Criteria for Natural Hierarchies

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Based on arXiv:1402.2658, with D. Hernandez and T.M.P. Tait.

Other references most closely related to what I will present (and there are several more):

- A. Casas, J.R. Espinoza, and I. Hidalgo, hep-ph/0410298 and hep-ph/0607279
- M. Farina, D. Pappadopulo, A. Strumia, arXiv:1303.7244

All of the results presented are obtained at the order-of-magnitude level, using naive perturbation theory and dimensional analysis. Much more quantitative results can be found in arXiv:1303.7244.

We only worry about one hierarchy problem at a time (this will become more clear. Landau poles will also be ignored.

THE HIERARCHY PROBLEM

Higgs potential: $V\left(H\right) = -\mu^2|H|^2 + \lambda|H|^4$

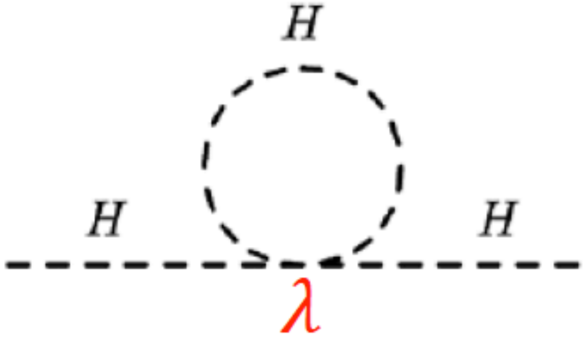
Classically: μ is equivalent to the mass of the Higgs

Quantum mechanics introduces corrections to this simple relation

$$\mu^2 \to \mu^2 + \delta\mu^2$$

THE HIERARCHY PROBLEM

Quantum corrections correspond to loop Feynman diagrams



$$\delta\mu^2 \propto \lambda\Lambda^2$$

unknown mass scale (regulator)

- This correction is infinite when $\Lambda \to \infty$
- It depends on the regulation procedure
- Renormalization gets rid of these

However, this may not be the right way to look at this. Quadratic divergences are not observable – they can be renormalized away. **They don't even show up if one uses dimensional regularization!** Some level of interpretation, often imply, is required in order to state that the presence of quadratic divergences implies that the weak scale is unstable.

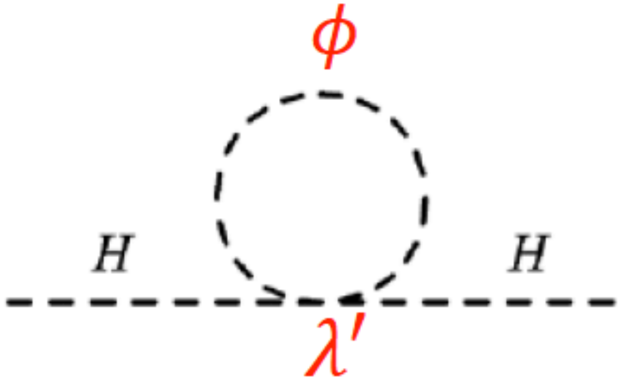
By itself, the Standard Model is a **one-mass-scale theory**. The weak scale, at the tree-level or the one-loop level, is the weak scale. And you can't predict it, it has to be measured.

Life is quite different if, on top of the weak scale, there is **another mass scale** $M_{\rm new}$. Is this case, there are finite corrections to the Higgs mass-squared parameter. These may de-stabilize the weak scale. We use this as the definition of the hierarchy problem:

Can the Weak Scale Co-Exist with Another Mass Scale?

The key point is that the answer **depends on the new physics** and **how it talks to the Higgs boson**. Estimating it from Standard Model parameters may be dangerous.

