A PRECISION ANALYSIS OF THE JET BROADENING DISTRIBUTIONS

[GUIDO BELL]

based on: T. Becher, GB, M. Neubert, Phys. Lett. B 704 (2011) 276
T. Becher, GB, Phys. Lett. B 713 (2012) 41
T. Becher, GB, JHEP 1211 (2012) 126
T. Becher, GB, Phys. Rev. Lett. 112 (2014) 182002
T. Becher, GB, P. Monni, H. Prager, work in progress

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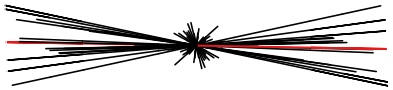
THE ROYAL SOCIETY

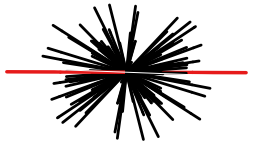
Jet broadening

Definition:

$$b_T = \frac{1}{2} \sum_i \left| \vec{p}_i \times \vec{n}_T \right|$$

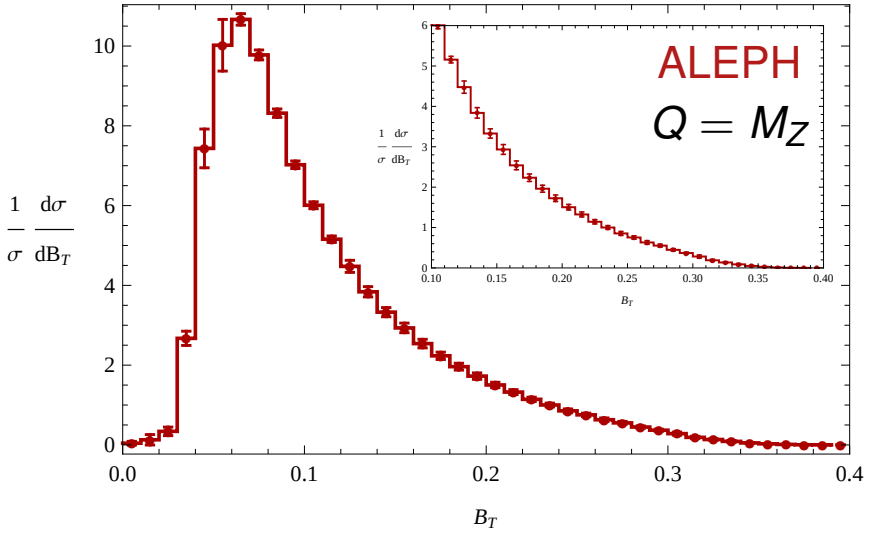


two-jet like: $b_T \simeq 0$



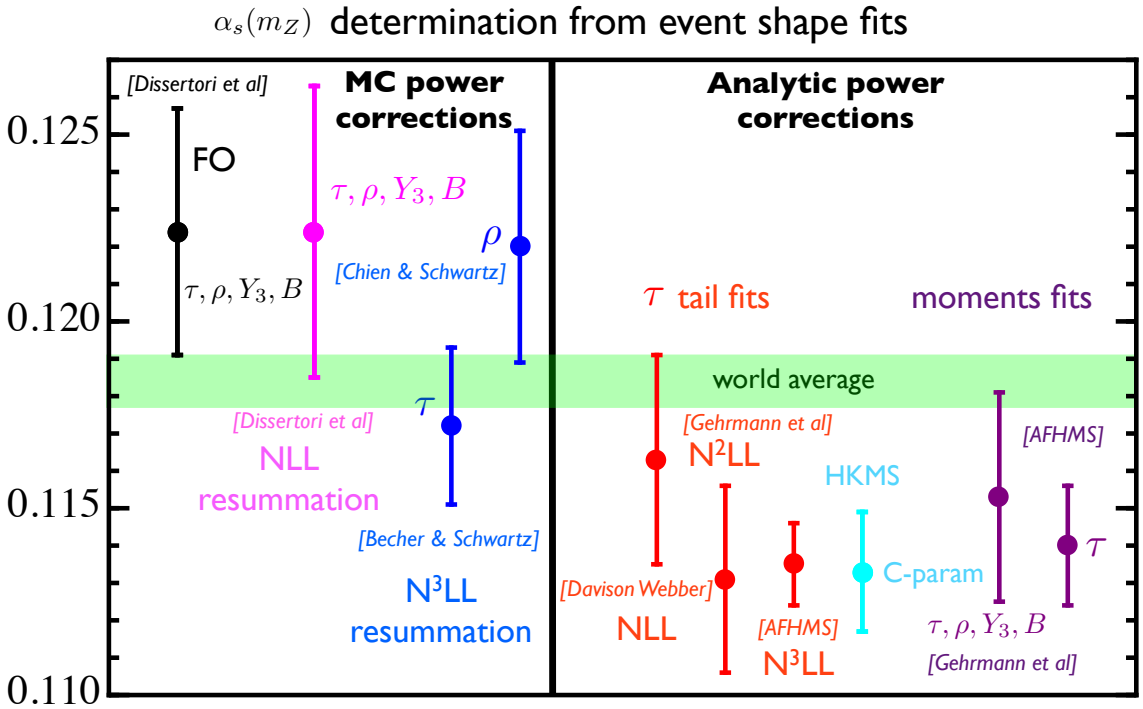
spherical: $b_T \simeq 0.4$

High-precision data from LEP, SLD, JADE, . . .



