Towards Natural Tuning

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Principal Technical Result

NB: Construction will be in (1+1)d. 2d theories are special in many respects, but not as far as the hierarchy problem goes

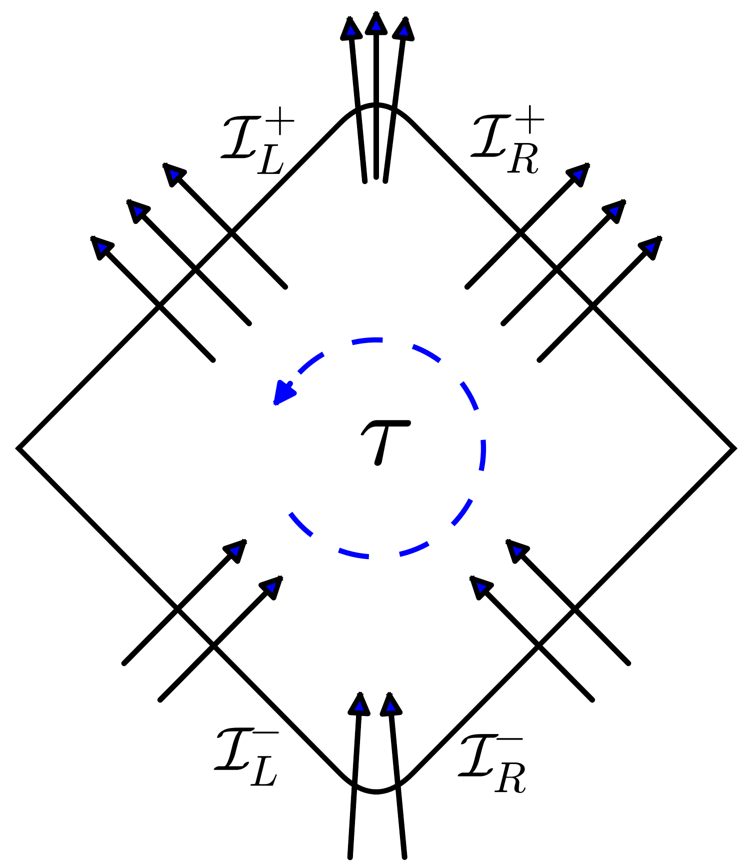
Start with an arbitrary UV complete natural QFT $\mathcal{L}(\psi, H)$
Non-protected scalars are allowed as soon as they are heavy

$\downarrow$

Calculate S-matrix $S_n(p_i)$

$\downarrow$

“Gravitational dressing” gives $\hat{S}_n(p_i, \ell)$



$$\hat{S}_n(p_i) = e^{i\ell^2/4\sum_{i<j} p_i * p_j} S_n(p_i)$$

Properties of gravitational dressing

- ✶Results in a well-to-do S-matrix
- ✶Physical spectrum remains the same
- ✶Low energy EFT description, tuned for $m\ell \ll 1$

$$\mathcal{L}(\psi, H) + \sum_{\Delta_i > 2} \ell^{\Delta_i - 2} \mathcal{O}_i$$

free massive scalar:

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2 + \frac{\ell^2}{8}\left((\partial\phi)^4 - m^4\phi^4\right) + \ldots$$

✶THIS CONSTRUCTION SHOULD NOT BE POSSIBLE !!!

This is how these theories should have been found:

What are possible integrable reflectionless massless theories in two dimensions?

Everything is determined by a two-particle phase shift:

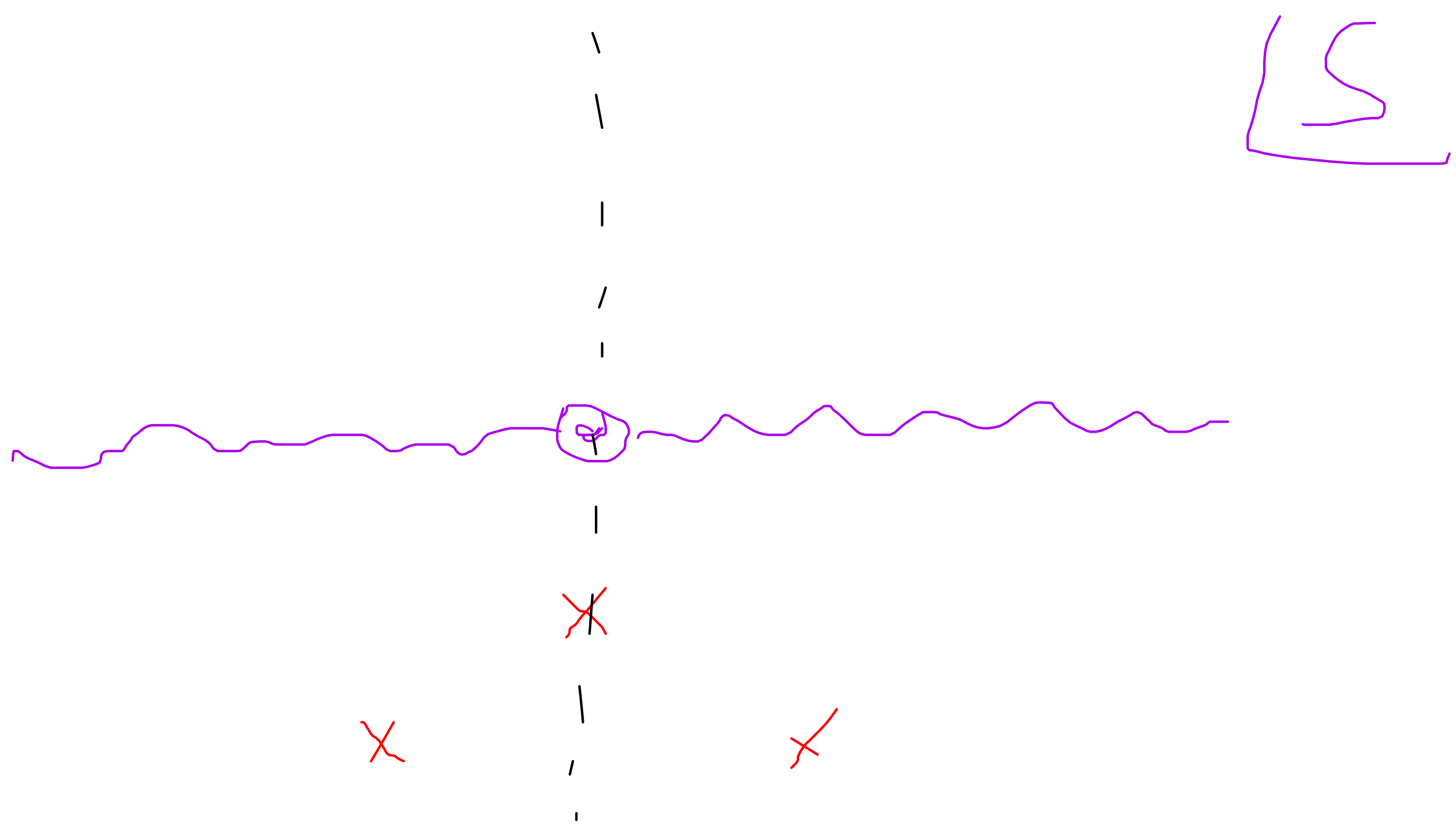
$$\boxed{S = e^{2i\delta(s)}\mathbf{1}}$$

