Luminosity Estimate in a Multi-TeV Muon Collider Using $e^+e^- \Rightarrow \mu^+\mu^-$ as the Muon Source

L. Keller

J. P. Delahaye

T. Markiewicz

U. Wienands

SLAC

MAP 2014 Spring Workshop, Fermilab

27-31 May, 2014

Some Previous work on this subject:

1. **D. Kaplan, P. Allport, T. Hart, http://arxiv.org/pdf/0707.1546.pdf**

   "Producing an Intense, Cool Muon beam via e^+e^- Annihilation", Nov. 2006

   Looked at ways to get the required muon rate of $10^{13} – 10^{14}$ /sec by boosting the positron beam current. Discussed colliding beams, high-power targets, e^+ storage ring, e^- gun , e^+ ERL.

   Concluded that "given the extraordinary beam and target parameters required, the cost effectiveness is far from clear".

2. **M. Antonelli, P. Raimondi, INFN-13-22/LNF and Snowmass 2013 Report**

   "Ideas for Muon Production from Positron Beam Interaction on a Plasma Target"

   Considered a plasma electron density of 10^{20} e^-/cm^3 and a plasma length of 10 m. Discussed the production cross section and beam degradation due to radiative Bhabha scattering. Muon production rate near threshold from an SLC –type machine ~0.5 x 10^6/sec. Try for very low emittance.

Now try looking at thin material targets to see if smaller emittance could make up for many orders of magnitude in lower muon production rate compared to a proton source.

I. Basic Method

Annihilate high energy positrons and atomic electrons in the reaction, $e^+e^- \Rightarrow \mu^+\mu^-$, to produce low emittance muon beams for injection into a multi-TeV muon storage ring. The fixed target threshold is a 43.7 GeV positron beam to produce two muons at rest in the CM.

To achieve the lowest possible muon emittance try to make sure, as much as possible, that it will only be determined by the positron beam size and the maximum muon production angle.

II. Choice of Positron Beam Energy

Consider 3 TeV muon beams with energy spread 0.1% at the IP (same as the MAP White paper at Snowmass 2013).

Starting with a muon central energy, E_0 = 22 GeV, take the muon source energy spread to be $\delta = (3000/22)(0.1) = \pm 13.5\%$. Assume this can be captured and damps linearly, so the source energy range becomes:

