GH2-filled 325 MHz Helical FOFO Snake for Initial Stage of 6D Ionization Cooling

Y. Alexahin
(FNAL APC)

Motivation

- GH2-filled RF offers a higher Gradient, gas cooling of Be windows particularly important for 325 MHz RF – use HCC for 6D cooling
- Direct matching of HCC to the Front End leads to high losses due to transition crossing with large momentum spread
- Charge separation will be much easier after initial 6D cooling
- With initial 6D cooling the muon acceleration for NF will be much easier than with 4D cooling

Differences with VRF HFOFO snake reported last October:

- Larger cell length (4.2m vs 3.72m) $\Rightarrow$ higher $<\beta_\perp>$

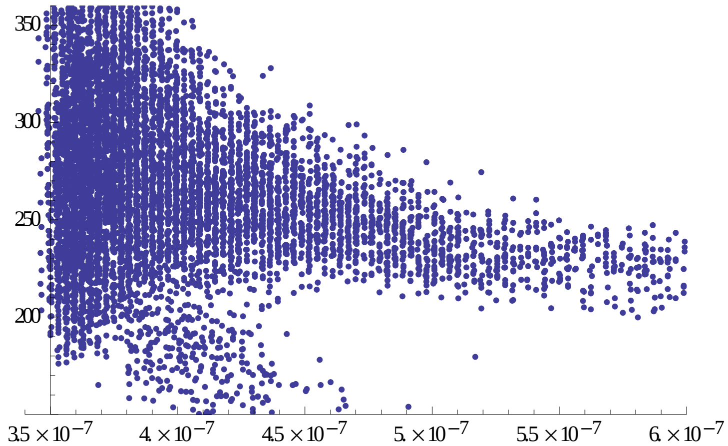
$\Rightarrow$ smaller beam angular spread
$\Rightarrow$ weaker the momentum-betatron amplitude correlation
$\Rightarrow$ lower current in solenoids

- Smaller solenoids pitch angle (2.5mrad vs 3mrad)
- Longer solenoids (30cm vs 24cm)

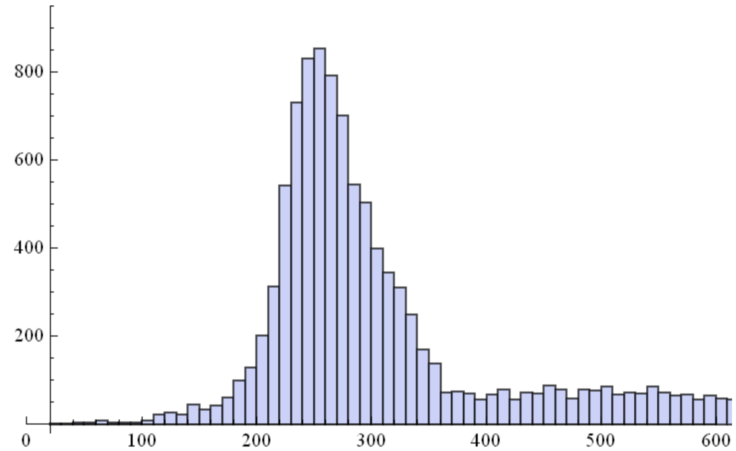
$\Rightarrow$ further reduction in current density in solenoids (94.6 A/mm^2)
$\Rightarrow$ smoother magnetic field

- Longer RF cavities (25cm vs 22cm)

Beam from Dave's new rotator



μ^+ longitudinal phase space (number of strong bunches increased ~ f_{RF})



μ^+ momentum distribution, MeV/c (μ^- look the same)

Muon beam parameters obtained with Gaussian fit*:

	N total	N 150<p<360	N core	$p^{(central)}$ MeV/c	σ_p MeV/c	$\sigma_\perp$ cm	ε_{mN} cm			ε_{6D} cm^3
μ^+	11755	7998	7329	248.0	29.8	7.6	1.2	2.2	2.4	6.2
μ^-	12396	9020	8248	248.8	28.2	7.4	1.2	2.1	2.2	5.6

$\beta_\perp$=82.8 cm for p=248.4 MeV/c Bz=2T

N.B.: There is a large imbalance in the transverse normal mode emittances, can it be used for better matching?