THE HIERARCHY PROBLEM
(Finite corrections)



Known mass scale

$$\delta\mu^2 \propto \lambda' m_\phi{}^2$$

- **Finite for $\Lambda \to \infty$. Hierarchy Problem when $m_\phi \gg \mu$.**
- **It is independent of the regulator**

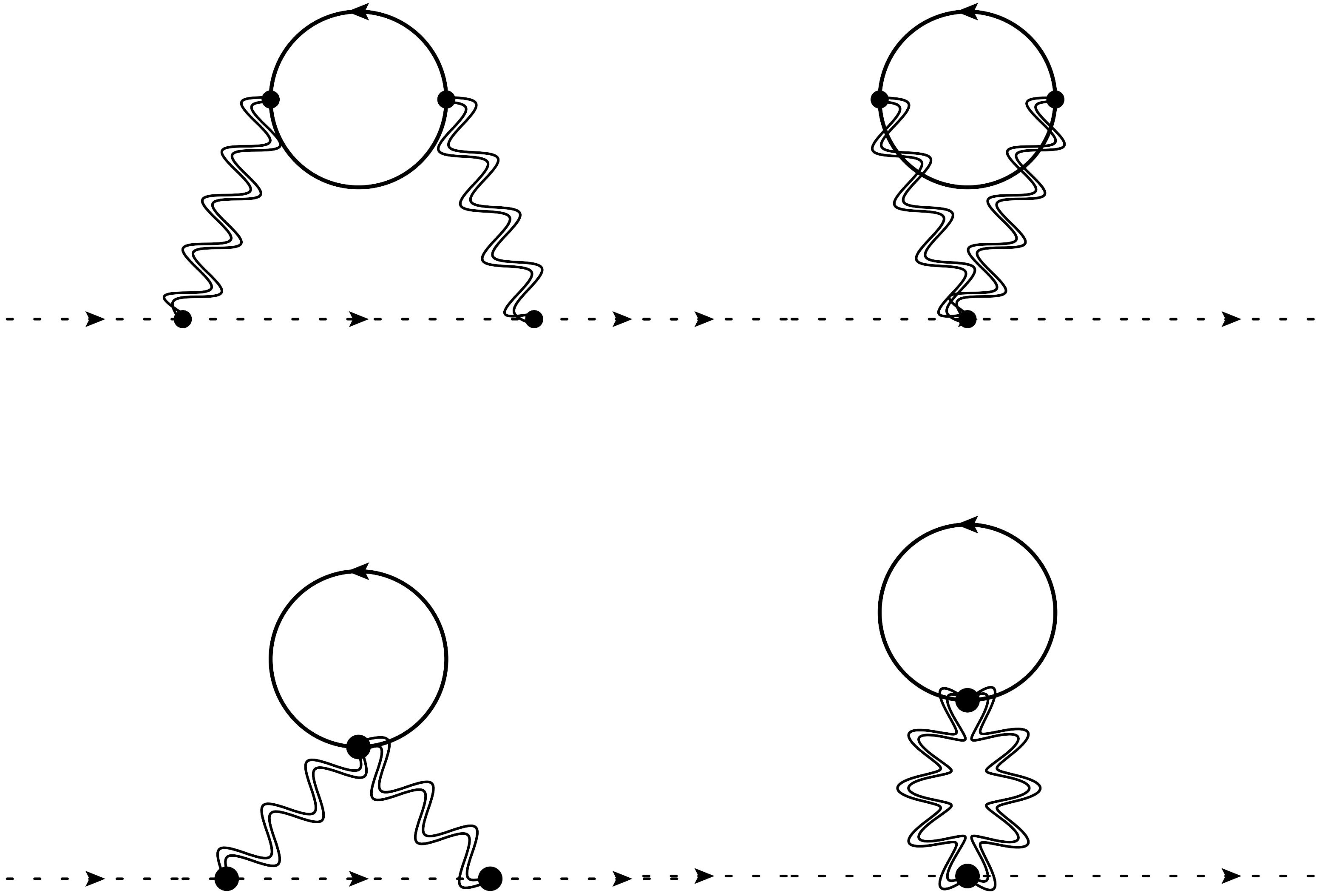
Case Study: New Fermion Ψ, uncoupled to the Standard Model

Complaint: Gravity exists, and the weak scale is way lower than the Planck scale. There is no way these two mass scales can co-exist.