Unitarity+Analyticity+Crossing:

Zamolodchikov '91

$$\boxed{e^{2i\delta(s)} = \prod_j \frac{\mu_j + s}{\mu_j - s} e^{iP(s)}}$$

$$\text{Im } s > 0$$



Expectation from Locality: $P(s) = 0$

Goldstino (Volkov-Akulov) Theory

$$\mathcal{L} = \psi\bar{\partial}\psi + \bar{\psi}\partial\bar{\psi} - \frac{1}{M^2}(\psi\partial\psi)(\bar{\psi}\bar{\partial}\bar{\psi}) + \ldots$$

$$\boxed{\mathrm{e}^{2i\delta_{\mathrm{Gold}}(s)} = \frac{iM^2 - s}{iM^2 + s}}$$

A simple example of "Asymptotic Safety":

naively non-renormalizable theory flows into a strongly coupled UV fixed point, no new stuff added

Corresponds to integrable RG flow between tricritical Ising model in the UV and Ising model in the IR

(equivalently, N=1 Wess-Zumino model in the UV and free fermion in the IR)

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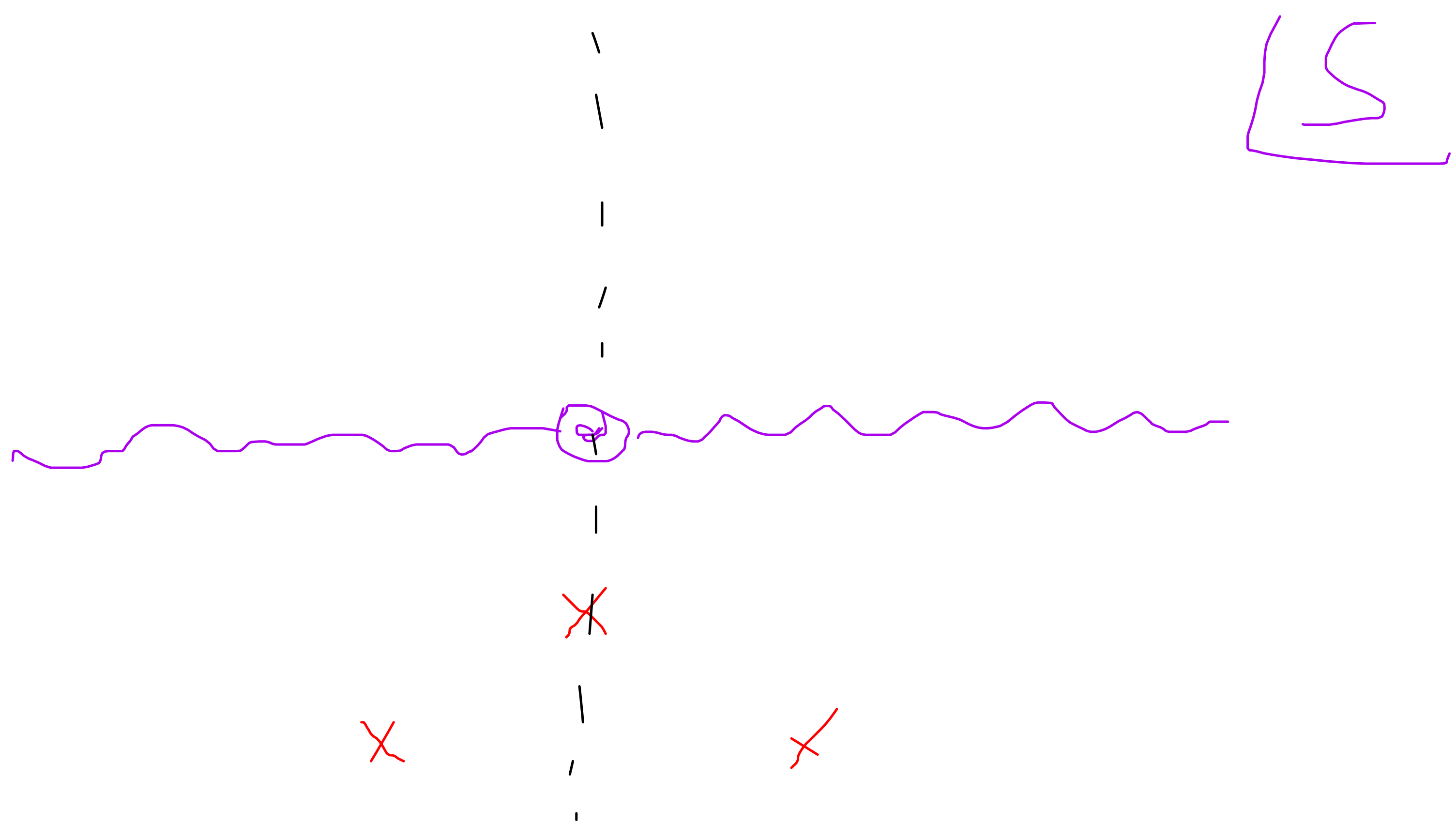