*) For comparison the r.m.s. emittances of the μ+ normal modes after 150 < p (MeV/c) < 360 cut are: 1.2, 2.1, 5.0 (cm); description of the fitting procedure can be found in MAP-doc-4358

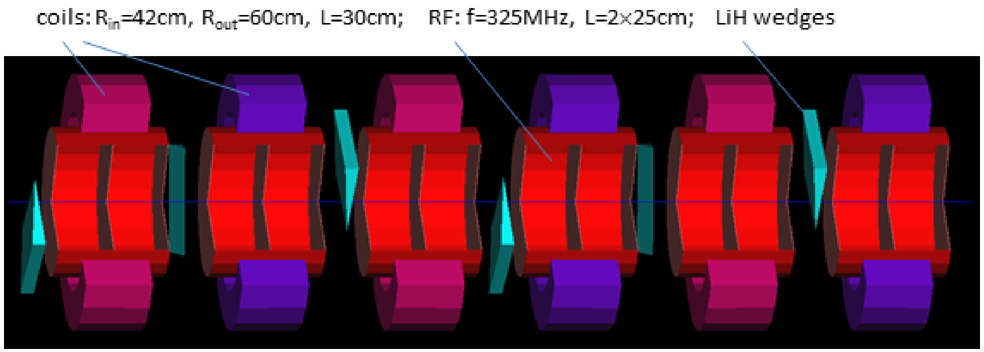
Procedure for HFOFO channel design

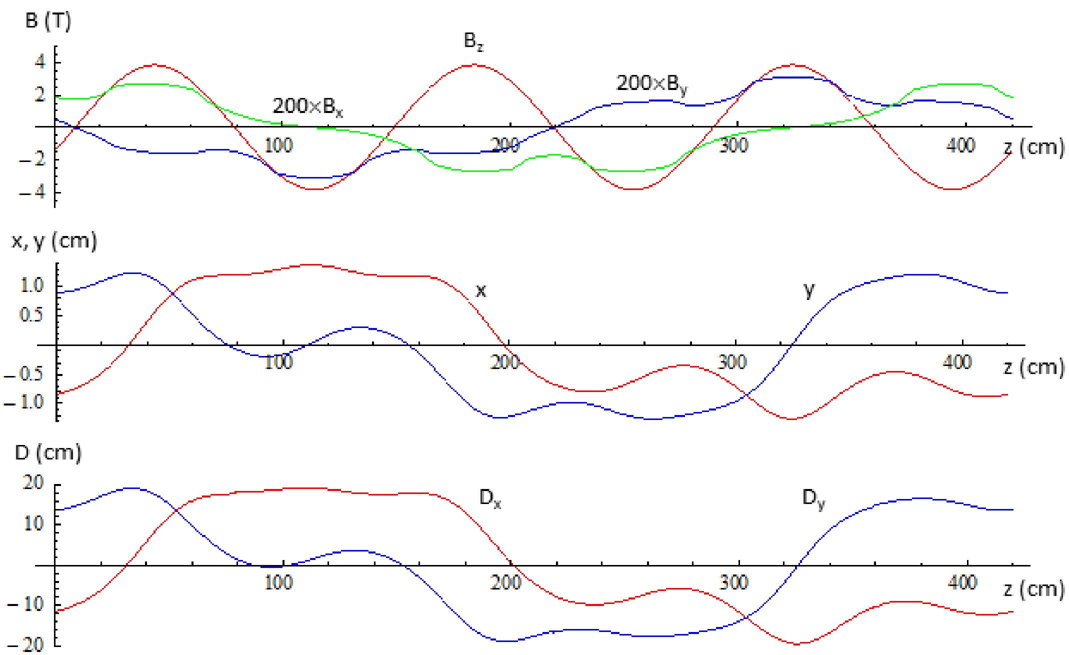
Major steps:

- **Using Methodical Ionization Cooling Channel Design (MICCD, a Mathematica code) determine parameters of a periodic channel :**
  – Geometry $\Rightarrow \beta_\perp$
  – Long. magnetic field $\Rightarrow$ betatron tunes
  – Solenoid pitch & LiH absorber wedge angle $\Rightarrow$ cooling rates and equilibrium emittances
- **Using Mathematica find satisfactory transition from constant Bz in the front end to alternating field in the snake**
  – match $\beta_\perp$ for as wide momentum range as possible
- **Create G4BL (ICOOL) description of the matching region and (quazi)periodic channel (including absorbers and RF)**
- **Adjust RF timing to achieve the desired (un-algorithmic) momentum vs z**
  – check $\beta_\perp$ -matching with varying momentum!
- **Adjust pitch and yaw of the first few solenoids to put both muon signs on their proper orbits in the periodic channel**
  – method for dispersion matching in a lattice with high synchrotron tune?
- **Launch distribution of particles from the front end and see the result:**

