

TEST OF OPTICAL STOCHASTIC COOLING AT FERMILAB

Valeri Lebedev
Fermilab

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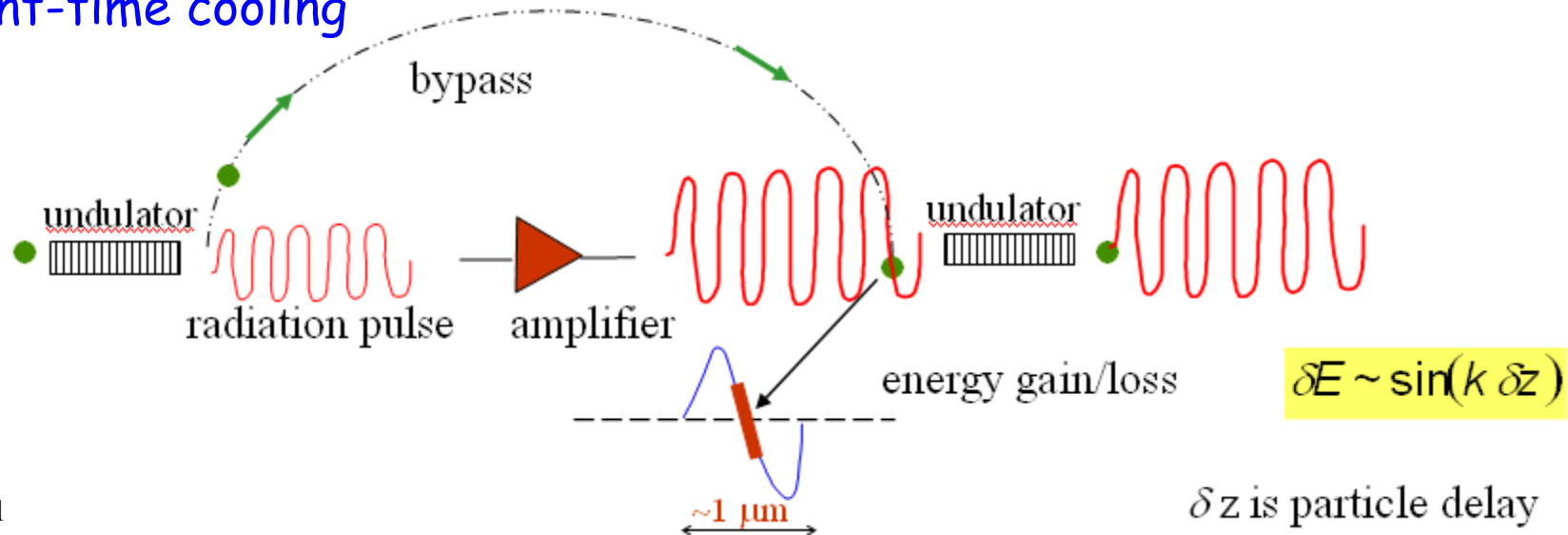
- Basics of Optical Stochastic Cooling
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Basics of Optical Stochastic Cooling

- The damping rate of stochastic cooling:

$$\lambda_{opt} \approx \frac{2\pi^2 W}{N n_\sigma^2} \frac{\sqrt{\pi} \sigma_s}{C}, \quad \begin{cases} W = f_{\max} - f_{\min}, & \text{for Rectangular gain shape} \\ W = 2\sqrt{\pi} \sigma_f, & \text{for Gaussian gain shape} \end{cases}$$

- OSC was suggested by Zolotarev, Zholents and Mikhailichenko (1994)
- Transition from the microwave SC to OSC increases the bandwidth by about 3 orders of magnitude ($\lambda \rightarrow 10^{-4} \lambda$, $\Delta f/f = 50\% \rightarrow \Delta f/f = 10\%$)
- Pickup and kicker must work in the optical range and support the same bandwidth as the amplifier
 - ◆ Undulators were suggested for both pickup and kicker
 - ◆ Bandwidth: OA bandwidth & inverse number of wiggles
 - ◆ Transient-time cooling



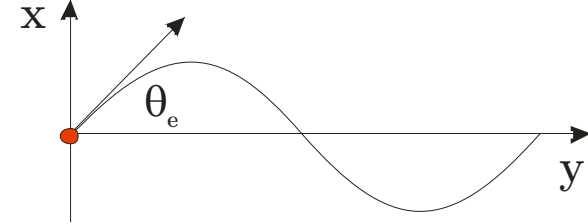
Basics of Optical Stochastic Cooling (continue)

- OSC can operate only with ultra-relativistic particles

- ◆ Slow particles do not radiate at optical frequencies

- Radiation wave length

$$\lambda = \frac{\lambda_{wgl}}{2\gamma^2} \left(1 + \gamma^2 \left(\frac{1}{2} \theta_e^2 + \theta^2 \right) \right) - \text{flat undulator}$$



- Radiation is concentrated in the angle $1/\gamma$

- Correction signal is proportional to a longitudinal position change on the travel from pickup to kicker

- Only longitudinal kicks are effective

- ◆ Requires s-x coupling for hor. cooling and x-y coupling for vert. cooling

- Non-zero dispersion in OSC pickup introduces difference between

M_{56} and partial slip-factor (\tilde{M}_{56}), and, consequently x-s coupling

⇒ Cooling rates (per turn)

$$\frac{\delta p}{p} = \xi_0 \sin(k \delta s), \quad \delta s = \tilde{M}_{56} \frac{\Delta p}{p}$$

$$\lambda_x = \frac{\xi_0}{2} k (M_{56} - \tilde{M}_{56})$$

$$\lambda_s = \frac{\xi_0}{2} k \tilde{M}_{56}$$

⇒

$$\lambda_x + \lambda_s = \frac{\xi_0}{2} k M_{56}$$

Basics of OSC - Cooling Range

- Cooling force depends on Δs nonlinearly

$$\frac{\delta p}{p} = k \xi_0 \Delta s \Rightarrow \frac{\delta p}{p} = \xi_0 \sin(k \delta s)$$

where $k \delta s = a_x \sin(\psi_x) + a_p \sin(\psi_p)$

and a_x & a_p are the amplitudes of longitudinal displacements in cooling chicane due to \perp and L motions measured in units of laser phase

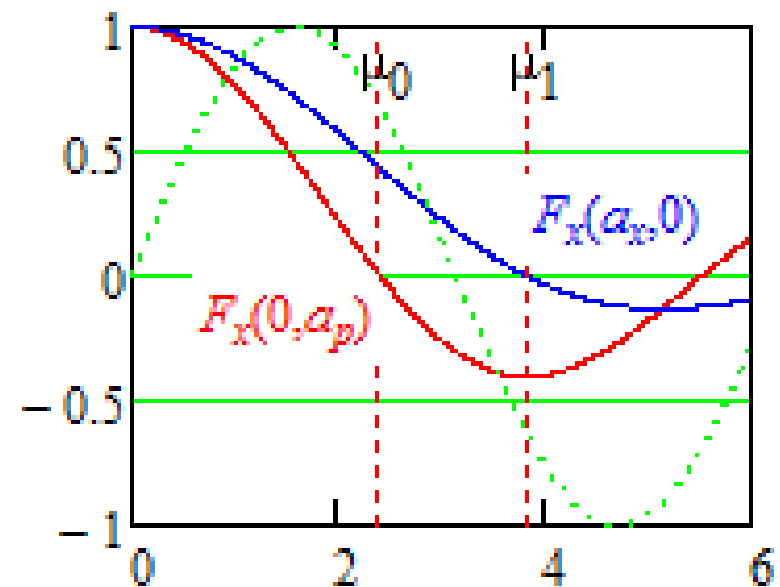
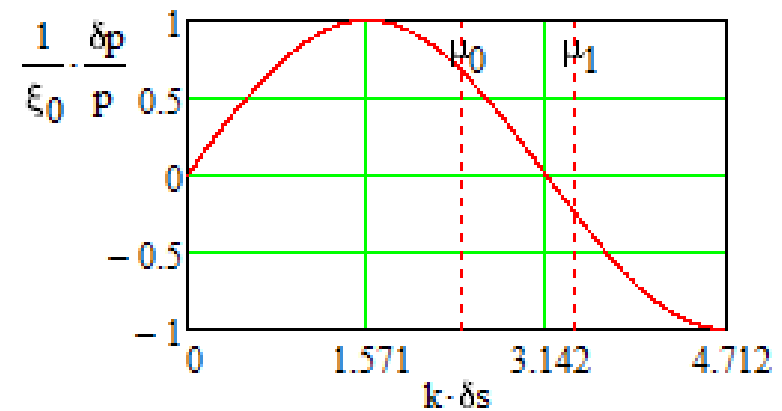
- Averaging yields the form-factors for damping rates

