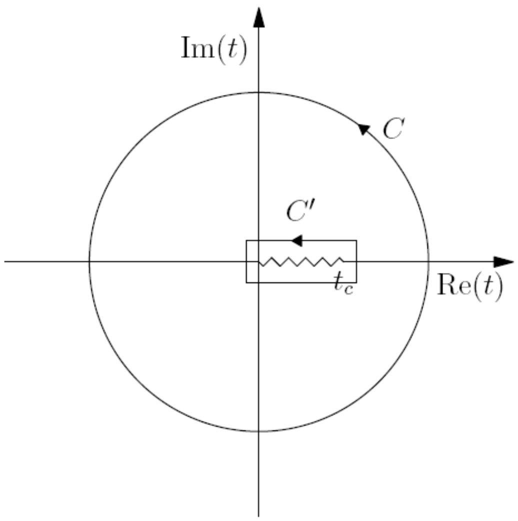
$$\Phi_2 = \int_0^{2\pi} \frac{d\phi_j}{2\pi} \frac{1}{1+\cos\theta_j\cos\theta_b - \cos\phi_j \sin\theta_j \sin\theta_b} \ln \frac{1+\cos\theta_j\cos\theta_b - \cos\phi_j \sin\theta_j \sin\theta_b}{2\cos\theta_j\cos\theta_b}$$

Change of variable $t = \exp(i\phi_j)$

$$\Phi_2 = -\frac{2}{\sin\theta_j \sin\theta_b} \oint_C \frac{dt}{2\pi i} \frac{1}{(t-t_+)(t-t_-)} \ln \frac{1+\cos\theta_j\cos\theta_b - \sin\theta_j\sin\theta_b\left(\frac{1}{2t}+\frac{t}{2}\right)}{2\cos\theta_j\cos\theta_b}$$

$$t_+ = \frac{1-\cos\theta_j\cos\theta_b + |\cos\theta_j - \cos\theta_b|}{\sin\theta_j\sin\theta_b}$$

$$t_- = \frac{1-\cos\theta_j\cos\theta_b - |\cos\theta_j - \cos\theta_b|}{\sin\theta_j\sin\theta_b}$$



- No single pole within C. A branch cut from the log
- Integral contour can be shrink to near the real axis
- Integral has the form of iterative integral after partial fractioning
- Can be easily integrated in terms of multiple polylogs

$$G(w_1,\ldots,w_n;x) = \int_0^x \frac{dt}{t-w_1} G(w_2,\ldots,w_n;t), \qquad G(\underbrace{0,\ldots,0}_{n};x) = \frac{1}{n!}\ln^n x$$

Integrating the azimuthal angle: II

- Partial azimuthal integral from one loop to four loop

$$\Phi_1 = \int_0^{2\pi} \frac{d\phi_j}{2\pi} \frac{1}{(jb)} = \frac{1}{|\cos\theta_j - \cos\theta_b|}$$

$$\Phi_2 = \int_0^{2\pi} \frac{d\phi_j}{2\pi} \frac{\ln(1+\langle jb\rangle)}{(jb)} = \frac{1}{\cos\theta_j - \cos\theta_b} \ln\frac{1+\langle b\bar{n}\rangle}{1+\langle j\bar{n}\rangle}$$

$$\Phi_3 = \int_0^{2\pi} \frac{d\phi_j}{2\pi} \frac{\ln\langle jb\rangle \ln(1+\langle jb\rangle) + \mathrm{Li}_2(-\langle jb\rangle)}{(jb)} = \frac{1}{\cos\theta_b - \cos\theta_j} \times \left[\ln\frac{1+\langle j\bar{n}\rangle}{1+\langle b\bar{n}\rangle} \ln\frac{\langle j\bar{n}\rangle + \langle b\bar{n}\rangle + |\langle j\bar{n}\rangle - \langle b\bar{n}\rangle|}{2} + \mathrm{Li}_2(-\langle j\bar{n}\rangle) - \mathrm{Li}_2(-\langle b\bar{n}\rangle)\right]$$

