Composite Higgs from Top Condensation

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HC, B. Dobrescu, and J. Gu, arXiv:1311.5928 HC and J. Gu, arXiv:1406.6689

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Introduction

- After the discovery of the Higgs boson, one of the most important jobs is to determine its properties.
- Because of the hierarchy problem, new physics is expected near the weak scale, which can modify the Higgs sector from the Standard Model.
- A crucial question is whether the Higgs boson is elementary or composite.

Composite Higgs

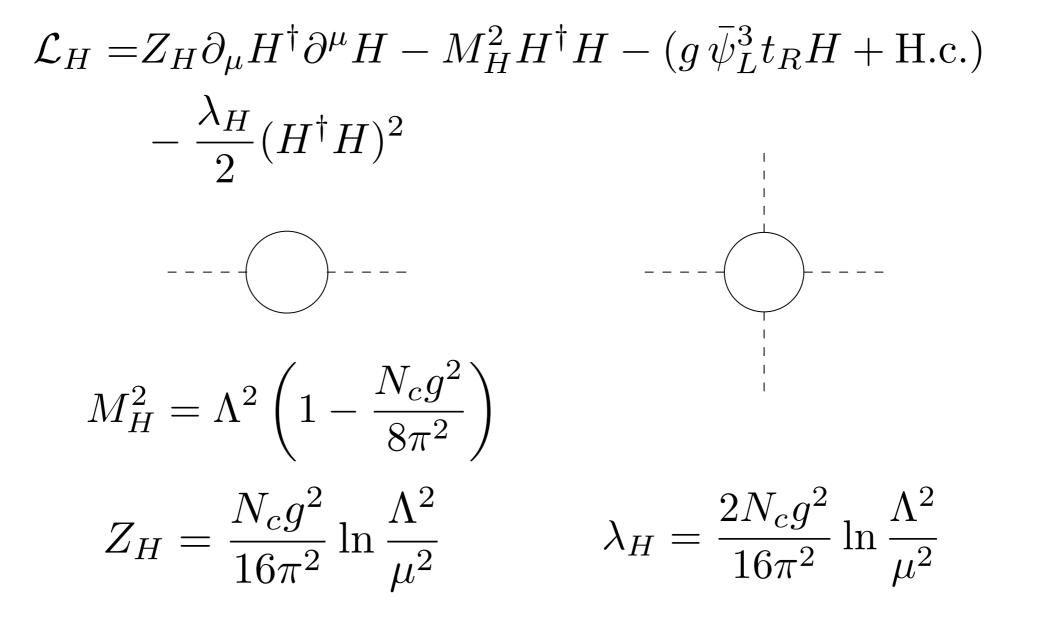
- A composite Higgs boson is generically expected to be heavy. To make it light, its mass should be protected by some symmetry, i.e., Higgs as a pseudo-Nambu-Goldstone boson (pNGB) (Kaplan & Georgi '84).
- The large top quark mass is a challenge for composite Higgs models. The top quark should (at least partially) participate in the strong dynamics which forms the composite Higgs.
 - An example is top condensation (Nambu '89, Miransky et al '89): Higgs is a bound state of $\bar{t}t$.

Top Condensation

$$\frac{g^2}{\Lambda^2} (\bar{t}_R \psi_L^3) (\bar{\psi}_L^3 t_R), \qquad \psi_L^3 = (t, b)_L$$

- The 4-fermion interaction may arise from integrating out new physics at high scale, e.g., topcolor (Hill, '91), SU(3)₁×SU(3)₂→SU(3)_C.
- Similar to the Nambu-Jona-Lasinio model (1961), for $g \gg 1$, it can form $\overline{t}_R \psi_L^3$ bound state, which has the same quantum number as the Higgs field.
- The 4-fermion interaction is not confining.

Top Condensation



• For $g > g_{crit} = \pi \sqrt{8/N_c} \Rightarrow$ bound state gets a VEV, breaking the chiral (EW) symmetry.

Top Condensation

• Top Yukawa coupling: $\xi \simeq \sqrt{\frac{8\pi^2}{N_c \ln(\Lambda/\mu)}}$

For Λ/μ not too large, $\xi \sim 3-4 \Rightarrow m_t \sim 600 \text{ GeV}$

• Higgs quartic coupling: in leading Nc (fermion bubble) approximation, $\lambda = 2\xi^2 \Rightarrow M_h = 2m_t$.

- M_h , m_t may be reduced by raising the compositeness scale at the expenses of fine tuning, but still too heavy. (Bardeen, Hill, Linder '90)

Top Seesaw Model

 An attractive solution to the top mass problem is to invoke the seesaw mechanism (Dobrescu & Hill '98): introducing vector-like singlet quarks χ_L, χ_R to mix with top quark.

$$\mathcal{L} = -(\bar{t}_L \ \bar{\chi}_L) \begin{pmatrix} 0 & \frac{\xi}{\sqrt{2}}v \\ m_{\chi t} & m_{\chi\chi} \end{pmatrix} \begin{pmatrix} t_R \\ \chi_R \end{pmatrix} + \text{H.c.},$$
$$v = 246 \text{ GeV}.$$

A light eigenstate ~ 173 GeV can be obtained which is identified as the top quark.

