Can concepts from Integrable Optics be applied to the nuSTORM decay ring design?

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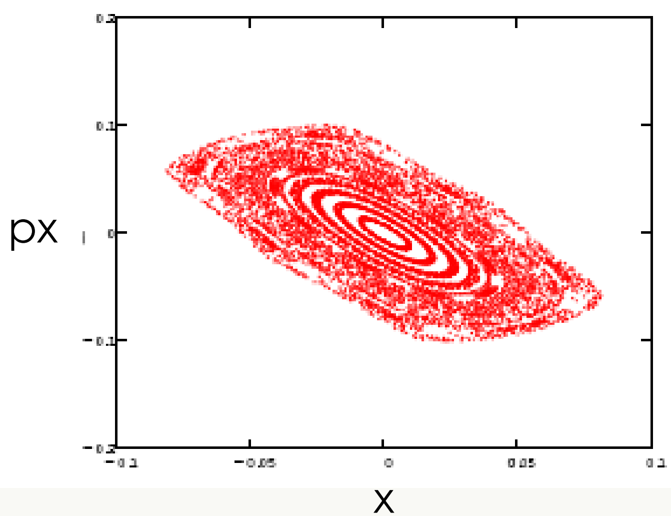
Fermilab

Outline

- Concept of Nonlinear Integrable Optics
  - Nonlinear lattice and potential
  - Maximum nonlinear tune shift
  - Numerical simulations
- IOTA Goals and Scope
- Discussion

Motivation for IO

- The main feature of all present accelerators – linear focusing lattice: particles have nearly identical betatron frequencies (tunes) by design.
  - Hamiltonian has explicit time dependence
  - All nonlinearities (both magnet imperfections and specially introduced) are perturbations and make single particle motion unstable due to resonant conditions



Typical phase space portrait (single octupole lens):
1. Regular orbits at small amplitudes
2. Resonant islands + chaos at larger amplitudes

Motivation

COLLIDING BEAMS: PRESENT STATUS; AND THE SLAC PROJECT*

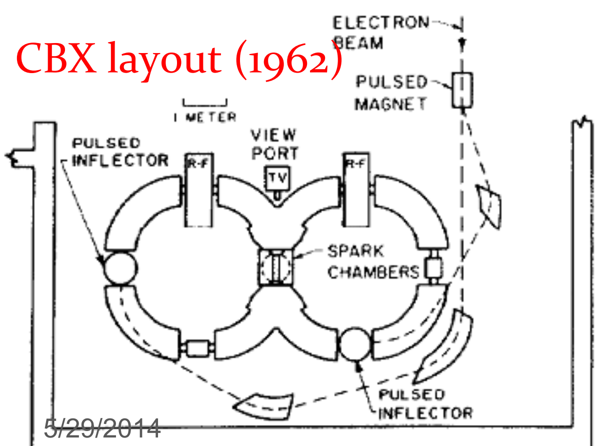
B. Richter

Stanford Linear Accelerator Center
Stanford University, Stanford, California 94305

Report at HEAC 1971

The discovery in the early '60's at the Princeton-Stanford ring of what was thought to be the resistive wall instability brought the realization that circular accelerators are fundamentally unstable devices because of the interaction of the beam with its environment. Stability is achieved only through Landau damping and/or some external damping system.

CBX layout (1962)



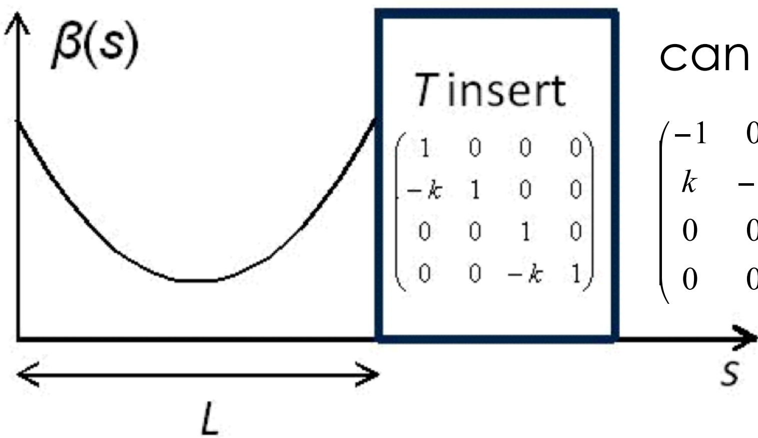
- 1965, Priceton-Stanford CBX: First mention of an 8-pole magnet
  - Observed vertical resistive wall instability
  - With octupoles, increased beam current from ~5 to 500 mA
- CERN PS: In 1959 had 10 octupoles; not used until 1968
  - At 10^{12} protons/pulse observed (1st time) head-tail instability. Octupoles helped.
  - Once understood, chromaticity jump at transition was developed using sextupoles.
  - More instabilities were discovered; helped by octupoles, fb