Answer: I don't know how to compute quantum gravity corrections to the Higgs mass-squared. I do know that, perturbatively and at low-energies (below the Planck scale), corrections are tiny since the coupling goes like $1/M_{Pl}^2$:

$$\delta\mu^2 \sim \mu^2 \left(\frac{\mu^2}{M_{Pl}^2}\right)^N$$

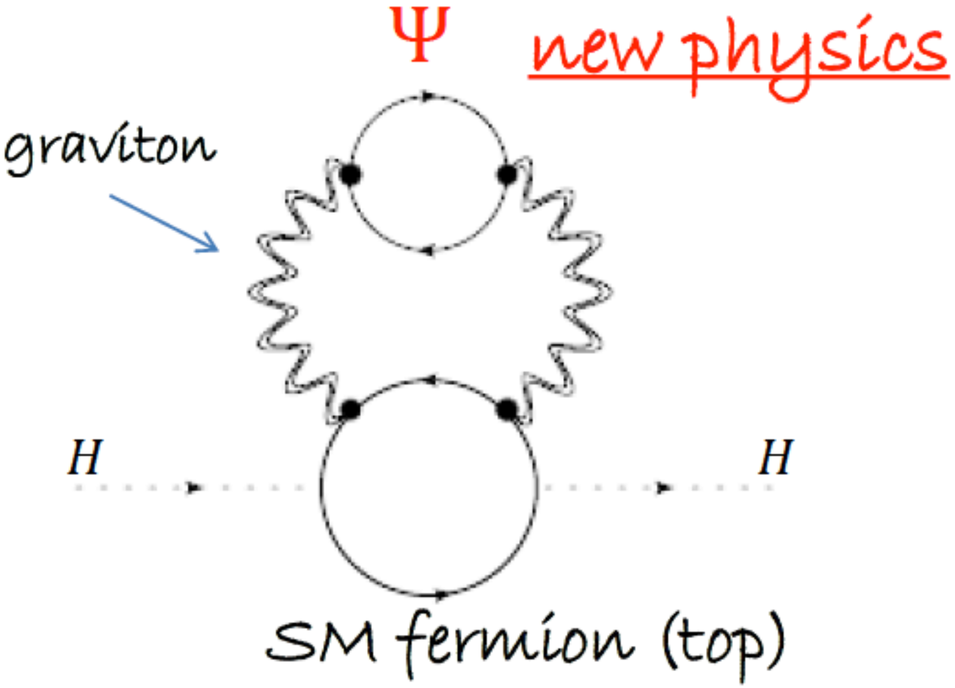
Toy-model: Add a new vector-like fermion with mass M_Ψ that does not couple to the Standard Model at all, except through gravity.



$$\delta\mu^2 \sim \frac{1}{(16\pi^2)^2}\frac{M_{\Psi}^4}{M_{Pl}^4} \times \mu^2.$$

Uncoupled:

(except for gravity)



$$\delta\mu^2 \sim \frac{\lambda_t^2}{(16\pi^2)^3} \frac{M_\Psi^4}{M_{Pl}^4} \times M_\Psi^2$$

$$\delta\mu^2 \lesssim 100^2 \ \text{GeV}^2 \quad \Rightarrow \quad M_\Psi \lesssim 10^{14} \ \text{GeV}$$

Other Case Studies with a new Fermion

Fermion charged under $SU(2)\times U(1)$:

$$\delta\mu^2 = \left(\frac{g^2}{16\pi^2}\right)^2 \times F\left(\frac{M^2_{W,Z}}{M^2_\Psi}\right) \times M^2_\Psi,$$

(two loops). Get in trouble for M_Ψ above tens of TeV.