- α_s determination
- testing ground for precision QCD techniques

α_s determinations



[compilation from A. Hoang, QCD workshop, Singapore, March 2013]

- NNLL / N^{3}LL resummations reduce uncertainties
- fits based on analytic power corrections lead to lower values
- tension between most precise determinations and world average

[AFHMS: Abbate, Fickinger, Hoang, Mateu, Stewart 10,12]

Status of jet broadenings

Early analyses revealed complex resummation structure

- recoil effects modify Sudakov exponent at NLL [Dokshitzer, Lucenti, Marchesini, Salam 98]
- non-perturbative effects do not only shift but squeeze the distribution

  (quantified in analytic coupling model) [Dokshitzer, Marchesini, Salam 98]

Recent progress using methods from Soft-Collinear Effective Theory

- first all-order factorisation theorem [Chiu, Jain, Neill, Rothstein 11; Becher, GB, Neubert 11]
- extension to NNLL accuracy [Becher, GB 12]
- model-independent treatment of non-perturbative effects [Becher, GB 13]

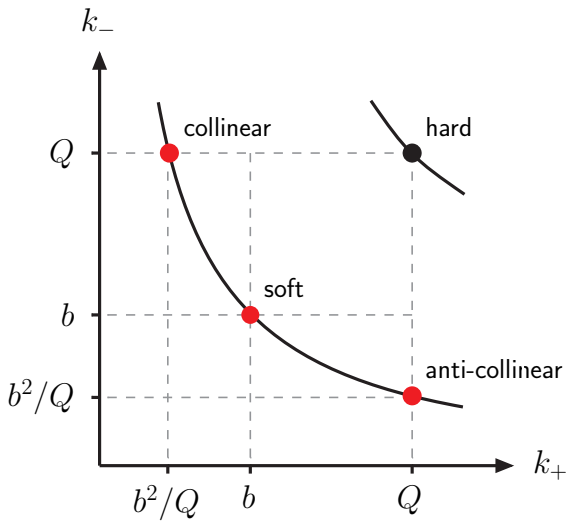
Rapidity divergences

Factorisation theorem

$$\frac{d\sigma}{db_T} \sim H(\mu_h)\ J(\mu_j) \otimes J(\mu_j) \otimes S(\mu_s)$$

$\Rightarrow$ jet and soft functions are not individually

well-definied in dimensional regularisation



Additional regulator that discriminates modes by their rapidities [Becher, GB 11]

$$\boxed{\int d^4k\ \delta(k^2)\ \theta(k^0) \quad \Rightarrow \quad \int d^dk\ \left(\frac{\nu}{k_+}\right)^{\alpha} \delta(k^2)\ \theta(k^0)}$$

- soft and jet functions contain divergences in α
- cancel in their product $\Rightarrow$ induces large rapidity logarithms

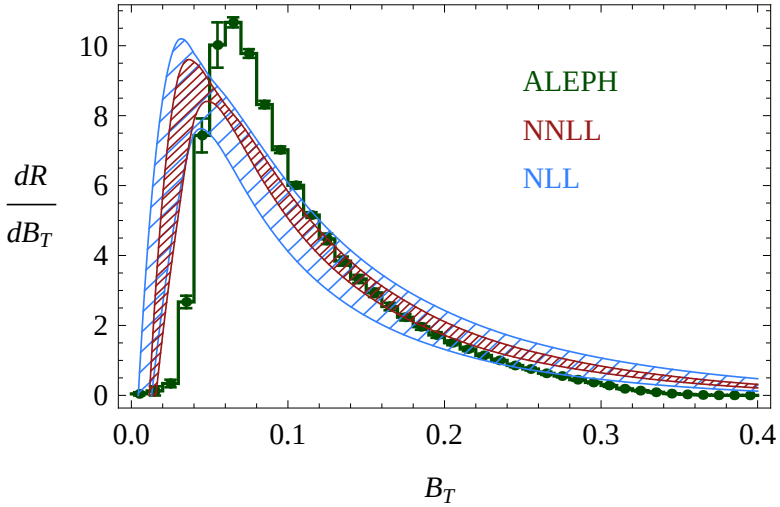
Collinear anomaly

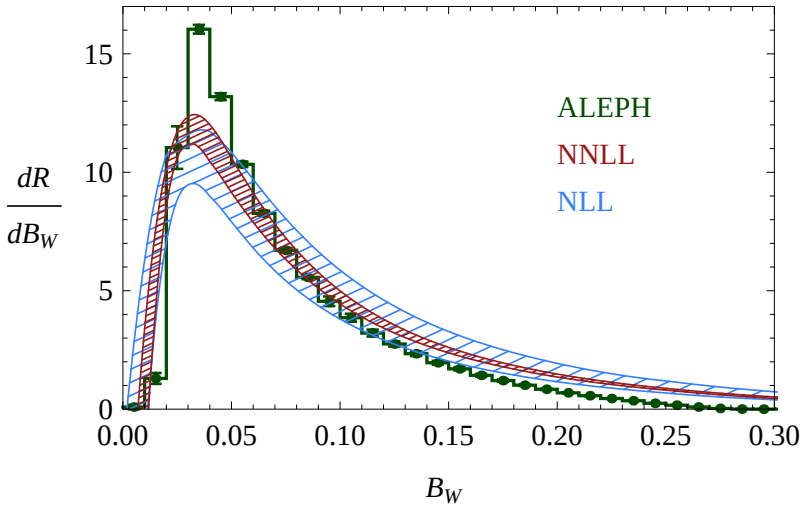
Rapidity logarithms exponentiate (in Laplace space) [Chiu, Golf, Kelley, Manohar 07; Becher, Neubert 10; Chiu, Jain, Neill, Rothstein 11]

$$\mathcal{J}(\mu)\ \mathcal{J}(\mu)\ \mathcal{S}(\mu)\ =\ \left(\frac{Q^2}{\mu^2}\right)^{-F(\mu)}\ W(\mu)$$

