Beyond Doublets

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Based on Hartling, Kumar & Logan, arXiv: 1410.5538, 1404.2640
and Hally, Logan & Pilkington, arXiv: 1202.5073

Outline

- **Motivation**: Why extend beyond doublets?
- **General limits**: How big can we go?
- **Difficulties with large scalar multiplets**: the ρ parameter.
- **Focus:** The Georgi-Machacek model
  - Model overview
  - Theoretical constraints
  - Decoupling limit
  - Experimental constraints
- **Conclusions and summary:** What next?

Why extend beyond scalar doublets?

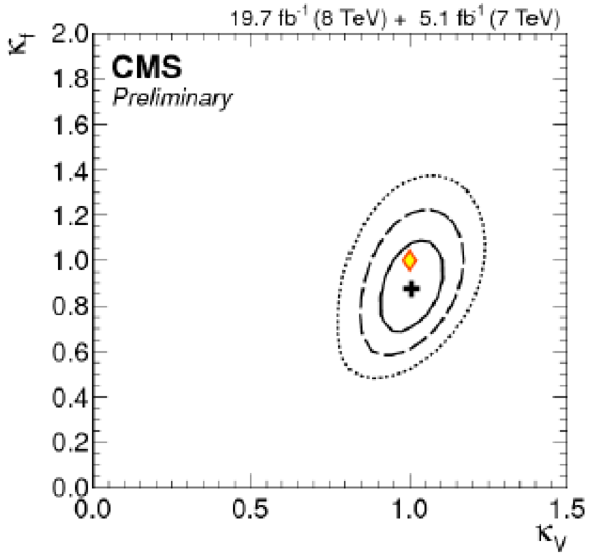
Larger multiplets can explain enhanced hVV couplings.

- Models with isospin doublets or singlets have hVV couplings **smaller than or equal to** those of the Standard Model (SM).

Table 1: Higgs couplings to WW (Q=T^3+Y/2)

SM	2HDM	SM + singlet	SM + general multiplet X
$i\frac{g^2}{2}g_{\mu\nu}$	$i\frac{g^2}{2}g_{\mu\nu}\sin(\beta-\alpha)$	$i\frac{g^2}{2}g_{\mu\nu}\cos\alpha$	$i\frac{g^2 v_X}{2}g_{\mu\nu}\cdot 2\left[T(T+1)-\frac{Y^2}{4}\right]$

- **Enhanced** hVV couplings require a scalar multiplet that:
  - Has isospin ≥ 1.
  - Has a non-negligible vev.
  - Mixes with the observed Higgs *h.*



Why extend beyond scalar doublets?

Enhanced Higgs couplings could hide new physics.

- If *all* of the *h* couplings are enhanced, they may compensate for new physics in the Higgs branching ratios.

$$\text{Rate} = \frac{\sigma_{\rm SM}\,\Gamma_{\rm SM}}{\Gamma^{\rm tot}_{\rm SM}} \rightarrow \frac{\kappa^2\,\sigma_{\rm SM}\,\kappa^2\,\Gamma_{\rm SM}}{\kappa^2\,\Gamma^{\rm tot}_{\rm SM} + \Gamma_{\rm new}} \quad \text{where} \quad \kappa = \frac{g_{\rm BSM}}{g_{\rm SM}}$$

- Rates measured at the LHC will be identical to SM predictions if

$$\kappa^2 = \frac{1}{1 - {\rm BR}_{\rm new}}$$

- Constraints on Γ_{tot} or κ from off-shell gg → h* → ZZ assume no new resonances in the s-channel (model-dependent assumption).

- Studies of concrete models are needed to provide benchmarks.

How big can scalar multiplets be?

- Consider an electroweak scalar multiplet of size n and hypercharge Y.

$$X = (\chi_n, ..., \chi_2, \chi_1)^T \qquad \text{(complex)}$$

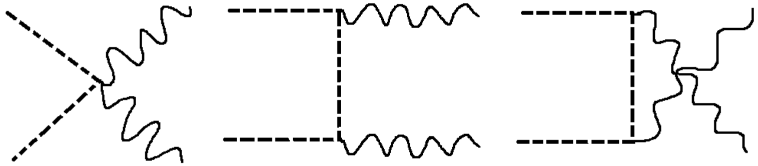
$$\xi_n = (\xi^Q, ..., \xi^0, ..., \xi^{-Q})^T \qquad \text{(real)}$$

- Tree-level unitarity requires that the partial-wave a_0 satisfy $|\mathrm{Re}(a_0)| < 1/2$.

- Applying this constraint to the eigenvalues of the matrix of coupled-channel 2 → 2 electroweak scattering processes can constrain *n* and *Y.*

$$n \leq \begin{cases} 8 \\ 9 \end{cases} \Rightarrow T \leq \begin{cases} \frac{7}{2} & \text{for a complex multiplet} \\ 4 & \text{for a real multiplet} \end{cases} \qquad |Y| \leq \frac{19.8}{n^{1/4}}$$

[Hally, Logan and Pilkington, 1202.5073]



Difficulties with scalar multiplets

- Higgs extensions with multiplets larger than doublets can result in large contributions to the ρ-parameter.

$$\rho = \frac{M_W^2}{M_Z^2 \cos\theta_W} = \frac{\sum_k 2\, v_k^2 \left[T_k(T_k+1) - Y_k^2/4\right]}{\sum_k Y_k^2 v_k^2}$$

where $Q = T^3 + Y/2$ and $\langle \phi_k^0 \rangle = v_k/\sqrt{2}\ (v_k)$.

- From global fits $\rho = 1.00040 \pm 0.00024$. [PDG 2014]

- **How can we avoid large contributions?**
  – ρ = 1 automatically for SM and septet with Y = 4. [Hisano & Tsumura, 1301.6455; Kanemura, Kikuchi, Yagyu, 1301.7303; Alvarado, Lehman, Ostdiek; 1404.3208]
  – Require that vevs be zero. [Earl, Hartling, Logan, Pilkington, 1303.1244, 1311.3656]
  – Require that vevs be very small (ex. Higgs Triplet). [Schechter & Valle, PRD 22, 2227 ('80); Cheng & Li, PRD 22, 2860 ('80)]
  – Balance contributions to charged and neutral weak currents using multiple multiplets (eg. Georgi-Machacek). [Georgi & Machacek, NPB262, 463 (1985)]

The Georgi-Machacek Model

- The Georgi-Machacek scalar sector contains a doublet and two triplets

$$\Phi = \begin{pmatrix} \phi^{0*} & \phi^+ \\ -\phi^{+*} & \phi^0 \end{pmatrix} \qquad X = \begin{pmatrix} \chi^{0*} & \xi^+ & \chi^{++} \\ -\chi^{+*} & \xi^0 & \chi^+ \\ \chi^{++*} & -\xi^{+*} & \chi^0 \end{pmatrix}$$

with $SU(2)_L$ x $SU(2)_R$ symmetry and where $s^{Q*} = (-1)^Q s^{-Q}$.