$$\lambda_{s,x}(a_x, a_p) = F_{s,x}(a_x, a_p) \lambda_{s,x}$$

$$F_x(a_x, a_p) = \frac{2}{a_x} J_0(a_p) J_1(a_x)$$

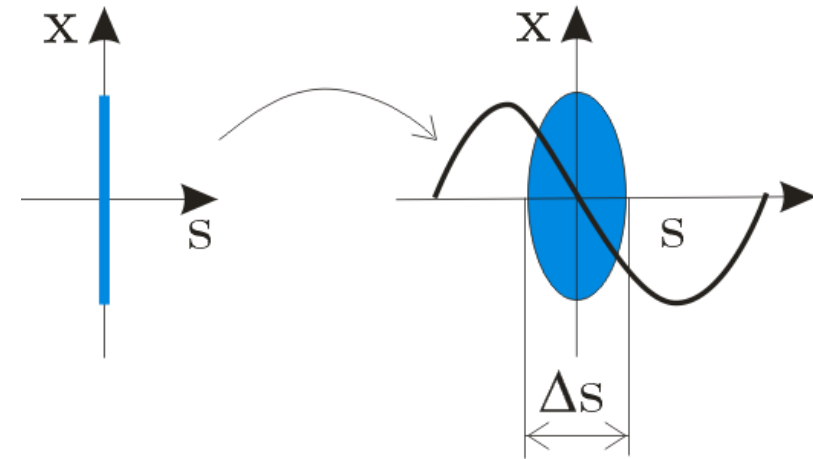
$$F_p(a_x, a_p) = \frac{2}{a_p} J_0(a_x) J_1(a_p)$$

- Damping requires both lengthening amplitudes (a_x and a_p) to be smaller than $\mu_0 \approx 2.405$



Basics of OSC - Sample Lengthening

- On the way from pickup to kicker a zero length sample lengthens on its way from pickup-to-kicker
 - ◆ Both $\Delta p/p$ and ε contribute to the lengthening



$$\sigma_{\Delta s}^2 = \sigma_{\Delta s \varepsilon}^2 + \sigma_{\Delta s p}^2$$

$$\sigma_{\Delta s \varepsilon}^2 = \varepsilon \left(\beta_p M_{51}^2 - 2\alpha_p M_{51} M_{52} + \gamma_p M_{52}^2 \right)$$

$$\sigma_{\Delta s p}^2 = \sigma_p^2 \tilde{M}_{56}^2$$

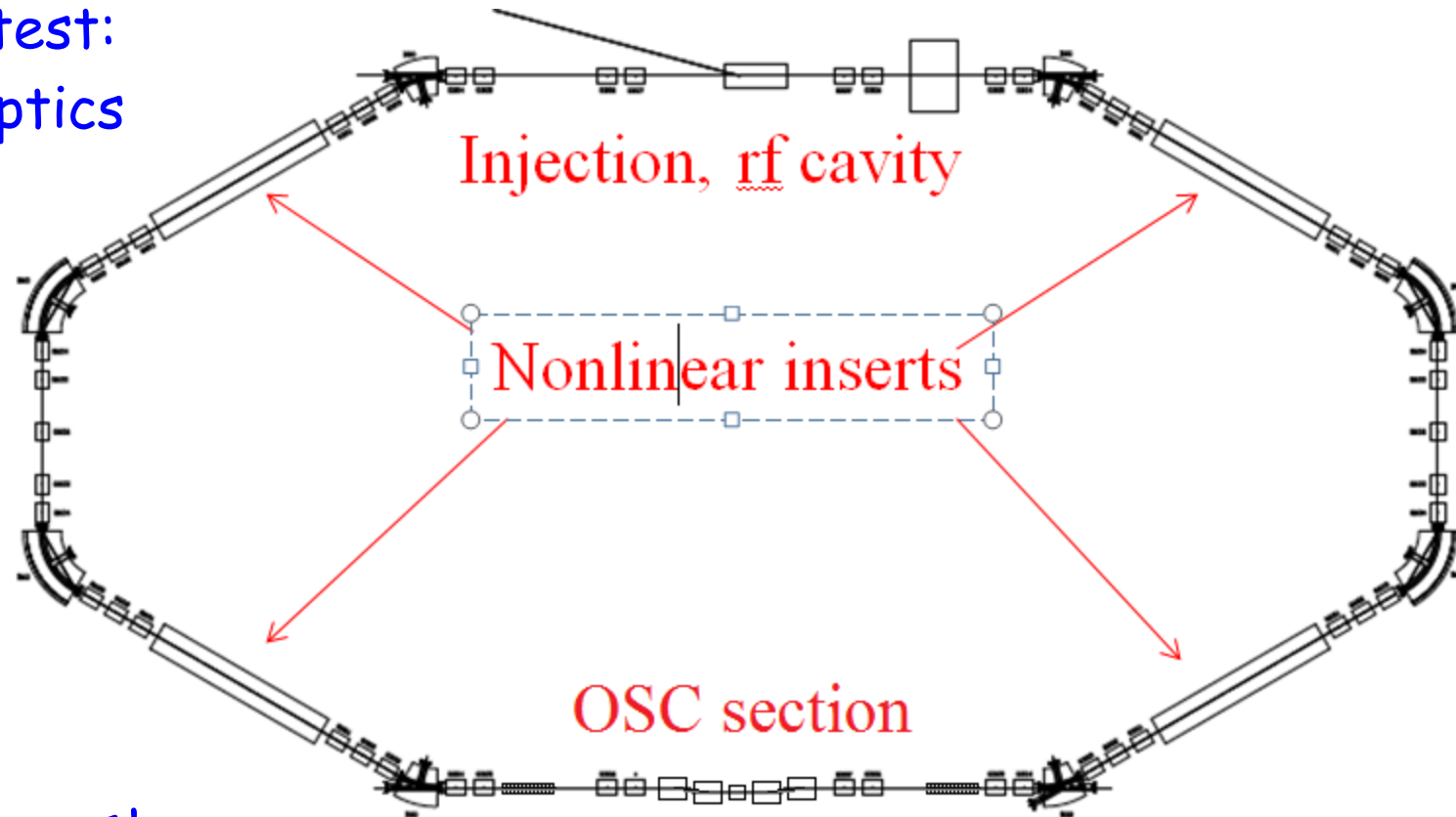
where $\tilde{M}_{56} \equiv M_{51} D_p + M_{52} D'_p + M_{56}$

- While in linear approximation β_p and α_p do not affect damping rates they affect sample lengthening due to beam horizontal emittance and, consequently, the horizontal cooling range

$$\begin{aligned} n_{\sigma \varepsilon} \sigma_{\Delta s \varepsilon} k &\leq \mu_0 \\ n_{\sigma p} \sigma_{\Delta s p} k &\leq \mu_0 \end{aligned} \quad \mu_0 \approx 2.405$$

Test of OSC in Fermilab

- First attempt to test the OSC in BATES, ~2007
 - ◆ Existing electron synchrotron
 - ◆ Did not get sufficient support
- Presently Fermilab is constructing a dual purpose small electron ring called IOTA to test:
 - ◆ Integrable optics
 - ◆ OSC
- Part of ASTA program
 - ◆ Full energy injection from SC linac
- Test in a small electron ring is a cost effective way to test the OSC

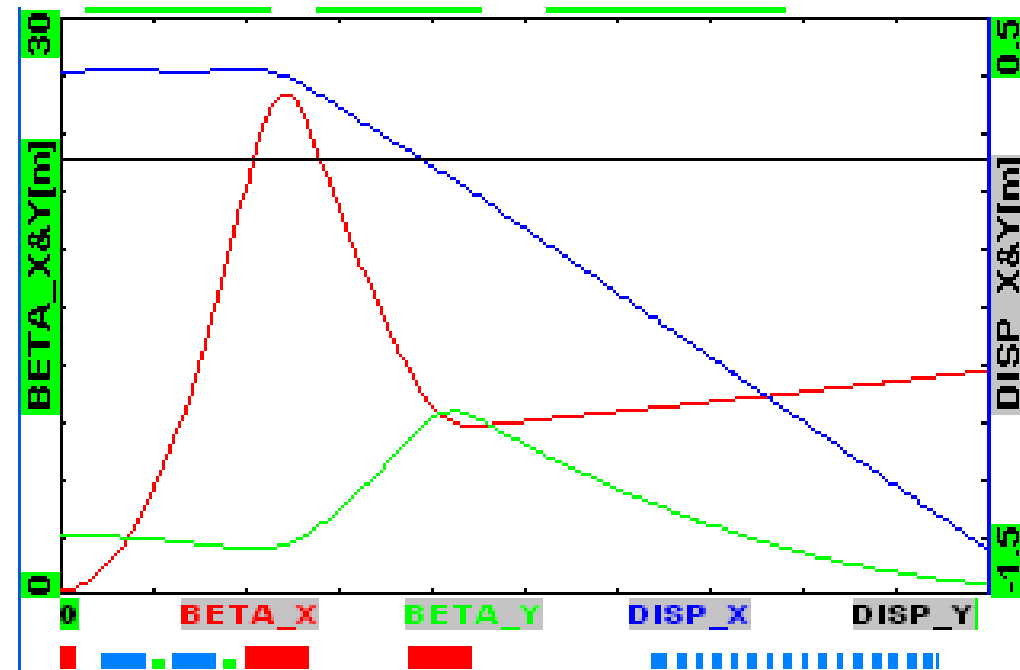
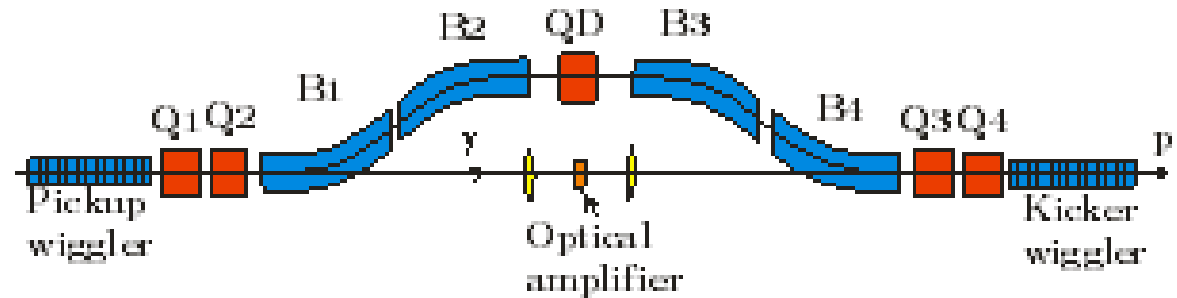


