Beam Instabilities for the Proton Driver

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DESIGN OF ACCUMULATOR AND COMPRESSOR RINGS FOR THE PROJECT-X BASED PROTON DRIVER*

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Table 1: FMC Accumulator and Compressor Parameters

Parameters	AR	CR
Circumference, m	308.23	308.23
Momentum compaction	-0.052	0.001
Slippage factor	-0.063	-0.01
RF frequency, MHz	3.87	3.87
RF voltage, kV	10	240
Synchrotron tune	$2.1 \cdot 10^{-4}$	$4.2 \cdot 10^{-4}$
Peak current, A	100	1040
Final r.m.s. bunch length, ns	29.2	3.2
Final r.m.s. energy spread	$5.2 \cdot 10^{-4}$	$6.9 \cdot 10^{-3}$
Threshold impedance, Ohm	20	$3 \rightarrow 53$
R.m.s. emittance, μm	5	5
Space charge tuneshift, h/v	0.02/0.02	0.14/0.16
Betatron tunes, h/v	7.94/6.91	6.76/8.44

AR, Longitudinal

$$\left.\frac{Z_{\parallel}}{n}\right|_{\mathrm{SC}} = i\frac{Z_0}{\gamma^2}\ln\left(\frac{b}{\sigma_x}\right) \approx 9\,\mathrm{Ohm};$$

Space charge impedance

$$\left.\frac{Z_{\parallel}}{n}\right|_{\mathrm{RW}} = (\mathrm{sgn}(\omega) - i)\frac{Z_0}{2}\frac{\delta(|n\omega_0|)}{b}; \;\; \delta(\omega) = \frac{c}{\sqrt{2\pi\sigma_{\mathrm{wall}}\omega}}$$

Resistive wall impedance

$$\left.\frac{\mathrm{Im}\,Z_{\parallel}}{n}\right|_{\mathrm{RW,\ single\ bunch}} \cong -\frac{Z_0}{2}\frac{c}{b\sqrt{2\pi\sigma_{\mathrm{wall}}/\sigma_{\tau}}} \approx -0.1\,\mathrm{Ohm}$$

$$\frac{\Delta Q_s}{Q_s} = \frac{N r_0 \eta R_0^2}{\sqrt{2\pi}Q_s^2\gamma\sigma_s^3}\frac{\mathrm{Im}\,Z_{\parallel}/n}{Z_0}$$

Relative synchrotron tune shift [2]

AR, Longitudinal results

With $N = 4.5 \cdot 10^{13}$, aperture radius $b = 4.8\,\mathrm{cm}$, this yields $\Delta Q_s / Q_s = -0.34$,

Meaning possible mismatch and loss of Landau damping.

However, with copper wall, the CB growth rate is too low:

$$\tau_{\parallel}^{-1} = \frac{N r_0 \eta R_0 c}{8 Q_s \gamma b d^{5/2}} \sqrt{\frac{c}{\sigma_{\mathrm{wall}}}} \Rightarrow \tau_{\parallel} = 30\,\mathrm{s}$$

Ref [3]

Thus, the only possible longitudinal problem is the mismatch.

AR, Transverse

$$\Delta Q_{\mathrm{sc}} = 0.018$$

SC tune shift

$$q_{\mathrm{sc}} = \Delta Q_{\mathrm{sc}} / (2Q_{\mathrm{s}}) = 42 \gg 1$$

SC parameter

$$\tau_{\mathrm{SB}}^{-1} \cong 0.1 \frac{N r_0 R_0 c}{\pi Q_x \gamma b^3 \sqrt{\sigma_\tau \sigma_{\mathrm{wall}}}} \Rightarrow \tau_{\mathrm{SB}}^{-1} = 30\,\mathrm{s}^{-1}$$

Single bunch growth rate

$$\tau_{\mathrm{CB}}^{-1} = \frac{M N r_0 c^2}{2\pi\gamma Q_x b^3 \sqrt{2\pi\sigma_{\mathrm{wall}}\omega_0 (1-\{Q_x\})}} \Rightarrow \tau_{\mathrm{CB}}^{-1} = 400\,\mathrm{s}^{-1}$$

Coupled bunch growth rate

$$k_{\mathrm{mode}} \cong 1.5\chi = 1.5\xi(\Delta p / p) / Q_{\mathrm{s}}$$

Lowest SB number of unstable modes

$$\xi_{\mathrm{th}} \cong \frac{Q_{\mathrm{s}} q_{\mathrm{sc}}^{3/4}}{\sigma_{\Delta p/p} (\omega_{\mathrm{s}} \tau_{\mathrm{CB}})^{1/4}} \approx 7; \quad \xi_{\mathrm{th}} \sigma_{\Delta p/p} \approx 0.0035$$