- The ρ-parameter will be 1 if the vevs of both triplets are equal,

$$\rho = \frac{v_\phi^2 + 4v_\xi^2 + 4v_\chi^2}{v_\phi^2 + 8v_\chi^2} = 1 \text{ if } v_\chi = v_\xi$$

  – Gauging hypercharge breaks custodial SU(2), leading to divergent radiative corrections to ρ at 1-loop. [Gunion, Vega & Wudka, PRD43, 2322 (1991)]

- Require that $v_\phi^2 + 8v_\chi^2 = v^2 = (246\,\mathrm{GeV})^2$, where $\langle\Phi\rangle = \frac{v_\phi}{\sqrt{2}}$ and $\langle X\rangle = v_\chi$.

[Georgi & Machacek, NPB262, 463 (1985), Chanowitz & Golden, PLB165, 105 (1985)]

The Georgi-Machacek Model

- Scalar mixing yields a physical fiveplet, triplet, and two singlets:

$$\Phi = \begin{pmatrix} \phi^{0*} & \phi^+ \\ -\phi^{+*} & \phi^0 \end{pmatrix} \qquad X = \begin{pmatrix} \chi^{0*} & \xi^+ & \chi^{++} \\ -\chi^{+*} & \xi^0 & \chi^+ \\ \chi^{++*} & -\xi^{+*} & \chi^0 \end{pmatrix} \longrightarrow \begin{pmatrix} H_5^{++} \\ H_5^+ \\ H_5^0 \\ H_5^- \\ H_5^{--} \end{pmatrix}, \begin{pmatrix} H_3^+ \\ H_3^0 \\ H_3^- \end{pmatrix}, H\,, h$$

- Generally *h* is assumed to the observed 125 GeV SM-like scalar boson.
- Scalars within each multiplet have identical masses: m_5, m_3, m_H, and m_h.
- The h, H, and H_3 scalars are mixtures of doublet φ and triplet χ, ξ states.
  - Mixing angle α controls how much *h* comes from the triplets.
  - Ratio $\tan\theta_H = 2\sqrt{2}\, v_\chi/v_\varphi$ controls how much H_3 comes from the triplets.
- H_5 states are composed of triplet χ, ξ states.

H_5 is vector-philic (H_5VV but no $H_5f\bar{f}$) and
H_3 is fermion-philic ($H_3f\bar{f}$ but no H_3VV).

The Decoupling Limit

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- The most general custodial-SU(2)-symmetric scalar potential is

$$V(\Phi, X) = \frac{\mu_2^2}{2}\mathrm{Tr}(\Phi^\dagger\Phi) + \frac{\mu_3^2}{2}\mathrm{Tr}(X^\dagger X) + \lambda_1[\mathrm{Tr}(\Phi^\dagger\Phi)]^2 + \lambda_2\mathrm{Tr}(\Phi^\dagger\Phi)\mathrm{Tr}(X^\dagger X)$$
$$+ \lambda_3\mathrm{Tr}(X^\dagger X X^\dagger X) + \lambda_4[\mathrm{Tr}(X^\dagger X)]^2 - \lambda_5\mathrm{Tr}(\Phi^\dagger\tau^a\Phi\tau^b)\mathrm{Tr}(X^\dagger t^a X t^b)$$
$$- M_1\mathrm{Tr}(\Phi^\dagger\tau^a\Phi\tau^b)(UXU^\dagger)_{ab} - M_2\mathrm{Tr}(X^\dagger t^a X t^b)(UXU^\dagger)_{ab}$$

[Aoki & Kanemura, 0712.4053; Chiang & Yagyu, 1211.2658; Chiang, Kuo & Yagyu, 1307.7526]

- The dimension-3 terms (M_1, M_2) are necessary for the model to decouple:
  - Masses are unbounded (without M_1, M_2 terms $M_{new} < 700$ GeV).
  - Masses become degenerate as M_{new} becomes large.
  - Couplings all become SM-like as v_χ and α approach 0.

Coupling	Decoupling	$M_{1,2} \sim v_{SM}$	$M_{1,2} \sim M_{new}$	2HDM
hVV	$M_1^2 v^2/M_{new}^4$	v^4/M_{new}^4	v^2/M_{new}^2	v^4/M_{new}^4
hff				v^2/M_{new}^2

[Hartling, Kumar & Logan, 1404.2640; Gunion & Haber, PRD67 075019 (2003)]

Theoretical Constraints

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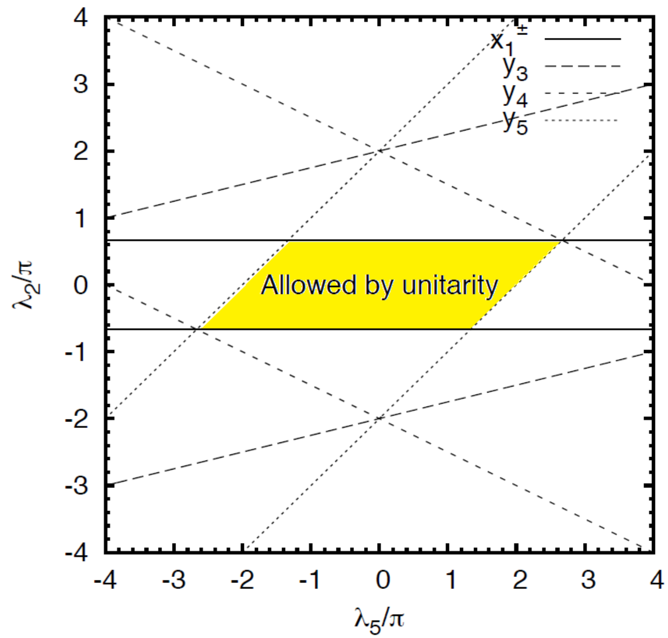
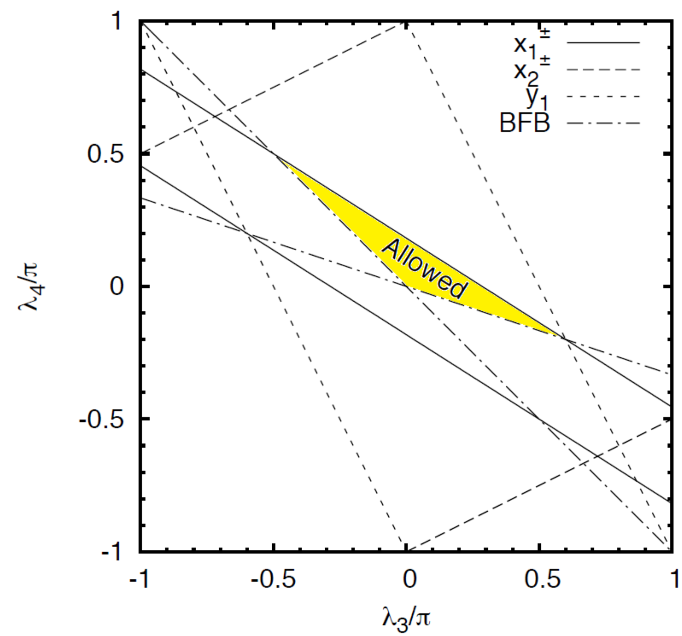
- **Perturbative unitarity** requires that the tree-level partial-wave a_0 from 2 → 2 scalar scattering satisfy $|\mathrm{Re}(a_0)| < 1/2$. [Aoki & Kanemura, 0712.4053]

