Beam Instabilities for the Higgs Factory

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Parameters of the Present Design

Parameter	Unit	Value
Circumference, C	m	299
β*	cm	2.5 (1.5-10)
Momentum compaction, α_p	-	0.0793
Betatron tunes	-	4.56 / 3.56
Bare lattice chromaticity	-	-124 / -197
Synchrotron tune* (100kV, 200MHz)	-	0.002
Number of muons / bunch	10^{12}	2
Normalized emittance, $\varepsilon_{\perp N}$	π·mm·rad	0.3
Long. emittance, $\varepsilon_{\|\|N}$	π·mm	1.0
Beam energy spread	%	0.003
Bunch length, σ_s	cm	5.64
Beam-beam parameter	-	0.0072
Repetition rate	Hz	30
Average luminosity	10^{31}/cm^2/s	2.5

*) Without wake-fields and longitudinal beam-beam effect

Transverse plane: formulas

- Head-tail growth rate, in the air-bag approximation:

$$\tau_l^{-1} = -\frac{N_\mu r_0 c}{4\pi\gamma T_0 \omega_x} \int_{-\infty}^{\infty} d\omega \operatorname{Re} Z_x(\omega) J_l^2\left(\omega \frac{\hat{z}}{c} - \chi\right)$$

$$Z_x(\omega) = \frac{\omega_x}{c} \oint Z_x(\omega, s) \beta_x(s) ds$$

- For a thick resistive wall, the local impedance scales as

$$Z_x(\omega, s) \propto 1/b^3(s) \propto 1/\beta_x^{3/2}$$

yielding the total impedance

$$Z_x(\omega) \propto \oint \frac{ds}{\sqrt{\beta_x(s)}} \cong C\sqrt{\frac{Q_x}{R}}$$

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Transverse plane: results

- For $N_\mu = 2 \cdot 10^{12};\ Q_x = 4.56;\ Q_s = 0.002;\ \eta = 0.08;\ \Delta p_{\mathrm{rms}} / p = 3 \cdot 10^{-5};$
- tungsten wall with the conductivity $\sigma_W = 1.8 \cdot 10^{17}\,\mathrm{s}^{-1},$
- the chromaticity $\xi = 5$, the aperture $b = 4.5\sigma_x$, the growth rate comes out:

$$\tau_1^{-1} \cong 0.5 \cdot 10^{-5}\,\omega_0 = 30\mathrm{s}^{-1}\ .$$

- The geometrical impedance can be estimated assuming its longitudinal value $Z_{\|} / n = 0.1\mathrm{Ohm}$
- After application of the Panofsky-Wenzel theorem, this leads to doubling of the total transverse impedance. Thus, with 1ms of the lifetime, there is a safety factor of ~100 for the transverse instabilities.
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Longitudinal plane: formulas

- Synchrotron frequency shift due to the potential well distortion:

$$\Delta\omega_s = \frac{N_\mu r_0 c\eta}{2\pi\omega_s \gamma C}\int_0^\infty d\omega \exp\left(-\omega^2\sigma_\tau^2/2\right)\omega \,\mathrm{Im}\, Z_\parallel(\omega)$$

- With the resistive wall impedance, this yields:

$$\frac{\Delta Q_s}{Q_s} \approx 0.2\frac{N_\mu r_0 \eta R^2\bar{\delta}}{Q_s^2\gamma\sigma_s^3\bar{b}}; \quad D = \frac{V_Z}{V_{RF}} \approx 2.5\frac{\Delta Q_s}{Q_s}$$

- where $\bar{\delta} = c/\sqrt{2\pi\sigma_W/\sigma_\tau}$ and $\bar{b} = 4.5\sqrt{R\varepsilon_n/(\gamma Q_x)}$ are the average skin depth and aperture.

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Longitudinal plane: results

- For the normalized rms emittance $\varepsilon_n = 0.2\mathrm{mm}$, this yields as high as

$$\Delta Q_s / Q_s \approx -0.6; \quad D \approx 1.5$$

- With the mentioned geometrical impedance, this number doubles. That high number of the potential well distortion means that one should expect up to ~50% of the bunch lengthening with some energy widening.

- This effect could be probably reduced by means of the second harmonic RF.

- It was shown by V. Balbekov [4] that Robinson instability should not play a role for the Higgs Factory.

References

- [1] Y. Alexahin, Preliminary Design of the mu+mu- Higgs Factory Ring Lattice, MAP-doc-4351-v1 (2012).
- [2] A. Chao, "Physics of collective beam instabilities in high energy accelerators", Eq. (6.213), p. 350, J. Wiley & Sons, 1993.
- [3] Ibid, Eq. (6.58), p. 291.
- [4] V. Balbekov, "Beam loading and longitudinal instabilities at the Neutrino Factory and Muon Collider", MAP Note 4363, 2013.

Many thanks!

Backup slides