☹ ☺ $\Rightarrow$ **Hawaii vacations!**

HFOFO period





- 325 MHz cavities easily fit inside solenoids
- The idea: create rotating $B_\perp$ field by periodically tilting solenoids, e.g. with 6-solenoid period.
- Periodic orbits for μ+ and μ– look exactly the same, just shifted by a half period (3 solenoids), as μ– in solenoids 4, 5, 6 see exactly the same forces as μ+ in solenoids 1, 2, 3 and vice versa.
- With tune $Q_\perp$>1 (per period) $\mathbf{r}\cdot\mathbf{D}>0$

⇒ muons with higher momentum make a longer path

⇒ longitudinal cooling achieved even with planar absorbers
- To enhance longitudinal cooling LiH wedge absorbers can be added in such a way that they work for both μ+ and μ– (here wedge angle = 0.18rad)

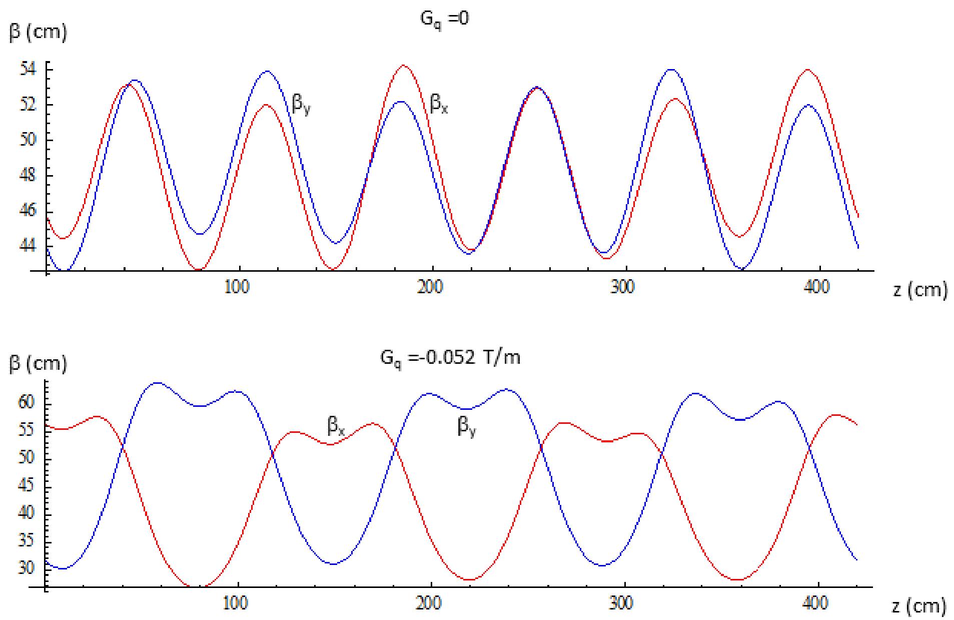
More detail can be found in MAP-doc-4377

Tunes and Emittances in a Periodic HFOFO Channel

Mode	I	II	III
Tune	1.2271 + 0.0100 i	1.2375 + 0.0036 i	0.1886 + 0.0049 i
Emittance (mm)	2.28	6.13	1.93

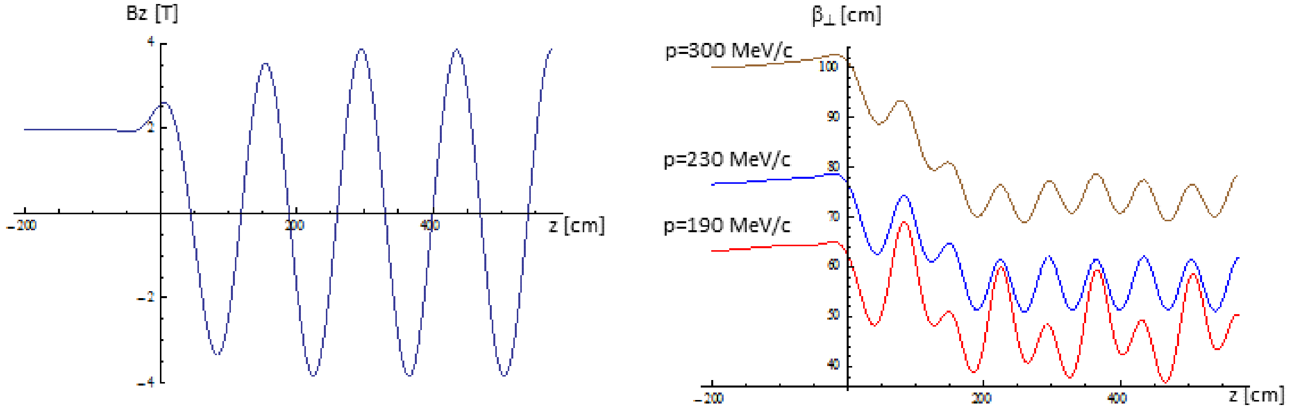
Table: normal mode tunes and normalized equilibrium emittances w/o equalization (analytics).

