

# Designing a Continuous-Wave RF Cavity for Bunch Rotation in Support of Experiments Mu2e and g-2

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## ABSTRACT

Experiments Mu2e and g-2 require RF bunch rotation cavities which are able to operate in continuous-wave mode while producing a 10kV accelerating gap voltage. Fermilab has previously used two different 2.5 MHz cavity designs in the Main Injector and p-bar ring for bunch rotation; however, neither of these cavity designs were able to meet the specifications of the new experiments. For this reason, a new water-cooled, ferrite loaded, bunch rotation cavity was designed by the RF Department. The following is a description of the methods used in planning the design and an account of the cavity parameters.

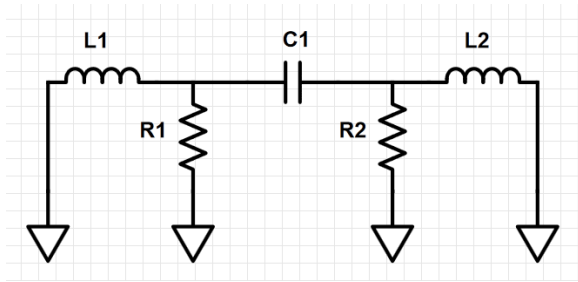
## INTRODUCTION

Fermilab is currently preparing for the start-up of their new muon experiments g-2 and Mu2e; these experiments have necessitated the design of seven new 2.5 MHz cavities, one 2.4 MHz cavity and one spare cavity. The 2.5 MHz cavities will be used in the Recycler Ring for bunch rotation and the 2.4 MHz cavity will be used in the Delivery Ring to maintain the beam's structure. All the cavities will share the same structural design. The variation in frequency will be accomplished by changing the amount of gap capacitance. The cavity intended for the Delivery Ring, and the spare cavity, will use newly purchased ferrite cores. The seven Recycler Ring cavities will primarily use ferrite cores reclaimed from decommissioned cavities. The new cavities will function as two transmission lines, operating in push-pull mode, with standing waves similar to the characteristics of a

quarter-wave resonator, though highly foreshortened, with the high voltage ends connected on either side of the accelerating gap. The RF department has started testing the new design by producing a shorter version of the final cavity. The new model utilizes a modular system which lends itself well to reduced scale testing. A test fixture was produced by using five ferrite cores and five aluminum housings—the full cavities will have 17 ferrite cores on either side of the accelerating gap. The test fixture was created to provide a proof of concept for the cavity design. Other modeling methods included a circuit model and CST Microwave Studio simulations. Using these methods we have established credibility for the heat transfer system and the cavity's ability to meet the requirements set forward by the g-2 and Mu2e. This paper will further explain those methods, the variables involved in the designing the cavity, and conclude with the cavity's current state.

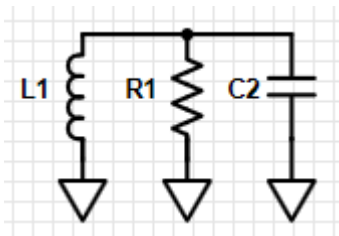
## METHODS

### A. Circuit/Mathematical Models



**Figure 1:** Circuit model of the cavity.  $L1$  and  $L2$  are derived from the solutions of short circuited transmission line impedance calculations,  $R1$  and  $R2$  are the cavity's two shunt resistances, and  $C1$  is the gap capacitance.

The cavity is two shorted coaxial transmission lines with their high voltage ends operating in opposite phase. The impedances of the transmission lines are inductive by design and are represented by  $L1$  and  $L2$  in the above circuit. We then added a capacitive load ( $C1$  above) across the cavity's two inner conductors and added parallel shunt resistors ( $R1$  and  $R2$  above) for each transmission line.

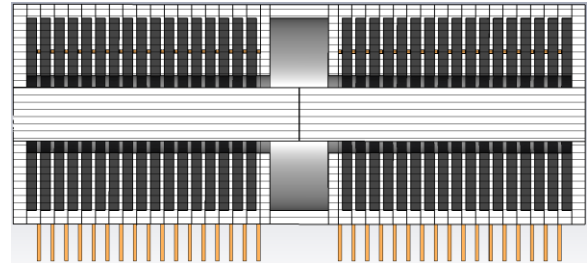


**Figure 2:** Simplified circuit model of the cavity.  $L1$  is derived from the solution of a short circuited transmission line impedance calculation,  $R1$  is the cavity's shunt resistance, and  $C2$  is two times the cavity's gap capacitance.