Do Accelerators Need to be Linear?

- Search for a lattice design that is strongly nonlinear yet stable
  - Orlov (1963)
  - McMillan (1967)
  - Perevedentsev, Danilov (1990)
  - Chow, Cary (1994)
- Nonlinear Integrable Optics: Danilov and Nagaitsev proposed a solution for nonlinear lattice with 2 invariants of motion that can be implemented with special magnets
  - Phys. Rev. ST Accel. Beams 13, 084002 (2010)

Integrable Optics: Time-independence 1st Integral of Motion

- Start with a round axially-symmetric *linear* lattice (FOFO) with the element of periodicity
  - Phase advance $0<\nu_0<0.5$ (2π) in drift *L*
  - n×0.5 in T insert
  - ν_0+0.5 total



$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ -k & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -k & 1 \end{pmatrix}$$

can be also

$$\begin{pmatrix} -1 & 0 & 0 & 0 \\ k & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & k & -1 \end{pmatrix}$$

- Add special nonlinear potential *V(x,y,s)* in the drift such that
  1. It satisfies the Laplace equation $\Delta V(x,y,s) \approx \Delta V(x,y) = 0$
  2. The Hamiltonian is time-independent in normalized variables

$$H = \frac{p_x^2}{2} + \frac{p_y^2}{2} + K(s)\cdot\left(\frac{x^2}{2} + \frac{y^2}{2}\right) + V(x,y,s) \qquad H_N = \frac{p_{xN}^2 + p_{yN}^2}{2} + \frac{x_N^2 + y_N^2}{2} + U(x_N, y_N, \psi)$$

$$U(x_N, y_N, \psi) = \beta(\psi) V\left(x_N\sqrt{\beta(\psi)}, y_N\sqrt{\beta(\psi)}, s(\psi)\right)$$

H is invariant of motion

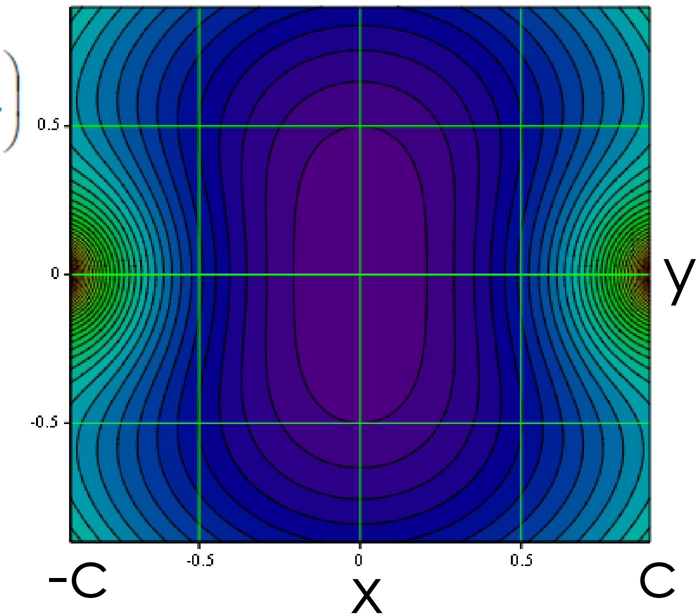
Integrable Optics: Special Potential 2nd Integral of Motion