The transverse normal modes (I and II) cooling rates and equilibrium emittances can be equalized with the help of a unipolar quadrupole field

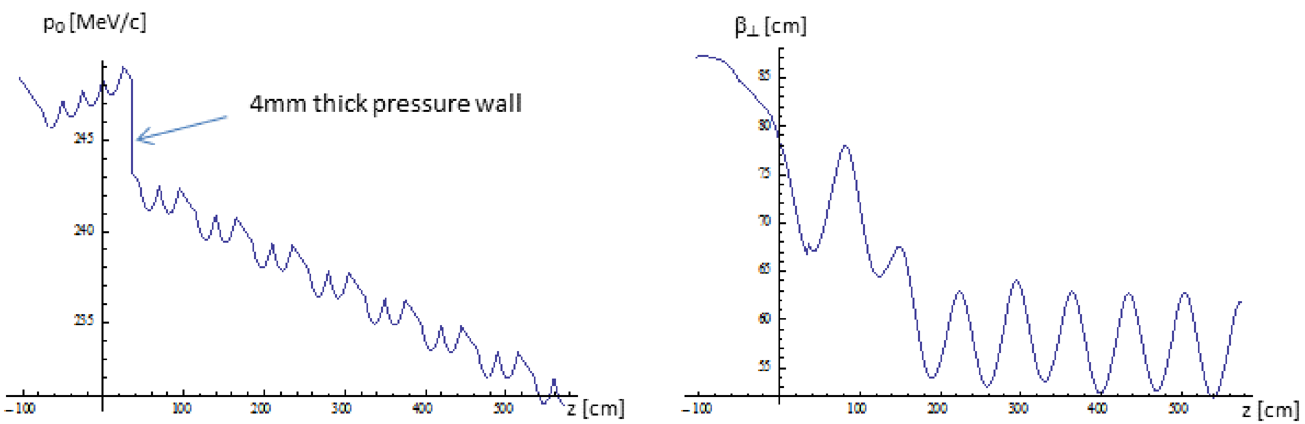


μ+ transverse β-functions with no (top) and with constant quadrupole field (bottom) of indicated strength. Strong β-beat is excited (Fig. 2) increasing slightly the 4D emittance

$\beta_\perp$-function matching

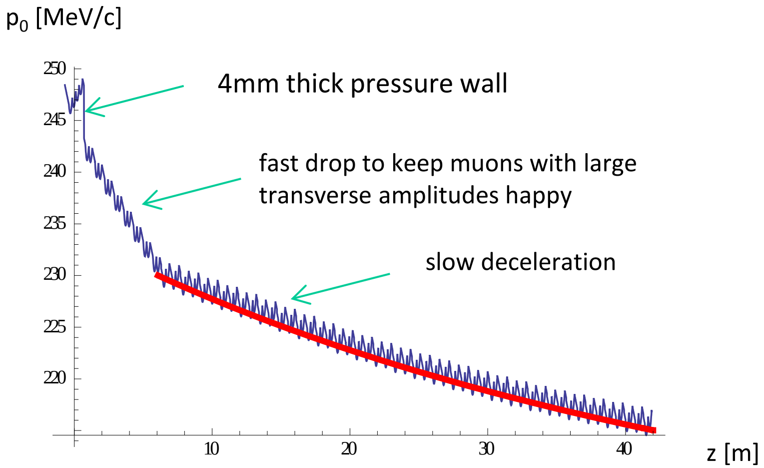


Magnetic field in the transition area (left) and β-function for constant momentum (right)