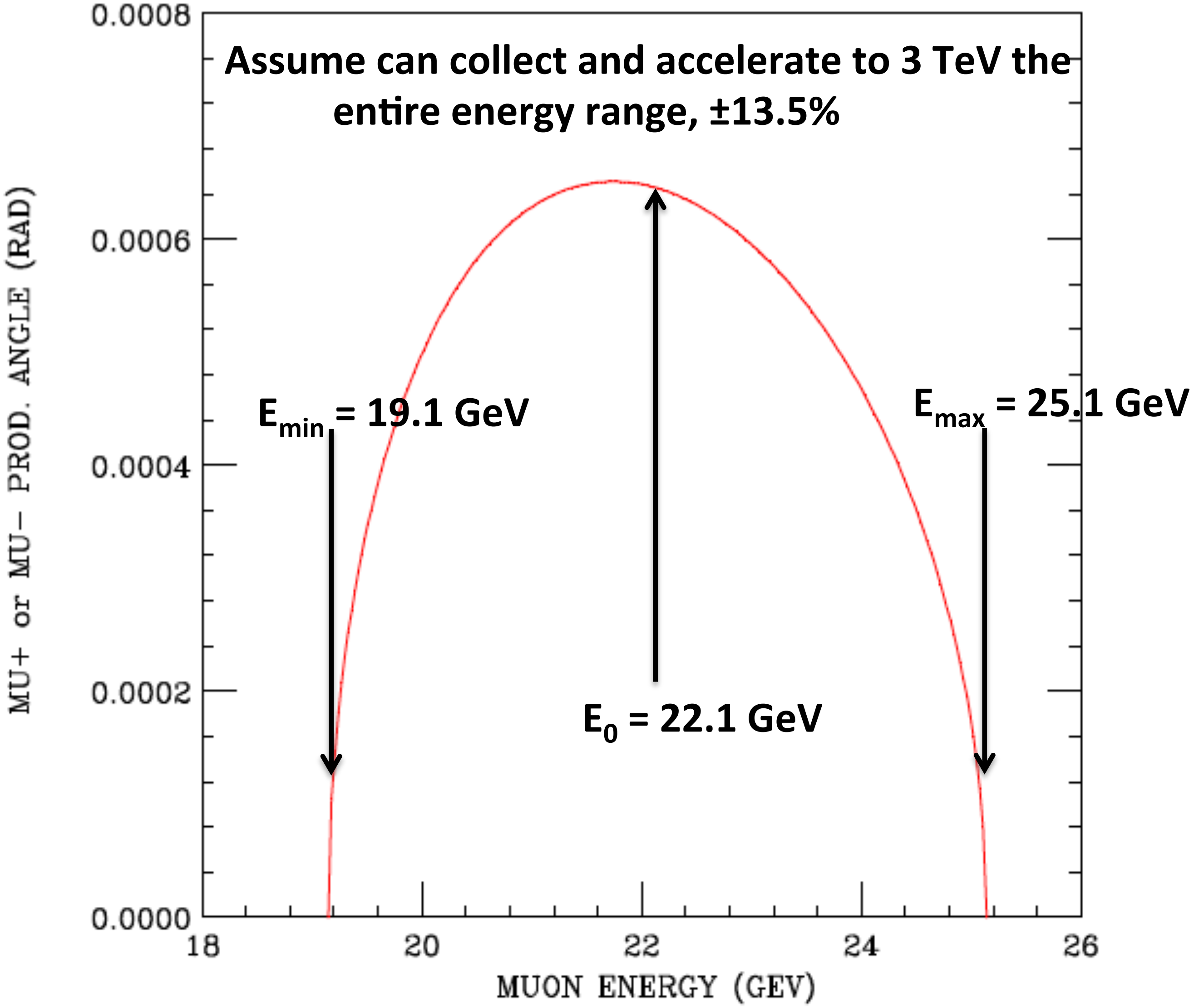
$$19.1 < E_0 < 25.1 \text{ GeV}$$

A 44.5 GeV positron beam produces muons that completely cover this energy range.