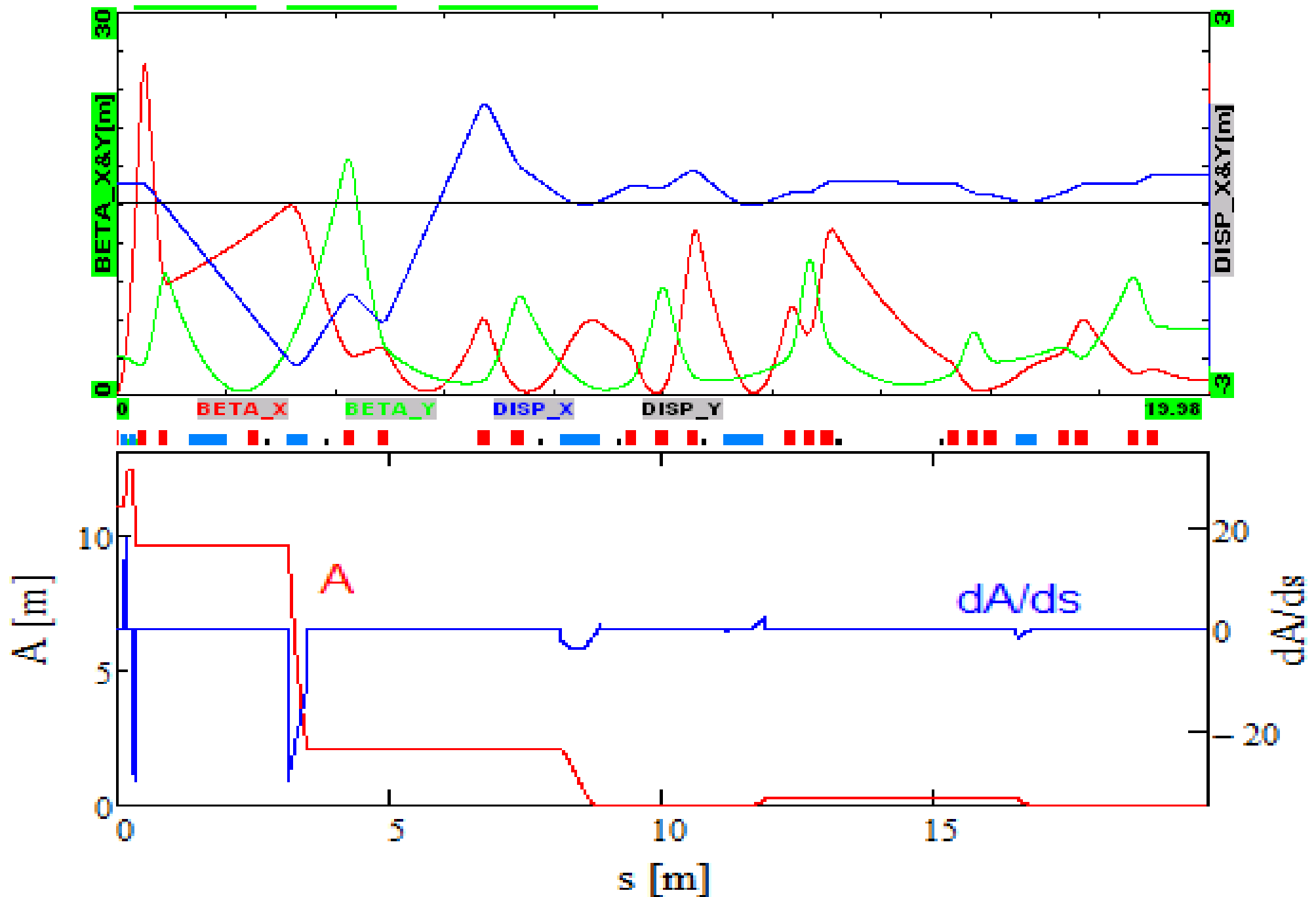
OSC Limitations on IOTA Optics

- Delay in OA amplifier determines delay in the chicane (Δs) $\Rightarrow M_{56} \approx 2\Delta s$
- D quad in the center and non-zero dispersion introduce xs-coupling: $\tilde{M}_{56} \approx M_{56} - \Phi D^* h$
 h - orbit offset in the chicane
- Sample lengthening minimization due to betat. motion requires collider type optics with small β^*
 \Rightarrow large dispersion invariant

$$A^* = D^{*2} / \beta^*$$

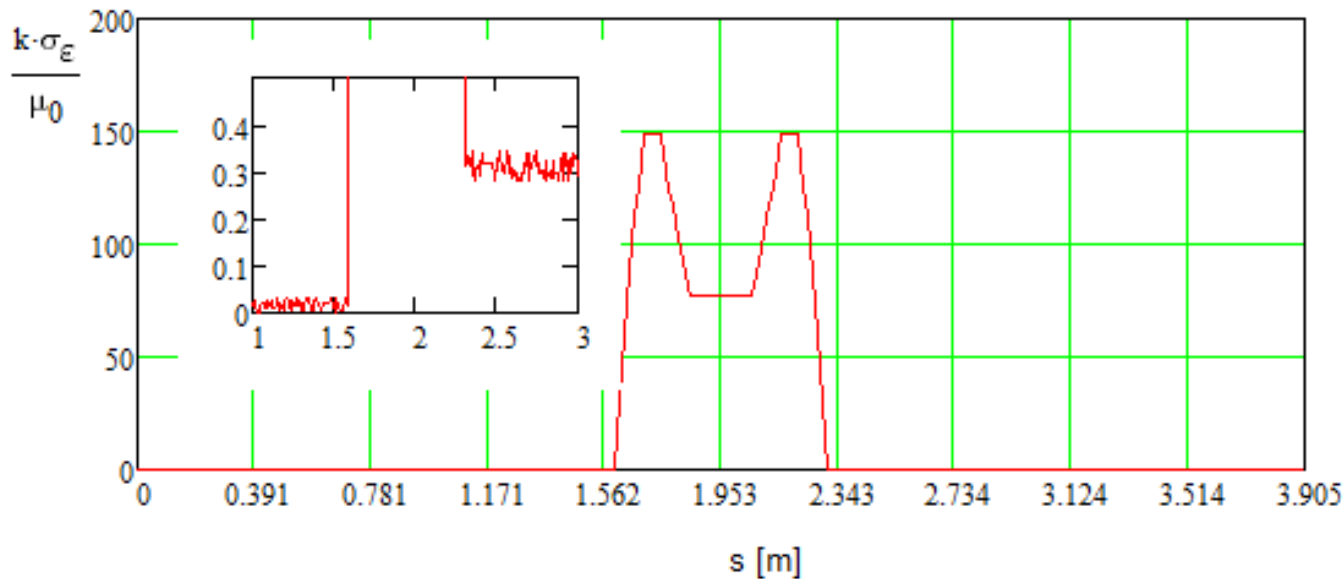
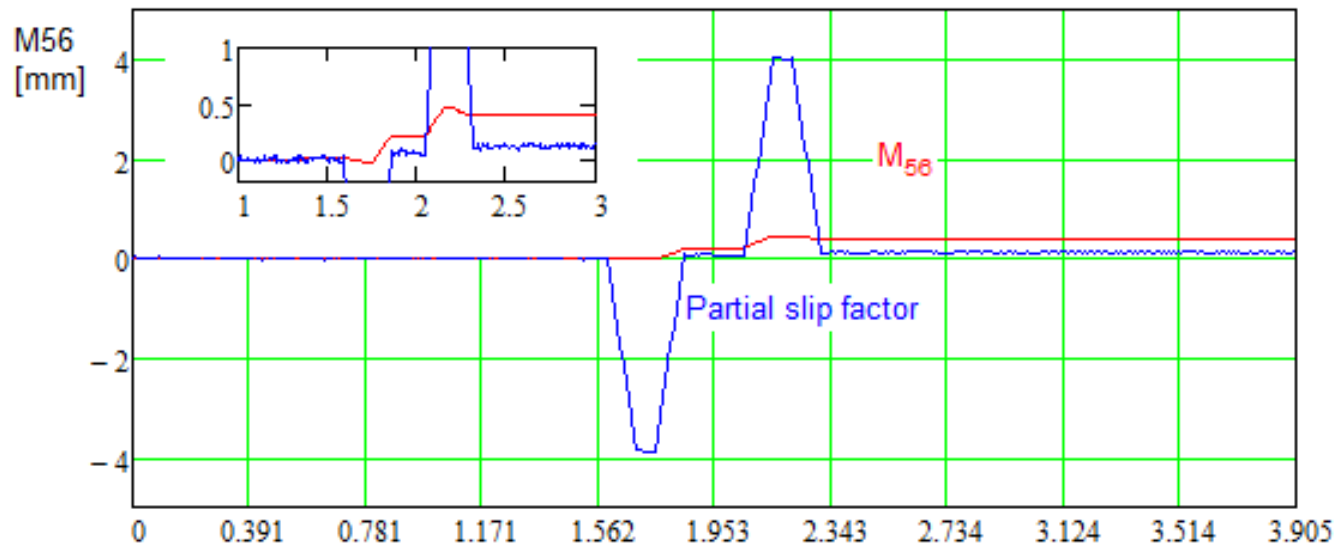
- An average value of A in dipoles determines the equilibrium emittance.
 - ◆ A^* is large and A needs to be reduced fast to get an acceptable value of the equilibrium emittance