- anomaly exponent $F(\mu)$, remainder function $W(\mu)$

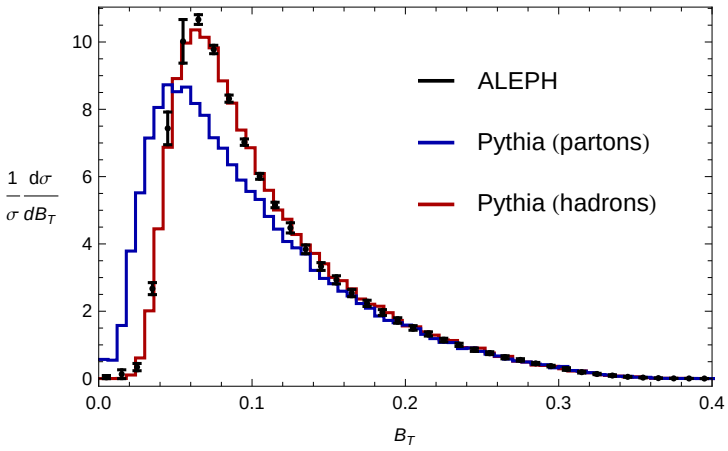
Computed 2-loop $F(\mu)$ and 1-loop $W(\mu)$ to achieve NNLL accuracy [Becher, GB 12]





Non-perturbative effects

Use hadronisation models implemented in Monte Carlos



- MC parton level defined with IR cutoffs
- tuning mixes pert. + non-pert. effects

$\Rightarrow$ cannot be used for precision studies

Instead use insights from factorisation

$$\mathcal{J}(\mu)\ \mathcal{J}(\mu)\ \mathcal{S}(\mu) \ = \ \left(\frac{Q^2}{\mu^2}\right)^{-F(\mu)} W(\mu)$$

$\Rightarrow$ NP effects associated with collinear anomaly are logarithmically enhanced [Becher, GB 13]

Operator product expansion

Expand soft function in the limit $b_{L,R} \sim |p^{\perp}_{L,R}| \gg \Lambda_{QCD}$

$$\mathcal{S}(b_L, b_R, p_L^{\perp}, p_R^{\perp}) = \sum\!\!\!\!\!\!\int_{X,\text{reg}} \delta\Big(b_L - \tfrac{1}{2}\sum_{i\in X_L} |p_i^{\perp}|\Big)\ \delta\Big(b_R - \tfrac{1}{2}\sum_{j\in X_R} |p_j^{\perp}|\Big)$$

$$\times\ \delta^{d-2}(p_L^{\perp} - p_{X_L}^{\perp})\ \delta^{d-2}(p_R^{\perp} - p_{X_R}^{\perp})\ \Big|\langle X|S_n^{\dagger}(0)\, S_{\bar{n}}(0)|0\rangle\Big|^2$$

Operator product expansion

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$$\times\ \delta^{d-2}(p_L^{\perp} - p_{X_L}^{\perp})\ \delta^{d-2}(p_R^{\perp} - p_{X_R}^{\perp})\ \Big|\langle X|S_n^{\dagger}(0)\, S_{\bar{n}}(0)|0\rangle\Big|^2$$

$$\simeq\ \delta^{d-2}(p_L^{\perp})\, \delta^{d-2}(p_R^{\perp})\ \Big[\delta(b_L)\,\delta(b_R) - \mathcal{M}_L\, \delta'(b_L)\,\delta(b_R) - \mathcal{M}_R\, \delta(b_L)\,\delta'(b_R)\Big]$$

- recoil corrections vanish due to rotation invariance
- leading non-perturbative effects are encoded in matrix element

$$\mathcal{M}_{L/R} = \sum\!\!\!\!\!\!\int_{X,\text{reg}} b_{X_{L/R}}\, \Big|\langle X|S_n^{\dagger}(0)\, S_{\bar{n}}(0)|0\rangle\Big|^2$$

Rapidity divergences (again)

Introduce transverse energy-flow operator $\mathcal{E}_T(\eta)$ [Lee, Sterman 06]

$$\Rightarrow \mathcal{M}_{L/R} = c_{L/R} \langle 0| S_{\bar{n}}^\dagger(0) S_n(0) \mathcal{E}_T(0) S_n^\dagger(0)\, S_{\bar{n}}(0)|0\rangle = c_{L/R}\, \mathcal{A}$$

- same NP matrix element $\mathcal{A}$ that drives thrust shift
- calculable coefficients $c_L = \frac{1}{2}\int_0^\infty d\eta$ $c_R = \frac{1}{2}\int_{-\infty}^0 d\eta$