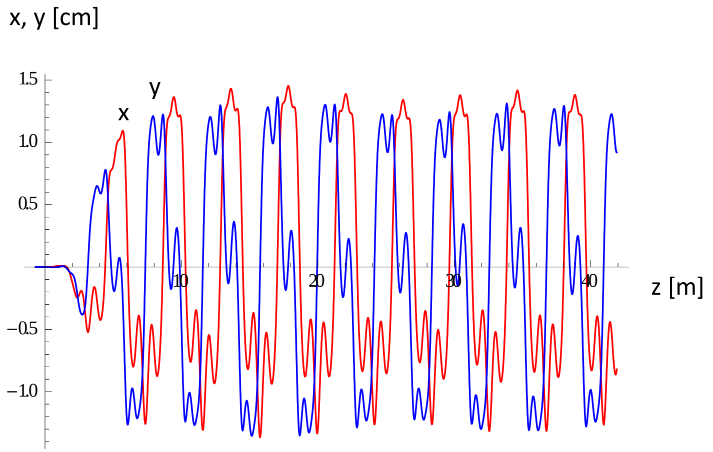


Total momentum of the reference particle in the beginning of the channel (left) and solution of Eq.(1) with such momentum dependence (right).

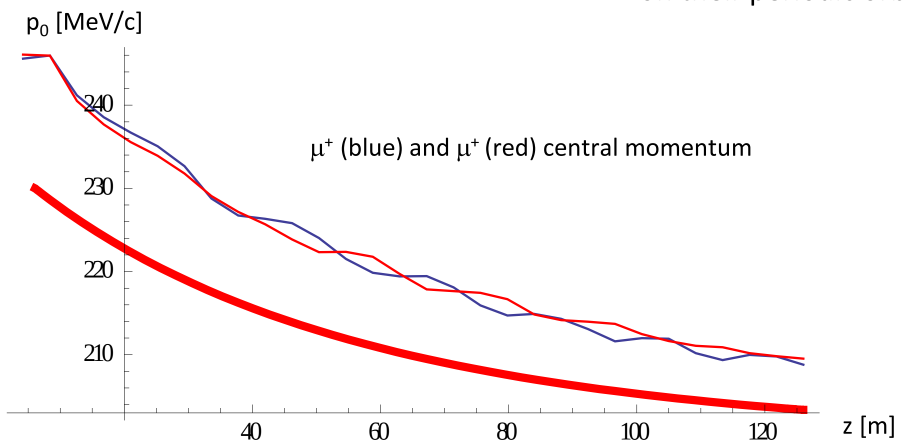
Momentum and orbit matching



Momentum of reference particle (blue) and target value (red). Adjustment made by RF timing (gradient fixed).

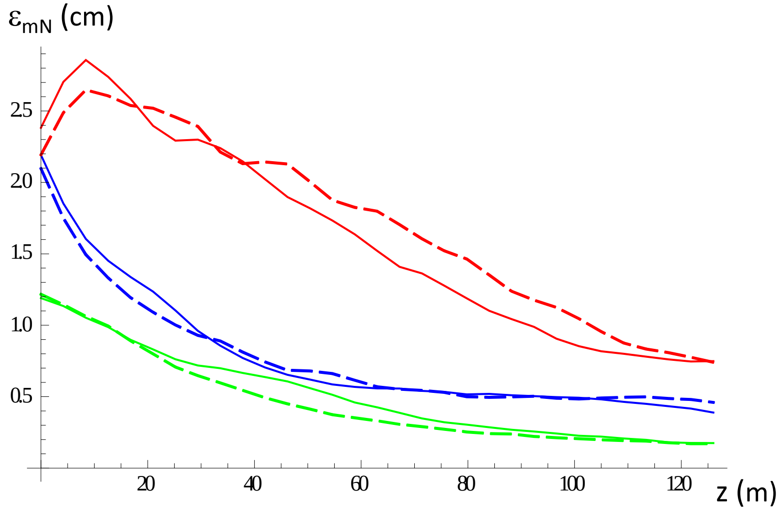


Reference particle trajectory (red – x, blue – y), inclination of solenoids 3-9 was used for placing μ^+ and μ^- on their periodic orbits simultaneously.