We assumed our cavity to be symmetric on either side of the accelerating gap, which allowed us to further simplify the model. When doing this, we modeled the capacitor as twice the cavity capacitance and assume any voltage across the shunt resistor to be half the voltage found in the cavity actual cavity (Griffin).

After the circuit model was determined, the circuit's mathematical equivalent, the lossless transmission line equations, and cavity parameters were entered into Mathematica. This allowed us to assess the impact of variable adjustments.

### B. CST Microwave Studio



**Figure 3:** CST Microwave Studio simulation of the cross section of a 34 ferrite cavity. In this simulation the inner conductor was used to create the gap capacitance.

CST Microwave Studio is a software tool which performs 3D EM simulations. Both the five ferrite test cavity and the 34 ferrite cavity were simulated. Several different simulations and models were used. The simplest models generated the gap capacitance by creating the inner conductor as a solid piece of aluminum, instead of hollow, and then using the parallel plate capacitor equations to determine the distance between the two faces. This was justified due to the center of the cavity being filled with air and therefore electrically short. The extra inches of inner conductor did not have a significant impact on the characteristics of the cavity and proved to be the easiest and most reliable method for creating the capacitor. The actual cavities will use vacuum capacitors for their gap capacitance.

CST MWS has several different solver methods, but our simulations only used the Eigenmode Solver. This solver does not take into consideration the input source for the cavity. Instead it assumes a signal internal to a closed structure, with perfect boundary conditions, and models the shell metals as

lossless. It is able to return the resonant frequencies in the cavity and also provides electric and magnetic field density maps.

### C. Test Cavity



**Figure 4:** Five ferrite test fixture. The RF input can be seen in the lower right side of the image.



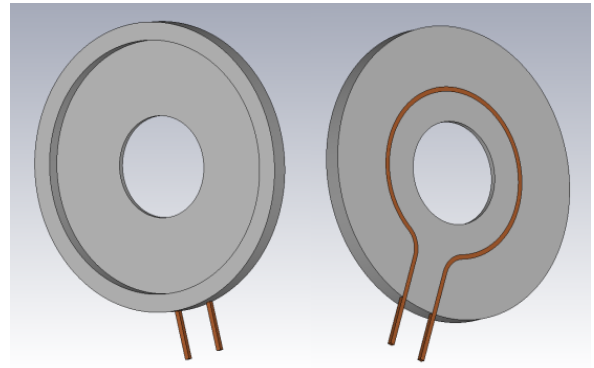
**Figure 5:** Reverse side of the test fixture. The copper water tubing can be seen on the left of the image.

In addition to our models and simulations we were able to gather data from a five ferrite test cavity. The test fixture was comprised of five aluminum housings, with attached copper water tubing, five ferrite cores, two aluminum end plates, and a 1500pF vacuum capacitor. The cavity had a tap point in the first ferrite, next to the shorted end of the cavity and was loaded with up to 500W of RF power.

The data we were able to obtain from the fixture included the shunt impedance, gap voltage, reflection efficiency, water temperature in/out difference and resonant frequency.

## CAVITY PARAMETERS

### A. Aluminum Components



**Figure 6:** Front and back CST MWS simulations of the aluminum housings. These repeating plates held the ferrite, acted as our cavity's outer conductor, and provided the heat dissipating surface. The copper water tubing was made flush with the aluminum surface.

Aluminum (T6061) was used to create repeating sections which act as the outer conductor for the cavity, as well as the heat dissipating surface for the ferrite. Each plate is a 22.5" OD X 7.916" ID X 1.41" thick toroid with a 19.71" OD X 1" thick recess milled out of each plate to make space for the ferrite cores. A hole with a diameter 7.916" is milled through the center of the plate. Also, around the outer lip of the aluminum, a small trench (not displayed in figure 4) is cut where a tin coated beryllium copper gasket is placed to ensure uniform electrical connection between the concentric plates. On the reverse side, a groove is cut into the face to make space for the copper water tubing. In total, each cavity will have 34 aluminum housings, with additional aluminum pieces (22.5" OD X 1.41" thick) forming the electrical short at the ends of the cavity. The inner conducting tubes (5.5" OD X

5" ID) will also be made of aluminum. Finally, the hollow center of the cavity will be formed with two aluminum plates and an aluminum cylinder. At the time of this writing the exact dimensions of the center have not been finalized. This vacancy will house the vacuum gap capacitors.