Threshold chromaticity, Ref. [5]

CR, Longitudinal

$$\left.\frac{Z_{\|}}{n}\right|_{\mathrm{SC}} = i\frac{Z_0}{\gamma^2}\ln\left(\frac{b}{\sigma_x}\right) \approx 9\,\mathrm{Ohm};$$

Space charge impedance

$$\left.\frac{Z_{\|}}{n}\right|_{\mathrm{RW}} = (\mathrm{sgn}(\omega) - i)\frac{Z_0}{2}\frac{\delta(|n\omega_0|)}{b};\ \ \delta(\omega) = \frac{c}{\sqrt{2\pi\sigma_{\mathrm{wall}}\omega}}$$

Resistive wall impedance

$$\left.\frac{\mathrm{Im}\,Z_{\|}}{n}\right|_{\mathrm{RW,\ single\ bunch}} \cong -\frac{Z_0}{2}\frac{c}{b\sqrt{2\pi\sigma_{\mathrm{wall}}/\sigma_\tau}} \approx -0.04\,\mathrm{Ohm}$$

$$D \equiv \frac{V_{\mathrm{SC}}}{V_{\mathrm{RF}}} \approx \frac{2\pi N r_0 R_0^2 \ln(b/\sigma_x)}{\gamma^2\sigma_s^3 h_{RF}}\frac{mc^2}{V_0}$$

Longitudinal space charge parameter

$$D \equiv \frac{V_{\mathrm{SC}}}{V_{\mathrm{RF}}} \approx 30.$$

Cannot be done!

CR, Longitudinal, results

- The big problem: $D \equiv \frac{V_{\text{SC}}}{V_{\text{RF}}} \approx 30.$

- This means the bunch cannot be compressed below

$$\sigma_{s0} D^{1/3} \approx 3\sigma_{s0} = 10\,\text{ns.}$$

Ref. [4]

- For a negative slippage, this would result in the instantaneous negative mass instability.

- Conclusion: to have the desired bunch compression in the CR, much higher RF Voltage (~10 times) is required.

- Better, the two-stage compression with increasing RF frequency be applied (Y. Alexahin)

CR, Transverse

$$\Delta Q_{\mathrm{sc}} = 0.18$$

SC tune shift

$$q_{\mathrm{sc}} = \Delta Q_{\mathrm{sc}} / (2Q_{\mathrm{s}}) = 220 \gg 1$$

SC parameter

$$\tau_{\mathrm{SB}}^{-1} \cong 0.1 \frac{N r_0 R_0 c}{\pi Q_x \gamma b^3 \sqrt{\sigma_\tau \sigma_{\mathrm{wall}}}} \Rightarrow \tau_{\mathrm{SB}}^{-1} = 120\,\mathrm{s}^{-1}$$

Single bunch growth rate

$$\tau_{\mathrm{CB}}^{-1} = \frac{M N r_0 c^2}{2\pi\gamma Q_x b^3 \sqrt{2\pi\sigma_{\mathrm{wall}}\omega_0 (1-\{\mathrm{Q}_x\})}} \Rightarrow \tau_{\mathrm{CB}}^{-1} = 270\,\mathrm{s}^{-1}$$

Coupled bunch growth rate

$$k_{\mathrm{mode}} \cong 1.5\chi = 1.5\xi(\Delta p / p) / Q_{\mathrm{s}}$$

Lowest SB number of Z-unstable mode

$$\xi_{\mathrm{th}} \cong \frac{Q_{\mathrm{s}} q_{\mathrm{sc}}^{3/4}}{\sigma_{\Delta p/p} (\omega_{\mathrm{s}} \tau_{\mathrm{CB}})^{1/4}} \approx 2.5; \quad \xi_{\mathrm{th}} \sigma_{\Delta p/p} \approx 0.017$$

Threshold chromaticity

Summary

Transversely, the beams can be stabilized with a sufficiently low chromaticity.

Longitudinally, in AR there is a moderate mismatch problem.

In CR, the RF focusing has to be significantly (~15-30 times) increased.

References

- [1] Y. Alexahin, D. Neuffer, IPAC'12
- [2] A. Chao, "Physics of collective beam instabilities in high energy accelerators", Eq. (6.58), p. 291, J. Wiley & Sons, 1993.
- [3] Ibid, Eq. (4.124), p. 210
- [4] Ibid, Eq. (6.43), p. 285
- [5] A. Burov, "Head-tail modes for strong space charge", PRST-AB, 12, 044202 (2009)

Many thanks!

Backup slides