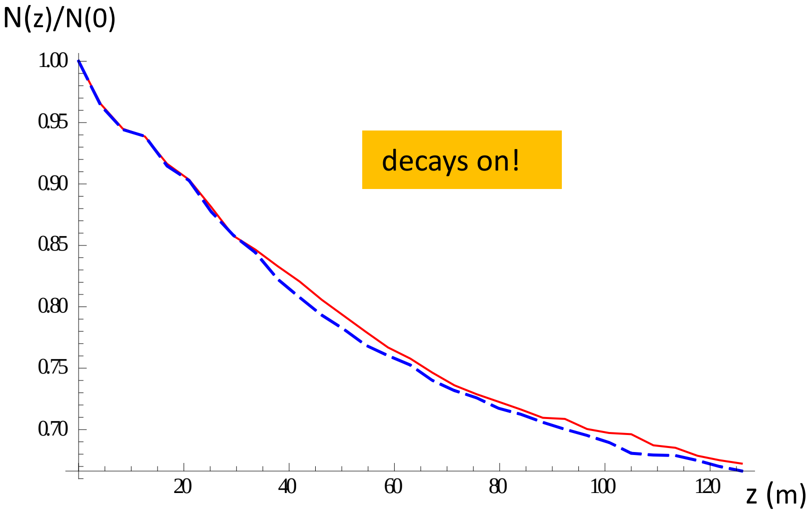


With real beam the RF timing had to be readjusted to compensate for the (residual) effect of momentum-betatron amplitude correlation

Cooling & Transmission (G4BL)



Normalized emittances (cm) from Gaussian fit:
μ^+ - solid lines, μ^- - dashed lines.

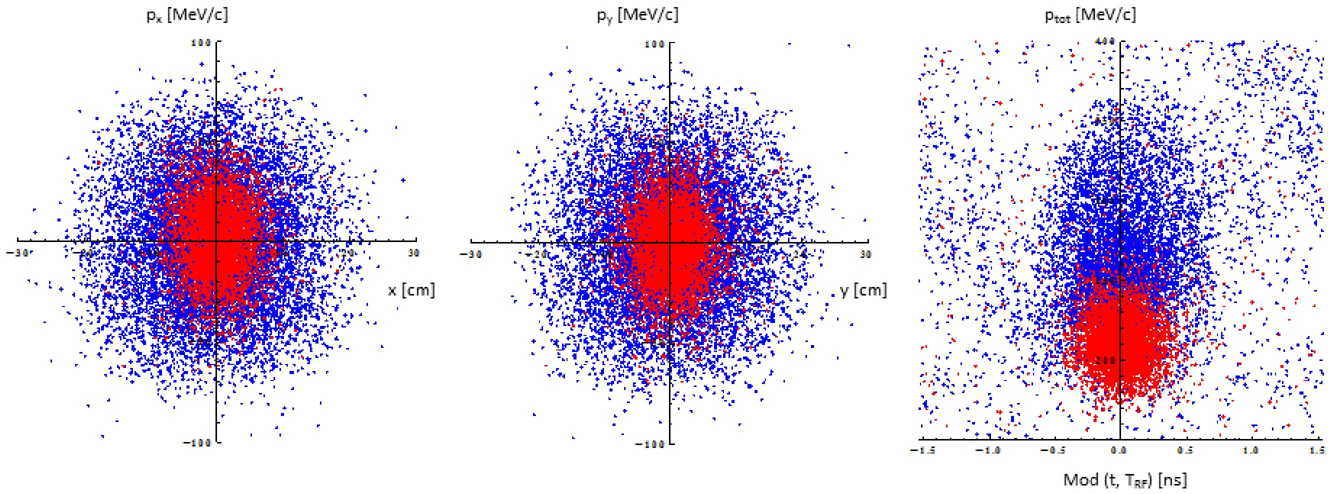


Transmission as a ratio of the number of muons in the Gaussian core: red solid line - μ^+, blue dashed line - μ^-.

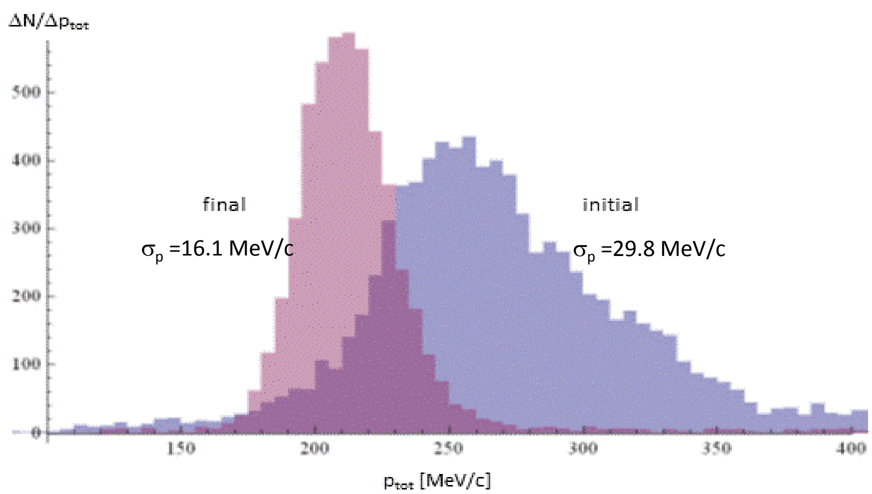
Final/Initial values (Gaussian fit):