- Scalar potential must be **bounded from below**.

- **Avoiding alternative minima** in the scalar potential: ensure that the global minimum preserves custodial SU(2).





[Hartling, Kumar & Logan, 1404.2640]

Theoretical Constraints

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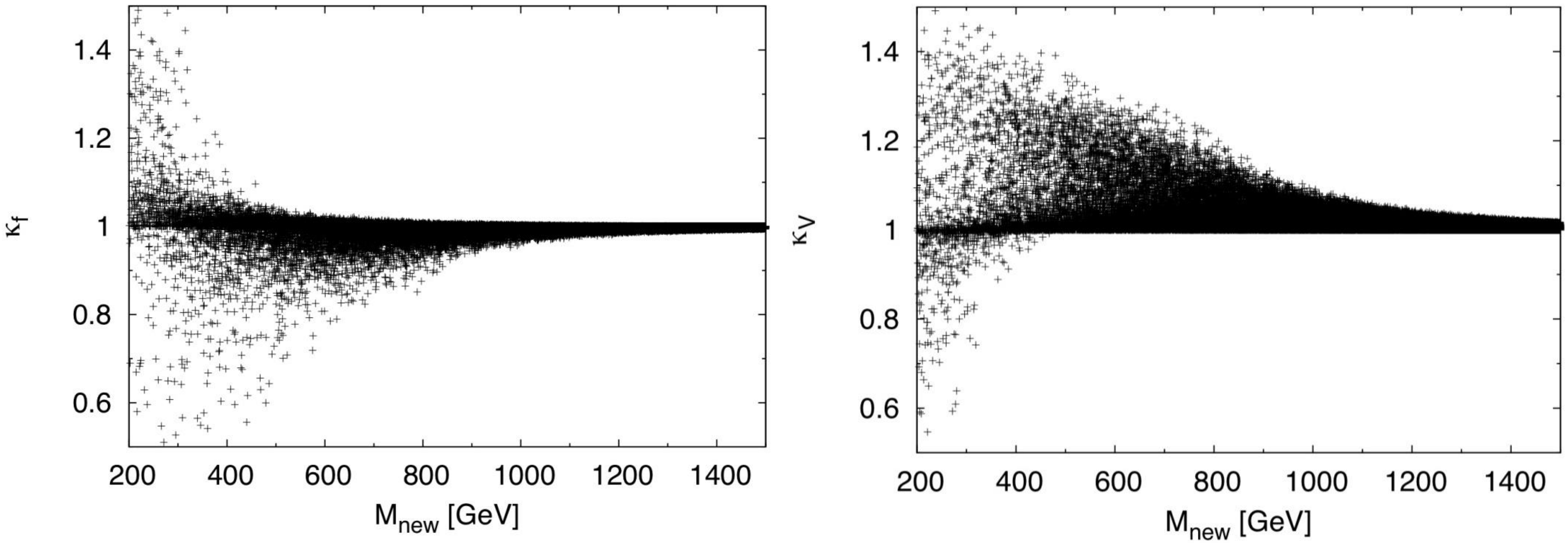

Higgs couplings as a ratio to the SM. Left: Fermion. Right: Vector boson.

[Hartling, Kumar & Logan, 1404.2640]

- **Decoupling**: coupling ratios to SM approach 1 as the mass scale increases.
- **Enhancement**: M_{new} < 500 GeV for all couplings to be enhanced.

Direct detection constraints

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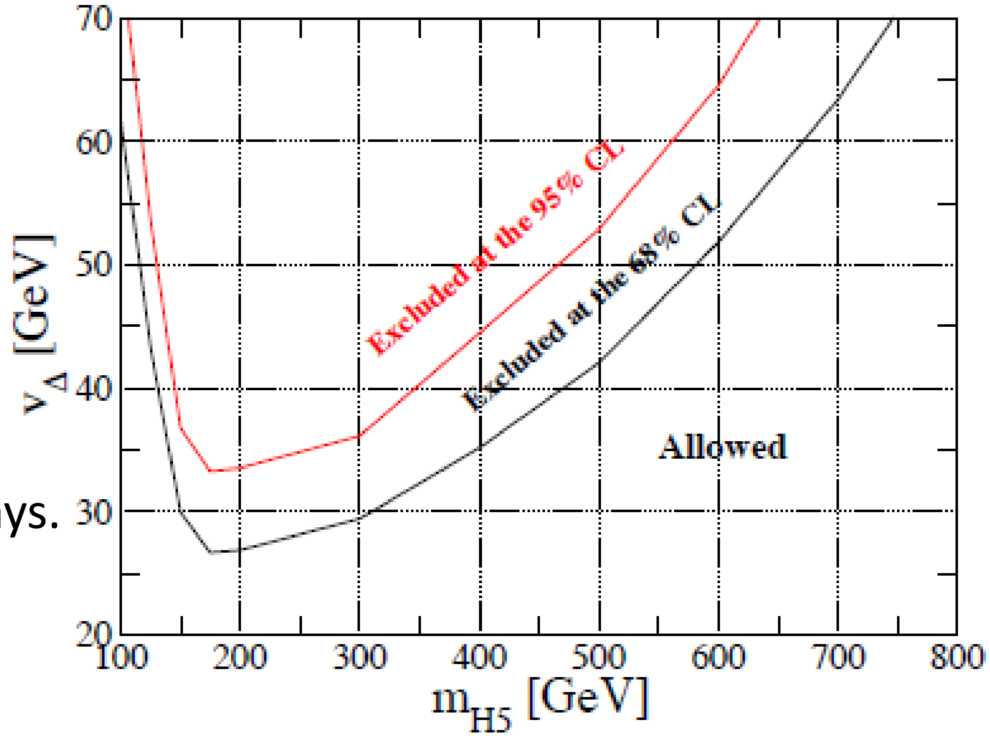
- **LEP results:**
  - Lower mass bound on a doubly-charged scalar: $M_{H\pm\pm} \gtrsim 43$ GeV

    [Kanemura, Yagyu & Yokoya, 1305.2383]

- **Collider searches**: applicability of bounds depend on preferred decays.
  - e.g. $M_{H\pm\pm} \gtrsim 400$ GeV if $H^{\pm\pm}$ decays dominantly to $l^{\pm}l^{\pm}$.