- Find potentials that result in the Hamiltonian having a second integral of motion. In elliptic variables ξ, η

$$U(x,y)=\frac{x^2}{2}+\frac{y^2}{2}+\frac{f_2(\xi)+g_2(\eta)}{\xi^2-\eta^2} \quad f_2(\xi)=\xi\sqrt{\xi^2-1}\left(d+t\,\mathrm{acosh}(\xi)\right) \quad g_2(\eta)=\eta\sqrt{1-\eta^2}\left(q+t\,\mathrm{acos}(\eta)\right)$$

- Multipole expansion of U for d=0, q=t $\pi/2$ is

$$U(x,y)\approx\frac{x^2}{2}+\frac{y^2}{2}$$
$$+t\,\mathrm{Re}\left((x+iy)^2+\frac{2}{3}(x+iy)^4+\frac{8}{15}(x+iy)^6+\frac{16}{35}(x+iy)^8+\ldots\right)$$



- Theoretical maximum nonlinear tune shift per cell is
  - 0.5 for mode 1, **or 50%**
  - 0.25 for mode 2, or 25%

Stability to Perturbations

- Are nonlinear integrable systems more stable to perturbations than linear?
- The first paper on the subject was written by Nikolay Nekhoroshev in 1971

Russian Math. Surveys 32:6 (1977), 1–65
From Uspekhi Mat. Nauk 32:6 (1977), 5–66

AN EXPONENTIAL ESTIMATE OF THE TIME OF STABILITY OF NEARLY-INTEGRABLE HAMILTONIAN SYSTEMS

N. N. Nekhoroshev

1.1 Nearly-integrable Hamiltonian systems. Perpetual stability and stability during finite intervals of time. In this article we investigate the behaviour of the variables I in the Hamiltonian system of canonical equations

$$\dot{I} = -\frac{\partial H}{\partial \varphi}, \quad \dot{\varphi} = \frac{\partial H}{\partial I}$$

with the Hamiltonian

(1.1) $$H = H_0(I) + \varepsilon H_1(I, \varphi),$$

where $\varepsilon \ll 1$ is a small parameter, the perturbation $\varepsilon H_1(I, \varphi)$ is 2π-periodic in $\varphi = \varphi_1, \ldots, \varphi_s$, and I is an s-dimensional vector, $I = I_1, \ldots, I_s$.