	$N^{(total)}$	$N^{(150<p<360)}$	$N^{(core)}$	$p^{(cnt)}$, MeV/c	ε_{mN}, cm			ε_{6D}, cm^3
μ^+	5378/11755	5167/7998	5010/7329	208.2/248.0	0.19/1.19	0.36/2.19	0.76/2.38	0.051/6.22
μ^-	5896/12396	5743/9020	5499/8248	207.7/248.8	0.16/1.22	0.46/2.10	0.72/2.19	0.051/5.59

Initial/Final Phase Space Distributions

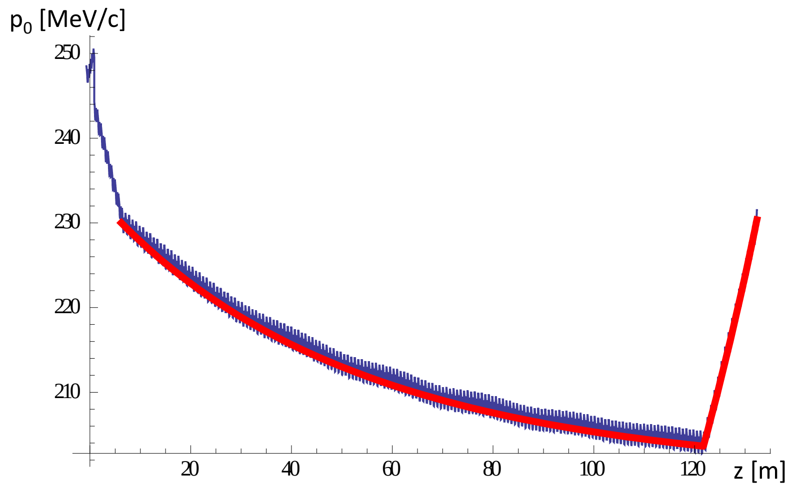


Initial μ+ beam (blue) and cooled beam in the 2T exit solenoid (red). All bunches were projected onto the same RF bucket in the right plot. No cuts applied



Reacceleration to 230 MeV/c

For charge separation and subsequent cooling in HCC a higher momentum can be beneficial.
A possibility of reacceleration after initial cooling in HFOFO was considered since keeping momentum ~ constant all the way increases the losses.