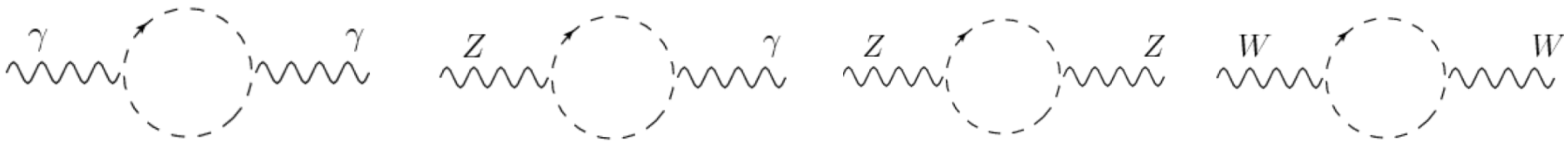
- **Bounds from like-sign WWjj**

  [Chiang, Kanemura & Yagyu, 1407.5053]

  - The like-sign $W^{\pm}W^{\pm}jj$ cross section measurement from ATLAS has been used to place limits on VBF $H^{\pm\pm} \rightarrow W^{\pm}W^{\pm}$.
  - Constrains the $v\chi$-m5 plane.

Constraints from S, T, U

- The Peskin-Takeuchi (or oblique) parameters S, T, U are defined as linear combinations of the vector boson self-energies.



- Global fits yield a correlated bound on S and T (assuming U=0):
  – S = 0.06 ± 0.09, T = 0.10 ± 0.07 with ρ = +0.91. [Baak et al (Gfitter), 1407.3792]

- T is divergent at 1-loop in the GM due to the gauging of hypercharge.
  – A counter-term is needed to calculate T. [Gunion, Vega & Wudka, PRD43, 2322 (1991); Englert, Re & Spannowski, 1302.6505]

- Calculate S and marginalize over T to obtain a conservative bound. [Chiang, Kuo & Yagyu, 1307.7526; Hartling, Kumar & Logan, 1410.5538]

- All states participate → constraint affects all free parameters.

Non-oblique and B-physics constraints

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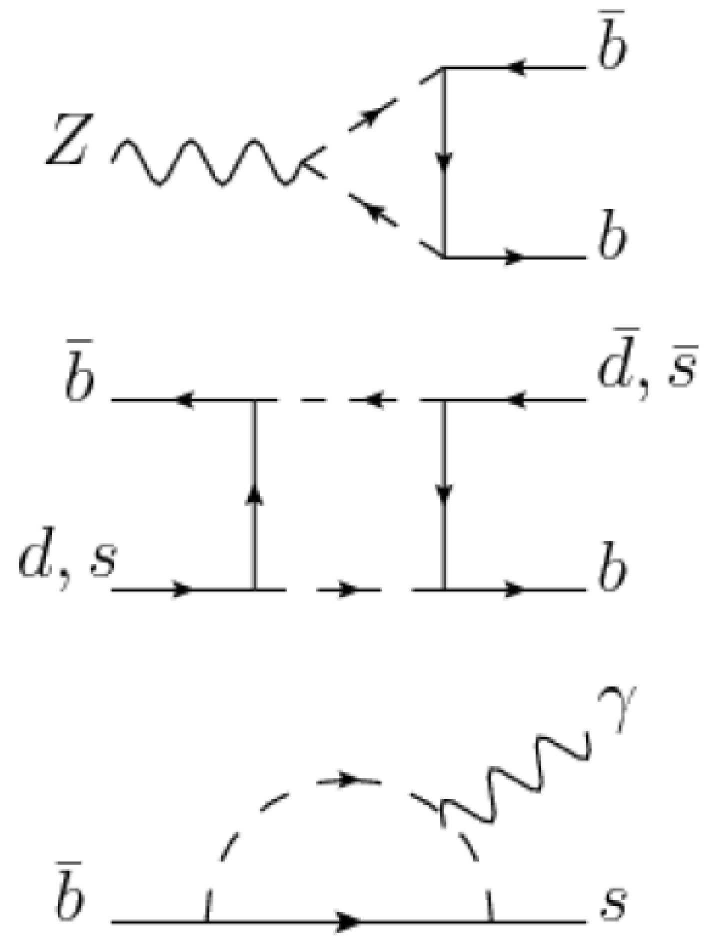
- **Non-oblique and B-physics processes:** GM scalars contribute at 1-loop to
  - R_b, from $Z \to b\bar{b}$ (LEP)
  - B_s meson mixing (HFAG)
  - $B_s \to \mu^+\mu^-$ (CMS, LHCb)
  - $b \to s\gamma$ (HFAG)
- **Constrain the v_χ-M_3 plane.**
- GM contributions are the same as those in the Type-I 2HDM.
  - Same fermion coupling structure.
  - $H_3^+H_3^-Z$ couplings are identical ($SU(2)_c$).
- **LO Type-I 2HDM formulas apply with the replacements $\cot_\beta \to \tan\theta_H$ and $M_{H+} \to M_3$.**



Non-oblique and B-physics constraints

- **Non-oblique and B-physics processes:** GM scalars contribute at 1-loop to
  - R_b, from $Z \rightarrow b\bar{b}$ (LEP) [Logan & Haber, hep-ph/9909335; Grant, hep-ph/9410267; Chiang & Yagyu, 1211.2658]
  - B_s meson mixing (HFAG) [Mahmoudi & Stal, 0907.1791]
  - $B_s \rightarrow \mu^+\mu^-$ (CMS, LHCb) [Li, Lu & Pich, 1404.5865]
  - $b \rightarrow s\gamma$ (HFAG) [Barger, Hewett & Phillips, PRD41, 3421 (1990); Mahmoudi, SuperIso v3.3]
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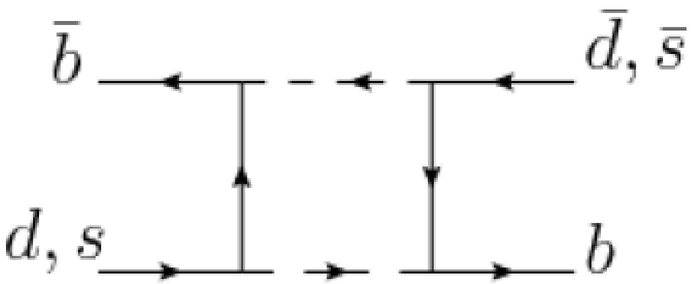