Expectation from Locality:

$$P(s) = 0 + \ell^2 s$$

Let us look at
at (D-2) bosons with

$$e^{2i\delta(s)} = e^{is\ell^2/4}$$

✶Polynomially bounded on the physical sheet

✶No poles anywhere. A cut all the way to infinity with an infinite number of broad resonances

✶One can reconstruct the entire finite volume spectrum using Thermodynamic Bethe Ansatz

$$E(N,\tilde{N}) = \sqrt{\frac{4\pi^2(N-\tilde{N})^2}{R^2} + \frac{R^2}{\ell^4} + \frac{4\pi}{\ell^2}\left(N+\tilde{N}-\frac{D-2}{12}\right)}$$

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A new type of RG flow behavior:

Asymptotic Fragility

Integrable theory of gravity

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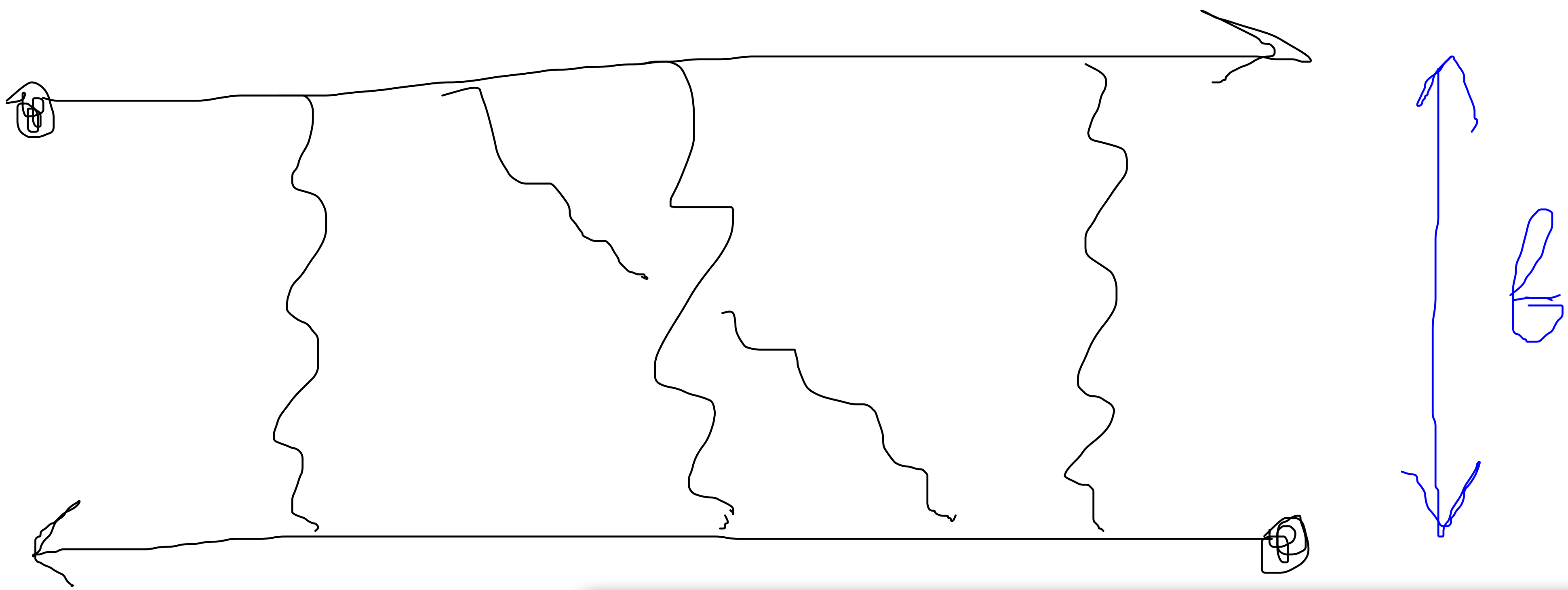
Integrable QG rather than QFT

Gravitational shock waves:

Dray, 't Hooft '85
Amati, Ciafaloni, Veneziano '88

$s >> M_{pl}^2$
$b >> R_s$



Eikonal phase shift:

$$\boxed{e^{i2\delta_{eik}(s)} = e^{i\ell^2 s/4}}$$

$$\ell^2 \propto G_N b^{4-d}$$

Some properties of the theory

classical action:

$$S_{NG} = -\ell^2 \int d^2\sigma \sqrt{-\det\left(\eta_{\alpha\beta} + \partial_\alpha X^i \partial_\beta X^i\right)}$$

- ✶Theory of gravitational shock waves.
- ✶No UV fixed point and central charge.
- ✶Maximal achievable (Hagedorn) temperature.
- ✶Integrable cousins of black holes.
- ✶Minimal length.
- ✶No local off-shell observables.