Top Seesaw with a Light Higgs

- What about the Higgs boson mass? If it's still heavy as one may naively expect, then it is ruled out by the discovery of a relatively light Higgs.
- A light Higgs boson arises naturally if the underlying strong dynamics preserves a U(3) symmetry among (t_L, b_L, χ_L).

Higgs field belongs to NGBs of U(3) \rightarrow U(2)

Scalar Potential

• Assuming the underlying (non-confining) strong dynamics is approximately U(3)_L X U(2)_R symmetric for (t_L , b_L , χ_L) and (t_R , χ_R), they form composite scalars, $\Phi = (\Phi_t \ \Phi_{\chi})$

$$\Phi_t = \begin{pmatrix} H_t \\ \phi_t \end{pmatrix} \sim \bar{t}_R \begin{pmatrix} \psi_L^3 \\ \chi_L \end{pmatrix}, \quad \Phi_\chi = \begin{pmatrix} H_\chi \\ \phi_\chi \end{pmatrix} \sim \bar{\chi}_R \begin{pmatrix} \psi_L^3 \\ \chi_L \end{pmatrix}.$$

Yukawa int:
$$\mathcal{L}_{\text{Yukawa}} = -\xi \left(\bar{\psi}_L^3, \bar{\chi}_L \right) \Phi \begin{pmatrix} t_R \\ \chi_R \end{pmatrix} + \text{H.c.}$$

Scalar potential:

$$V_{\Phi} = \frac{\lambda_1}{2} \operatorname{Tr}[(\Phi^{\dagger}\Phi)^2] + \frac{\lambda_2}{2} (\operatorname{Tr}[\Phi^{\dagger}\Phi])^2 + M_{\Phi}^2 \Phi^{\dagger}\Phi$$

Symmetry Breaking

- We assume that there are large U(2)_R breaking effects: $V_{U(2)_R} = \delta M_{tt}^2 \Phi_t^{\dagger} \Phi_t + \delta M_{\chi\chi}^2 \Phi_{\chi}^{\dagger} \Phi_{\chi} + (M_{\chi t}^2 \Phi_{\chi}^{\dagger} \Phi_t + \text{H.c.})$ which split M_{tt}^2 and $M_{\chi\chi}^2$ such that $M_{\chi\chi}^2 < 0 < M_{tt}^2$ Nonzero $\langle \Phi_{\chi} \rangle$ is induced and U(3)_L is spontaneously broken.
- The U(3)_L is also explicitly broken by the quark mass terms: L_{mass} = -μ_{χt} χ̄_Lt_R μ_{χχ} χ̄_Lχ_R + H.c. They map into scalar tadpole terms in low energy EFT, V_{tadpole} = -(0,0, C_{χt})Φ_t - (0,0, C_{χχ})Φ_χ + H.c. C_{χt} ≈ μ_{χt}Λ²/ξ, C_{χχ} ≈ μ_{χχ}Λ²/ξ.

We can use U(2) rotation to set $C_{\chi\chi} = 0$.

Minimizing the Scalar Potential

• The tadpole $C_{\chi t}$ induces a nonzero $\langle \phi_t \rangle$, $\langle H_{\chi} \rangle$ is then induced by $M_{\chi t}^2$, $M_{\chi \chi}^2$, breaking EW sym.

Minimizing the potential, we obtain:

$$\begin{array}{ll} \langle H_t \rangle = 0, & \langle \phi_t \rangle = u_t \equiv u \sin \gamma = u s_{\gamma}, \\ \langle H_{\chi} \rangle = v, & \langle \phi_{\chi} \rangle = u_{\chi} \equiv u \cos \gamma = u c_{\gamma}, \\ f \equiv \sqrt{u^2 + v^2} \ (\approx u \gg v), \\ M_{H^{\pm}}^2 = M_{tt}^2 + \frac{\lambda_1}{2} u^2 s_{\gamma}^2 + \frac{\lambda_2}{2} \ (u^2 + v^2) \\ \sqrt{2} C_{\chi t} = u \, s_{\gamma} \, M_{H^{\pm}}^2 & M_{\chi t}^2 = -\frac{\lambda_1}{2} u^2 s_{\gamma} c_{\gamma}, \\ C_{\chi \chi} = 0 & M_{\chi \chi}^2 = -\frac{\lambda_1}{2} \left(u^2 c_{\gamma}^2 + v^2 \right) - \frac{\lambda_2}{2} \left(u^2 + v^2 \right) \end{array}$$

Top Quark Mass

• Charge-2/3 fermion mass matrix:

$$-\frac{\xi}{\sqrt{2}}(t_L,\chi_L)\begin{pmatrix} 0 & v\\ us_\gamma & uc_\gamma \end{pmatrix}\begin{pmatrix} t_R\\ \chi_R \end{pmatrix} + \text{H.c.}$$

Light eigenvalue: $m_t \approx \frac{\xi}{\sqrt{2}} v \, s_\gamma \Rightarrow s_\gamma \approx \frac{y_t}{\xi} \approx \frac{1}{4} \sim \frac{1}{5}.$