**B. Ferrite**  
**I. General**



**Figure 7:** In this image are several of the ferrite cores harvested from decommissioned cavities.

In total the new cavities will require 306 ferrite cores. Previous bunch rotation cavities were disassembled and we have reclaimed 275 cores from those cavities. The majority of the ferrite used in the 7 Recycler Ring cavities will be ferrite reclaimed from previous cavities. The reclaimed ferrite cores (19.69" OD X 7.874" ID X 1" thick) have the following average values:

@ 2.5 MHz

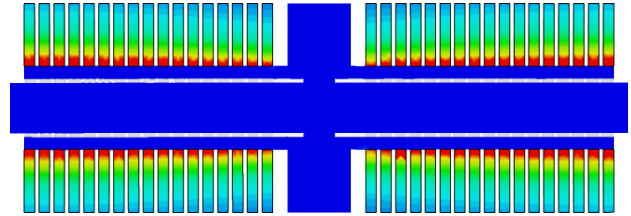
- 155 Relative Permeability
- 11 Relative Permittivity
- 125 Q Factor

72 new Ferroxcube 4M2 ferrite cores (19.69" OD X 7.874" ID X .984" thick) were purchased to replace damaged ferrite cores, to fill the Delivery Ring cavity, and also to fill the spare cavity. The new ferrite cores have the following average values:

@ 2.5 MHz

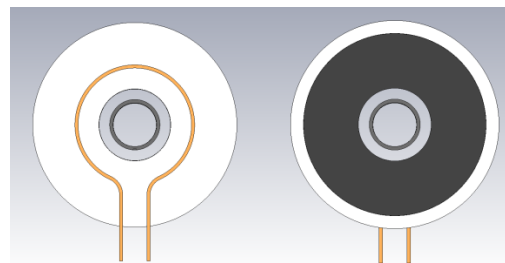
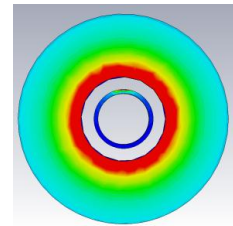
- 130 Relative Permeability
- 13 Relative Permittivity
- 97 Q Factor @ 2.5 MHz

**II. Heating**



**Figure 8:** CST MWS fundamental mode simulation of the cross section of the magnetic field density in the 34 ferrite cavity.

The CST MWS simulations verified that the due to the cavity only rotating though 28° on a given side of the accelerating gap, the induced magnetic heating in the ferrite was nearly constant along the Z axis of the cavity. With this, we feel comfortable placing the copper tubing at the same position on every aluminum plate.



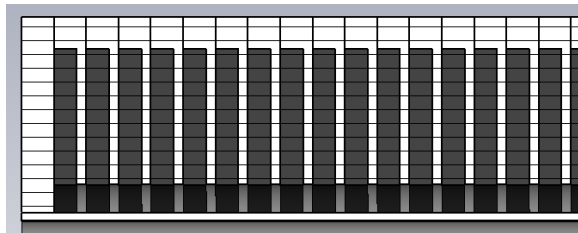
**Figure 9:** (Top) CST MWS simulation of the magnetic field density in a ferrite core. (Bottom) Simulations of the front and back of the aluminum plates with ferrite inserted.

The CST MWS simulation also reinforced our mathematical projections that the highest

magnetic flux would happen near the inner radius of the ferrite cores and demonstrated that the cooling tubing is well positioned to extract heat from the system. In our five ferrite test cavity experiments, we input up to 500W in the cavity and found that we were able to stabilize the temperature with 2 gpm of water, and maintained a steady resonant frequency while running CW. We are confident that the 34 ferrite cavities will be able to maintain thermal equilibrium while running CW, as those cavities we will have a significantly lower power distribution across their ferrite cores.

### C. Characteristic Impedance

The equations for the characteristic impedance of a coaxial transmission line are well known; however, our dielectric is not radially uniform and warrants special consideration. Beyond this, there are two different repeating sections in the cavity with two different characteristic impedances.



**Figure 10:** Magnified perspective of the upper left-hand corner of Figure 3. The copper tubing was removed for clarity.

In the section where the outer conductor's ID is 19.71", there are three different dielectric transitions—1.187" of air to 5.908" of ferrite to .01" of air. When calculating the inductance per unit length for this section the three values were summed together as inductors in series would be summed. The capacitance per unit length calculation added the three values as you would add three capacitors in series.

In the section where the outer conductor's ID is 7.916", there is only an air dielectric from the

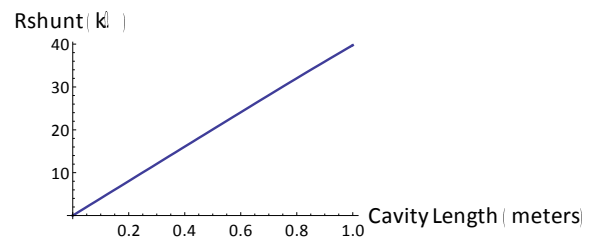
inner conductor's OD to the outer conductor's ID.