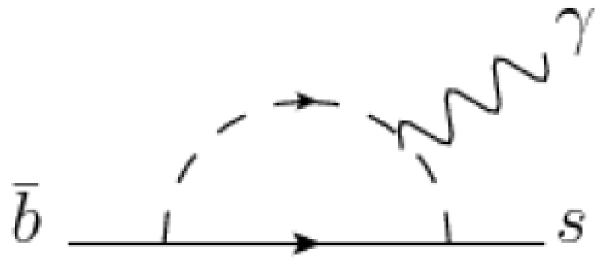

Indirect Z-pole and B-physics constraints

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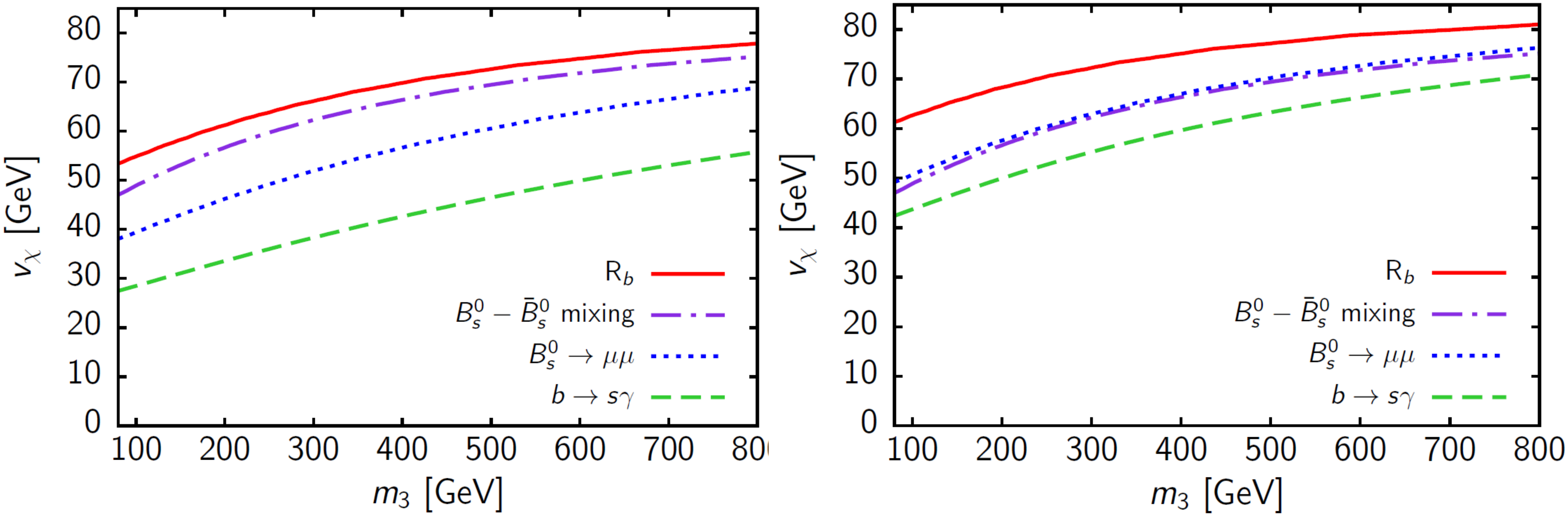


Left: 2σ bound from experiment. Right: 2σ worse than SM prediction.

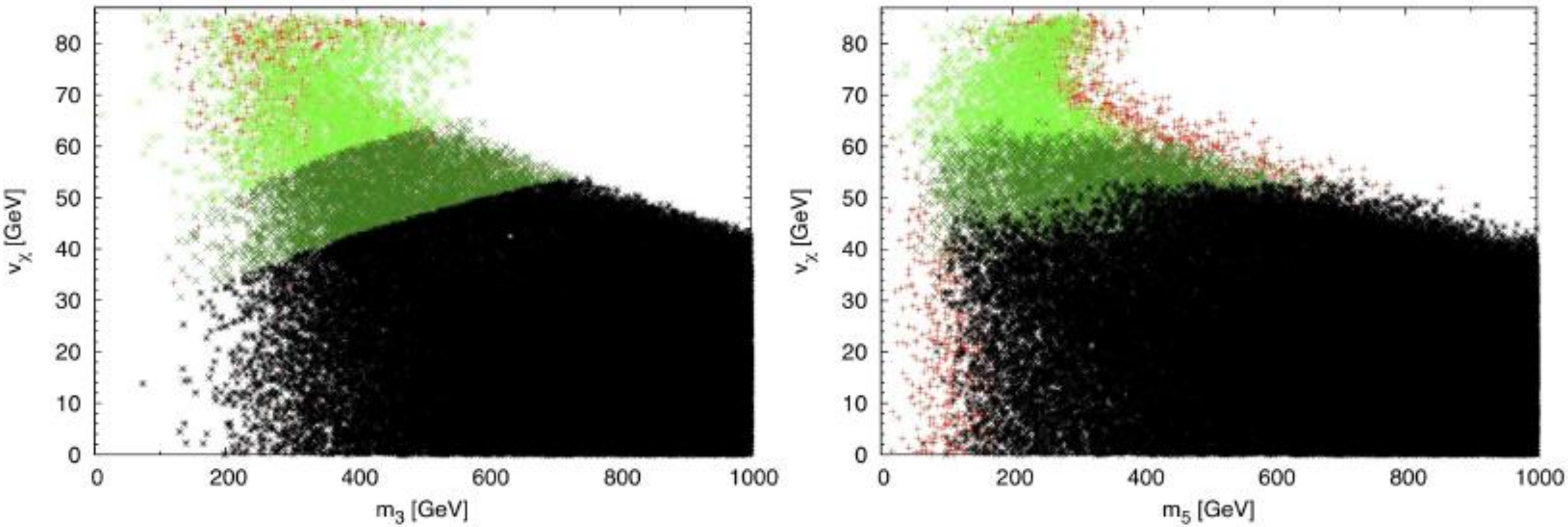
[Hartling, Kumar & Logan, 1410.5538]