Heavy *t*' fermion:
$$m_{t'} \approx \frac{\xi}{\sqrt{2}} u ~(\sim 2.5 f)$$

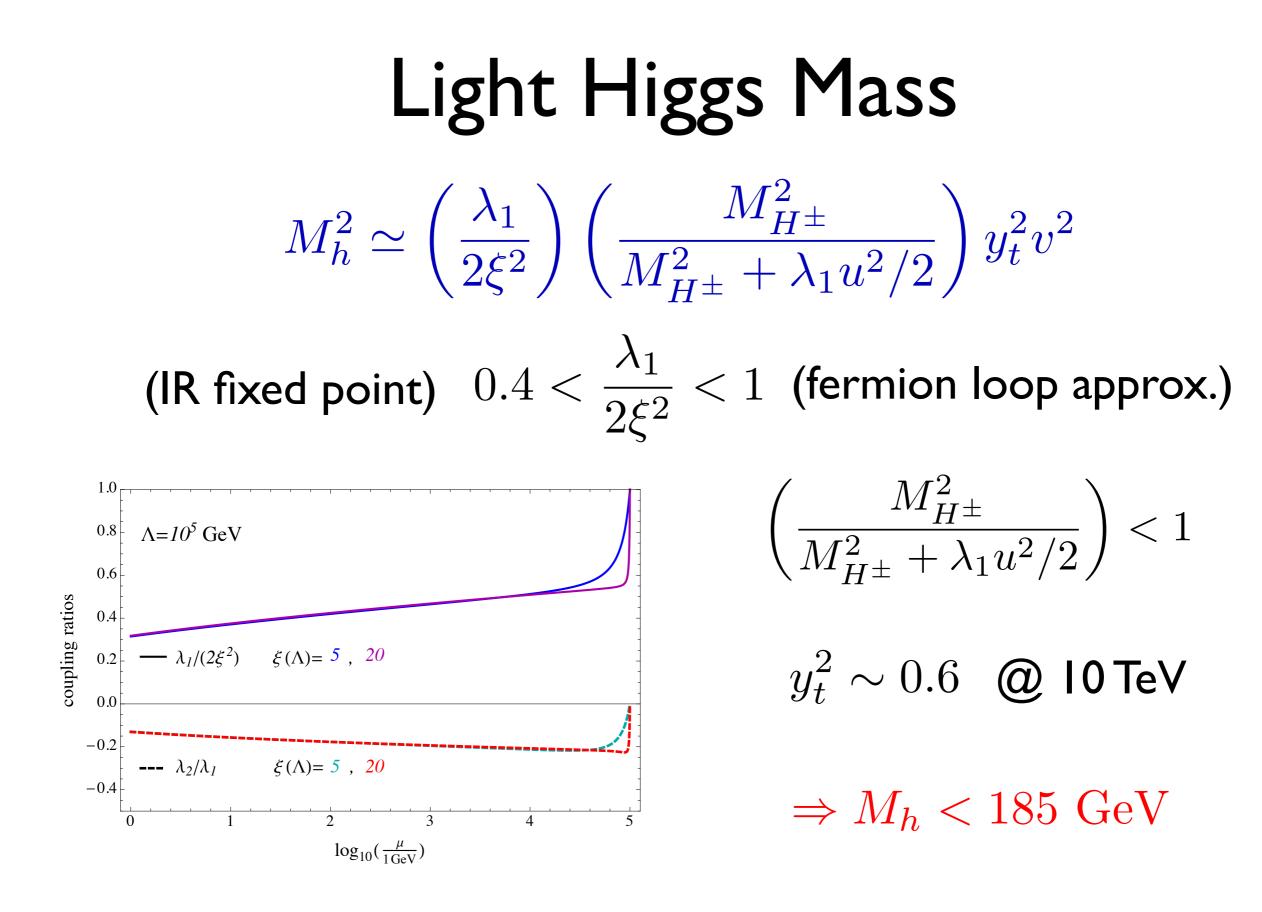
Light Higgs Mass

• CP-even scalar mass matrix: $(h_t, h_{\chi}, \phi_t, \phi_{\chi})$

$$\begin{pmatrix} M_{H^{\pm}}^{2} + \frac{\lambda_{1}}{2}v^{2} & 0 & -\frac{\lambda_{1}}{2}uvc_{\gamma} & -\frac{\lambda_{1}}{2}uvs_{\gamma} \\ 0 & (\lambda_{1} + \lambda_{2})v^{2} & \lambda_{2}uvs_{\gamma} & (\lambda_{1} + \lambda_{2})uvc_{\gamma} \\ -\frac{\lambda_{1}}{2}uvc_{\gamma} & \lambda_{2}uvs_{\gamma} & M_{H^{\pm}}^{2} + \left[\lambda_{1}\left(1 - \frac{c_{\gamma}^{2}}{2}\right) + \lambda_{2}s_{\gamma}^{2}\right]u^{2} & \left(\frac{\lambda_{1}}{2} + \lambda_{2}\right)u^{2}s_{\gamma}c_{\gamma} \\ -\frac{\lambda_{1}}{2}uvs_{\gamma} & (\lambda_{1} + \lambda_{2})uvc_{\gamma} & \left(\frac{\lambda_{1}}{2} + \lambda_{2}\right)u^{2}s_{\gamma}c_{\gamma} & \left[\lambda_{1}\left(1 - \frac{s_{\gamma}^{2}}{2}\right) + \lambda_{2}c_{\gamma}^{2}\right]u^{2} \end{pmatrix}$$

Lightest eigenvalue: $M_h^2 \simeq \left(\frac{\lambda_1}{2\xi^2}\right) \left(\frac{M_{H^{\pm}}^2}{M_{H^{\pm}}^2 + \lambda_1 u^2/2}\right) y_t^2 v^2$

In the limit $\xi \to \infty$ or $m_t \to 0$, $\sin \gamma \to 0$ and $C_{\chi t} \to 0$, there is no explicit U(3) breaking, Higgs becomes an exact NGB.