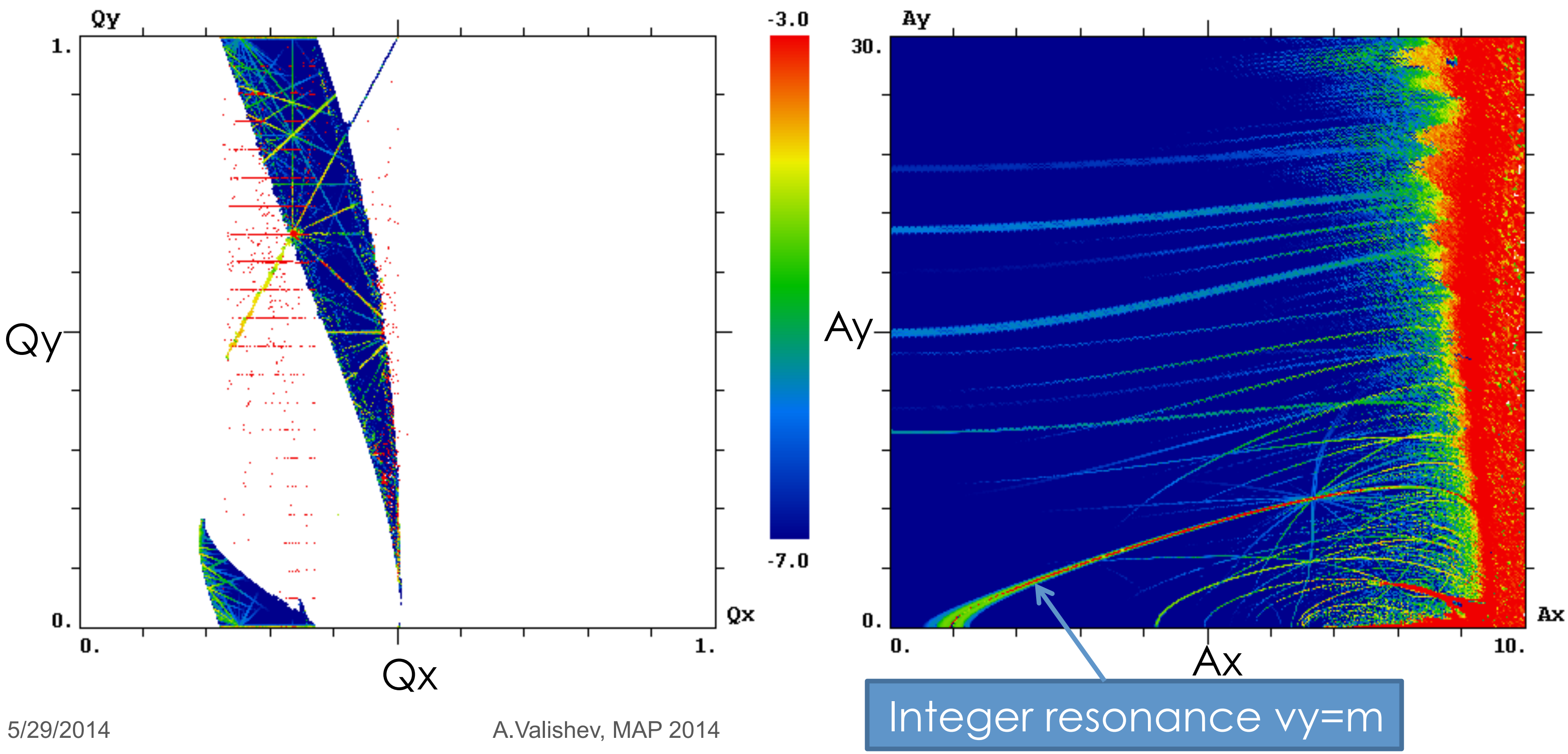
$$\|I(t) - I(0)\| \leq R_* \epsilon^b \quad \text{for} \quad |t| \leq T_* \exp\left(\epsilon^{-a}\right)$$

- He proved that for sufficiently small ε provided that $H_0(I)$ meets certain conditions know as steepness
  - Convex and quasi-convex functions $H_0(I)$ are the steepest
- An example of a NON-STEEP function is a linear function