- **Strongest bound is from b → sγ.**
  - Implemented in numerical scan using SuperIso. [Mahmoudi, 0710.2067]

$$\mathrm{BR}(B \to X_s\gamma)_{SM} = (3.11 \pm 0.23) \times 10^{-4} \quad \text{(SuperIso)}$$

$$\mathrm{BR}(B \to X_s\gamma)_{exp} = (3.55 \pm 0.24 \pm 0.09) \times 10^{-4} \quad \text{(HFAG)}$$

Constraint results

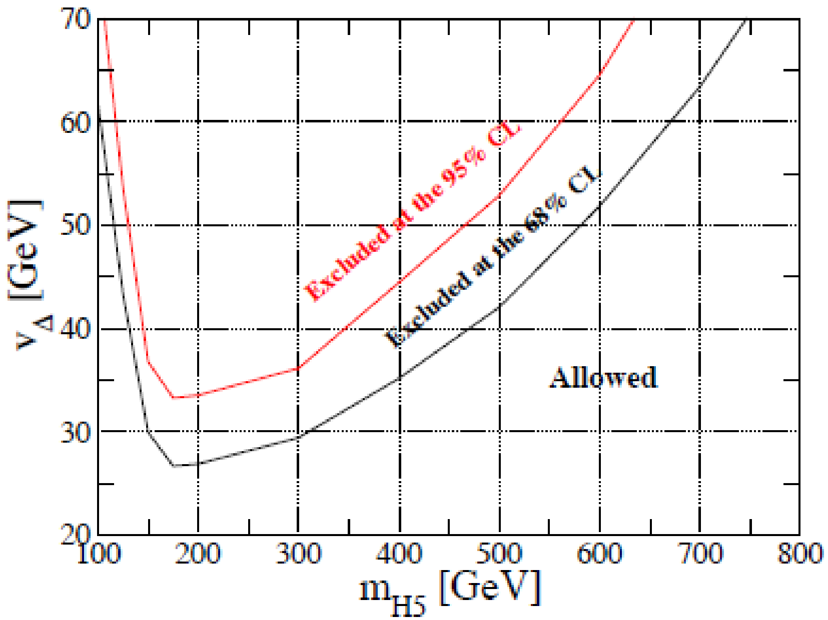


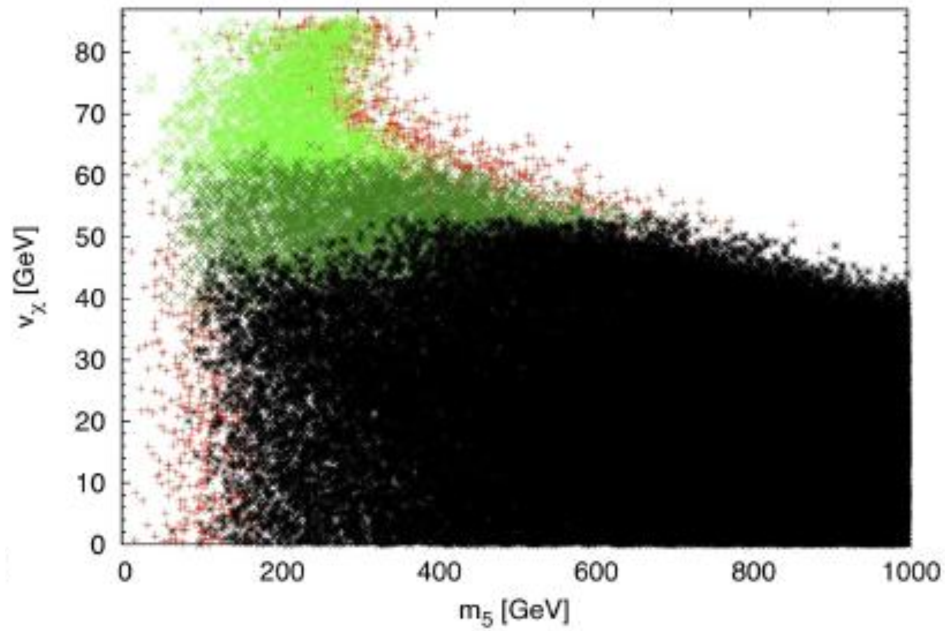
Red: eliminated by S. Green: eliminated by b → sγ. Black: allowed.

[Hartling, Kumar & Logan, 1410.5538]

- Tight b → sγ experimental bound constrains $v_\chi < 54$ GeV.
- Loose SM bound constrains $v_\chi < 65$ GeV.
- Complements like-sign WWjj bound.

Constraint results





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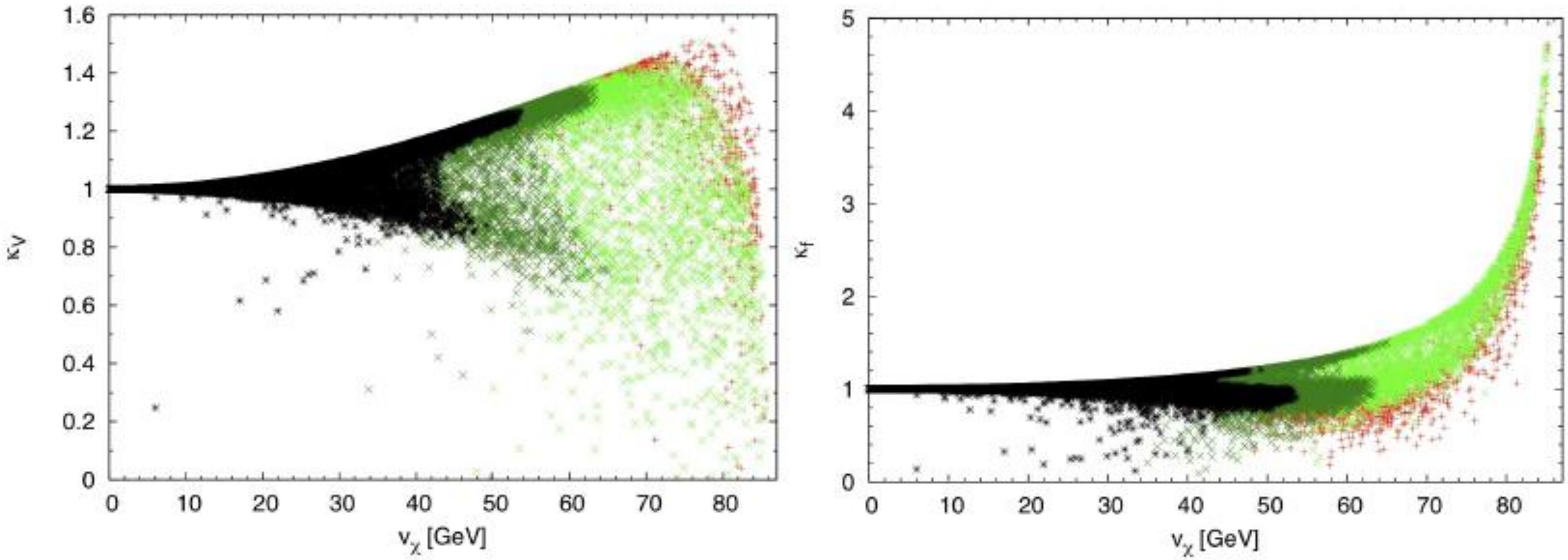