Electroweak Interactions

• Explicit U(3) breaking electroweak interaction can further decreases the Higgs boson mass.

$$\Delta m_{h\,(\text{mass})}^{2} = \frac{9g_{2}^{2} + 3g_{1}^{2}}{64\pi^{2}} \frac{M_{\rho}^{2}}{u^{2}} v^{2} \approx -0.16v^{2} \frac{M_{\rho}^{2}}{(5u)^{2}} \quad \text{(mass splitting)}$$

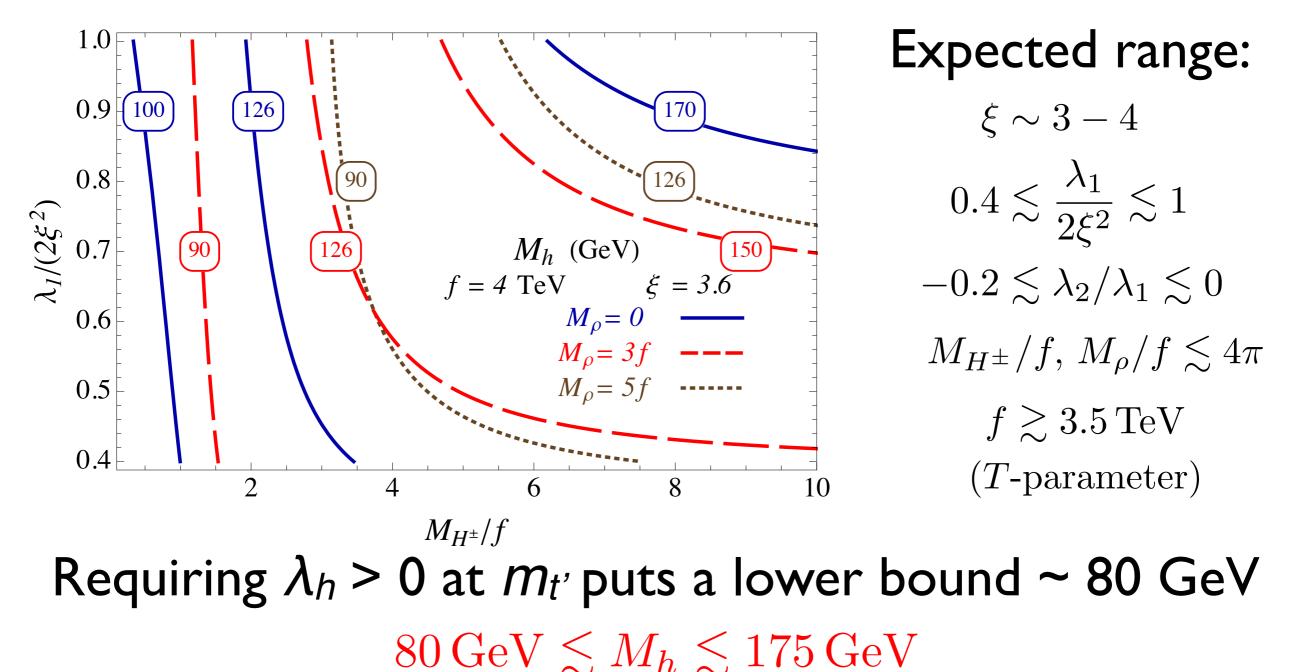
$$\Delta m_{h\,(\text{quartic})}^{2} = -\frac{9g_{2}^{2} + 3g_{1}^{2}}{64\pi^{2}} \lambda_{1} v^{2} \ln \frac{M_{\rho}}{\mu} \approx -0.16v^{2} \left(\frac{\lambda_{1}}{2\xi^{2}}\right) \left(\frac{\xi}{3.6}\right)^{2} \ln \frac{M_{\rho}}{\mu}$$
(quartic splitting)

where M_{ρ} is the cutoff the EW gauge loop.

• $M_h = 125$ GeV corresponds to $\lambda_h = 0.14$ @ 10 TeV.

Numerical Results

• Higgs mass depends on $\lambda_1/(2\xi^2)$, $M_{H^{\pm}}/f$, M_{ρ}/f , but is insensitive to ξ, λ_2, f .



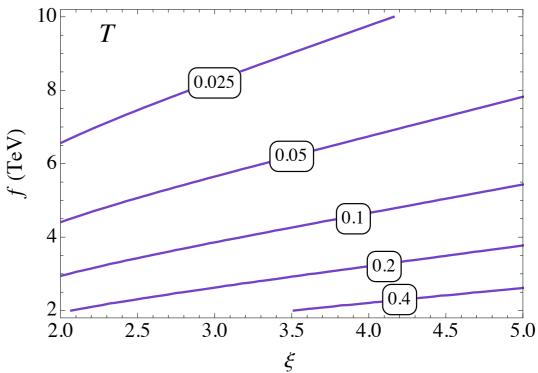
Weak-isospin violation

• The main constraint comes from weak isospin violating T parameter from the t' loop contribution. $[U(3)_{L}$ doesn't contain a custodial SU(2) symmetry.]

$$T = \frac{3s_L^2}{16\pi^2 \alpha v^2} \left[s_L^2 m_{t'}^2 + 4(1 - s_L^2) \frac{m_{t'}^2 m_t^2}{m_{t'}^2 - m_t^2} \ln\left(\frac{m_{t'}}{m_t}\right) - (2 - s_L^2) m_t^2 \right]$$

 $s_L = \sin \theta_L \simeq v/f$

T < 0.1 (0.15) [68% (95%) CL] $\Rightarrow f \gtrsim 4.3 (3.5) \text{ TeV for } \xi = 3.6$ if there is no cancellation with other contributions.