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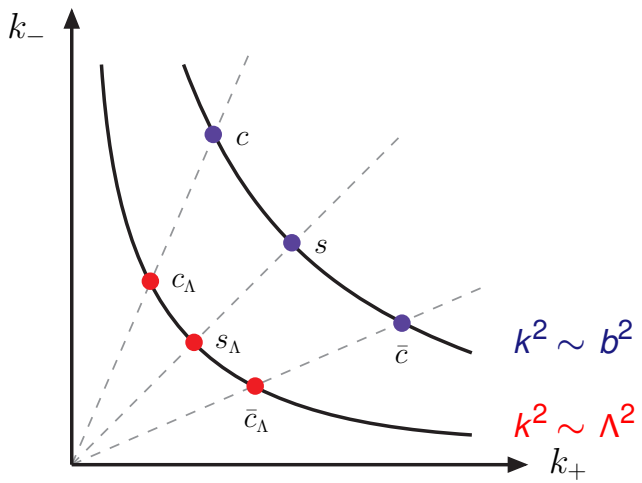
Presence of other non-perturbative modes



perturbative modes	NP modes
$k_s^\mu = (b, b, b)$	$k_{s_\Lambda}^\mu = (\Lambda, \Lambda, \Lambda)$
$k_c^\mu = (\frac{Q}{b}\, b, b, \frac{b}{Q}\, b)$	$k_{c_\Lambda}^\mu = (\frac{Q}{b}\, \Lambda, \Lambda, \frac{b}{Q}\, \Lambda)$

Non-perturbative anomaly

Rapidity divergences find their counterpart in the jet functions

$$\overline{\mathcal{J}}_L(\tau_L, z_L)\ \overline{\mathcal{J}}_R(\tau_R, z_R)\ \overline{\mathcal{S}}(\tau_L, \tau_R, z_L, z_R) \ = \ \overline{\mathcal{J}}_L^{\text{pert}}(\tau_L, z_L)\ \overline{\mathcal{J}}_R^{\text{pert}}(\tau_R, z_R)\ \overline{\mathcal{S}}^{\text{pert}}(\tau_L, \tau_R, z_L, z_R)$$

$$\times \left\{ 1 + \frac{\mathcal{A}}{2} \left[\left(\qquad\qquad + \frac{1}{\alpha} + \ln\left(\frac{\nu}{\Lambda}\right) \right) \tau_L + \left(\qquad\qquad - \frac{1}{\alpha} - \ln\left(\frac{\nu}{\Lambda}\right) \right) \tau_R \right] \right\}$$

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$$\times \left\{ 1 + \frac{\mathcal{A}}{2} \left[\left(-\frac{1}{\alpha} - \ln\left(\frac{\nu Q \tau_L}{\Lambda}\right) + \frac{1}{\alpha} + \ln\left(\frac{\nu}{\Lambda}\right) \right) \tau_L + \left(+\frac{1}{\alpha} + \ln\left(\frac{\nu}{\Lambda Q \tau_R}\right) - \frac{1}{\alpha} - \ln\left(\frac{\nu}{\Lambda}\right) \right) \tau_R \right] \right\}$$

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$$\times \left\{ 1 + \frac{\mathcal{A}}{2} \left[\left(\qquad - \ln\left(Q\tau_L\right) \qquad \right) \tau_L + \left(\qquad - \ln\left(Q\tau_R\right) \qquad \right) \tau_R \right] \right\}$$

$\Rightarrow$ NP effects are enhanced by a rapidity logarithm

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$\Rightarrow$ NP effects are enhanced by a rapidity logarithm

At higher orders the leading NP corrections exponentiate

$$\overline{\mathcal{J}}_L(\tau_L, z_L)\, \overline{\mathcal{J}}_R(\tau_R, z_R)\, \overline{\mathcal{S}}(\tau_L, \tau_R, z_L, z_R) \ = \ (Q^2\tau_L^2)^{-F_B(\tau_L, z_L)}\ (Q^2\tau_R^2)^{-F_B(\tau_R, z_R)}\ W(\tau_L, \tau_R, z_L, z_R)$$

- NP correction to anomaly exponent: $F_B(\tau, z) = F_B^{\text{pert}}(\tau, z) + \frac{1}{4}\,\tau\,\mathcal{A}$