Optics functions and dispersion invariant for IOTA half ring

Sample Lengthening on the Travel through Chicane



*Sample lengthening due to momentum spread (top)
and due to betatron motion (bottom)*

- Very large sample lengthening on the travel through chicane
- High accuracy of dipole field is required to prevent uncontrolled lengthening,
 $\Delta(BL)/(BL)_{\text{dipole}} < 10^{-3}$

Second Order Contributions to Sample Lengthening

- Linear part of long. displacement due to bet. motion:

$$\Delta L_{\max X} = (a_x / \sigma_x) \times 97 \text{ nm}$$

- Major non-linear contribution comes from angle: $\Delta L = \int (\theta^2(s) / 2) ds$
- Integration over trajectory yields:

$$I_1 = \int (1 + \alpha^2) d\mu$$

$$\Delta L = \frac{\varepsilon}{4} (I_1 + (I_2 + I_3) \cos(2\psi) + (I_4 + I_5) \cos(2\psi)), \quad I_2 = 2 \int \alpha \sin(2\mu) d\mu, \quad I_3 = - \int (1 - \alpha^2) \cos(2\mu) d\mu$$

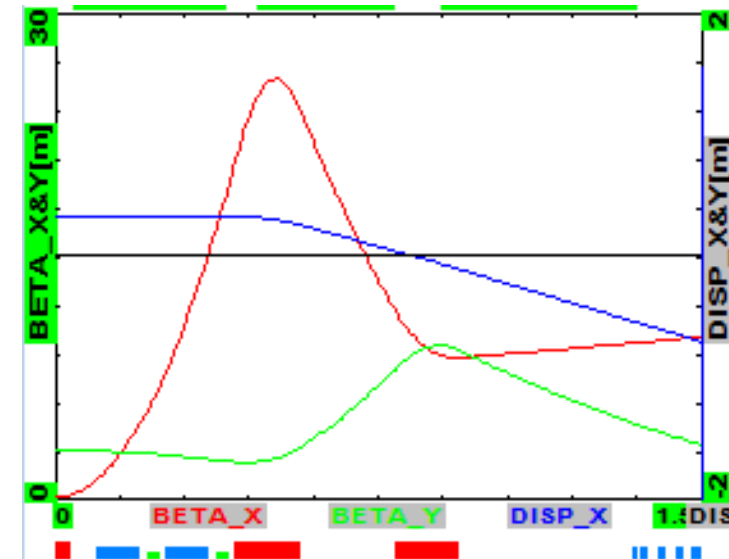
$$I_4 = 2 \int \alpha \cos(2\mu) d\mu, \quad I_5 = \int (1 - \alpha^2) \sin(2\mu) d\mu$$

ψ - the phase of betatron motion

- Performing numerical integration from pickup to kicker results in:

$$\Delta L_{\max X} = \left(\frac{a_x}{\sigma_x} \right)^2 \times 690 \text{ nm}, \quad \Delta L_{\max Y} = \left(\frac{a_y}{\sigma_y} \right)^2 \times 140 \text{ nm}$$

- These values are too large and need to be compensated
- Sextupole correction
 - ◆ 1-st Sext - correction ΔL
 - ◆ 2-nd sext - major correction non-linearity
 - ◆ ... correction of chromaticity & RDT - work in progress (next presentation)



IOTA Optics and Parameters

Main Parameters of IOTA storage ring for OSC

Circumference	40 m
Nominal beam energy	100 MeV
Bending field	4.79 kG
Transverse emittances, $\varepsilon = \varepsilon_x = \varepsilon_y$, rms	11 nm
Rms momentum spread, σ_p	$1.21 \cdot 10^{-4}$
SR damping times (ampl.), $\tau_s / (\tau_x = \tau_y)$	1.36 / 1.58 s

Main parameters of cooling chicane

Delay in the chicane, Δs	2 mm
Horizontal beam offset, h	20.1 mm
M_{56}	3.95 mm
D^* / β^*	307 mm / 8.59 mm
Cooling rates ratio, $(\lambda_x = \lambda_y) / \lambda_s$	1.18
Cooling ranges (before OSC), n_{ox} / n_{os}	2.1 / 3.2
Dipole: magnetic field * length	4.22 kG * 10 cm
Strength of central quad, GdL	1.58 kG

- Energy is reduced 150→100 MeV to reduce ε , σ_p and undulator period and length
- Operation on coupling resonance $Q_x/Q_y = 5.83/3.83$ reduces horizontal emittance and introduces vertical OSC damping

- Small β^* is required to minimize sample lengthening due betatron motion

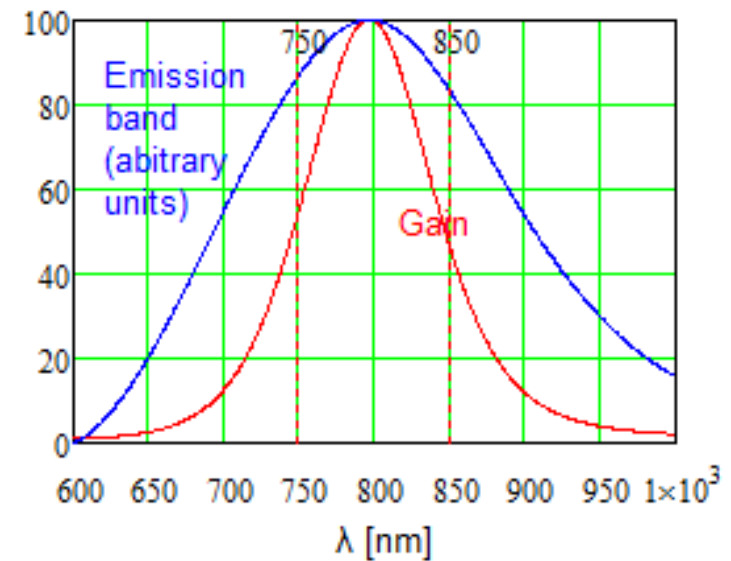
RF, Touschek, IBS and Gas scattering

Slip-factor	-0.065
RF harmonic	4
RF voltage	100 V
SR loss	14 V/turn
RF bucket height	$1.4 \cdot 10^{-3}$
Rms bunch length,	25 cm
Number of particles	$2.5 \cdot 10^6$
Emittance growth rate due to SR (H)	36 nm/s
Emittance growth rate due to IBS (H) ($\varepsilon_x = \varepsilon_y$)	3.6 nm/s
Growth rate for $(\Delta p/p)^2$ due to SR	$2.1 \cdot 10^{-8}$
Growth rate for $(\Delta p/p)^2$ due to IBS ($\varepsilon_x = \varepsilon_y$)	$2.7 \cdot 10^{-9}$
Touschek lifetime (set by bucket height)	4.3 hour
Machine acceptance (set by dynamic aperture)	1 μ m
Average vacuum (H_2 equivalent)	$1.5 \cdot 10^{-10}$
Emittance growth due to gas scattering (H/V)	2.5/1.8 nm
Gas scattering lifetime	17 min.