Yukawa coupled ($y_{\rm new}(\psi H)\Psi$):

$$\delta\mu^2 \sim C\ \frac{y^2_{\rm new}}{16\pi^2} \times M^2_\Psi.$$

(one loop). Get in trouble for M_Ψ above ...? Depends on the Yukawa coupling!

Hints for New Mass Scales

We know there is new physics beyond the standard model!

1. Nonzero neutrino masses. New mass scale → maybe. Not necessarily very high.
2. Dark matter. New mass scale → most likely. Not necessarily very high.
3. Gauge coupling unification. New mass scale → certainly. Most likely very high.
4. Flavor symmetries. New mass scale → certainly. Probably pretty high.
5. Inflation. New mass scale → most likely. Not necessarily very high.
6. Baryogenesis. New mass scale → probably. Not necessarily very high.
7. Dark Energy. New mass scale → ???

Type-I Seesaw

$$\mathcal{L}_{SM} + \bar{N}_i \bar{\sigma}^\mu \partial_\mu N^i - \frac{M_R^{ij}}{2} N_i N_j - y_{ij} L^i N^j H + H.c.,$$

where i, j are family indices, M_R is the right-handed neutrino mass-matrix, and y is the neutrino Yukawa coupling matrix.

$$\delta\mu^2 = -\frac{1}{4\pi^2} \sum_{ij} |y_{ij}|^2 \times M_j^2 \quad \rightarrow M < 10^4 \text{ TeV}.$$

The weak scale can co-exist with the right-handed neutrino mass-scale if the right-handed neutrino masses are below 10 TeV. Is this a problem? No, except that vanilla leptogenesis does not work.

Flavor Model Example

$$\mathcal{L} \supset i\overline{D}\not{\partial}D - M_D\overline{D}D + y_1\overline{Q}_3DH + y_2\overline{D}b_R\phi + H.c.,$$

where Q_3 is the third generation quark doublet, t_R and b_R are the quark singlets, ϕ is the scalar whose VEV breaks the $U(1)$ global symmetry, and D is a vector-like quark with the same SM gauge interactions as b_R.

b-Yukawa coupling only after integrating out D,

$$y_b^{\rm eff} = y_1y_2\frac{\langle\phi\rangle}{M_D}\ .$$

$$\delta\mu^2 = -\frac{6|y_1|^2}{8\pi^2}\times M_D^2, \rightarrow M_D < \frac{900\ \rm GeV}{|y_1|}\ .$$

(Quantum corrections to μ^2 proportional to M_ϕ^2 are also generically expected. We discuss these momentarily)

Flavor scale low, unless new Yukawa couplings are small. Defeats the purpose of the flavor model? “All the same but small” still better than hierarchical couplings?

Case Study: New Scalar Φ

Scalars are qualitatively different, since the marginal coupling

$$\mathcal{L}_{SM+\Phi} \supset \lambda_{\rm new}|H|^2|\Phi|^2.$$

is always allowed. At the one-loop level, this interaction allows the mass scale M_Φ to contribute to the Higgs boson mass-squared,

$$\delta\mu^2 \sim \frac{\lambda_{\rm new}}{16\pi^2} \times M_\Phi^2.$$

Grand Unified Theories

$$\delta\mu^2 = \frac{C}{16\pi^2} \times M^2_{\rm GUT},$$

where C is a coefficient of order (at least) the known gauge couplings that depends on the detailed physics at the GUT scale. Since limits from proton decay require $M_{\rm GUT} \sim 10^{16}$ GeV, it is clear that this correction to μ^2 is highly unnatural, and requires a seemingly magical cancellation between the tree-level Higgs mass-squared parameter and all of its higher order quantum corrections.

Higgs Portal Dark Matter

While there is a new mass scale, it is of order the weak scale (so you get the relic density right). Not problem here.

