Higgs naturalness and Scale Invariance

W. Skiba (Yale U)

Phys. Rev. D89 (2014) 015009

with G. Marques Tavarez and M. Schmaltz

Outline

- Higgs mass in scale invariant theories
- Threshold corrections
- Comments on different regulators
  (is dim reg special?)
- Scale invariance all the way
  and asymptotically free theories

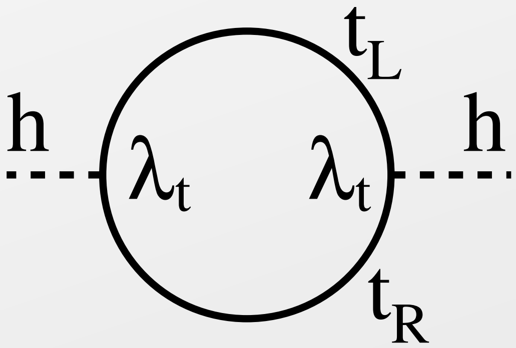
One idea to address naturalness: scale-invariant theories.

There are two possible philosophies:

(i) SM is embedded in a scale-invariant theory up to the Planck scale and then gravity is somehow different

(ii) SM and gravity merge into a scale-invariant theory (asymptotic safety)

It is easy to see how fine tuning might be avoided using dimensional regularization:



$$\mathcal{L} = -\lambda_t\, h\, \bar{t} P_L t + \text{H.c.}$$
$$= -\lambda_t\, h\, \mathcal{O} + \text{H.c.}$$

$$\delta m_h^2 = \frac{N_c \lambda_t^2}{4\pi^2} m_t^2 \left(\frac{1}{\epsilon} - \gamma - \log \frac{m_t^2}{4\pi\mu^2} + \frac{1}{2} \right)$$

After minimal subtraction corrections to the Higgs mass are proportional to small mass scales.

Two essential points:

(i) $$\delta m_h^2 \propto \int \frac{d^d p}{(2\pi)^d} \frac{1}{p^2} = 0$$

$$[\ \delta m_h^2 \propto \int \frac{d^d p}{(2\pi)^d} \frac{1}{(p^2)^\alpha} = 0\ ]$$

(ii) there are no heavy particles

Threshold corrections

SM is nearly scale invariant classically, but cannot be so quantum mechanically at high energies because of the hypercharge U(1) and gravity.

Let's assume that SM merges into a CFT at an energy scale M.

Threshold corrections

SM is nearly scale invariant classically, but cannot be so quantum mechanically at high energies because of the hypercharge U(1) and gravity.

Let's assume that SM merges into a CFT at an energy scale M.

[If there are heavy particles at M, the usual fine-tuning arguments apply. Perhaps the CFT phase can be entered without heavy states present as a result of coupling evolution. Banks-Zaks fixed point a good example.]

A useful way of computing Higgs mass corrections:

$$\delta m_h^2 = i(-i\lambda_t)^2 \mu^{4-d} \int d^d x \langle 0|T\,\mathcal{O}^\dagger(x)\mathcal{O}(0)|0\rangle.$$

In a CFT

$$\langle 0|T\mathcal{O}^\dagger(x)\mathcal{O}(0)|0\rangle = C\left(\frac{1}{-x^2+i\epsilon}\right)^{d-1+\gamma_{\mathcal{O}}}$$

Parameterize different behaviors in the IR and UV with an intermediate threshold M:

$$\langle 0|T\,\mathcal{O}^\dagger(x)\mathcal{O}(0)|0\rangle = \left(\frac{1}{-x^2}\right)^{d-1} f(-x^2M^2)$$

$$\langle 0|T\,\mathcal{O}^\dagger(x)\mathcal{O}(0)|0\rangle = \left(\frac{1}{-x^2}\right)^{d-1} f(-x^2 M^2)$$

$$f(y) \to \begin{cases} 1 & \text{as} \quad y \to \infty \;\; (\text{IR}), \\ y^{-\gamma_{\text{UV}}} & \text{as} \quad y \to 0 \quad (\text{UV}). \end{cases}$$

$$\langle 0|T\,\mathcal{O}^\dagger(x)\mathcal{O}(0)|0\rangle = \left(\frac{1}{-x^2}\right)^{d-1} f(-x^2M^2)$$

$$f(y) \to \begin{cases} 1 & \text{as} \quad y \to \infty \ \ (\mathrm{IR}), \\ y^{-\gamma_{\mathrm{UV}}} & \text{as} \quad y \to 0 \quad (\mathrm{UV}). \end{cases}$$

$$\delta m_h^2 \propto \mu^{4-d} \int d^d x \left(\frac{1}{-x^2}\right)^{d-1} f(-x^2M^2)$$

$$\propto \ M^2 \frac{\pi^{d/2}}{\Gamma(d/2)} \left(\frac{\mu^2}{M^2}\right)^{2-d/2} \int_0^\infty \frac{dy}{y^{d/2}}\, f(y)$$