- Number of particles per bunch is set so that IBS would be ~10% of growth rates set by SR
- Very good vacuum is required to support good lifetime in the presence of strong limitation of dynamic aperture

Optical Amplifier

- Ti:Sapphire Optical Amplifier has a few advantages
 - ◆ Quite wide bandwidth
 - 10% FWHM at $G_0=100$
 - ◆ Allows operation in the CW regime
 - Decay time due to sp. rad. $\sim 3.15 \mu\text{s}$
 - ◆ Can deliver significant amplification with only $\sim 1 \text{ mm}$ signal delay.
- We bought a highly doped (0.5%wt Ti_2O_3) **2 mm thick** Ti: Sapphire crystal from GT Crystal Systems for a prototype of OA
- An estimated low power gain is ~ 100 (20 Db) with pumping power density of 1.8 MW/cm^2
- Pumping along the direction of amplified radiation
 - ◆ $P = 50 \text{ W}$, square profile with $r = 30 \mu\text{m}$
- Cooling the OA to the liquid nitrogen temperature is required.
 - ◆ It increases the crystal thermal conductivity
 - \Rightarrow an acceptable ΔT across the crystal ($\sim 8\text{K}$) and thermal stress
 - ◆ It reduces $dn/dT \Rightarrow$
 - reduces optics distortions related to high pumping power



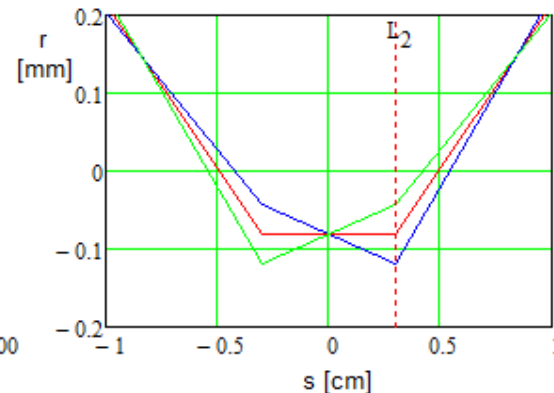
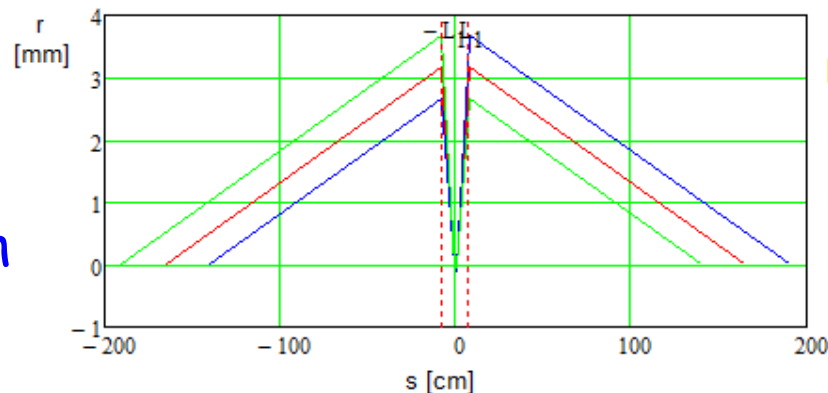
Focusing of Beam Radiation to OA and Kicker

■ Two lens system ($F=8$ cm, radius - 3.5 mm)

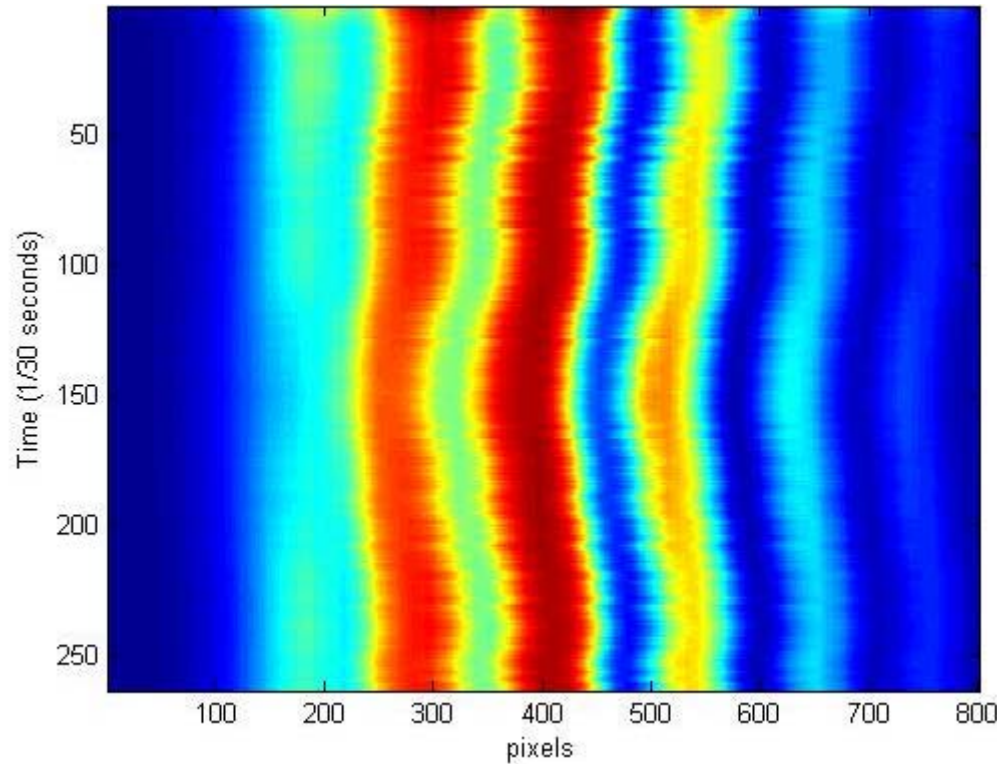
- ◆ Reasonable compromise between major requirements
- ◆ The spot size in OA to be sufficiently small: $r < 30 \mu\text{m}$
 - diffraction limited size in OA: $\text{HWHM}=6 \mu\text{m}$ or total size $r \approx 15 \mu\text{m}$
 - size due to beam convergence/divergence at OA input/exit $\approx 25 \mu\text{m}$
- ◆ Requirements to suppress Depth of field effects in kicker wiggler
 - diffraction limited size in kicker wiggler: $\text{HWHM}=120 \mu\text{m}$ or total size $r \approx 300 \mu\text{m}$
 - Size increase due to the depth of field for radiation radiated at the entrance or exit of pickup wiggler: $170 \mu\text{m}$
- ◆ To mitigate the depth of field effects the wigglers are moved from the chicane by ~ 50 cm

■ For OSC tests without OA the 4 lens telescope will be used

- ◆ Complete suppression of depth of field
- ◆ Larger bandwidth



Test of Optical Amplifier Prototype (continue)



Interference picture displacements on time[#]

■ Interferometer is assembled

- ◆ first tests started
- ◆ Working on
 - the stabilization of interference picture
 - electronics to measure displacement of interference pattern with wave-length change

Courtesy of Matt Andorf

Cooling Rates

- Undulator period was chosen so that $\lambda|_{\theta=0}=750 \text{ nm}$
- Cooling rates were computed using earlier developed formulas(HB2012)
 - ◆ Averaging over amplifier band yielded additionally ~20% reduction of rates.
- 2 mrad angular acceptance of optical system (aperture $r=3.5 \text{ mm}$)
 - \Rightarrow upper boundary of the band = 850 nm
- E.-m. wave dispersion in the OA amplifier is included into the gain
 - ◆ $G = 10$ implies an amplitude amplification of 10
 - \Rightarrow Dispersion makes the power gain to be somewhat larger than G^2 .
- Undulator parameter $K=0.6$ is close to the optimal for chosen bandwidth and aperture

Main parameters of OSC

Undulator parameter, K	0.6
Undulator period	4.92 cm
Radiation wavelength at zero angle	750 nm
Number of periods, m	10
Total undulator length, L_w	0.50 m
Length from OA to undulator center	1.65 m
Amplifier gain (amplitude)	10
Telescope aperture, $2a$	7 mm
Lens focal length, F	80 mm
Damp. rates ($x=y/s$)	160/140 s^{-1}