Integrable Black Hole Precursors

Time Delay

$$\boxed{\Delta t_{cms} = \frac{1}{2} \ell_s^2 E_{cms}}$$

c.f. $\Delta t_H = \ell_{Pl}^4 E_{cms}^3$ for Hawking evaporation in 4d

Equivalence Principle at work

Δt is the same for a single hard particle and for a bunch of soft ones

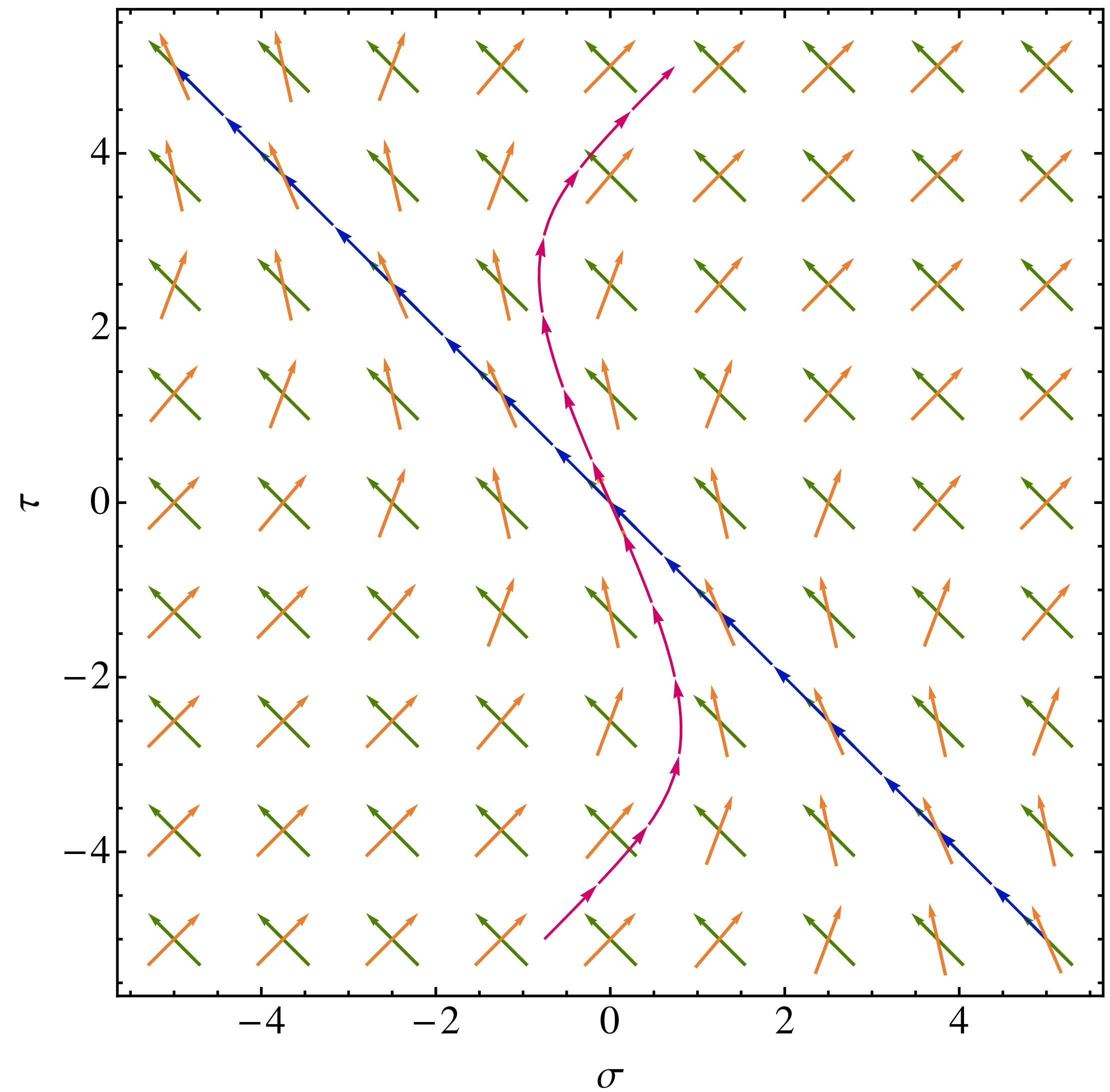
String uncertainty principle

$$\boxed{\Delta x_L \Delta x_R \geq \ell_s^2}$$

for identical packets $\Delta x_{out}^2 = \Delta x_{in}^2 + \dfrac{\ell_s^4}{\Delta x_{in}^2}$

Classical Origin of the Time Delay

$X^i_{cl}(\tau + \sigma)$ is a solution



$$\Delta t = \int_{-\infty}^{\infty} dz X'^2_{cl} = \ell^2 E$$

exactly reproduces the quantum answer

This was an integrable QG coupled to (D-2) massless bosons.

Is there a generalization to other (non-integrable) theories?