$$\begin{aligned}
\Phi_4 =& (\cos\theta_b - \cos\theta_j) \int_0^{2\pi} \frac{d\phi_j}{2\pi(jb)} \left[-\frac{1}{6} r_{jb}^3 + \frac{1}{L^3} g_{jb}^{(3)}(L) - \frac{1}{L^2} r_{jb} g_{jb}^{(2)}(L)\right] \\
=& -\frac{1}{12} G(-1,0,\langle b\bar{n}\rangle;\langle j\bar{n}\rangle) - \frac{1}{12} G(0,-1,\langle b\bar{n}\rangle;\langle j\bar{n}\rangle) + \frac{1}{8} G(-1,0,-1;\langle j\bar{n}\rangle) \\
& - \frac{1}{24} G(0,-1;\langle j\bar{n}\rangle) G(-1;\langle b\bar{n}\rangle) - \frac{1}{12} G(0,-1;\langle j\bar{n}\rangle) G(0;\langle b\bar{n}\rangle) + \frac{1}{12} G(-1;\langle j\bar{n}\rangle) G(0,0;\langle b\bar{n}\rangle) \\
& + \frac{1}{24} G(-1;\langle j\bar{n}\rangle) G(0,-1;\langle b\bar{n}\rangle) - \frac{1}{12} G(-1,0,0;\langle b\bar{n}\rangle) - \frac{1}{24} G(-1,0,-1;\langle b\bar{n}\rangle) \\
& + \frac{\pi^2}{36} G(-1;\langle b\bar{n}\rangle) - \frac{\pi^2}{36} G(-1;\langle j\bar{n}\rangle), \qquad (155)
\end{aligned}$$

- Results look quite simple
- Major complication at higher orders: increasingly complicated branch cut structure from multiple polylogs
- Alternative approach to the azimuthal integral?

Integrating the polar angle

- The polar angle integral also has the form of iterative integral

A two-loop example $$\int_b^c dc_j \frac{1-c_b}{2(1-c_j)(c_b-c_j)}\left[\ln\frac{1+c_b}{2c_b}-\ln\frac{1+c_j}{2c_j}\right] \qquad c_i = \cos\theta_i$$

- Thanks to study in scattering amplitudes, we have many tools to deal with such integrals: symbols, coproduct, hyperlogarithms …
- Compact results to four loops for a, b point int the same hemisphere

$$\begin{aligned}\frac{1}{L^2}g_{ab}^{(2)}(L) =& -\frac{1}{4}G(-1,-1;x)+\frac{1}{4}G(-1,0;x)\\ =&\frac{1}{4}\ln x\ln(1+x)-\frac{1}{8}\ln^2(1+x)+\mathrm{Li}_2(-x)\end{aligned}$$

$$\begin{aligned}\frac{1}{L^3}g_{ab}^{(3)}(L) =&\frac{\pi^2}{36}G(-1;x)-\frac{1}{4}G(-1,-1,-1;x)+\frac{1}{4}G(-1,-1,0;x)+\frac{1}{12}G(-1,0,-1;x)\\ &-\frac{1}{12}G(-1,0,0;x)\end{aligned}$$

$$\begin{aligned}\frac{1}{L^4}g_{ab}^{(4)}(L) =&\frac{\pi^2}{36}G(-1,-1;x)-\frac{\pi^2}{144}G(-1,0;x)-\frac{3}{16}G(-1,-1,-1,-1;x)+\frac{3}{16}G(-1,-1,-1,0;x)\\ &+\frac{1}{12}G(-1,-1,0,-1;x)-\frac{1}{12}G(-1,-1,0,0;x)+\frac{1}{48}G(-1,0,-1,-1;x)\\ &-\frac{1}{96}G(-1,0,-1,0;x)-\frac{1}{32}G(-1,0,0,-1;x)+\frac{1}{48}G(-1,0,0,0;x)-\frac{\zeta(3)}{16}G(-1;x)\end{aligned}$$

Connection with Buffer mechanism??