$$\langle 0|T\,\mathcal{O}^\dagger(x)\mathcal{O}(0)|0\rangle = \left(\frac{1}{-x^2}\right)^{d-1} f(-x^2M^2)$$

$$f(y) \to \begin{cases} 1 & \text{as} \quad y\to\infty \ \ (\mathrm{IR}), \\ y^{-\gamma_{\mathrm{UV}}} & \text{as} \quad y\to 0 \quad (\mathrm{UV}). \end{cases}$$

$$\delta m_h^2 \propto \mu^{4-d}\int d^dx \left(\frac{1}{-x^2}\right)^{d-1} f(-x^2M^2)$$

$$\propto\ M^2\frac{\pi^{d/2}}{\Gamma(d/2)}\left(\frac{\mu^2}{M^2}\right)^{2-d/2}\int_0^\infty \frac{dy}{y^{d/2}}\,f(y)$$

Cannot calculate f(y). It is inherently non-perturbative.

$$f(y) = \begin{cases} 1 & \text{for} \quad y > 1, \\ y^{-\gamma_{\mathrm{UV}}} & \text{for} \quad 1 > y > 0. \end{cases}$$

$$f(y) = \begin{cases} 1 & \text{for} \quad y > 1, \\ y^{-\gamma_{\rm UV}} & \text{for} \quad 1 > y > 0. \end{cases}$$

UV subtraction

$$\delta m_h^2 \propto M^2 \int_0^\infty \frac{dy}{y^2} \left[f(y) - f_{CFT}(y) \right]$$
$$= M^2 \int_1^\infty \frac{dy}{y^2} \left[1 - \frac{1}{y^{\gamma_{\rm UV}}} \right]$$

Finite, can be computed in 4d

$$f(y) = \begin{cases} 1 & \text{for} \quad y > 1, \\ y^{-\gamma_{\rm UV}} & \text{for} \quad 1 > y > 0. \end{cases}$$

$$\delta m_h^2 \propto M^2 \int_1^\infty \frac{dy}{y^2} \left[1 - \frac{1}{y^{\gamma_{\rm UV}}} \right] = M^2 \frac{\gamma_{\rm UV}}{1 + \gamma_{\rm UV}}$$

UV subtraction,
(= 0 in dim reg)

(The abruptness of the transition is not crucial to the result.)

$$f(y) = \begin{cases} 1 & \text{for} \quad y > c \\ (y/c)^{-\gamma_{\mathrm{UV}}/2} & \text{for} \quad c > y > 1/c \\ y^{-\gamma_{\mathrm{UV}}} & \text{for} \quad 1/c > y > 0 \end{cases}$$

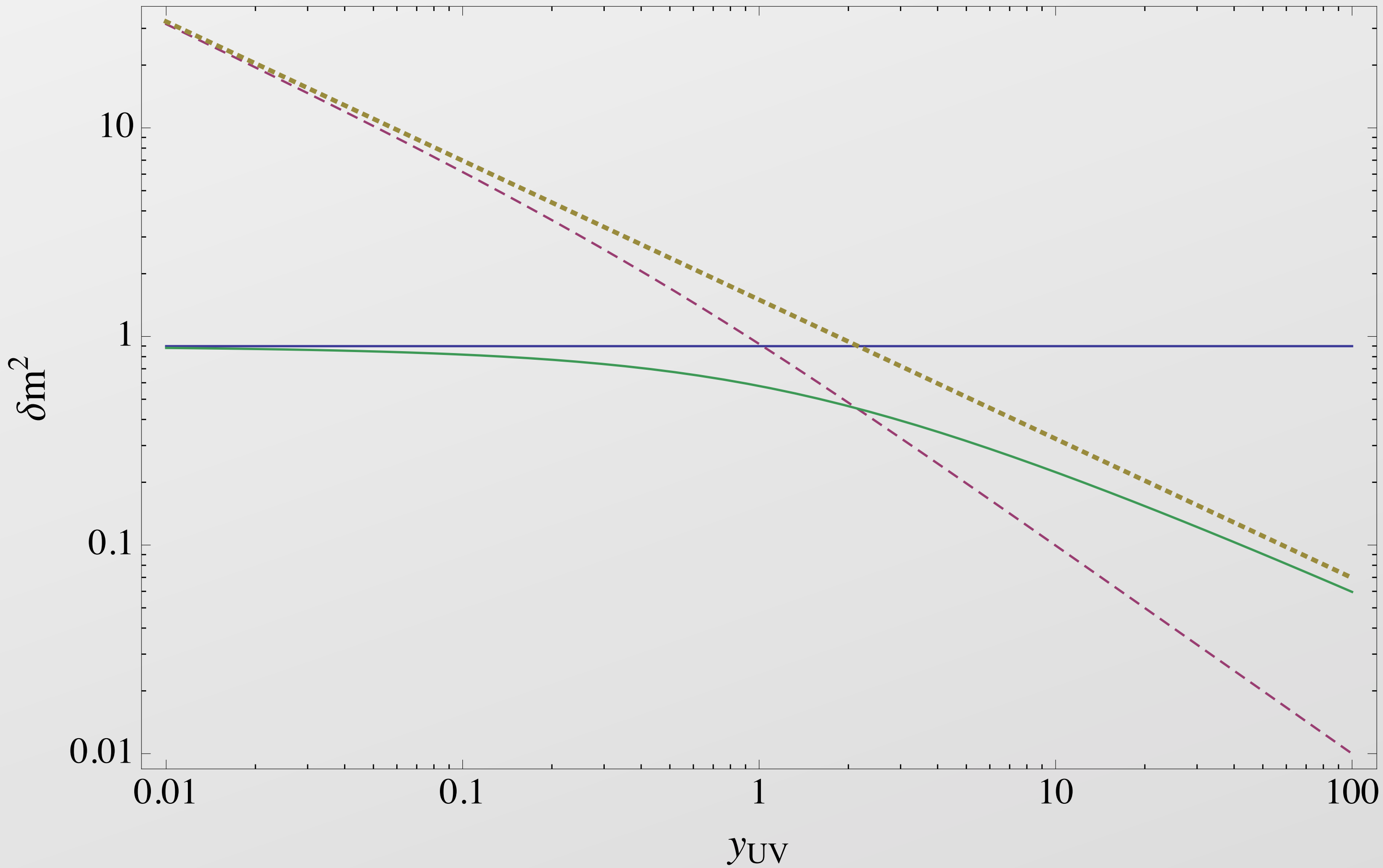
$$f(y) = \left(\frac{1}{y^{n\gamma_{\mathrm{UV}}} + y^{n\gamma_{\mathrm{IR}}}} \right)^{1/n}$$