Phenomenology

- Large f implies fine tuning (~v²/f²) of the EW scale, also makes all other states very heavy
 [except the 5th PNGB, A₁, with a mass ~ (f/v) M_h]. They are beyond of reach of the LHC.
- It also means the model is near the decoupling limit. Corrections to the Higgs coupling to SM fields is ~ $v^2/(2f^2) \leq 0.2\%$, difficult even for a future lepton collider. However, *T*-parameter can be precisely measured at a future *Z* factory
- t' of mass up to ~10 TeV may be reachable at a 100 TeV collider, through $t' \rightarrow Wb$, tZ, th, tA_1 .

Custodial SU(2) Extensions (HC and J. Gu, arXiv:1406.6689)

- The *T* constraint could be alleviated if the model can be extended to include a custodial SU(2).
- Extension to bottom seesaw is strongly constrained by $Z \to b\overline{b}$.
- To avoid the constraint from $Z \rightarrow b\overline{b}$, we need to assign $(t, b)_L$ as (2, 2) under SU $(2)_L \times SU(2)_R$ with a P_{LR} symmetry. (Agashe et al, hep-ph/0605341)
 - Introduce a vector-like hypercharge +7/6 doublet quark (X,T) in addition to the singlet χ . The weak isospin can be protected by the O(5)_L \subset U(5)_L among $\Psi_L = (t_L, b_L, X_L, T_L, \chi_L)$.

Weak-isospin T Parameter

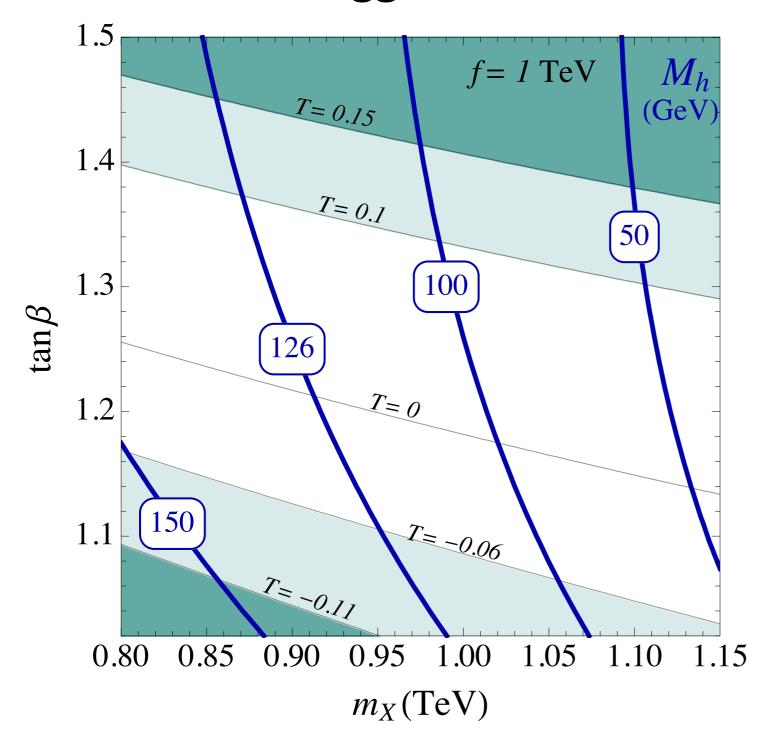
Explicit $O(5)_L$ breaking masses:

 $\mathcal{L}_{\text{fermion masses}} = -\mu_t \overline{\chi}_L t_R - \mu_{\chi\chi} \overline{\chi}_L \chi_R - \mu_Q \left(\overline{X}_L \quad \overline{T}_L \right) \begin{pmatrix} X_R \\ T_R \end{pmatrix} + \text{H.c.}$

- In the limit $\mu_Q \rightarrow 0$ ($M_X \rightarrow 0$), adding $(X, T)_L$ cancels the SM $(t, b)_L$ contribution to T, resulting in a negative T.
- In the limit $\mu_Q \rightarrow \infty$ $(M_X \rightarrow \infty)$, (X,T) decouples and we recover the minimal model. There is a large positive contribution to T if f is low.
- For a suitable range of $\mu_Q(M_X \leq f)$ we expect to get a T value compatible with the EW precision measurement.

Higgs Boson Mass

Current bound $M_X > 800$ GeV (CMS). Small T and correct Higgs mass can be obtained for $M_X \leq f$.