- Limit of NGLs when one of the Wilson line close to hemisphere edge

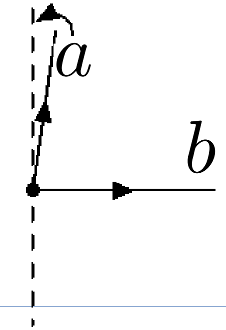
$$\lim_{x\to\infty} g_{ab}^{(2)}(L) = -\frac{\pi^2}{24}L^2 + \mathcal{O}\left(\frac{1}{x}\right),$$

$$\lim_{x\to\infty} g_{ab}^{(3)}(L) = \frac{\zeta(3)}{12}L^3 + \mathcal{O}\left(\frac{1}{x}\right),$$

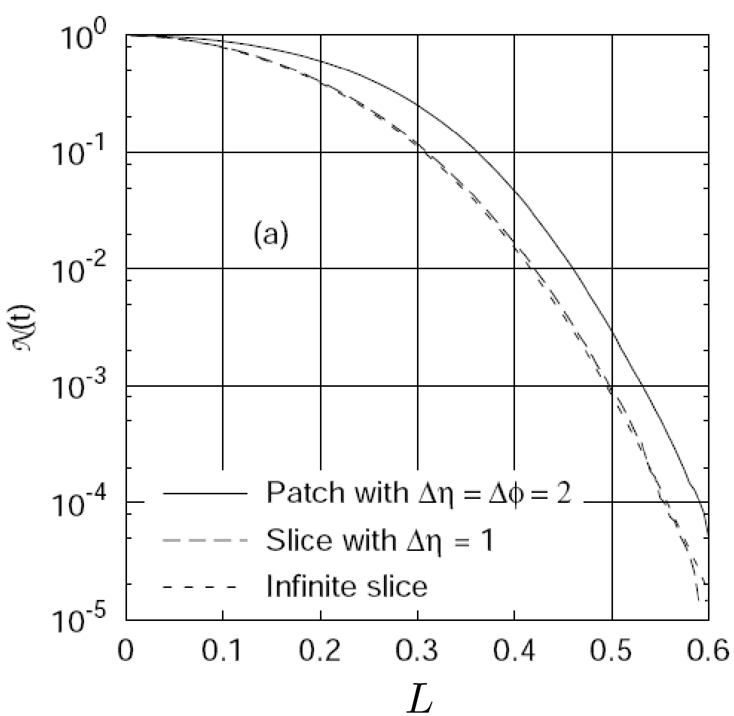
$$\lim_{x\to\infty} g_{ab}^{(4)}(L) = \left(-\frac{\pi^4}{5760} - \frac{\zeta(3)}{48}\ln x\right)L^4 + \mathcal{O}\left(\frac{1}{x}\right),$$

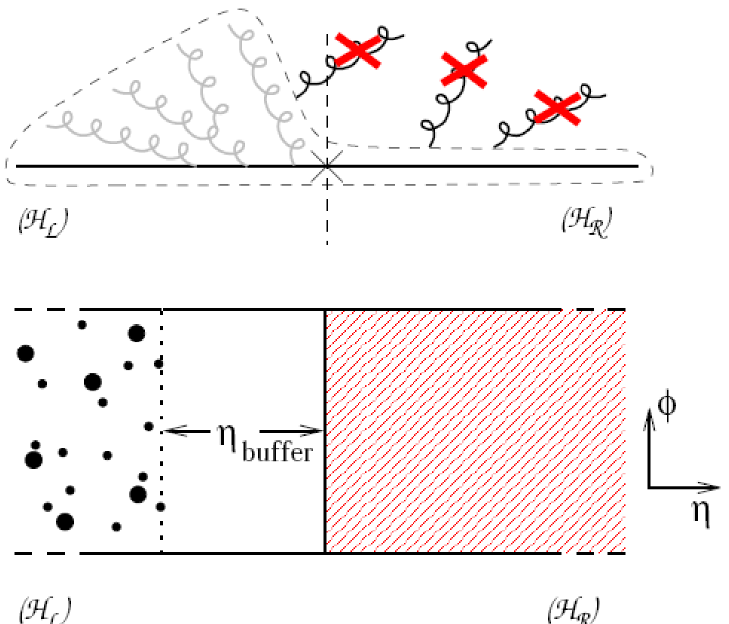
$x \to \Rightarrow$ a or b close to hemisphere edge

$$x = \frac{1-\cos\theta_{ab}}{2\cos\theta_a\cos\theta_b}$$



- Asymptotic independence of measured-region geometry shape
- The easiest way to suppress emission at large L is to suppress emission close to hemisphere edge: a empty buffer [Dasgupta, Salam, hep-ph/0203009]





[Salam, Moriond 2003]

- Can the Log(x) behavior responsible for buffer mechanism? Need higher order calculation!

