

Muon Emittance Exchange with a Skew Quad Triplet

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6D Muon Cooling Status

- 6D channels cool muons by almost enough overall.

More than enough longitudinally.

But not enough transversely.

High field YBCO solenoids cut transverse emittance more by mostly exchanging transverse for longitudinal emittance.

But getting all the way to 25 mm-mrad is hard.

And getting below 25 mm-mrad is very hard.

An additional way of exchanging emittance might be useful.

Large Muon Beam Radius in Unflipped Cooling Solenoids

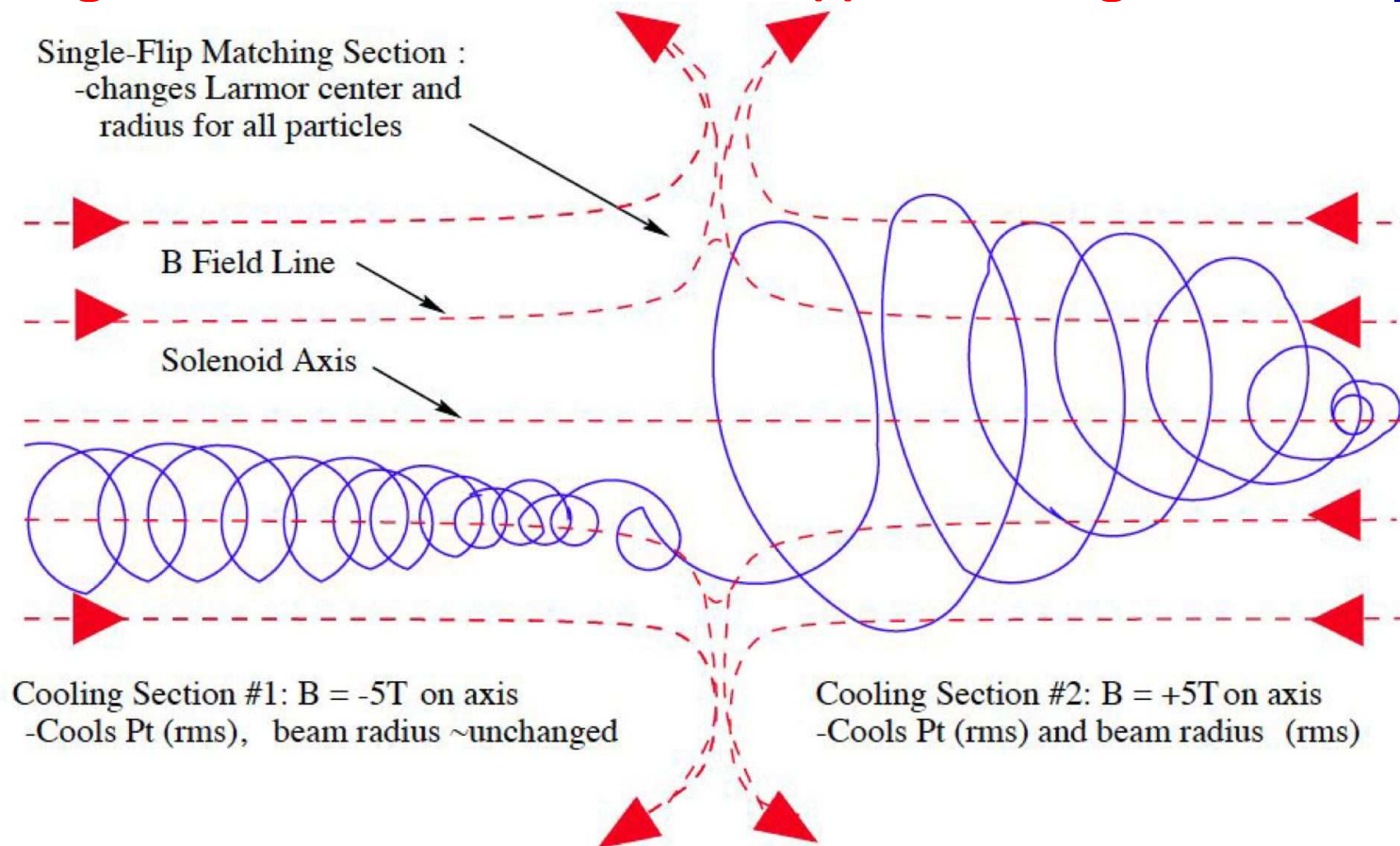
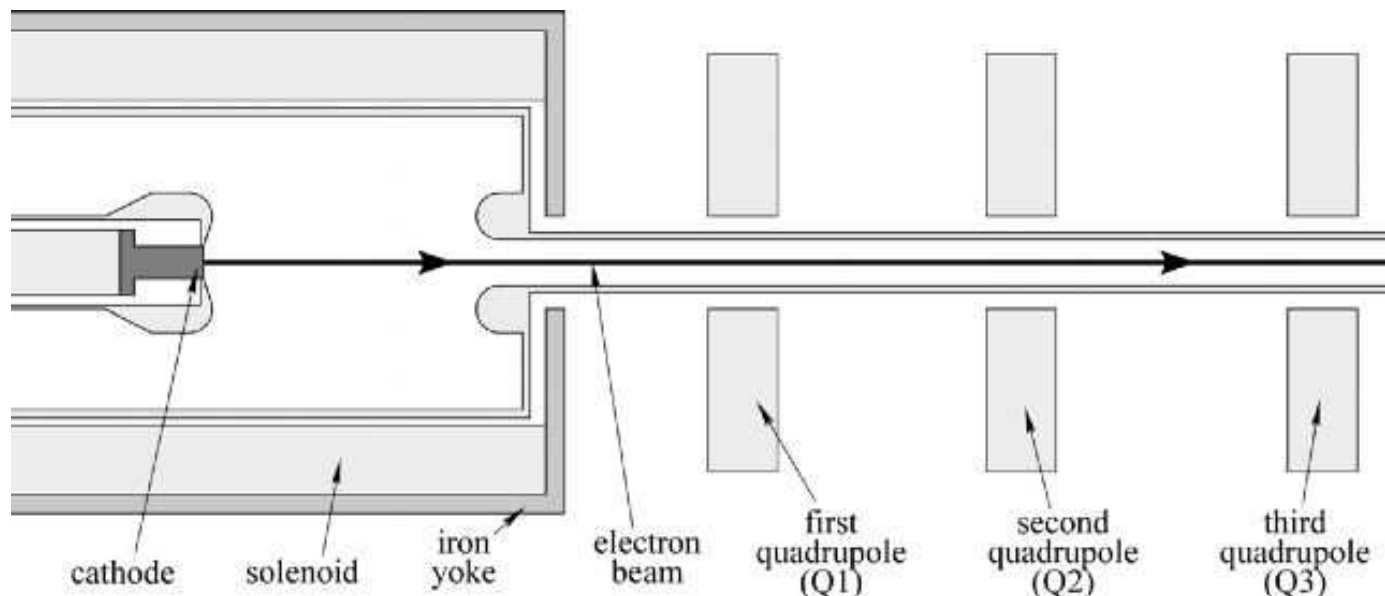