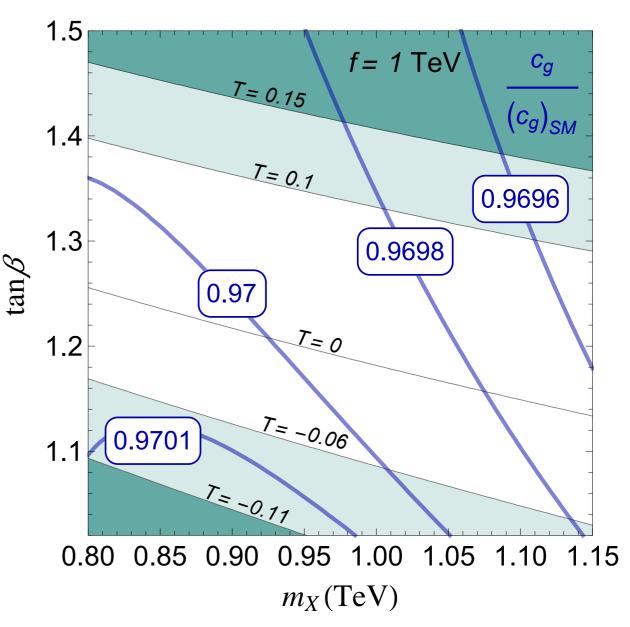


f = 1 TeV, $\xi = 3.6,$ $\lambda_1/(2\xi^2) = 0.7,$ $\lambda_1/\lambda_2 = 0,$ $M_{\rho} = 3f,$ $M_{\Sigma_{X,T,t}} = 10f$

Higgs Couplings

 Most tree-level Higgs couplings are suppressed by 1-v²/(2f²). Corrections are ~3% for f=1 TeV.

 hgg coupling receives loop contributions from top partners. However, they are small within the allowed parameter space.



Compared with other models

- Holographic composite Higgs: (Agashe, Contino, Pomarol, ...)
 - The global symmetry is preserved by the strong sector exactly. Explicit breaking comes from coupling to SM fermions, which are not complete multiplets.
 - Higgs boson mass related to the top partner masses which cut off the radiative contributions.
- Top seesaw model:
 - Top and new quarks form a complete multiplet of the global symmetry. Explicit breaking comes from fermion mass terms, similar to QCD.
 - Higgs boson mass related to the top quark mass through the top seesaw mechanism.

Conclusions

- A light Higgs boson of 125 GeV can arise naturally in a composite Higgs model from top condensation with the top seesaw mechanism.
- The simplest model based on U(3) symmetry requires a large f ⇒ fine-tuned and probably out of the reach of LHC.
- Extension to O(5) can reduce fine-tuning. It requires relatively light exotic top partners $(X^{\frac{5}{3}}, T^{\frac{2}{3}})$. The I4 TeV LHC can significantly extend their reach.
- Most heavy states (scalars and singlet top partner) require a higher energy machine beyond LHC.

Backup Slides

Light Fermion Masses and FCNC

- The light SM fermion masses come from 4-fermion interactions at the compositeness scale.
- There are 2 Higgs doublets. Large tree-level FCNCs can be induced if they have general couplings (not type I or II) to fermions.
- However, fermion masses and mixings are hierarchical. It's likely there is some approximate flavor symmetry which controls the 4-fermion interactions. In that case FCNC constraints can be satisfied if the other Higgses are heavier than ~ I TeV. (Cheng, Sher, '84, ,Antaramian, Hall, Rasin, '92, ...)

• We introduce a vector-like hypercharge +7/6 doublet quark (X,T) in addition to the singlet χ . The strong dynamics has approximate U(5)_L ×U(4)_R symmetry among $\Psi_L=(t_L, b_L, X_L, T_L, \chi_L)$ and $\Psi_R=(X_R, T_R, t_R, \chi_R)$.

$$\mathcal{L} = \mathcal{L}_{\text{kinetic}} + G(\overline{\Psi}_{L_i}\Psi_{R_j})(\overline{\Psi}_{R_j}\Psi_{L_i})$$

They form scalar bound states:

$$\Phi = \begin{pmatrix} \sigma_{tX}^{-} & \sigma_{tT}^{0} & \sigma_{tt}^{0} & \phi_{t\chi}^{0} \\ \sigma_{bX}^{--} & \sigma_{bT}^{-} & \sigma_{bt}^{-} & \phi_{b\chi}^{-} \\ \sigma_{XX}^{0} & \sigma_{XT}^{+} & \sigma_{Xt}^{+} & \phi_{X\chi}^{+} \\ \sigma_{TX}^{-} & \sigma_{TT}^{0} & \sigma_{Tt}^{0} & \phi_{T\chi}^{0} \\ \sigma_{\chi X}^{-} & \sigma_{\chi T}^{0} & \sigma_{\chi t}^{0} & \phi_{\chi\chi}^{0} \end{pmatrix} \equiv \left(\Sigma_{X} \quad \Sigma_{T} \quad \Sigma_{t} \quad \Phi_{\chi} \right).$$

with Yukawa interaction $\mathcal{L}_{Yukawa} = -\xi \overline{\Psi}_L \Phi \Psi_R + H.c.$

The vector-like fermion can have gauge invariant masses,

$$\mathcal{L}_{\text{fermion masses}} = -\mu_t \overline{\chi}_L t_R - \mu_{\chi\chi} \overline{\chi}_L \chi_R - \mu_Q \left(\overline{X}_L \quad \overline{T}_L \right) \begin{pmatrix} X_R \\ T_R \end{pmatrix} + \text{H.c.}$$