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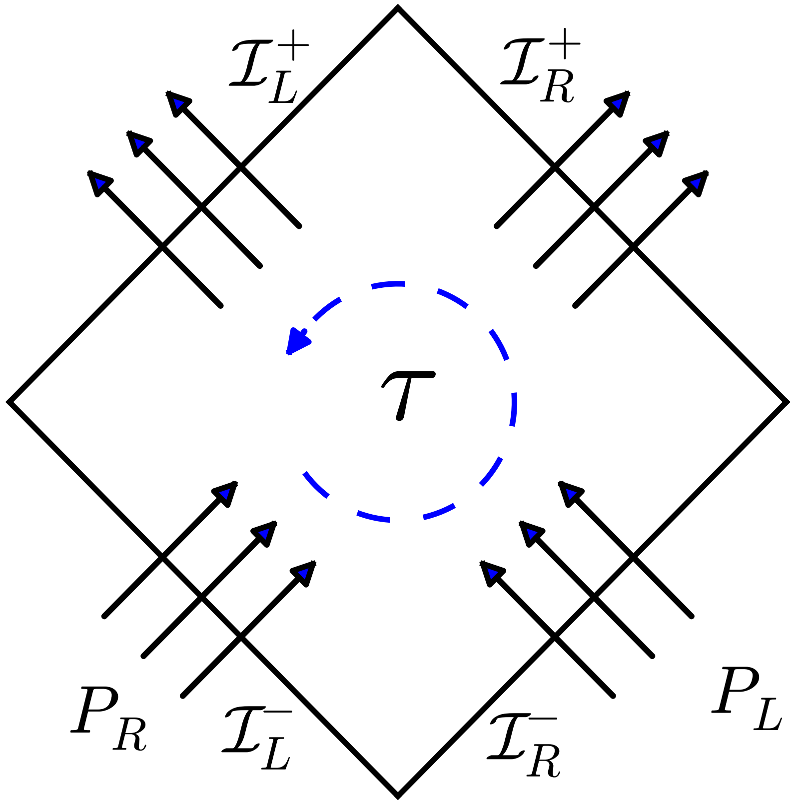
Eikonal Scattering From Boundary Quantum Mechanics

Verlinde & Verlinde '91

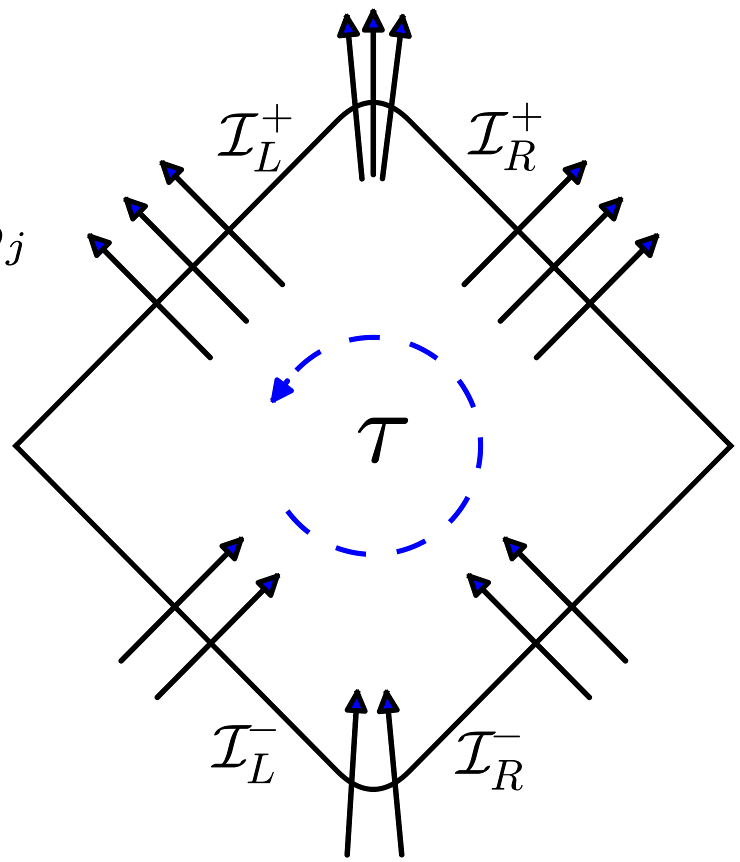
$$g_{\mu\nu} = \begin{pmatrix} e^{\phi}\partial_\alpha X^a \partial_\beta X^b \eta_{ab} & 0 \\ 0 & h_{ij} \end{pmatrix}$$

$$S_{CS}[X] = \ell^{-2} \oint d\tau \epsilon_{\alpha\beta} X^\alpha \partial_\tau X^\beta$$



$$S_{eik} = \int \mathcal{D}X e^{iS_{CS}[X]+i\left(\sum_i p^\alpha_{iR} X_\alpha(\tau_i) + \sum_j p^\alpha_{jL} X_\alpha(\tau_j) + \sum_i \bar{p}^\alpha_{iR} X_\alpha(\bar{\tau}_i) + \sum_j \bar{p}^\alpha_{jL} X_\alpha(\bar{\tau}_j)\right)}$$