Figure 2: Particle Motion Cartoon in the Single Flip Channel.

- **Balbekov, Lebrun, Monroe, Spentzouris, MuCool 125 (2000)**
Unflipped solenoid only cools p_T .
Muon moves from left to right.
Large radius is almost unchanged if the field is not flipped.

Electron Beam: Round spinning to flat non-spinning

- R. Brinkmann, Ya. Derbenev, and K. Flottmann, Low emittance, flat-beam electron source for linear colliders, Phys.Rev.ST Accel.Beams 4 (2001) 053501;
B.E. Carlsten, Kip Bishofberger, New J.Phys. 8 (2006) 286



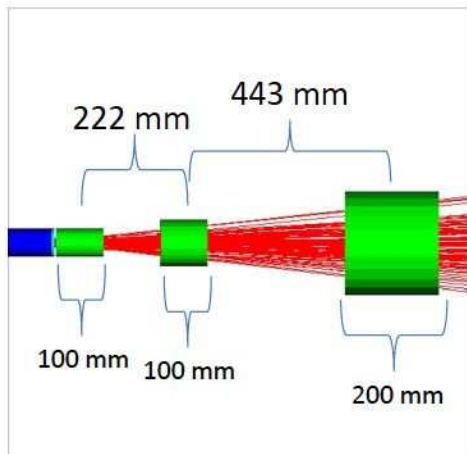
$$\epsilon_{\text{larger}} / \epsilon_{\text{smaller}} \approx (2L / \epsilon_{\text{intrinsic}}, N)^2$$

$$L = e B_{\text{cathode}} R_{\text{cathode}}^2 / 8mc$$

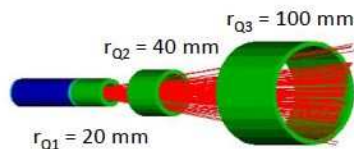
Ratio = 100, if L is 5x normalized intrinsic emittance

G4Beamline μ : Round spinning to 100:1 flat non-spinning

- 25 \rightarrow 2.5, 250 mm-mr + dilution, $B = 5\text{T}$, $\sigma_x = R/2 = 4.2\text{mm}$
 $p = 115\text{ MeV}/c$, Quad pole tip fields = 0.54, 0.12, 0.05T

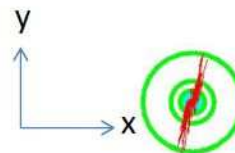


$B_z = 5\text{ T}$ in solenoid
 $B_r = (r/2)(\Delta B_z/\Delta z) = (r/2)(5\text{ T}/5\text{ mm})$
 for 5 mm after solenoid end