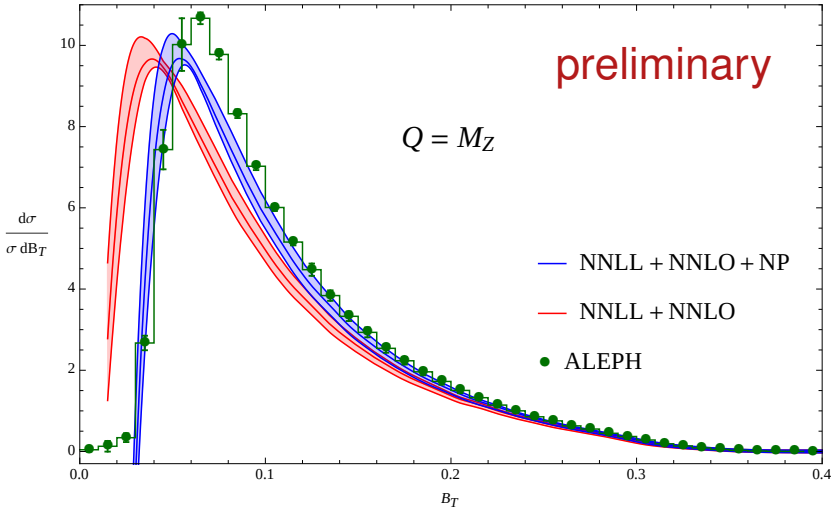
Implementation

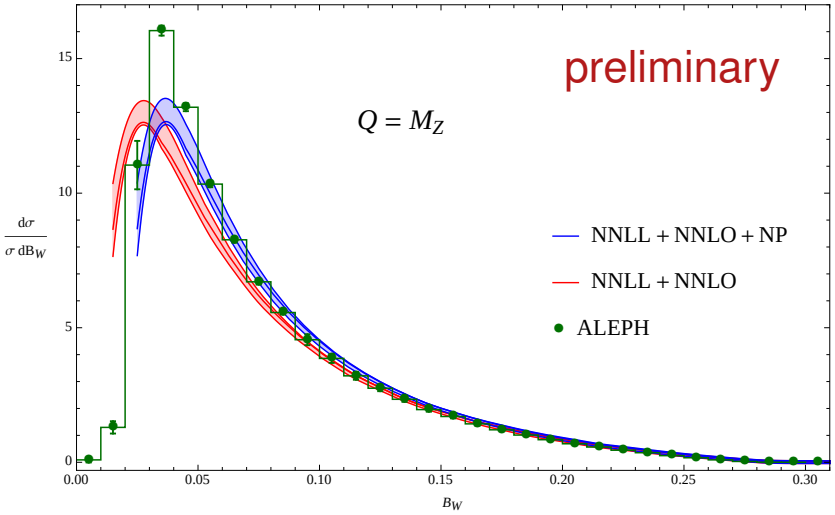
The leading NP effect takes the form

$$\frac{d\sigma}{de}(e) = \frac{d\sigma_{\text{pert}}}{de}\left(e - c_e \frac{\mathcal{A}}{Q}\right) \qquad c_{B_T} \simeq -\ln B_T \qquad c_{B_W} \simeq -1/2 \ln B_W$$

- seen before in effective coupling model [Dokshitzer, Marchesini, Salam 98]

  (where non-log. corrections are also associated with $\mathcal{A}$ – model-dependent!)





Drell-Yan production

The low q_T cross section is also affected by a collinear anomaly [Becher, Neubert 10; Chiu, Jain, Neill, Rothstein 12]

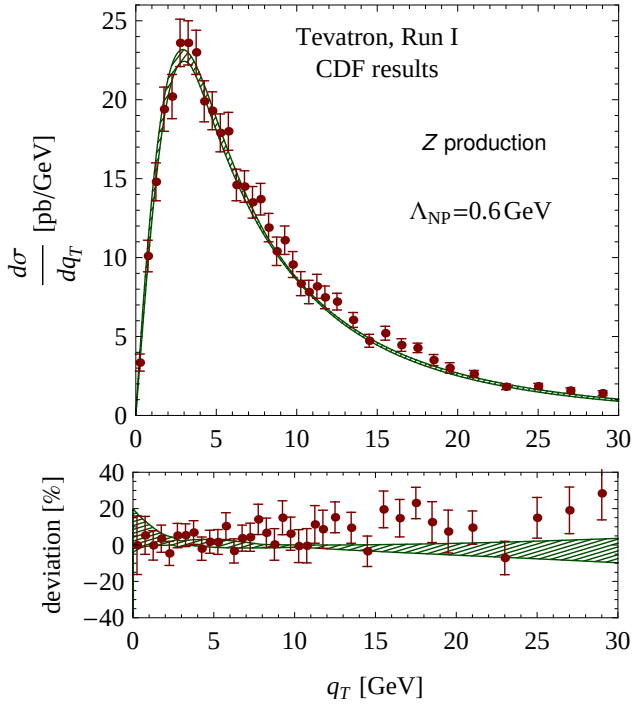
$$\frac{d^2\sigma}{dq_T^2 d\eta} = \sum_{ij} H_{ij}(M^2) \int d^2x_\perp \; e^{-ix_\perp q_\perp} \; (x_T^2 M^2)^{-F_{ij}(x_T^2)} \; B_{i/N_1}(\xi_1, x_T^2) \; B_{j/N_2}(\xi_2, x_T^2)$$

- equivalent to Collins-Soper-Sterman formalism
- anomaly exponent $F_{ij}(x_T^2)$ known to two loops
- transverse-momentum-dependent PDFs $B_{i/N}(\xi, x_T^2)$

  computed to two loops [Gehrmann, Lübbert, Yang 12,14]

$\Rightarrow$ all ingredients for NNLL resummation available

[Becher, Neubert, Wilhelm 11]



Non-perturbative effects

NP effects are often modelled with a Gaussian cutoff or a variant thereof

$$\int d^2x_\perp \; e^{-ix_\perp q_\perp} \; (x_T^2 M^2)^{-F_{ij}(x_T^2)} \; B_{i/N_1}(\xi_1, x_T^2)\, e^{-\Lambda_{NP}^2 x_T^2} \; B_{j/N_2}(\xi_2, x_T^2)\, e^{-\Lambda_{NP}^2 x_T^2}$$