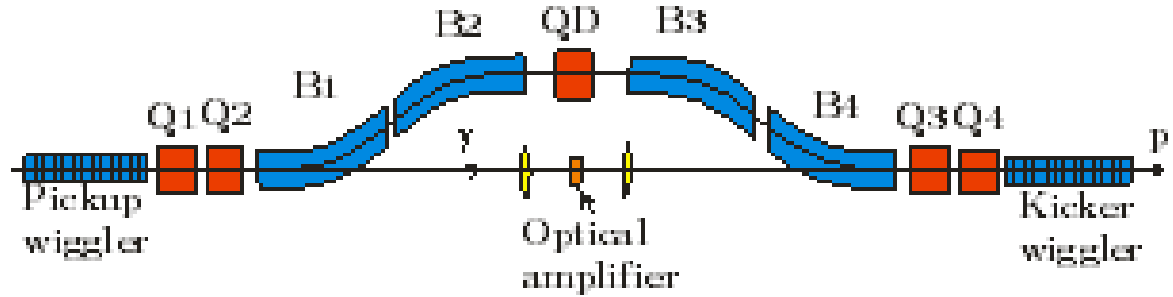
Conclusions

- Optical stochastic cooling looks as a promising technique for future hadron colliders
- Experimental study of OSC in Fermilab is in its initial phase
 - ◆ It is aimed to validate cooling principles and to demonstrate cooling with and without optical amplifier
 - Even in the absence of amplification (passive system, $G = 1$) the OSC damping exceeds SR damping by more than an order of magnitude
- The beam intensity ranges from a single electron to the bunch population limited by operation at the optimum gain (10^8)
 - ◆ Single electron cooling - localization of electron wave function and essence of quantum mechanics
 - Quantum noise for passive cooling
 - ◆ Cooling at the optimal gain (ultimate cooling) gets us to otherwise hidden details of OSC, in particular, to signal suppression

Backup Slides

OSC Limitations on IOTA Optics

- In the first approximation the orbit offset in the chicane (h), the path lengthening (Δs) and the defocusing strength of chicane quad (Φ) together with dispersion and beta-function in the chicane center (D^*, β^*) determine the entire cooling dynamics
- Δs is set by delay in the amplifier $\Rightarrow M_{56}$
- $\Phi D^* h$ is determined by the ratio of decrements \Rightarrow for known ε we obtain the dispersion invariant (A^*)
- An average value of A in dipoles determines the equilibrium emittance. A^* is large and A needs to be reduced fast to get an acceptable value of the emittance (ε)



$$M_{56} \approx 2\Delta s,$$

$$\tilde{M}_{56} \approx 2\Delta s - \Phi D^* h,$$

$$\lambda_x / \lambda_s \approx \Phi D^* h / (2\Delta s - \Phi D^* h),$$

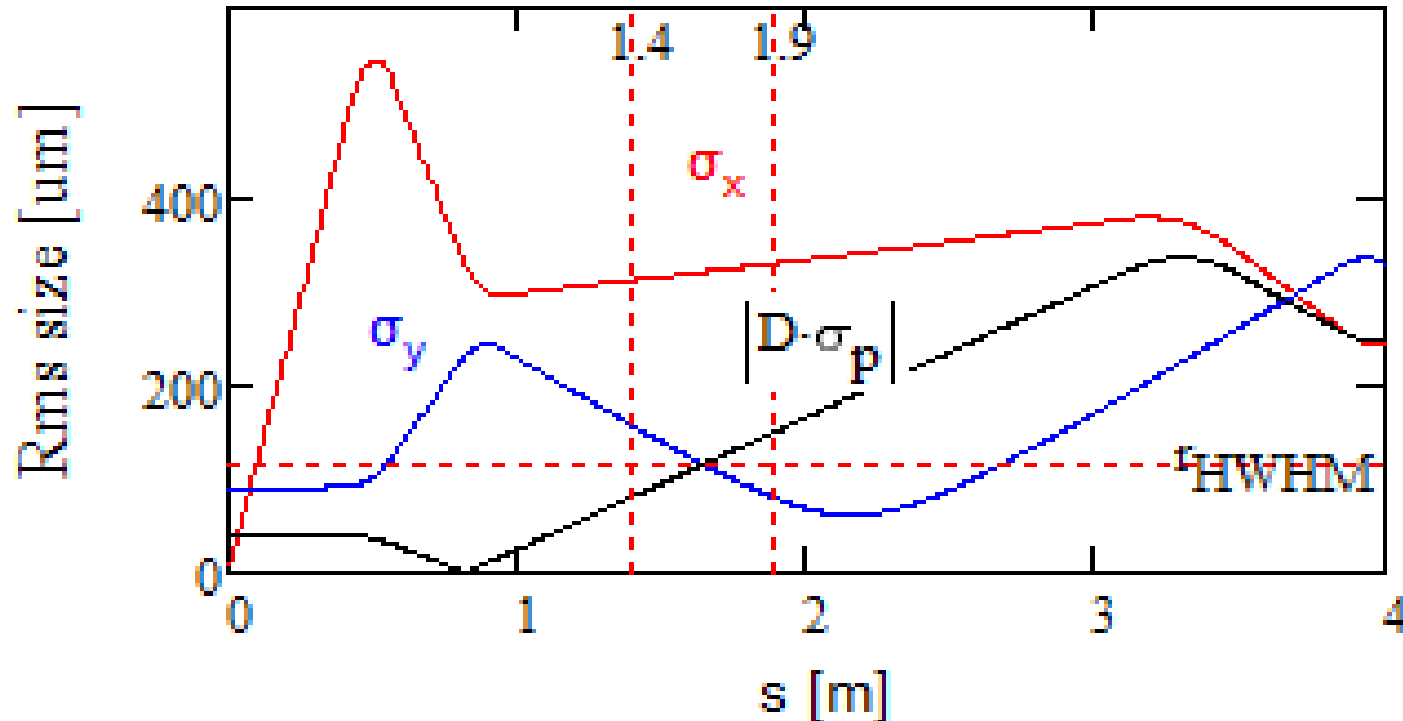
$$n_{\sigma p} \approx \mu_0 / \left((2\Delta s - \Phi D^* h) k \sigma_p \right),$$

$$n_{\sigma x} \approx \mu_0 / \left(2kh\Phi \sqrt{\varepsilon \beta^*} \right),$$

$$\Rightarrow \Phi D^* h \approx \frac{\mu_0}{2kn_{\sigma x}} \sqrt{\frac{A^*}{\varepsilon}}, \quad A^* \equiv \frac{D^{*2}}{\beta^*}$$

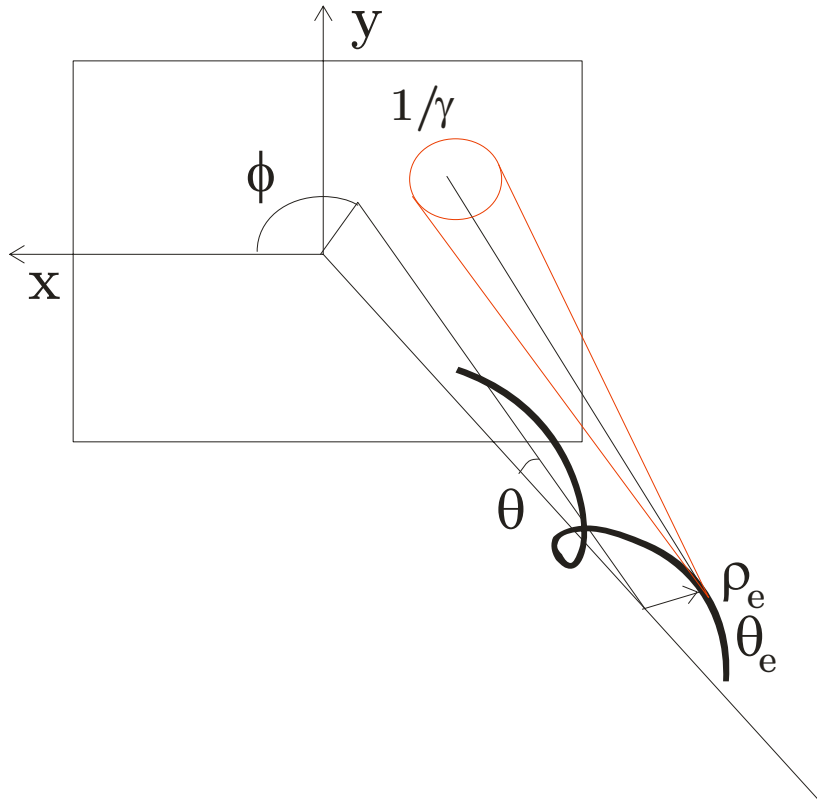
Effect of Beams Overlap on Cooling Rates

- In computation of cooling rates we neglected incomplete overlap of light and particle beams in the kicker undulator at the beginning of cooling process when the e-beam size is determined by SR.
- The problem is negligible for cooled beam
 - ◆ Factor of 5 reduction at the cooling beginning



Rms beam sizes (horizontal - σ_x , vertical - σ_y , and due to momentum spread - $|D\sigma_p|$) in vicinity of cooling chicane starting from the center of OSC section