Most simple-minded generalization:



$$\mathcal{D}(p_i) = \int \mathcal{D}X e^{iS_{CS}[X] + i\sum_i p_i^\alpha X_\alpha(\tau_i)} = e^{i\ell^2/4 \sum_{i<j} p_i * p_j}$$

$$p_i * p_j = \epsilon_{\alpha\beta} p_i^\alpha p_j^\beta$$

does not produce a consistent S-matrix,

but allows to dress:

$$\boxed{\hat{S}_n(p_i) = e^{i\ell^2/4 \sum_{i<j} p_i * p_j} S_n(p_i)}$$

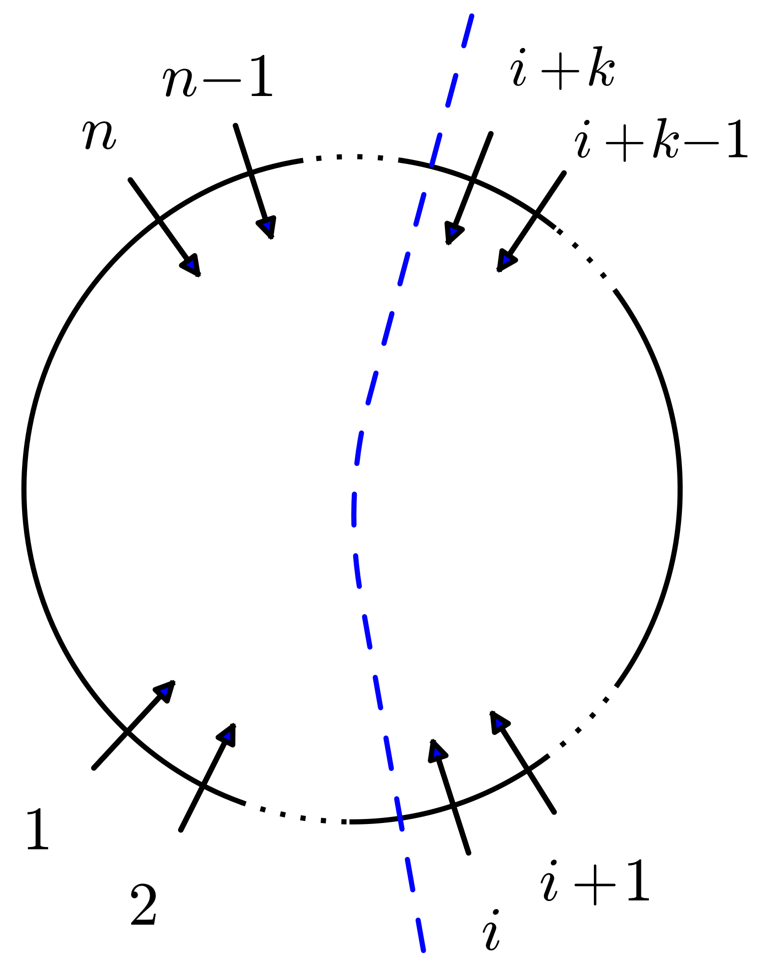
✓ Crossing Symmetry

✓ Analyticity

? Unitarity

? Factorization

✓ Unitarity from Factorization



$$\mathcal{D}(p_i) = e^{i\ell^2/4\sum_{a<a'} k_a * k_{a'}} e^{i\ell^2/4\sum_{b<b'} q_b * q_{b'}}$$

$$\boxed{\hat{S} = USU}$$

$$U|\{k_a\}\rangle = e^{i\ell^2/4\sum_{a<a'} k_a * k_{a'}} |\{k_a\}\rangle$$

The whole story is a bit similar to non-commutativity.

Two crucial differences:

✶Dressing of the full S-matrix, rather than of the tree amplitudes.

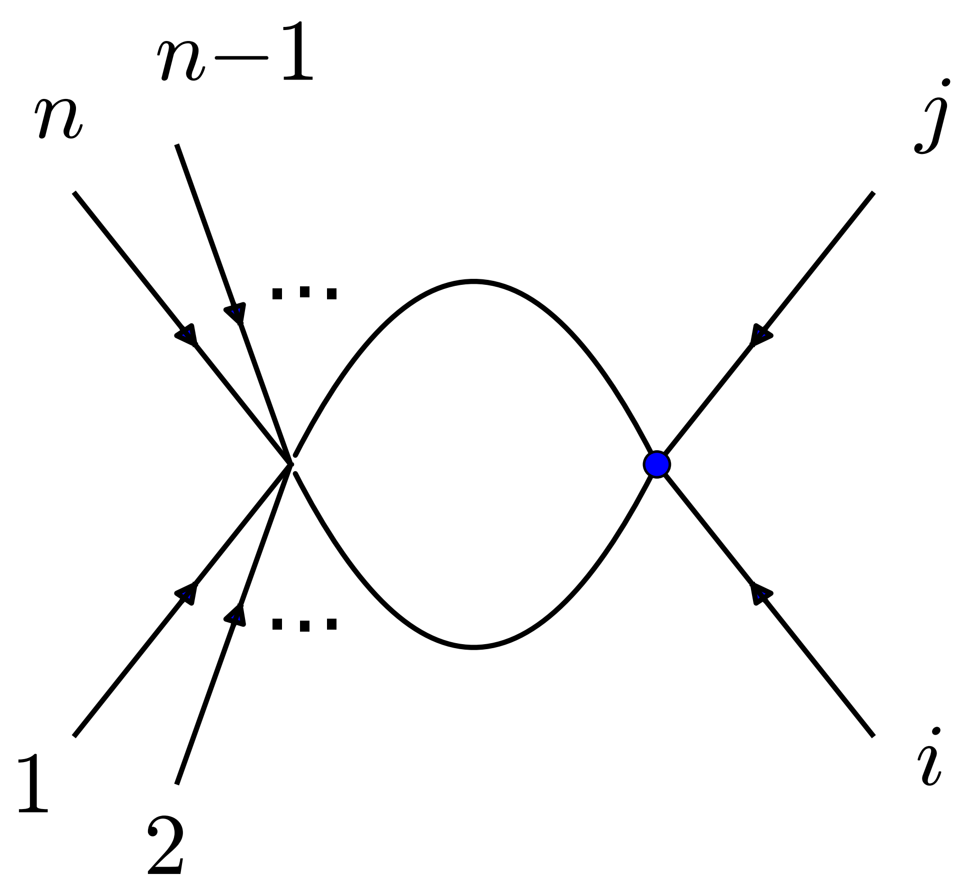
✶No summation over different cycling orderings. Preserves causality.