The total inductance per unit length was then the linear combination of the two values weighted by their proportionate length (1"/1.41" or .41"/1.41") in the Z direction. The same method was applied to total capacitance per unit length. Finally, the characteristic impedance was calculated using these numbers.

### D. Quality Factor

The Q factor of our cavity is dominated by the ferrite. There are slight losses (<5) due to the resistivity of the aluminum and the tin, but these values are not significant compared to the ferrite itself. The Q factors measured in the test cavity were 130. The Q factor was not calculated using mathematical models; instead our mathematical models used measured Q values as an input parameter.

### E. Shunt Impedance



**Figure 11:** Graph showing the shunt impedance as a function of cavity length. The relationship becomes non-linear outside the scope of this graph due to capacitive foreshortening.

The previous bunch rotation cavities used 17 ferrite cores on both sides of the accelerating gap and it was decided that the new cavities would use the same. In our mathematical models, this resulted in a shunt impedance of 24.1kΩ. With this shunt impedance we need to supply roughly 500W of RF power into each side of the cavity, or 1kW per cavity in total. This can easily be done with the 8kW amplifiers scheduled for use with these



cavities. With 5 ferrites, our test fixture was able to produce a shunt impedance of  $4.2\text{k}\Omega$  and generated a gap voltage of  $2\text{kV}$  with  $\sim 475\text{W}$  of input power.

### E. Power Reflection

Due to the aluminum plates in the cavity, we have a discrete set of available tap points which are spaced every  $1.41''$ , starting  $.5''$  from the shorted end of the cavity. To insert the RF input into the cavity we will drill a hole through the outer conductor and down the center of one of the ferrite cores, perpendicular to the Z axis. We project that the best match will be in the second ferrite from the shorted end of the cavity. However, due to the modular design of the cavity, we are well suited to move this tap plate in either direction along the Z axis. We are confident that the cavity will have a transfer efficiency of greater than 70%. In our test fixture we were able to obtain transfer efficiencies of 80%. To enhance our match, we will also use a shorted or open stub tuner in parallel with the input source transmission line.

### F. Harmonic Modes

The fundamental modes of the 34 ferrite cavities will need to be either 2.5 MHz or 2.4 MHz depending on their accelerating ring. Using CST MWS we simulated five total modes and found their frequencies at: 2.5, 6.5, 11.5, 12.7, 13.9 MHz. The fundamental mode of the test cavity was found to be 2.0 MHz.

## DISCUSSION

During our testing with the test fixture, it was noticed that the test cavity was showing higher than expected Q values. It is believed that this is due to losses in ceramic capacitors that were used during ferrite testing. Finding that the Q of the ferrite is higher than expected will result in a larger shunt impedance and is not a major concern.

We noticed that there is a variation between CST MWS's projected resonant frequency (2.28

MHz), the circuit models projected resonant frequency (2.15 MHz) and the measured results (2.0 MHz) of the test cavity. At the time of this writing we are unable to reconcile these differences. We do not expect that this will be an issue as our variable vacuum capacitor will be able to tune the 34 ferrite cavities to the correct frequency. The most significant issue that this could cause would be the down time of waiting for a different sized variable capacitor to be delivered, should the capacitors be unable to adequately adjust the cavity.

There was disagreement between CST MWS's higher order mode frequencies and the test cavities measured mode frequencies. This could potentially cause higher than expected induced beam voltages in the full sized cavities. If beam induced voltages become an issue, higher order mode dampeners will need to be created and installed in the cavities. We do not expect this to be an issue, and the solution for a damper is relatively simple.

## CONCLUSION

The primary motivation for the new cavity design was to increase heat extraction while maintaining a continuous wave with a  $10\text{kV}$  gap potential. These tests confirmed the design's capabilities in this regard. At this point in the project, the cavity design is well understood and production will begin in January of 2015 with a projected completion date of September 2016. This project is done under the Recycler RF Accelerator Improvement Plan (AIP).

## REFERENCES

J.E. Griffin, "A Numerical Example of an RF Accelerating System", in R.A. Carrigan, F.R. Huson, and M. Month, ed., *Physics of High Energy Particle Accelerators (Fermilab Summer School, 1981)*, American Institute of Physics, New York, 1982, p. 564-582.

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