Basics of OSC – Radiation from Undulator

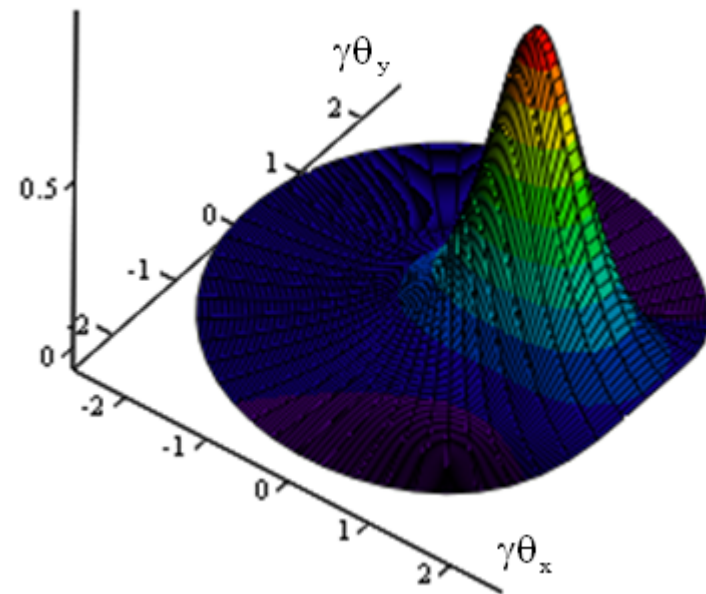


- Liénard-Wiechert potentials and E-field of moving charge in wave zone

$$\begin{cases} \varphi(\mathbf{r}, t) = \frac{e}{(R - \boldsymbol{\beta} \cdot \mathbf{R})} \Big|_{t-R/c} \\ \mathbf{A}(\mathbf{r}, t) = \frac{e\mathbf{v}}{(R - \boldsymbol{\beta} \cdot \mathbf{R})} \Big|_{t-R/c} \end{cases} \Rightarrow$$

$$\mathbf{E}(\mathbf{r}, t) = \frac{e}{c^2} \frac{(\mathbf{R} - \boldsymbol{\beta} \cdot \mathbf{R})(\mathbf{a} \cdot \mathbf{R}) - \mathbf{a}R(R - \boldsymbol{\beta} \cdot \mathbf{R})}{(R - \boldsymbol{\beta} \cdot \mathbf{R})^3} \Big|_{t-R/c}$$

where $\mathbf{a} \equiv \frac{d\mathbf{v}}{dt}$



E_x for K=1

- Radiation of ultra-relativistic particle is concentrated in $1/\gamma$ angle

- Undulator parameter:

$$K \equiv \gamma\theta_e = \frac{\lambda_{wgl}}{2\pi} \frac{eB_0}{mc^2}$$

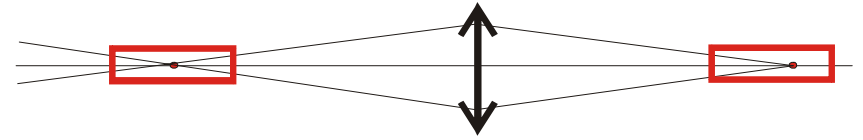
- For $K \geq 1$ the radiation is mainly radiated into higher harmonics

Basics of OSC – Radiation Focusing to Kicker Undulator

■ Modified Kirchhoff formula

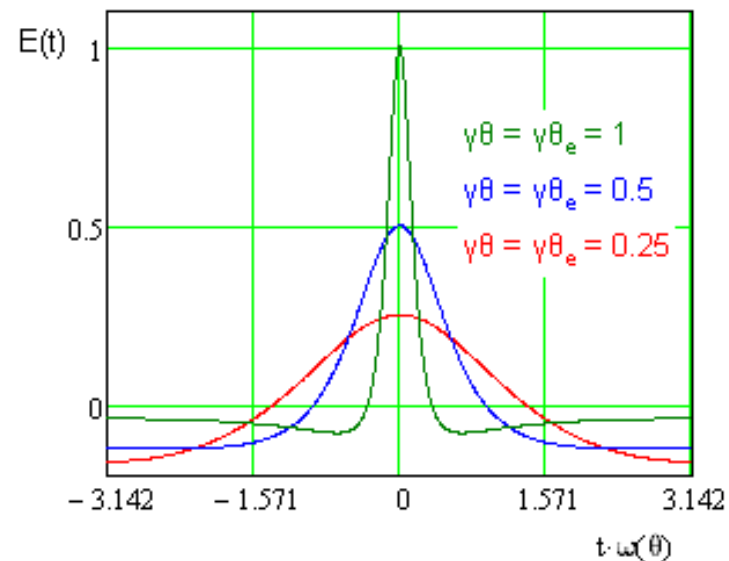
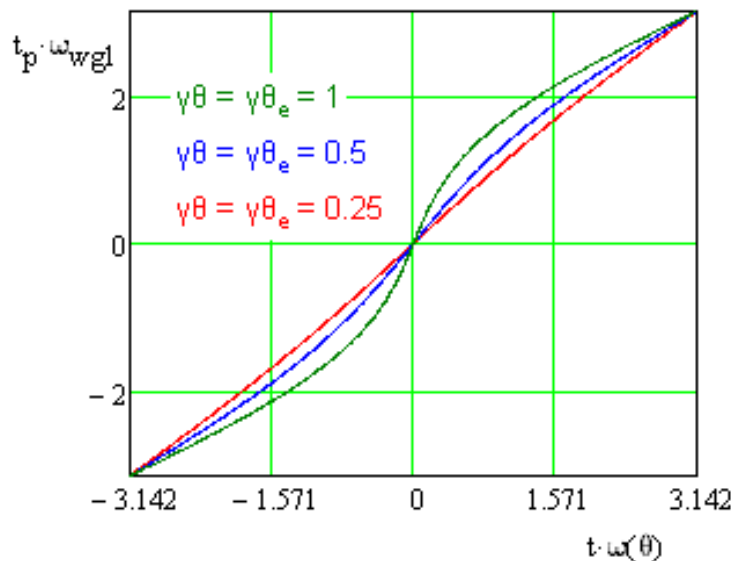
$$E(r) = \frac{\omega}{2\pi ic} \int_s \frac{E(r')}{|r - r'|} e^{i\omega|r - r'|} ds'$$

$$\Rightarrow E(r) = \frac{1}{2\pi ic} \int_s \frac{\omega(r') E(r')}{|r - r'|} e^{i\omega|r - r'|} ds'$$



■ Effect of higher harmonics

- ◆ Higher harmonics are normally located outside window of optical lens transparency and are absorbed in the lens material

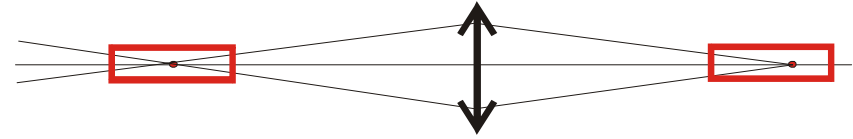


Dependences of retarded time (t_p) and E_x on time for helical undulator

■ Only first harmonic is retained in the calculations presented below

Basics of OSC – Longitudinal Kick for $K \ll 1$

- For $K \ll 1$ refocused radiation of pickup undulator has the same structure as radiation from kicker undulator. They are added coherently:

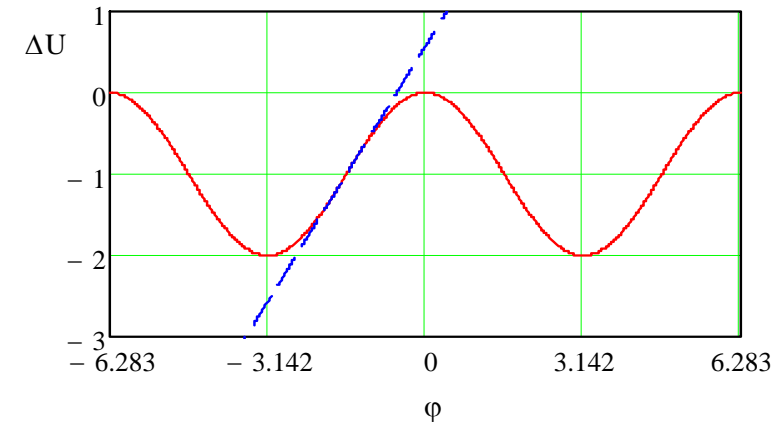


$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 e^{i\phi} \xrightarrow{\mathbf{E}_1 = \mathbf{E}_2} 2 \cos(\phi / 2) \mathbf{E}_1 e^{i\phi/2}$$