Our analysis shows that leading NP effects are associated with collinear anomaly

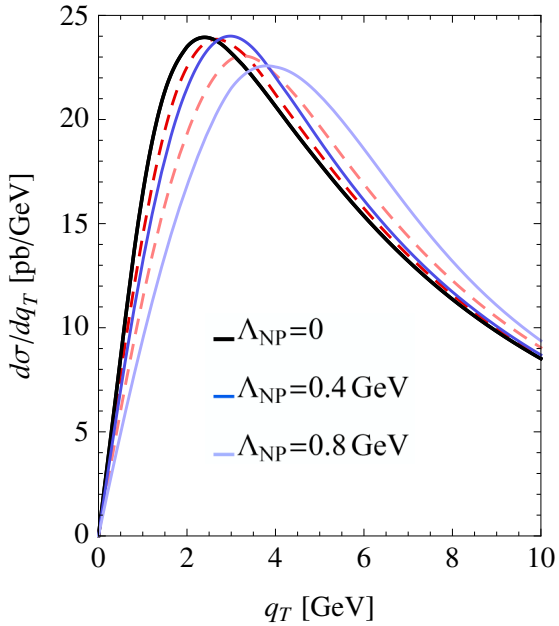
$$\int d^2x_\perp \; e^{-ix_\perp q_\perp} \; (x_T^2 M^2)^{-F_{ij}(x_T^2)-\Lambda_{NP}^2 x_T^2} \; B_{i/N_1}(\xi_1, x_T^2) \; B_{j/N_2}(\xi_2, x_T^2)$$

- Λ_{NP}^2 can be extracted from the matrix element

$$\mathcal{M}_\perp = \sum\!\!\!\!\!\!\int_{X,\mathrm{reg}} \; p_{X_\perp}^2 \; \left|\langle X | S_n^\dagger(0)\, S_{\bar{n}}(0) | 0 \rangle\right|^2$$

- depends on colour representation, but not on flavour i, j
- leading NP effects exponentiate

[Becher, GB 13]



Conclusions

Systematic formalism to perform p_T resummation in SCET

$$\boxed{\int d^d k \;\; \delta(k^2)\, \theta(k^0) \quad \Rightarrow \quad \int d^d k \;\; \left(\frac{\nu_+}{k_+}\right)^{\alpha} \delta(k^2)\, \theta(k^0)}$$

- efficient regulator to calculate resummation ingredients to higher orders

Field-theoretical understanding of NP effects to p_T-dependent observables

- logarithmic enhancement from collinear anomaly

High-precision analysis of jet broadening distributions

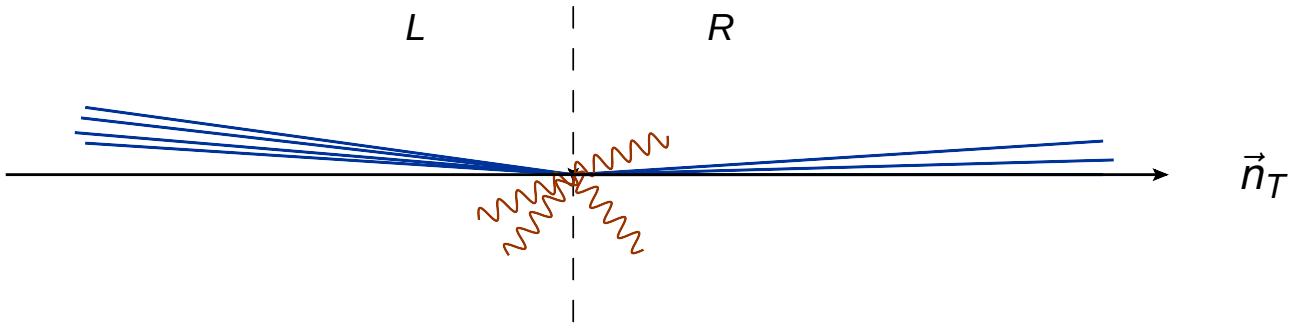
- perturbative input at NNLL+NNLO accuracy
- leading non-perturbative correction related to thrust shift

Backup slides

Factorisation

In the two-jet limit $b_L \sim b_R \to 0$ the broadening distribution factorises

$$\frac{1}{\sigma_0}\frac{d^2\sigma}{db_L\, db_R} = H(Q^2,\mu) \int db_L^s \int db_R^s \int d^{d-2}p_L^\perp \int d^{d-2}p_R^\perp$$
$$\mathcal{J}_L(b_L - b_L^s, p_L^\perp, \mu)\ \mathcal{J}_R(b_R - b_R^s, p_R^\perp, \mu)\ \mathcal{S}(b_L^s, b_R^s, -p_L^\perp, -p_R^\perp, \mu)$$



- jet recoils against soft radiation [Dokshitzer, Lucenti, Marchesini, Salam 98]
- relevant scales: $Q^2 \gg b_L^2 \sim b_R^2 \sim (p_L^\perp)^2 \sim (p_R^\perp)^2$

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Convenient to work in Laplace-Fourier space

- Laplace transform $b_{L,R} \to \tau_{L,R}$
- Fourier transform $p_{L,R}^\perp \to x_{L,R}^\perp$ and define $z_{L,R} = \frac{2|x_{L,R}^\perp|}{\tau_{L,R}}$

$$\frac{1}{\sigma_0}\frac{d^2\sigma}{d\tau_L\, d\tau_R} = H(Q^2,\mu) \int dz_L \int dz_R\; \overline{\mathcal{J}}_L(\tau_L, z_L, \mu)\; \overline{\mathcal{J}}_R(\tau_R, z_R, \mu)\; \overline{\mathcal{S}}(\tau_L, \tau_R, z_L, z_R, \mu)$$

$H(Q^2,\mu) =$ square of on-shell vector form factor

Analytic regularisation in SCET

Our prescription amounts to

$$\boxed{\int d^4k \;\; \delta(k^2)\;\theta(k^0) \quad \Rightarrow \quad \int d^dk \;\; \left(\frac{\nu_+}{k_+}\right)^{\alpha} \delta(k^2)\;\theta(k^0)}$$