Perturbative Check

$$\mathcal{L}_{QFT} = \frac{1}{2}\left(\sum_i \partial\phi_i \partial\phi_i - m_i^2\phi_i^2\right) - \lambda\phi_1\phi_2\ldots\phi_n$$

Free Field Dressing:

$$\Delta\mathcal{L}_2 = -\frac{\ell^2}{8}\left((\partial_\alpha\phi_i\partial^\alpha\phi_i)^2 - 2(\partial_\alpha\phi_i\partial^\alpha\phi_j)^2 + m_i^2 m_j^2\phi_i^2\phi_j^2\right)$$



Reproduces $\mathcal{O}(\lambda\ell^2)$-dressing up to local polynomial terms

Hierarchy Problem

Directly in terms of properties of the RG flow,
without ever mentioning quadratic divergencies

For concreteness, let us place the discussion in the context of non-SUSY GUTs

$$m_H \ll E \ll m_{GUT} \ : \mathcal{L} = CFT_{321} + \underbrace{m_H^2 H^2}_{\text{relevant}} + \sum_i \underbrace{\frac{\mathcal{O}_i}{M_{GUT}^{\Delta_i - 4}}}_{\text{irrelevant}}$$

How comes $m_H \ll m_{GUT}$ given no symmetry?

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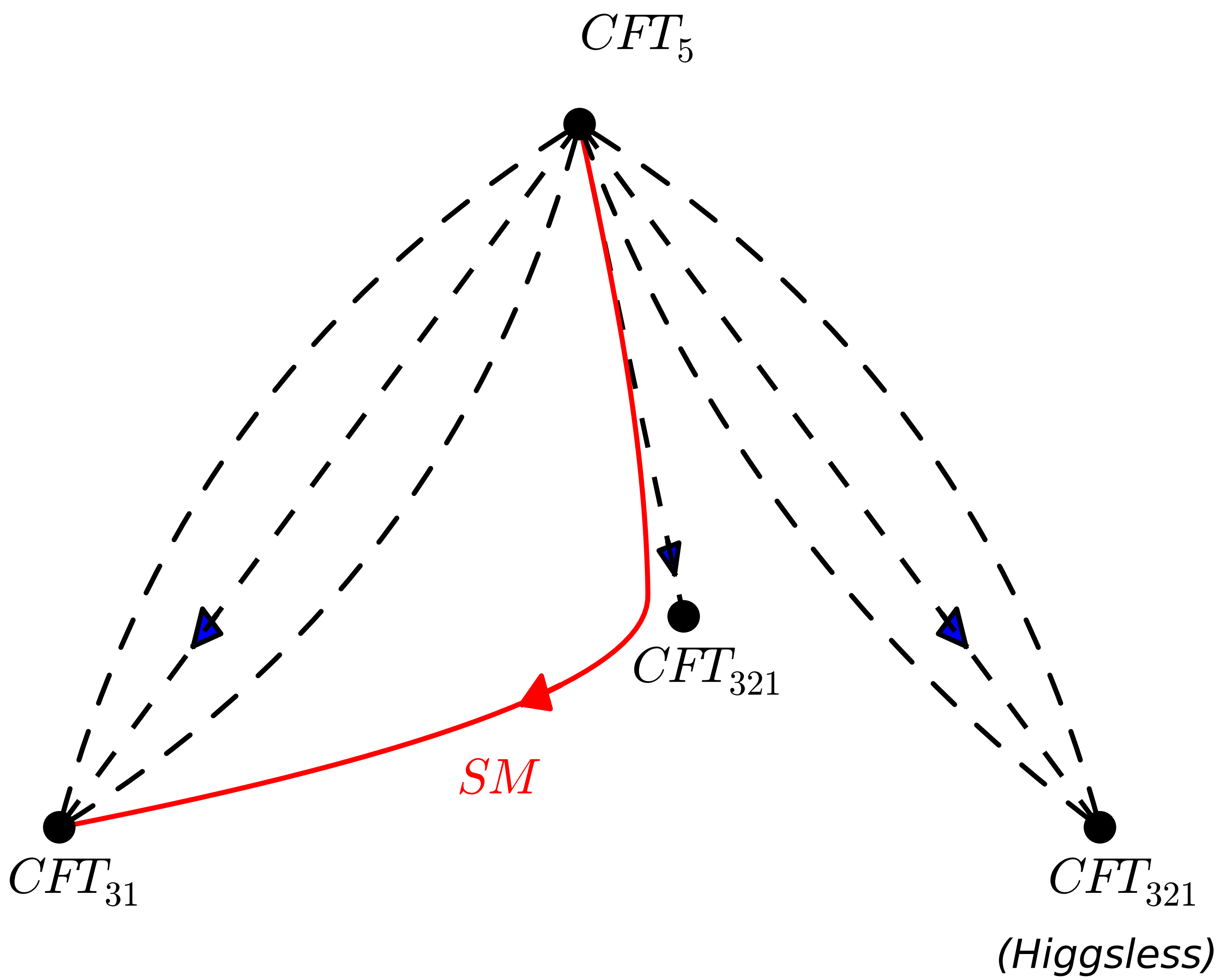
However, fine-tuning is truly manifest only as seen from higher energies:

$$m_{GUT} \ll E : \mathcal{L} = CFT_5 + \underbrace{g_h m_{GUT}^2 H^2}_{\text{relevant}} + \underbrace{g_\Sigma m_{GUT}^2 \Sigma^2}_{\text{relevant}} + \dots$$



No picture like that in our example. Energy scale does not correspond to a threshold. No scale invariance and no Wilsonian RG above the scale.