⇒ Energy loss after passing 2 undulators

$$\Delta U \propto |E^2| = 4 \cos^2(\phi / 2) |\mathbf{E}_1|^2 = 2(1 + \cos \phi) |\mathbf{E}_1|^2 = 2 \left(1 + \cos \left(kM_{56} \frac{\Delta p}{p} \right) \right) |\mathbf{E}_1|^2$$

- Large derivative of energy loss on momentum amplifies damping rates and creates a possibility to achieve damping without optical amplifier



- SR damping: $\lambda_{||-SR} \approx \frac{2\Delta U_{SR}}{pc} f_0$

- OSC: $\lambda_{||-OSC} \approx f_0 \frac{2\Delta U_{wgl}}{pc} (GkM_{56}) \xrightarrow{kM_{56}(\Delta p/p)_{\max} = \pi} f_0 \frac{2\Delta U_{wgl}}{pc} \left(\frac{G}{(\Delta p/p)_{\max}} \right)$

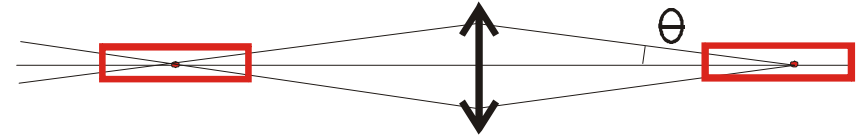
where G - optical amplifier gain, $(\Delta p/p)_{\max}$ - cooling system acceptance

⇒ $\lambda_{||-OSC} \propto B^2 L \propto K^2 L$ - but cooling efficiency drops with K increase above ~ 1

Basics of OSC – Longitudinal Kick for $K \ll 1$ (continue)

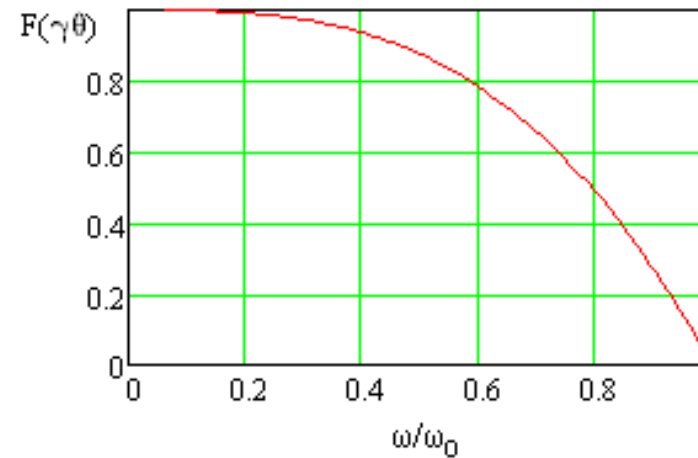
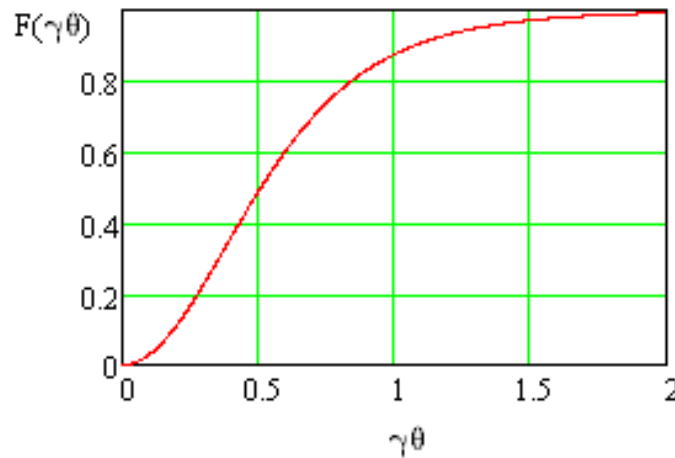
- Radiation wavelength depends on θ as

$$\lambda = \frac{\lambda}{2\gamma^2} (1 + \gamma^2 \theta^2)$$



Limitation of system bandwidth by (1) optical amplifier band or (2) subtended angle reduce damping rate

$$\lambda_{||_SR} = \lambda_{||_SR0} F(\gamma\theta_m), \quad F(x) = 1 - \frac{1}{(1+x^2)^3}$$



- For narrow band: $\Delta U_{wgl} = \Delta U_{wgl0} \left(\frac{3\Delta\omega}{\omega} \right), \quad \frac{3\Delta\omega}{\omega} \ll 1$

where $\Delta U_{wgl0} = \frac{e^4 B^2 \gamma^2 L}{3m^2 c^4} \begin{cases} 1, & \text{Flat wiggler} \\ 2, & \text{Helical wiggler} \end{cases}$ the energy radiated in one undulator

Basics of OSC – Radiation from Flat Undulator

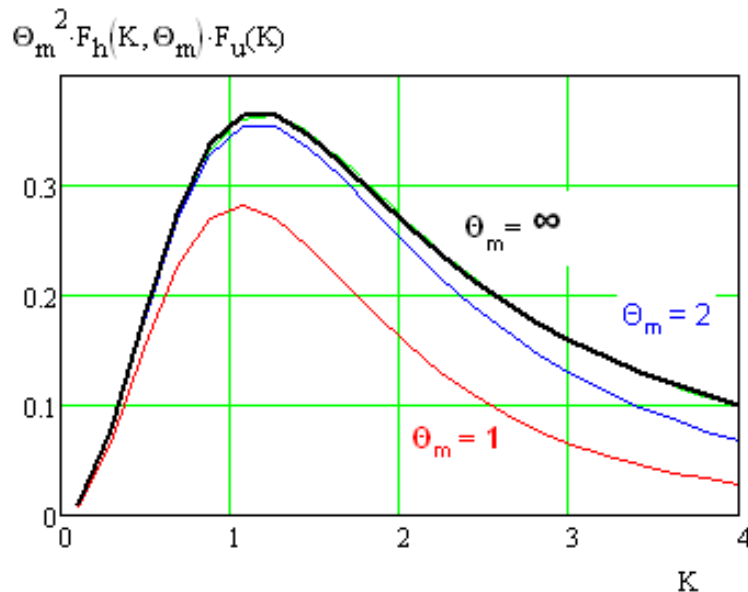
- For arbitrary undulator parameter we have

$$\Delta U_{OSC_F} = \frac{1}{2} \frac{4e^4 B_0^2 \gamma^2 L}{3m^2 c^4} GF_f(K, \gamma\theta_{\max}) F_u(\kappa_u)$$

$$F_u(\kappa_u) = J_0(\kappa_u) - J_1(\kappa_u), \quad \kappa_u = K^2 / \left(4(1 + K^2/2)\right)$$

Fitting results of numerical integration yields:

$$F_h(K, \infty) \approx \frac{1}{1 + 1.07K^2 + 0.11K^3 + 0.36K^4}, \quad K \equiv \gamma\theta_e \leq 4$$



- Dependence of wave length on θ :

$$\lambda \approx \frac{\lambda_{wgl}}{2\gamma^2} \left(1 + \gamma^2 \left(\theta^2 + \frac{\theta_e^2}{2} \right) \right)$$

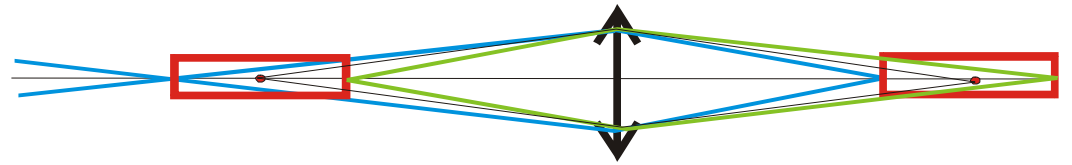
$K \equiv \gamma\theta_e$

- Flat undulator is “more effective” than the helical one
- For the same K and λ_{wgl} flat undulator generates shorter wave lengths

- For both cases of the flat and helical undulators and for fixed B a decrease of λ_{wgl} and, consequently, λ yields kick increase
 - ◆ but wavelength is limited by both beam optics and light focusing

Basics of OSC – Correction of the Depth of Field

- It was implied above that the radiation coming out of the pickup undulator is focused



on the particle during its trip through the kicker undulator

- ◆ It can be achieved with lens located at infinity

$$\frac{1}{2F + \Delta s} + \frac{1}{2F - \Delta s} = \frac{1}{F} \rightarrow \frac{1}{F - \Delta s^2 / 4F} = \frac{1}{F} \xrightarrow{F \rightarrow \infty} \frac{1}{F} = \frac{1}{F}$$

- ◆ but this arrangement cannot be used in practice

- A 3-lens telescope can address the problem within limited space

$$\begin{bmatrix} 1 & L \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -F_1^{-1} & 1 \end{bmatrix} \begin{bmatrix} 1 & L_1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -F_2^{-1} & 1 \end{bmatrix} \begin{bmatrix} 1 & L_1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -F_1^{-1} & 1 \end{bmatrix} \begin{bmatrix} 1 & L \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