They turn into scalar tadpole terms in low energy EFT. We also assume that U(4)_R is strongly broken so that only Φ_X have negative mass-squared, while $\Sigma_X, \Sigma_T, \Sigma_t$ remain heavy. The scalar potential at low energy is given by

$$V = \frac{\lambda_1}{2} \operatorname{Tr}[(\Phi^{\dagger}\Phi)^2] + \frac{\lambda_2}{2} (\operatorname{Tr}[\Phi^{\dagger}\Phi])^2 + M_{\Sigma_{X,T}}^2 \Sigma_X^{\dagger} \Sigma_X + M_{\Sigma_{X,T}}^2 \Sigma_T^{\dagger} \Sigma_T + M_{\Sigma_t}^2 \Sigma_t^{\dagger} \Sigma_t + M_{\Phi_{\chi}}^2 \Phi_{\chi}^{\dagger} \Phi_{\chi} - C_Q \sigma_{XX}^0 - C_Q \sigma_{TT}^0 - C_{\chi t} \sigma_{\chi t}^0 - C_{\chi \chi} \phi_{\chi \chi}^0 + \text{H.c.}$$

For heavy $\Sigma_X, \Sigma_T, \Sigma_t$, they can be integrated out and we focus on Φ_X . The leading effects of $\Sigma_X, \Sigma_T, \Sigma_t$ are their VEVs induced by the tadpole terms.

$$V = \frac{\lambda_1}{2} \operatorname{Tr}[(\Phi^{\dagger}\Phi)^2] + \frac{\lambda_2}{2} (\operatorname{Tr}[\Phi^{\dagger}\Phi])^2 + M_{\Phi_{\chi}}^2 \Phi_{\chi}^{\dagger} \Phi_{\chi} - C_{\chi\chi}(\phi_{\chi} + \phi_{\chi}^{\dagger}),$$
$$\Phi = \begin{pmatrix} 0 & 0 & 0 & \phi_t^0 \\ 0 & 0 & 0 & \phi_b^- \\ \frac{w}{\sqrt{2}} & 0 & 0 & \phi_X^+ \\ 0 & \frac{w}{\sqrt{2}} & 0 & \phi_T^0 \\ 0 & 0 & \frac{u_t}{\sqrt{2}} & \phi_\chi^0 \end{pmatrix}, \quad \Phi_{\chi} = \begin{pmatrix} \phi_t^0 \\ \phi_b^- \\ \phi_X^+ \\ \phi_X^0 \\ \phi_\chi^0 \\ \phi_\chi^0 \end{pmatrix},$$

$$\langle \sigma_{XX} \rangle = \langle \sigma_{TT} \rangle \equiv \frac{w}{\sqrt{2}} , \qquad \langle \sigma_{\chi t} \rangle \equiv \frac{u_t}{\sqrt{2}} .$$

To avoid 2 light Higgs doublets and to protect the custodial symmetry, we include a scalar mass term which breaks $U(5)_{L}$ down to $O(5)_{L}$,

$$V_{U(5)} = \frac{1}{2} K^2 \left(\operatorname{Tr}[\Sigma'^{\dagger}\Sigma'] + A_{\chi}^2 \right),$$

$$\Sigma \equiv \begin{pmatrix} \phi_t^{0*} & \phi_X^+ \\ \phi_b^- & \phi_T^0 \end{pmatrix}, \quad \Sigma' \equiv \frac{1}{\sqrt{2}} (\Sigma - \epsilon \Sigma^* \epsilon^T) = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_t^{0*} - \phi_T^{0*} & \phi_X^+ + \phi_b^+ \\ \phi_X^- + \phi_b^- & -\phi_t^0 + \phi_T^0 \end{pmatrix}$$

$$A_{\chi} = \sqrt{2} \operatorname{Im} \phi_{\chi}$$

 $O(5)_{L}$ is then spontaneously broken by negative $M_{\Phi_{\chi}}^{2}$, generating light Goldstone bosons as the Higgs.

Due to the tadpoles, ϕ_t^0 , ϕ_T^0 also obtain VEVs in addition to ϕ_{χ}^0 , breaking EW symmetry.

$$\phi_t^0 = \frac{v_t + h_t + iA_t}{\sqrt{2}}, \qquad \phi_T^0 = \frac{v_T + h_T + iA_T}{\sqrt{2}}, \qquad \phi_\chi^0 = \frac{u_\chi + h_\chi + iA_\chi}{\sqrt{2}}$$
$$v = \sqrt{v_t^2 + v_T^2} = 246 \text{ GeV}, \qquad \tan \beta \equiv \frac{v_t}{v_T} > 1$$
$$(\tan \beta \to 1 \text{ as } K^2 \to \infty)$$

Chiral symmetry breaking scale: $f = \sqrt{v_t^2 + v_T^2 + u_\chi^2}$

Generalized Top Seesaw

$$\mathcal{L} \supset -\frac{\xi}{\sqrt{2}} \begin{pmatrix} \overline{t_L} & \overline{T_L} & \overline{\chi_L} \end{pmatrix} \begin{pmatrix} 0 & 0 & v_t \\ 0 & w & v_T \\ u_t & 0 & u_\chi \end{pmatrix} \begin{pmatrix} t_R \\ T_R \\ \chi_R \end{pmatrix} - \frac{\xi w}{\sqrt{2}} \overline{X_L} X_R .$$
$$m_{\rm top}^2 \approx \frac{\xi^2 v_t^2}{2} \frac{u_t^2}{f^2} \quad \Rightarrow \quad \frac{u_t}{f} \approx \frac{y_t}{\xi \sin \beta}$$

The heavy top partners:

$$m_{t_2} \approx m_X = rac{\xi w}{\sqrt{2}}$$
 (< f to minimize T)
 $m_{t_3} \approx rac{\xi f}{\sqrt{2}} \sim (2-3)f$

Current experimental bound: $m_X > 800$ GeV (CMS).