Two notions of a naturalness:

1) *If a natural theory possesses unprotected relevant operators (scalar masses), the corresponding energy scale should be the highest*

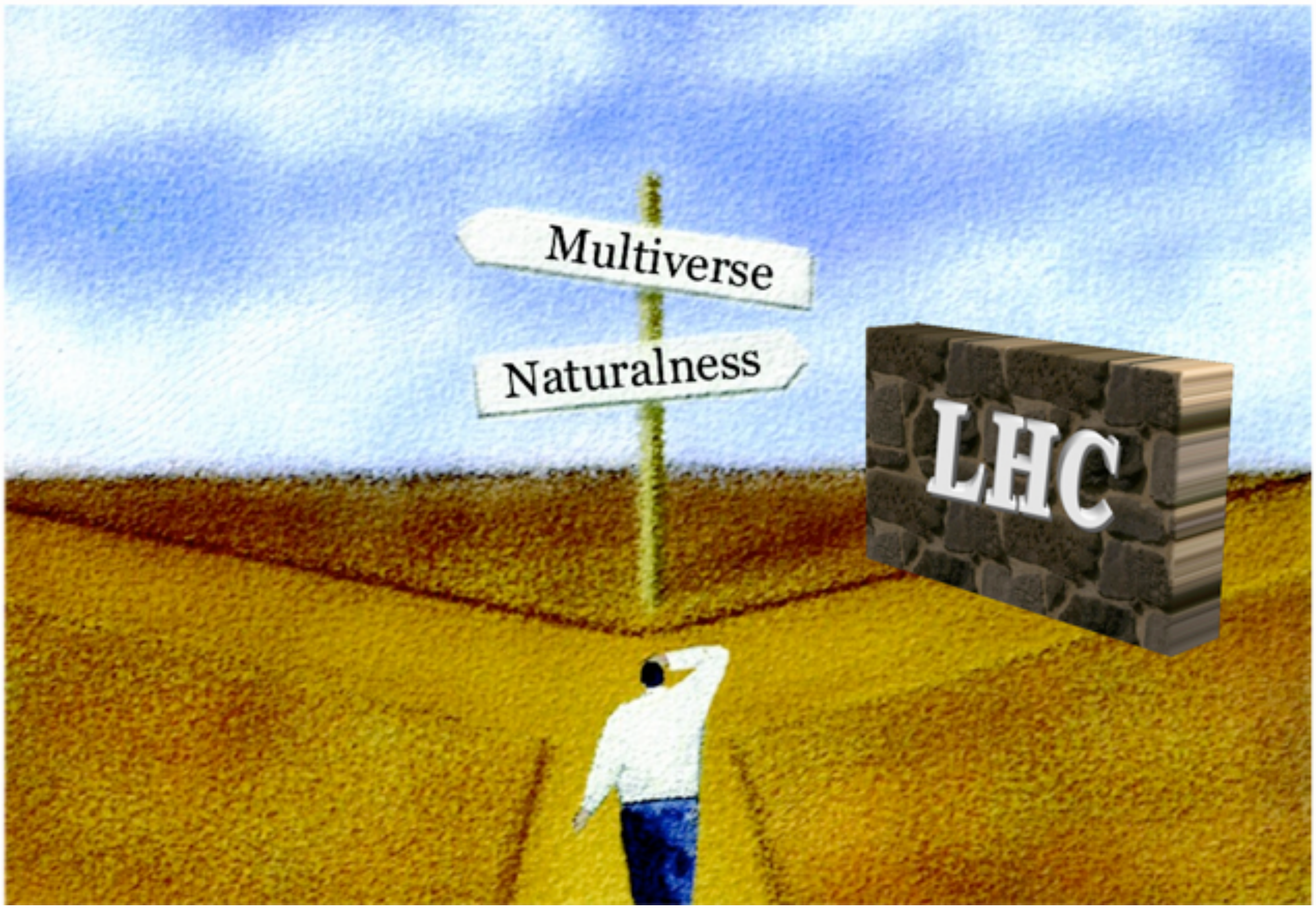
2) *Among all possible scales set by relevant operators unprotected operators should correspond to the highest scale*

✳Agree for QFT = UV CFT perturbed by relevant operators.

✳May disagree in the presence of gravity.

Indeed, disagree in the gravitational dressing construction.

Is there a place for this scenario within the “standard” picture/string theory?



The moment we talk about naturalness we are in the Landscape/Multiverse

Two canonical regions in the Landscape capable of producing a light Higgs:

✳An island where the Higgs mass is protected by a symmetry (SUSY...)

✳Among "10^{100}" or so of random vacua with randomly distributed values of the Higgs mass

Is there a third one?

✳Dragonland: A (small) set of strongly coupled vacua: $g_s = 1$ and Planckian extra dimensions

Possible lesson:

Should we be more serious about thinking on-shell when gravity is involved?

CC:

- ✴Off-shell: nothing special about zero vacuum energy
- ✴On-shell: zero CC is extremely special:

AdS:CFT, Minkowski:S-matrix, de Sitter: ???

Another possible lesson/alternative definition of naturalness:

Every natural QFT is an answer to some question.
Perhaps we should learn to ask more questions.

c.f. the following naturalness question:

31415926535897932384626433832795028841971693993...

is this sequence of digits “natural”?