$$H_0(J_1, J_2) = \omega_1 J_1 + \omega_2 J_2$$

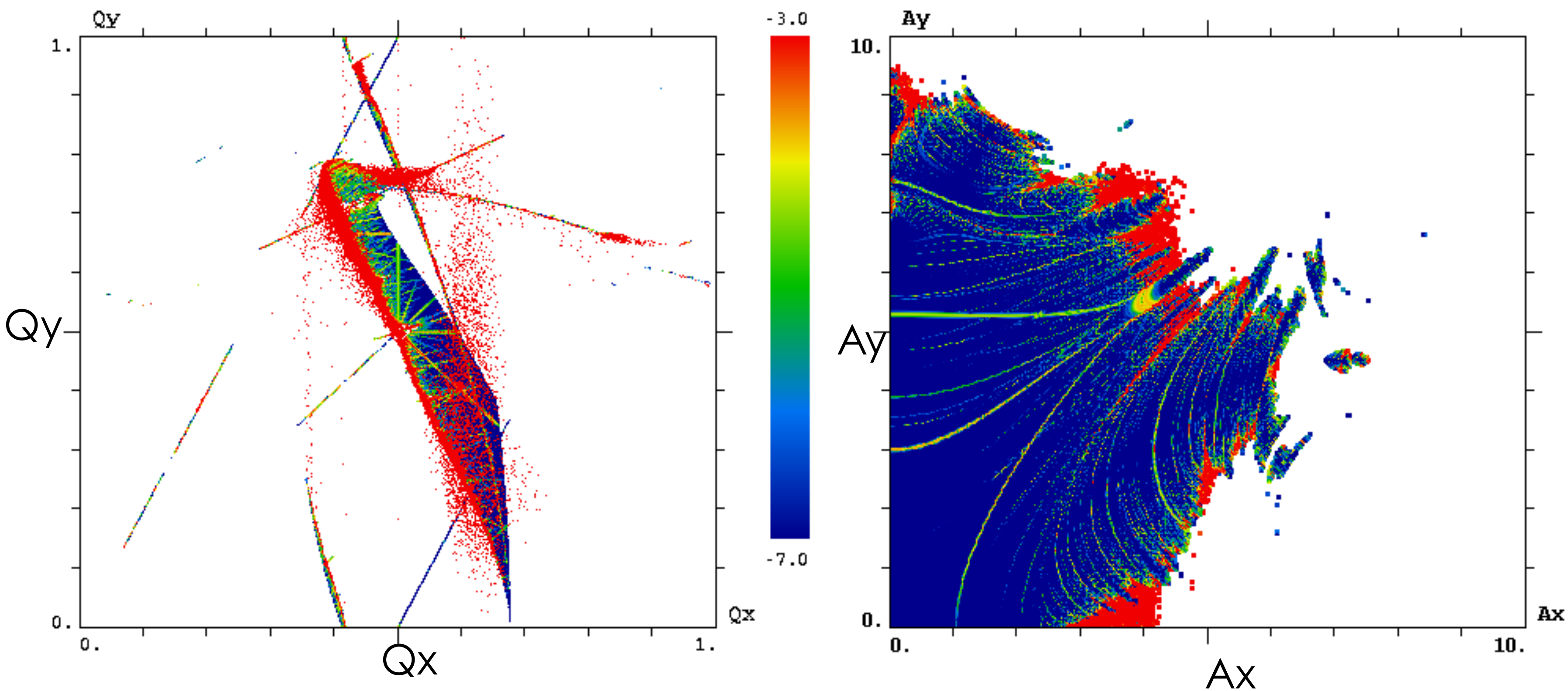
FMA of Integrable Optics

- ν_0=0.3, *t*=0.15, 40 thin elements per drift, 4 elements of periodicity



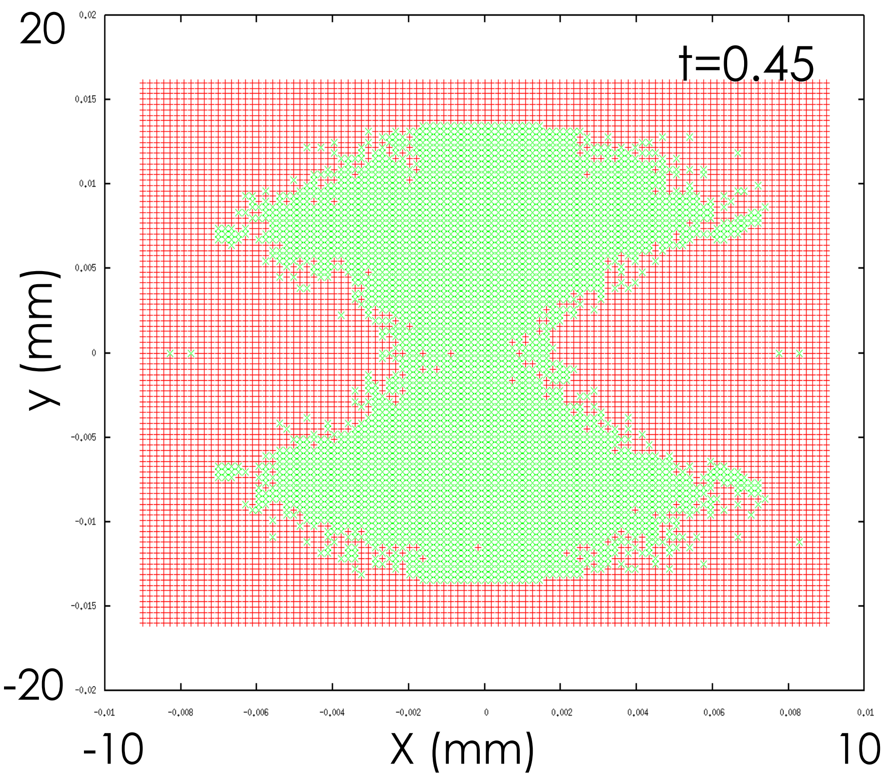
Integrable Optics vs. Conventional Multipoles