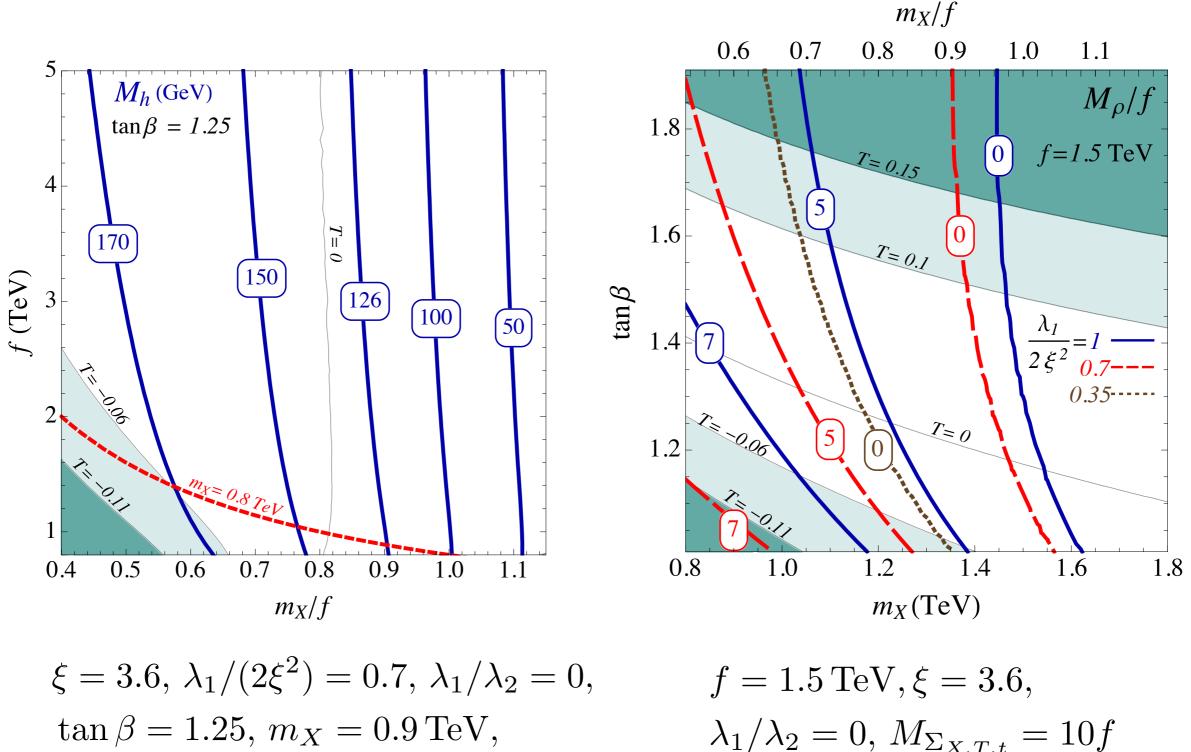
CP-even scalars: $\begin{pmatrix} h_1 \\ h_2 \\ h_\chi \end{pmatrix} = \begin{pmatrix} \frac{v_t}{v} & \frac{v_T}{v} & 0 \\ -\frac{v_T}{v} & \frac{v_t}{v} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} h_t \\ h_T \\ h_\chi \end{pmatrix},$

$$M_{\text{scalar}}^{2} = \begin{pmatrix} (\lambda_{1} + \lambda_{2})v^{2} & 0 & (\lambda_{1} + \lambda_{2})u_{\chi}v \\ 0 & \frac{\lambda_{1}w^{2}v^{2}}{2(v_{t}^{2} - v_{T}^{2})} & 0 \\ (\lambda_{1} + \lambda_{2})u_{\chi}v & 0 & (\lambda_{1} + \lambda_{2})u_{\chi}^{2} + \frac{\lambda_{1}u_{t}^{2}}{2} - \frac{\lambda_{1}w^{2}v_{T}}{2(v_{t} + v_{T})} \end{pmatrix}$$

$$M_h^2 \approx \frac{\lambda_1 v^2}{2f^2} (u_t^2 - \frac{w^2 v_T}{v_t + v_T}) \approx \frac{\lambda_1}{2\xi^2} (\frac{y_t^2}{\sin^2 \beta} - \frac{m_X^2}{f^2} \frac{2}{1 + \tan \beta}) v^2$$

EW gauge loops further reduce the Higgs mass. 125 GeV Higgs ($\lambda_h = M_h^2/V^2 = 0.26$) is typically obtained for $m_X \leq f$.

Higgs mass dependence on model parameters



 $\tan \beta = 1.25, m_X = 0.9 \,\text{TeV},$ $M_{\rho} = 3f, M_{\Sigma_{X,T,t}} = 10f$