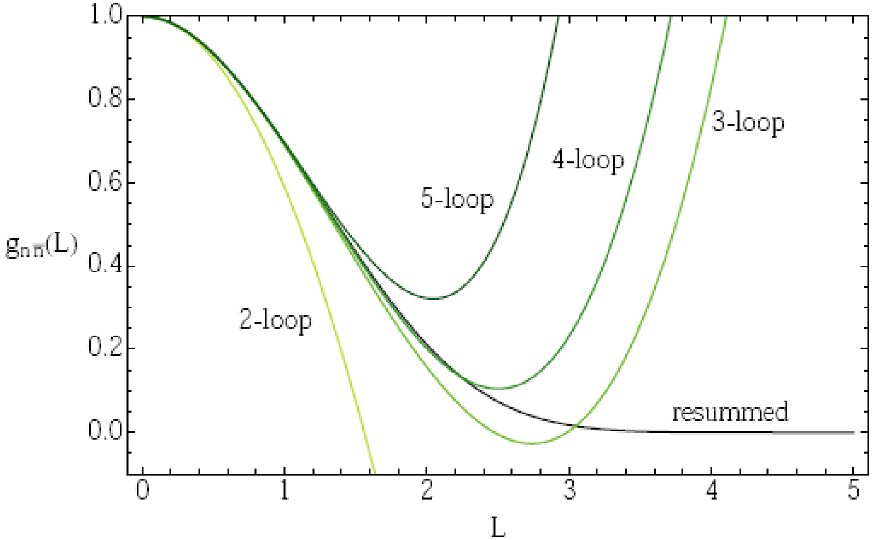
Hemisphere NGLs through to five loops

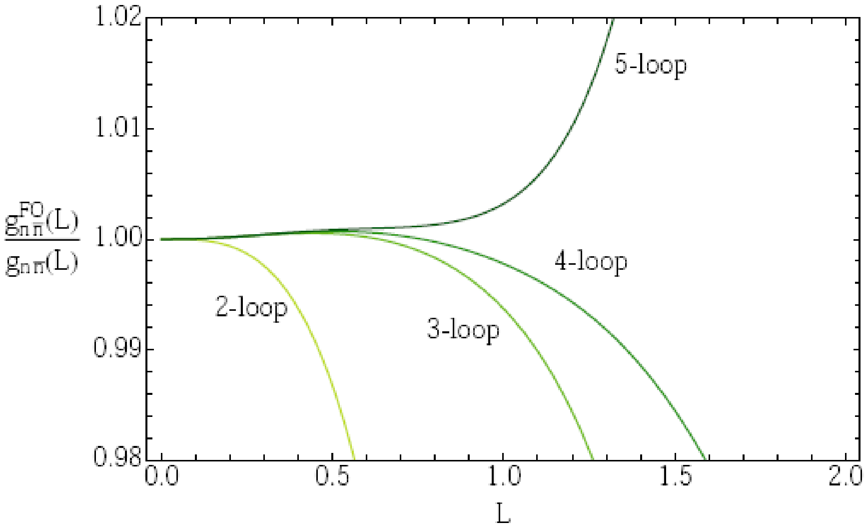
- Using the results for NGLs $g_{ab}(L)$ with a, b point to same hemisphere, we get hemisphere NGLs through to five loops analytically

$$g_{n\bar{n}}(L) = 1 - \frac{\pi^2}{24}L^2 + \frac{\zeta(3)}{12}L^3 + \frac{\pi^4}{34560}L^4 + \left(-\frac{\pi^2\zeta(3)}{360} + \frac{17\zeta(5)}{480}\right)L^5 + \ldots$$

- We can also solve BMS equation on a grid, result in good agreement with Monte Carlo fit of [Dasgupta, Salam, 2001]

- Comparison of perturbative expansion with all order numerical resummed results





- Seems to converges for L<0.5

Conclusion

- Analytically compute NGLs to four loops for a, b pointing to the same hemisphere, five loops for hemisphere NGLs
- Symmetry of BMS equation of great help in the calculation
- Perturbative expansion works well for L<0.5
- It would be good if we can
  - Find an easier way to do the azimuthal integral?
  - Bootstrapping $g_{ab}(L)$? (uniform weight, first entry condition, alphabet={0,1})
  - Interpolation between fixed order expansion (small L) and all order resummation (large L)?
  - Are there hidden symmetry in the subleading logarithmic terms?

Thank you!