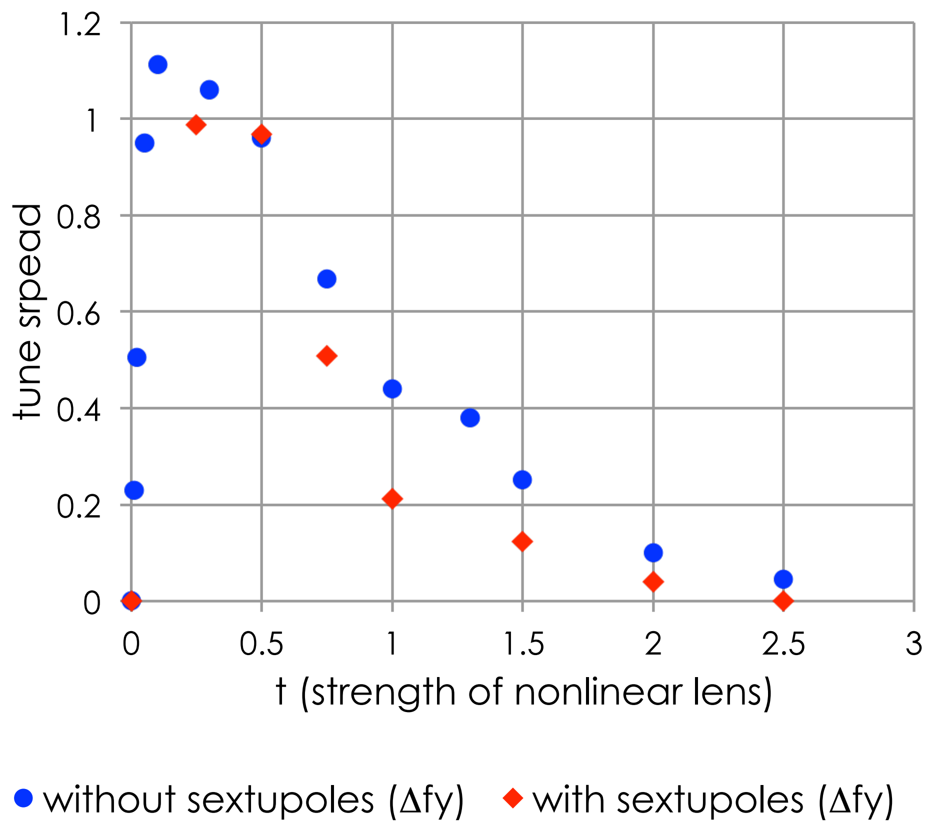
- Nonlinear potential approximated by multipole expansion up to 9th order



IO in a lattice with Sextupoles

- Track a bunch of macro-particles in integrable lattice with aperture limitations + chromaticity sextupoles





Integrable Optics Test Accelerator
Motivation

- An experimental test of the idea would benefit the worldwide accelerator physics community as well as the field of nonlinear dynamics in general
- Fermilab has a unique opportunity to take lead in this research as ASTA provides means to perform the experiment quickly and at low cost
- The ring can be used to perform other advanced accelerator R&D