Case Study: New Real Scalar Φ, $|H|^2\Phi$ Coupling

A gauge-singlet scalar can also couple singly to a pair of Higgs fields via

$$\mathcal{L}_{SM+\Phi} \supset \kappa_{\rm new}|H|^2\Phi,$$

where $\kappa_{\rm new}$ is a coupling constant with dimensions of mass. In the limit $\kappa_{\rm new} \to 0$, there is an enhanced Z_2 symmetry, indicating that any value of $\kappa_{\rm new}$ is natural in the sense of 't Hooft.

$$\delta\mu^2 \sim -\frac{\kappa^2_{\rm new}}{16\pi^2} \times \log\left(\frac{M^2_\Phi}{|\mu^2|}\right),$$

whose size is characterized by $\kappa_{\rm new}$, and depends only logarithmically on M_Φ. Obviously the theory will be finely tuned unless $\kappa_{\rm new} <$ TeV. However, the super-renormalizable interaction **shields the Higgs mass from the heavy mass scale** M_Φ, despite allowing for relatively large coupling between the Higgs and Φ sectors.

($\lambda_{\rm new}\Phi^2|H|^2$ is still around, but its contribution is negligible if $\lambda_{\rm new} \ll 1$. This is not pretty, but technically natural.)

Case Study: Mixed Fermion/Scalar (Scalar Portal)

$$\mathcal{L}_{SM+\Phi+\Psi} \supset \kappa_{\rm new}|H|^2\Phi + y\Phi\bar{\Psi}\Psi,$$

and

$$\delta\mu^2 = \frac{y^2\kappa_{\rm new}^2 M_\Psi^2}{(16\pi^2)^2 M_\Phi^2}\ F\left(\frac{M_\Psi^2}{M_\Phi^2}\right)$$

where $F(M_\Psi^2/M_\Phi^2)$ is $\mathcal{O}(1)$. The finite corrections proportional to M_Ψ^2 do not destabilize the weak scale as long as $M_\Psi^2 < M_\Phi^2$ (provided $\kappa_{\rm new} <$ TeV and $\lambda_{\rm new} \ll 1$). This feature extends to diagrams with any number of loops, since in the limits $\kappa_{\rm new} \to 0$ or $M_\Phi \to \infty$, the SM and Ψ must decouple → all contributions to $\delta\mu^2$ equal to $\kappa_{\rm new}^2$ multiplied by an analytic function of the ratio M_Ψ^2/M_Φ^2.

Natural model (?) of heavy fermionic dark matter (played by Ψ in the discussion above) which communicates primarily with the Higgs via exchange of Φ. At low energies, Φ exchange results in an operator of the form $|H|^2\bar{\Psi}\Psi$. This is relatively free from fine-tuning.

Some Final Remarks

- We advocate a "QFT-only" approach to the hierarchy problem. It is less ambitious but very concrete. And keep in mind that the standard interpretation [quadratic divergences imply new physics at the weak scale] may be a red herring.
- The standard model by itself has only one mass scale. It is "natural." So we are addressing the following question: **can the weak scale "co-exist" with other new physics scales?**
- The answer depends dramatically on how the new physics talks to the Higgs boson! Beware of small couplings. There is nothing wrong with them (remember that most couplings we know are small) and they allow different scales to co-exist.
- Standard model parameters play no or a limited role as far as deciding whether an extension of the standard model is natural. → Maybe the only thing special about the top quark is that it is the heaviest one!

- Quantum gravity. Perturbatively and at low energies, graviton loops are safe (whatever that means). A more definitive answer requires a concrete model.

- Nature has already revealed that there is physics beyond the standard model. Dark matter and nonzero neutrino masses require new degrees of freedom and, **perhaps**, new mass scales.

- More indirect hints like the unification of gauge couplings and the fermion particle content (GUTs), the need for a mechanism of baryogenesis, the strong CP problem, and the flavor puzzle also suggest the existence of new, usually very heavy, new states.

- SUSY allows different mass scales to co-exist. Broken SUSY, of course, brings about its own naturalness problem (a bunch of new fermions and bosons with mass $M_{\rm SUSY}$ and SM couplings). This appears to be the only way forward if GUTs are real. Same applies for many models that require ultra-high mass scales.