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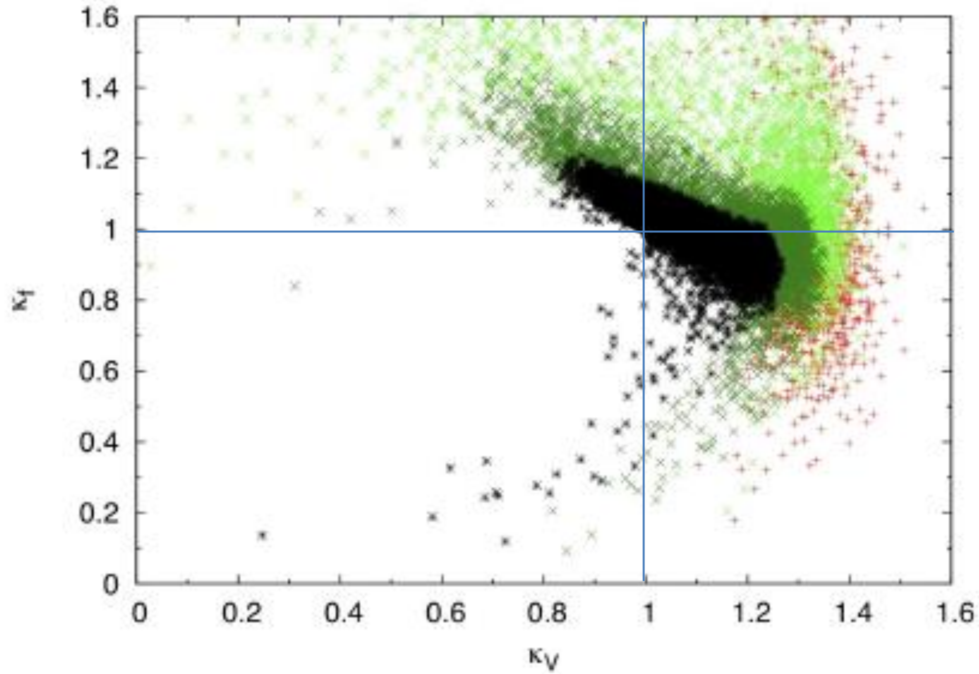
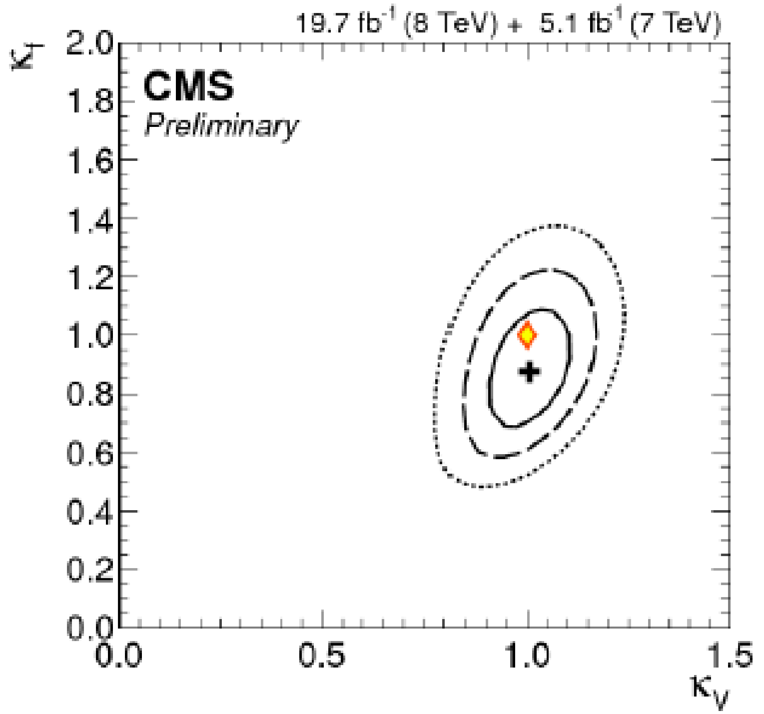
[Hartling, Kumar & Logan, 1410.5538]

- Loose $b \to s\gamma$ experimental bound constrains $\kappa_V < 1.36$ and $\kappa_f < 1.49$.
- The like-sign WWjj bound will strengthen the constraint.

Constraint results





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[Hartling, Kumar & Logan, 1410.5538]

- **GM can accommodate simultaneous enhancement of κ_v and κ_f .**
- Simultaneous enhancement requires $\kappa_v \approx \kappa_f < 1.18$ from the loose constraint (1.09 for tight).

Summary and outlook

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- **Summary**
  - Beyond doublets → interesting phenomenology and benchmarks!
  - The Georgi-Machacek model can enhance Higgs couplings to VV.
- **Outlook**
  - Still a lot of work to be done beyond doublets!
  - Georgi-Machacek:
    - Calculator and FeynRules file under development.
    - Constraints from direct searches.
    - Collider studies of effect of enhanced couplings.
  - Other models: septet, generalized GM, etc.