- virtual corrections do not need regularisation

  matrix elements of Wilson lines in QCD $\Rightarrow$ the same for thrust and broadening

  technical reason: $\int d^{d-2}k_\perp \;\; f(k_\perp, k_+) \;\sim\; k_+^{-\epsilon}$

- required for observables sensitive to transverse momenta

  $f(k_\perp, k_+) \sim \delta^{d-2}(k_\perp - p_\perp) \quad \Rightarrow$ factor $k_+^{-\epsilon}$ absent $\quad \Rightarrow$ reinstalled as $k_+^{-\alpha}$

  can show that the prescription regularises all LC singularities in SCET [Becher, GB 11]

- not sufficient for cases where virtual corrections are ill-defined

  examples: electroweak Sudakov corrections, Regge limits

Collinear anomaly

Can show that the Q dependence exponentiates using and extending arguments from

- electroweak Sudakov resummation [Chiu, Golf, Kelley, Manohar 07]
- p_T resummation in Drell-Yan production [Becher, Neubert 10]

Start from the logarithm of the product of jet and soft functions

$$\ln P = \ln \overline{\mathcal{J}}_L\Big(\ln\,(Q\nu_+\bar{\tau}_L^2);\ \tau_L, z_L\Big) + \ln \overline{\mathcal{J}}_R\Big(\ln\,(\tfrac{\nu_+}{Q});\ \tau_R, z_R\Big) + \ln \overline{\mathcal{S}}\Big(\ln\,(\nu_+\bar{\tau}_L);\ \tau_L, \tau_R, z_L, z_R\Big)$$

collinear: $k_+ \sim \frac{b^2}{Q}$ anticollinear: $k_+ \sim Q$ soft: $k_+ \sim b$

- use that product does not depend on ν_+ and that it is LR symmetric

$$\Rightarrow \ln P = \frac{k_2(\mu)}{4}\ln^2\left(Q^2\,\bar{\tau}_L\bar{\tau}_R\right) - F_B(\tau_L, z_L, \mu)\,\ln\left(Q^2\bar{\tau}_L^2\right) - F_B(\tau_R, z_R, \mu)\,\ln\left(Q^2\bar{\tau}_R^2\right) + \ln W(\tau_L, \tau_R, z_L, z_R, \mu)$$

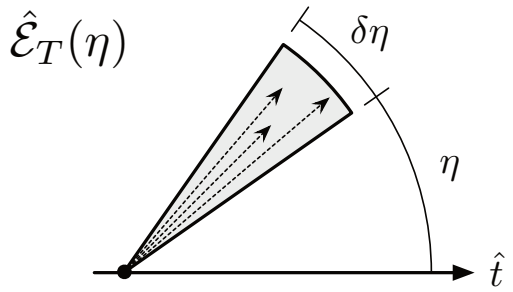
- RG invariance implies $k_2(\mu) = 0$ to all orders

$$\Rightarrow \boxed{P(Q^2, \tau_L, \tau_R, z_L, z_R, \mu) \;=\; \left(Q^2\bar{\tau}_L^2\right)^{-F_B(\tau_L, z_L, \mu)}\,\left(Q^2\bar{\tau}_R^2\right)^{-F_B(\tau_R, z_R, \mu)}\,W(\tau_L, \tau_R, z_L, z_R, \mu)}$$

Transverse energy-flow operator

Introduce transverse energy-flow operator [Lee, Sterman 06]

$$\mathcal{E}_T(\eta)\,|X\rangle = \sum_{i\in X} |p_i^\perp|\,\delta(\eta-\eta_i)\,|X\rangle$$



- measures total transverse momentum

  flowing into rapidity interval $\eta + d\eta$

Perform Lorentz boost along thrust axis

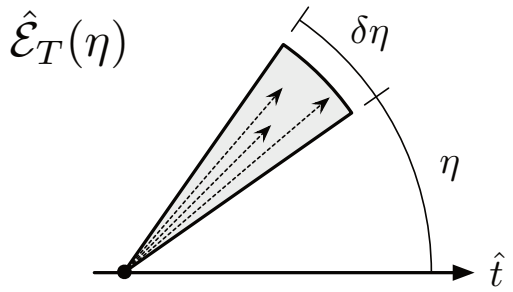
$$\Rightarrow \mathcal{M}_{L/R} = c_{L/R}\,\langle 0|\,S_{\bar{n}}^\dagger(0)S_n(0)\mathcal{E}_T(0)S_n^\dagger(0)\,S_{\bar{n}}(0)|0\rangle = c_{L/R}\,\mathcal{A}$$

- rapidity dependence $c_L = \frac{1}{2}\int_0^\infty d\eta$ $\qquad c_R = \frac{1}{2}\int_{-\infty}^0 d\eta$

Transverse energy-flow operator

Introduce transverse energy-flow operator [Lee, Sterman 06]

$$\mathcal{E}_T(\eta)\,|X\rangle = \sum_{i\in X} |p_i^\perp|\,\delta(\eta-\eta_i)\,|X\rangle$$



- measures total transverse momentum

  flowing into rapidity interval $\eta + d\eta$

Perform Lorentz boost along thrust axis

$$\Rightarrow \mathcal{M}_{L/R} = c_{L/R}\,\langle 0|\,S_{\bar{n}}^\dagger(0)S_n(0)\mathcal{E}_T(0)S_n^\dagger(0)\,S_{\bar{n}}(0)|0\rangle = c_{L/R}\,\mathcal{A}$$