IOTA Goals

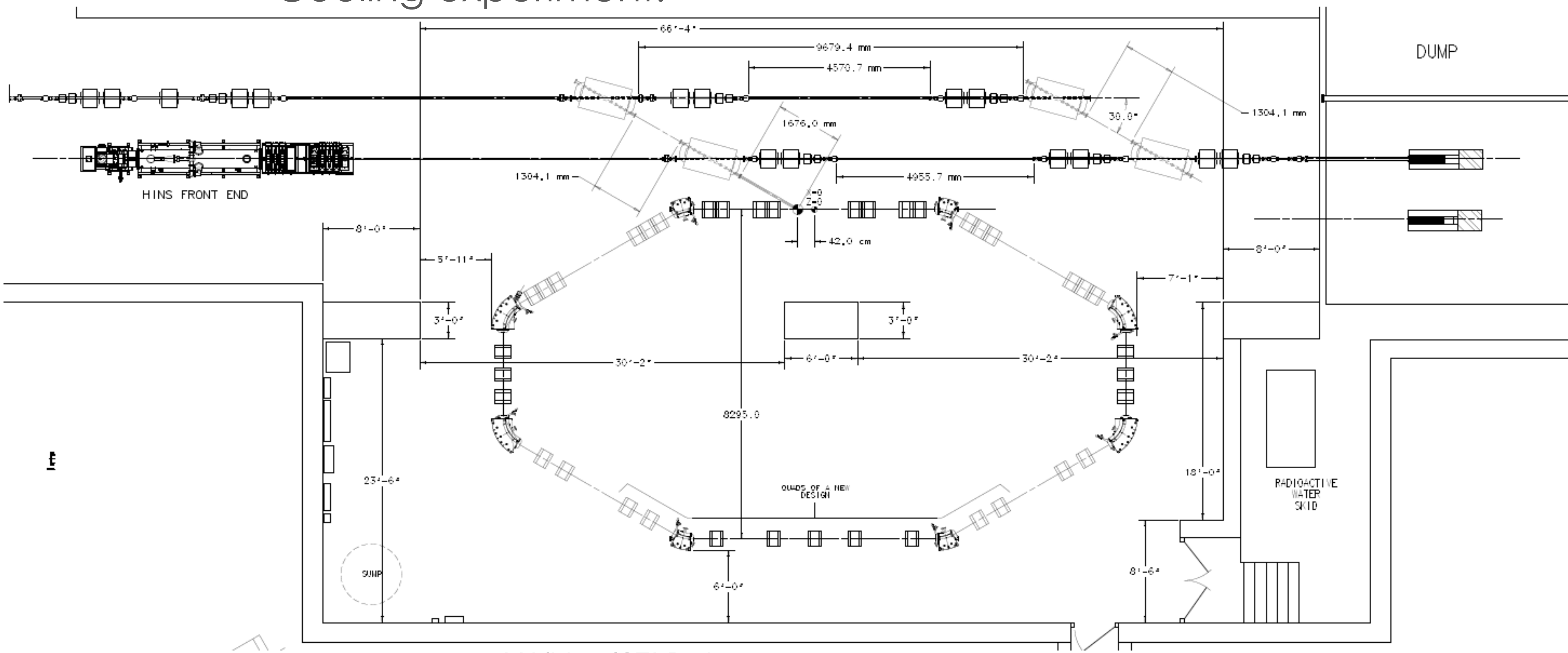
- We are constructing the Integrable Optics Test Accelerator ring, which would use the beam from e^- SRF linac with the ***goal to demonstrate the possibility to implement nonlinear integrable optics in a realistic accelerator design***
  - Only concentrate on the academic aspect of single-particle motion stability, leaving the studies of collective effects and attainment of high beam current to future research
  - Achieve large nonlinear tune shift/spread without degradation of dynamic aperture by "painting" the accelerator aperture with a "pencil" beam
  - Suppress strong lattice resonances = cross the integer resonance by part of the beam without intensity loss
  - Investigate stability of nonlinear system to perturbations
- The measure of success will be achievement of high nonlinear tune shift = 0.25

IOTA Goals 2

- After the proof-of-principle demonstration, further work will be directed towards
  - Achievement of large tune spread within a circulating beam
  - Achievement of space charge suppression in a nonlinear accelerator lattice
- We collaborate with ORNL, BINP, John Adams Institute (Oxford), RadiaSoft (SBIR phase I), Radiabeam (SBIR phase II), both on the current design and further development
- In addition to the primary goal, the ring can accommodate other Advanced Accelerator R&D experiments and/or users
  - Only portion of circumference is occupied with nonlinear magnets
  - Current design accommodates Optical Stochastic Cooling

IOTA Layout

- 4 - 2m drifts for nonlinear magnets, Electron Lens
- T-insert tunable to allow a wide range of phase advances and beta-functions in the drift space in order to study different betatron tune working points
- One 5m-long straight section for the Optical Stochastic Cooling experiment.



Discussion

- Can concepts from Integrable Optics be applied to the nuSTORM decay ring design?