Production Kinematics

Muon Production Angle vs. Muon Energy

E_{beam} = 44.5 GeV



Assume can collect and accelerate to 3 TeV the entire energy range, ±13.5%

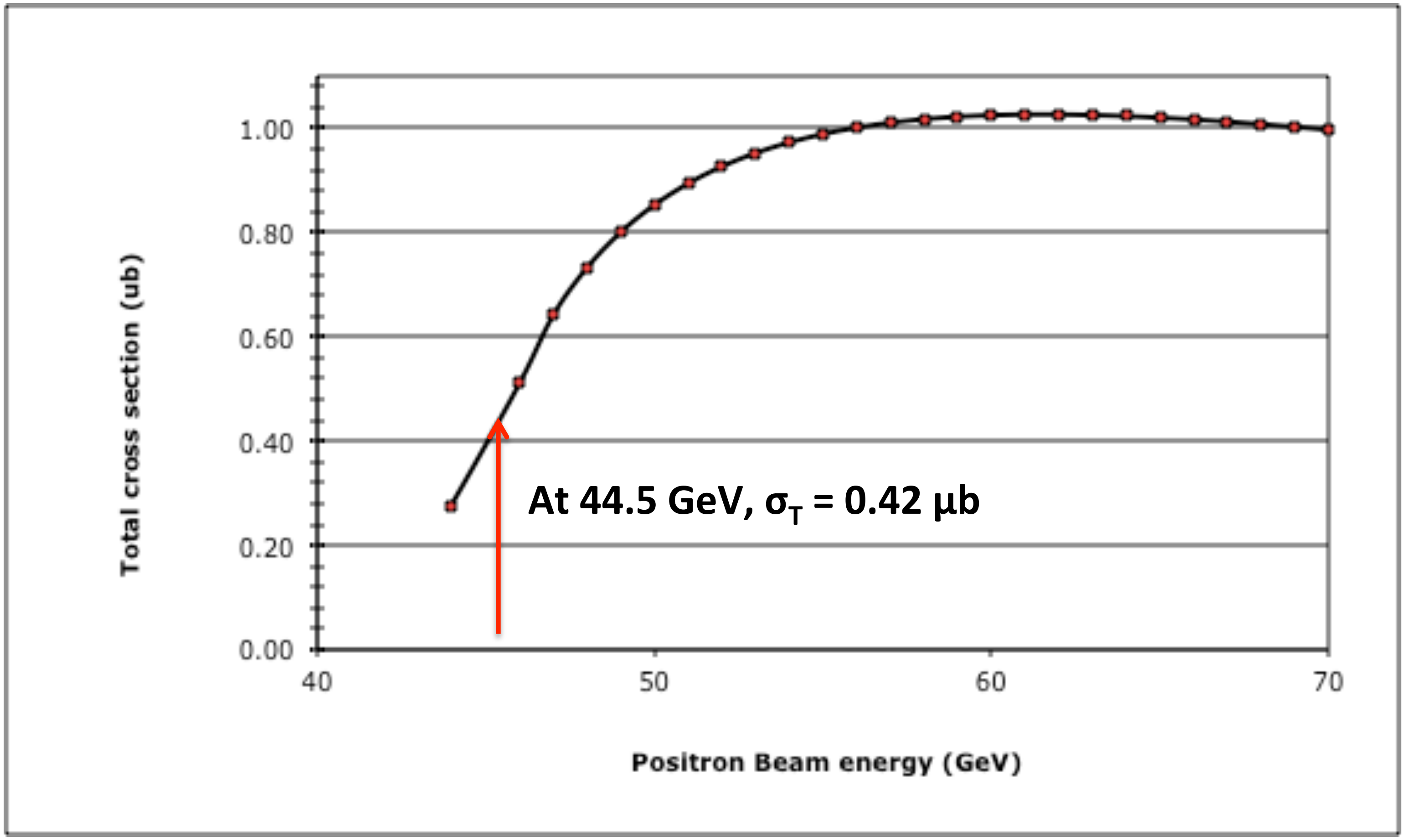
E_{min} = 19.1 GeV

E_0 = 22.1 GeV

E_{max} = 25.1 GeV

Positron Fixed Target Cross Section, $e^+e^- \longrightarrow \mu^+\mu^-$

Total Annihilation Cross Section vs. Beam Energy


At 44.5 GeV, $\sigma_T = 0.42\ \mu b$

Choosing beam energies > 44.5 GeV only results in producing muons outside the usable energy range and therefore get only part of the total cross section.

Target Choice

- For a positron beam near the muon production threshold the target thickness is limited by beam degradation from radiative Bhabha scattering, $e^+e^- \Rightarrow e^+e^- \gamma$ and bremsstrahlung.

- For equivalent thickness, nt ≥ 10^{24} e-/cm^2, and beam energy near production threshold a substantial fraction of the beam positrons begin to fall below threshold.

- For ~1 radiation length targets the muon production is dominated by the two-stage Bethe-Heitler process, $e^+ \Rightarrow \gamma \Rightarrow \mu^+\mu^-$, which gives a much larger angular spread as well as more MCS.

In summary, after trying C, Ti, Cu, and W, ended up using 0.4 cm of copper (0.28 rl and nt = 10^{24} e-/cm^2).

Muon Beam Emittance

Start with the ILC TDR positron beam parameters scaled to a round spot at 44.5 GeV:

=> $\sigma_x = \sigma_y = 0.5\mu$, $\sigma_{x'} = \sigma_{y'} = 50\ \mu\text{rad}$, $\sigma_L = 0.25$ mm

Muon kinematics give, $\theta_{max} = 0.6$ mrad, which will be increased by the angular spread of the positron beam, MCS of the incoming positron beam, and MCS of the outgoing muons. Taking half of the maximum angle and including MCS gives a divergence (non-Gaussian) = 0.37 mrad.

The effective source size is given by the positron beam size but has to also include a contribution from the production angle extended over the length of the target. This gives an effective source size of 0.7μ.

Multiplying by muon $\gamma_{22\ \text{GeV}}$ gives a normalized emittance, $\varepsilon_x = \varepsilon_y = 5.4 \times 10^{-5}\ \pi$ mm-rad

Results for 6 TeV CM

The ILC positron rate is 2.6 x 10^{14} /sec in 10 bunch trains. => 41 μA average current.
Using the target equivalent thickness, nt = 10^{24} e^-/cm^2 and σ = 0.42 μb,
gives R_μ = 1.1 x 10^8 /sec

	Proton Source	e^+e^- Source
Muon rate	2.4 x 10^{13}/sec	1.1 x 10^8/sec
Muons/bunch	2 x 10^{12}	1.1 x 10^7
β^*	0.005 m	0.00025 m
δ	0.1%	0.1%
$\gamma\epsilon_T$	25x10^{-6} πm	5.4x10^{-8} πm
σ_L^*	0.005 m	0.00025 m
$\sigma_x^* = \sigma_y^*$	1.5 μ	0.022 μ
$\sigma_{x'}^* = \sigma_{y'}^*$	420 μrad	90 μrad
Luminosity	8.8 x 10^{34} cm^{-2}s^{-1}	2.0 x 10^{28} cm^{-2}s^{-1}

Summary and Comments

1. Because of the relatively large divergence in muon production angles, even near threshold, could only get about a factor of 450 reduction in normalized emittance compared to the proton source design, and cutting on the production angle costs cross section linearly.
2. If compared to the Higgs Factory design, the emittance reduction factor becomes about 8,000, but the requirement of a very small energy spread severely limits the fraction of usable muon production cross section.
3. The obvious way to achieve a luminosity comparable to the proton source design is to increase the positron beam current by a factor $(8.8 \times 10^{34}/2.0 \times 10^{28})^{1/2} \approx 2{,}000$, so 41 μA => 80 mA. An ERL might be an option, but is beyond current technology.
4. To achieve $nt = 10^{24}\ e^-/cm^2$ in plasma would need either a very long plasma or a series of short plasmas with optics in between to control the effective source size (Raimondi).
5. Survival of the 4 mm copper target has not been addressed here, but simple dE/dx shows that at 80 mA, 450 kW is deposited in the copper alone.

We conclude that, even with the substantial emittance reduction estimated here, significant developments would be needed to reach the proton source luminosity design in a TeV class muon collider.