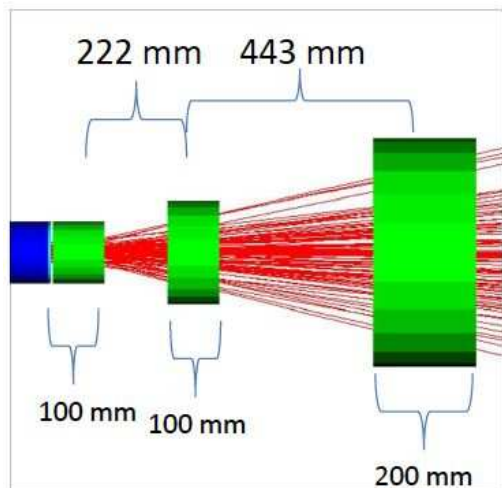
Q1 pole tip field = 0.54 T
 Q2 pole tip field = 0.12 T
 Q3 pole tip field = 0.05 T

$p = 115\text{ MeV}/c$ muon beam
 $\epsilon_{\text{TR},N} = 25\text{ mm-mrad}$ with $\sigma_{x,y} = 4.2\text{ mm}$
 $\epsilon_{\text{mag},N} = 125\text{ mm-mrad}$ when leaving solenoid

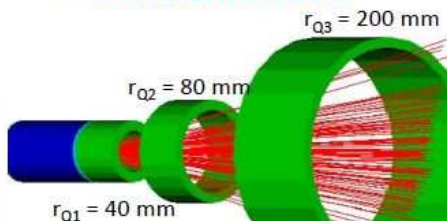


Try for $(\epsilon_{y,N}/\epsilon_{x,N}) = [2(125)/25]^2$
 = 100 after skew-quadrupole triplet

- 100 \rightarrow 10, 1000 mm-mr + dilution, $B = 5\text{T}$, $\sigma_x = R/2 = 8.4\text{mm}$
 $p = 115\text{ MeV}/c$, Quad pole tip fields = 1.08, 0.24, 0.10T

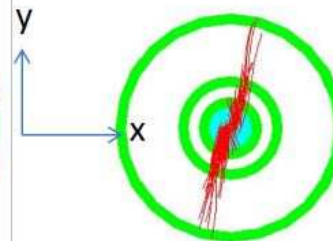


$B_z = 5\text{ T}$ in solenoid
 $B_r = (r/2)(\Delta B_z/\Delta z) = (r/2)(5\text{ T}/5\text{ mm})$
 for 5 mm after solenoid end



Q1 pole tip field = 1.08 T
 Q2 pole tip field = 0.24 T
 Q3 pole tip field = 0.10 T

$p = 115\text{ MeV}/c$ muon beam
 $\epsilon_{\text{TR},N} = 100\text{ mm-mrad}$ with $\sigma_{x,y} = 8.4\text{ mm}$
 $\epsilon_{\text{mag},N} = 500\text{ mm-mrad}$ when leaving solenoid



Try for $(\epsilon_{y,N}/\epsilon_{x,N}) = [2(500)/100]^2$
 = 100 after skew-quadrupole triplet

Septa slice wide beams with little loss

- Don Edwards and Mike Syphers, page 126
“An Introduction to the Physics of High Energy Accelerators”
1/3 integer resonant extraction ring
0.1mm thick electrostatic septa, magnetic septa, Lambertson...
- $\text{Loss} = 4w / [x_{\max} \sqrt{\beta_s / \beta_0} \cos \theta]$
 $\text{Loss} = 4 \times 0.1\text{mm} / [20\text{mm} \sqrt{2.3} \cos(45^\circ)] = 0.02$
- See if the width is OK for muons
 $\text{Width} = \sqrt{\epsilon_{x,N} \beta_x / (\beta \gamma)} = \sqrt{(100\mu\text{m})(10\text{m})/2} = 22 \text{ mm}$
Yes, it is OK.
- Emittance is divided by the number of non-Hamiltonian slices
Half integer resonance extracts in every other ring orbit
A ring only needs one septa, but at the price of muon decay.
A linear channel needs many septa, but fewer muons decay.
- Slices must be recombined longitudinally.

XYZ Emittance Exchange Summary

- Muons need to be at a large enough radius.
Round to 100:1 flat muon beam with a skew quad triplet
Slice wide beam direction with low loss electrostatic septa...
Recombine transversely sliced bunches longitudinally.
- If $\Delta E/E = \Delta p/p = 10^{-3}$ @ 750 GeV and $\Delta z = 10$ mm,
 $\epsilon_{L,N} = (\Delta p/p) \Delta z \beta \gamma = 10^{-3} 10 \text{ mm } 7500 = 75 \text{ mm-rad.}$
Collider collision chromaticity sets longitudinal emittance.