Extra Slides

Scalar potential

$$V(\Phi, X) = \frac{\mu_2^2}{2}\mathrm{Tr}(\Phi^\dagger\Phi) + \frac{\mu_3^2}{2}\mathrm{Tr}(X^\dagger X) + \lambda_1[\mathrm{Tr}(\Phi^\dagger\Phi)]^2 + \lambda_2\mathrm{Tr}(\Phi^\dagger\Phi)\mathrm{Tr}(X^\dagger X)$$
$$+ \lambda_3\mathrm{Tr}(X^\dagger X X^\dagger X) + \lambda_4[\mathrm{Tr}(X^\dagger X)]^2 - \lambda_5\mathrm{Tr}(\Phi^\dagger\tau^a\Phi\tau^b)\mathrm{Tr}(X^\dagger t^a X t^b)$$
$$- M_1\mathrm{Tr}(\Phi^\dagger\tau^a\Phi\tau^b)(UXU^\dagger)_{ab} - M_2\mathrm{Tr}(X^\dagger t^a X t^b)(UXU^\dagger)_{ab}$$

Here $\tau^a = \sigma^a/2$ with σ^a being the Pauli matrices, and

$$t^1 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad t^2 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \quad t^3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix},$$

The matrix U rotates X into the Cartesian basis, and is given by

$$U = \begin{pmatrix} -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ -\frac{i}{\sqrt{2}} & 0 & -\frac{i}{\sqrt{2}} \\ 0 & 1 & 0 \end{pmatrix}.$$

Scalar potential

$$
\begin{aligned}
V(\Phi, X) = &\frac{\mu_2^2}{2}\mathrm{Tr}(\Phi^\dagger\Phi) + \frac{\mu_3^2}{2}\mathrm{Tr}(X^\dagger X) + \lambda_1[\mathrm{Tr}(\Phi^\dagger\Phi)]^2 + \lambda_2\mathrm{Tr}(\Phi^\dagger\Phi)\mathrm{Tr}(X^\dagger X) \\
&+ \lambda_3\mathrm{Tr}(X^\dagger X X^\dagger X) + \lambda_4[\mathrm{Tr}(X^\dagger X)]^2 - \lambda_5\mathrm{Tr}(\Phi^\dagger\tau^a\Phi\tau^b)\mathrm{Tr}(X^\dagger t^a X t^b) \\
&- M_1\mathrm{Tr}(\Phi^\dagger\tau^a\Phi\tau^b)(UXU^\dagger)_{ab} - M_2\mathrm{Tr}(X^\dagger t^a X t^b)(UXU^\dagger)_{ab}
\end{aligned}
$$

Constraint	Sets parameter
$v = 246$ GeV	v_ϕ
$\partial V/\partial v_\chi = 0$	v_χ
$\partial V/\partial v_\phi = 0$	μ_2
$m_h = 125$ GeV	λ_1

Free parameters: λ_2, λ_3, λ_4, λ_5, M_1, M_2 and mass scale μ_3.

Particle mixing

Goldstone boson, triplet and fiveplet states ($c_H \equiv \cos\theta_H = \frac{v_\phi}{v}$):

$$G^+ = c_H \phi^+ + s_H \frac{(\chi^+ + \xi^+)}{\sqrt{2}}$$

$$G^0 = c_H \phi^{0,i} + s_H \chi^{0,i}$$

$$H_3^+ = -s_H \phi^+ + c_H \frac{(\chi^+ + \xi^+)}{\sqrt{2}}$$

$$H_3^0 = -s_H \phi^{0,i} + c_H \chi^{0,i}$$

$$H_5^{++} = \chi^{++}$$

$$H_5^+ = \frac{(\chi^+ - \xi^+)}{\sqrt{2}}$$

$$H_5^0 = \sqrt{\frac{2}{3}}\xi^0 - \sqrt{\frac{1}{3}}\chi^{0,r}$$

The two custodial SU(2) singlets:

$$H_1^0 = \phi^{0,r} \qquad H_1^{0\prime} = \sqrt{\frac{1}{3}}\xi^0 + \sqrt{\frac{2}{3}}\chi^{0,r}.$$

These states mix by an angle α to form the singlets h and H:

$$h = \cos\alpha\, H_1^0 - \sin\alpha\, H_1^{0\prime} \qquad H = \sin\alpha\, H_1^0 + \cos\alpha\, H_1^{0\prime}$$

Theoretical constraints

Requiring unitarity and V bounded-from-below constrains λ_i:

$$\lambda_1 \in \left(0, \frac{1}{3}\pi\right) \simeq (0, 1.05) \qquad \text{(Unitarity, BFB)}$$

$$\lambda_2 \in \left(-\frac{2}{3}\pi, \frac{2}{3}\pi\right) \simeq (-2.09, 2.09) \qquad \text{(Unitarity)}$$

$$\lambda_3 \in \left(-\frac{1}{2}\pi, \frac{3}{5}\pi\right) \simeq (-1.57, 1.88) \qquad \text{(Unitarity, BFB)}$$

$$\lambda_4 \in \left(-\frac{1}{5}\pi, \frac{1}{2}\pi\right) \simeq (-0.628, 1.57) \qquad \text{(Unitarity, BFB)}$$

$$\lambda_5 \in \left(-\frac{8}{3}\pi, \frac{8}{3}\pi\right) \simeq (-8.38, 8.38) \qquad \text{(Unitarity, BFB)}$$

$$|M_1|/\sqrt{\mu_3^2} \lesssim 3.3 \qquad (\lambda_1 \text{ Unitarity})$$

$$|M_2|/\sqrt{\mu_3^2} \lesssim 1.2 \qquad (8\, v_\chi^2 \leq v^2)$$

Desired vacuum is a global minimum if λ_3, λ_5, M_1 and M_2 are > 0.

Otherwise must be checked numerically.

Decoupling

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M_i can scale with μ_3^n, but unitarity bound on λ_1 constrains $n \leq 1$.

$$\lambda_1 \approx \frac{1}{8}\left[\frac{m_h^2}{v^2} + \frac{3}{4}\frac{M_1^2}{\mu_3^2}\left(1 - 3(2\lambda_2 - \lambda_5)\frac{v^2}{\mu_3^2} + \frac{3M_1M_2v^2}{\mu_3^4} + \frac{5m_h^2}{3\mu_3^2}\right)\right]$$

implies $|M_1|/\sqrt{\mu_3^2} \lesssim 3.3$

Sensible minimum of the potential implies $|M_2|/\sqrt{\mu_3^2} \lesssim 1.2$.

Georgi-Machacek couplings in the decoupling limit

$$\kappa_V = \cos\alpha\frac{v_\phi}{v} - \frac{8}{\sqrt{3}}\sin\alpha\frac{v_\chi}{v} \simeq 1 + \frac{3}{8}\frac{M_1^2v^2}{\mu_3^4}$$

$$\kappa_f = \cos\alpha\frac{v}{v_\phi} \simeq 1 - \frac{1}{8}\frac{M_1^2v^2}{\mu_3^4}$$

2HDM couplings in the decoupling limit

$$\kappa_V^{2\mathrm{HDM}} \simeq 1 - \frac{\hat{\lambda}^2v^4}{2\,m_A^4},$$

$$\kappa_f^{2\mathrm{HDM}} \simeq 1 + \frac{\hat{\lambda}v^2}{m_A^2} \times \begin{cases} \cot\beta & \text{for up type fermions} \\ -\tan\beta & \text{for down type fermions.} \end{cases}$$