- rapidity dependence $c_L = \frac{1}{2}\int_0^\infty d\eta$ $\qquad c_R = \frac{1}{2}\int_{-\infty}^0 d\eta$

  phase-space regulator: $(k_{1,+}\cdots k_{n,+})^{-\alpha/n} \to e^{\alpha\eta}\,(k_{1,+}\cdots k_{n,+})^{-\alpha/n}$

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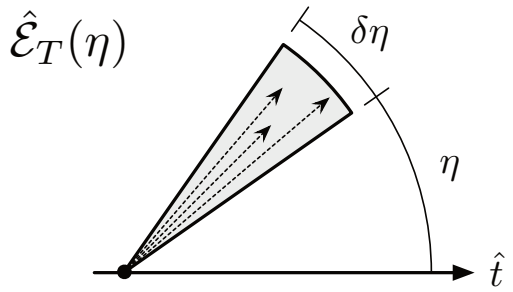

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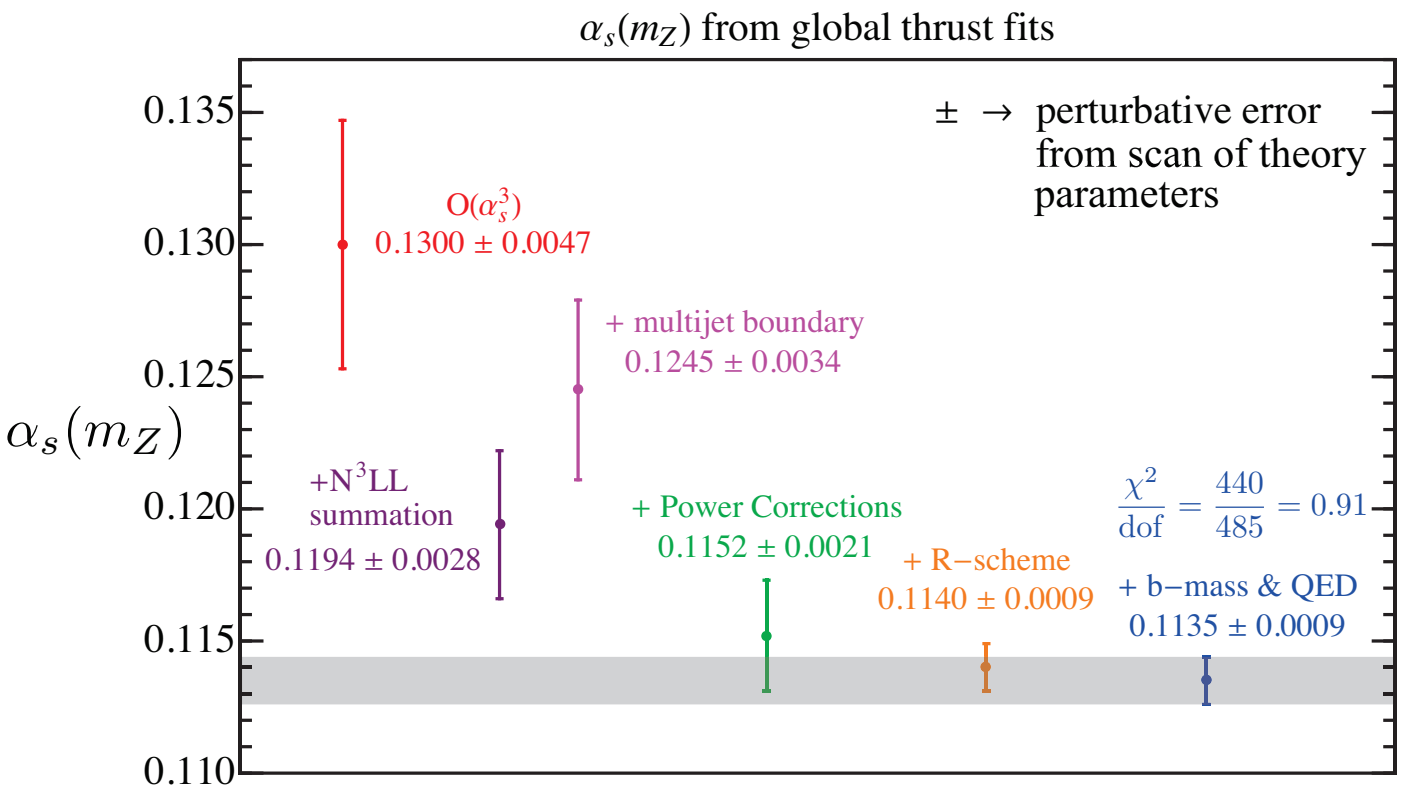
Precision thrust analysis



distribution: $\alpha_s(M_Z) = 0.1135 \pm 0.0002\,(\text{exp}) \pm 0.0005\,(\text{had}) \pm 0.0009\,(\text{pert})$ [Abbate et al 10]

moment: $\alpha_s(M_Z) = 0.1140 \pm 0.0004\,(\text{exp}) \pm 0.0013\,(\text{had}) \pm 0.0007\,(\text{pert})$ [Abbate et al 12]

NNLO + NNLL: $\alpha_s(M_Z) = 0.1131\,^{+0.0028}_{-0.0022}$ [Monni, Gehrmann, Luisoni 12]