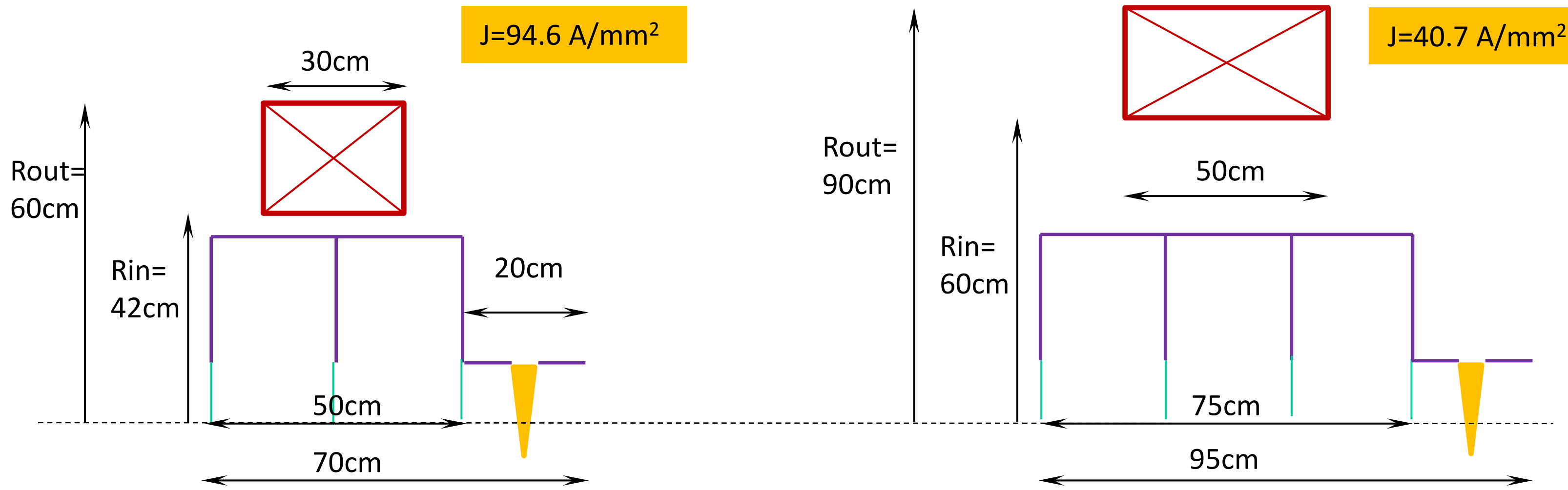


Momentum of reference particle (blue) and target value (red).
Adjustment made by RF timing (gradient fixed).

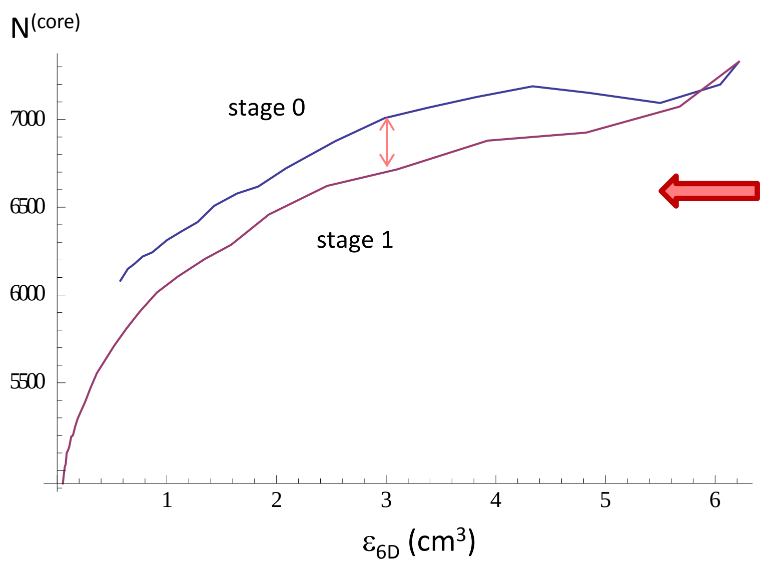
	$N^{(total)}$	$N^{(150<p<360)}$	$N^{(core)}$	$p^{(cnt)}$, MeV/c	σ_p, MeV/c	ε_{6D}, cm^3
μ+ no reacceleration	5378	5167	5010	208.2	16.1	0.051
μ+ with reacceleration	5320	5107	4925	236.5	17.2	0.050

Still there is a noticeable hit on Gaussian core

Section with Higher $\beta_\perp$ (first look)



Composition of considered above stage 1 (left) and new “stage 0” with higher $\beta_\perp$ (right)



Using the present first version of “stage 0” for cooling down to ε_{6D} ~ 3 cm^3 will allow for ~5% increase in the Gaussian core intensity.
Hopefully this number can be further increased by optimization.

The main difficulty with “stage 0” is very high synchrotron tune $Q_{||}$~ 0.3 (now there is 6 × 3 = 18 RF cavities / period)

Summary & Outlook

- The present 1-stage HFOFO design cools both $\mu+$ and $\mu-$ providing
  - √ 6D emittance reduction by two orders of magnitude in ~ 130m
  - √ transmission ~47% if long high-momentum tails are counted and ~68% for Gaussian core
- Optimum transmission requires the muon momentum to be low, muons can be re-accelerated after cooling if needed
- Transmission can be improved by a few more %% by adding a "stage 0" with higher $\beta_\perp$
- A "stage 2" can be appended with shorter RF cavities (say, 15 cm in length)
  $\Rightarrow$ smaller period length $\Rightarrow$ smaller $\beta_\perp$-function
  $\Rightarrow$ 6D emittance reduction by another factor 3 to 5
- GH2-filled Front End with earlier HFOFO snake introduction to increase momentum acceptance?