$$f(y) = \left(\frac{1}{y^{n\gamma_{\mathrm{UV}}} + y^{n\gamma_{\mathrm{IR}}}} \right)^{1/n}$$

$$\delta m_h^2 \propto M^2 \frac{\Gamma\left(\frac{1}{n} + \frac{1}{n\gamma_{UV}}\right)\Gamma\left(1 - \frac{1}{n\gamma_{UV}}\right)}{\Gamma\left(\frac{1}{n}\right)}$$

$$f(y) = \left(\frac{1}{y^{n\gamma_{\rm UV}} + y^{n\gamma_{\rm IR}}}\right)^{1/n}$$

$$\delta m_h^2 \propto M^2 \frac{\Gamma\left(\frac{1}{n} + \frac{1}{n\gamma_{UV}}\right)\Gamma\left(1 - \frac{1}{n\gamma_{UV}}\right)}{\Gamma\left(\frac{1}{n}\right)}$$



$$n = 4$$
$$\gamma_{\rm UV} = -1/3$$

Choice of regulator

- Dimensional regularization seems to go well with scale invariance because it seems to break the scale invariance minimally compared to other regulators.

Choice of regulator

- Dimensional regularization seems to go well with scale invariance because it seems to break the scale invariance minimally compared to other regulators.

- Clearly physics should not depend on the regulator, so we need to reach the same conclusions using any regularization scheme.

Choice of regulator

- Dimensional regularization seems to go well with scale invariance because it seems to break the scale invariance minimally compared to other regulators.

- Clearly physics should not depend on the regulator, so we need to reach the same conclusions using any regularization scheme.

- It is the symmetries of the quantum theory that dictate the form of counter-terms via the Ward identities and choosing specific regulators should not matter.

Well-known example: fermion masses are not tuned due to chiral symmetry $\delta m_\psi \propto m_\psi$

Well-known example: fermion masses are not tuned due to chiral symmetry $\delta m_\psi \propto m_\psi$

What happens if we choose a regulator that breaks chiral symmetry, e.g. higher-derivative regulator ?

$$\mathcal{L} = i\overline{\psi}\not{D}\psi - m\overline{\psi}\psi + \overline{\psi}\frac{\partial^2}{\Lambda}\psi + \ldots$$

$$\delta m_\psi \propto \Lambda$$

Well-known example: fermion masses are not tuned due to chiral symmetry $\delta m_\psi \propto m_\psi$

What happens if we choose a regulator that breaks chiral symmetry, e.g. higher-derivative regulator ?

$$\mathcal{L} = i\overline{\psi}\not{D}\psi - m\overline{\psi}\psi + \overline{\psi}\frac{\partial^2}{\Lambda}\psi + \ldots$$

$$\delta m_\psi \propto \Lambda$$

Fermion mass is still natural even though the regulator obscures that fact. The Ward identities need to be enforced and the choice of the counter-term to satisfy such identities does not constitute fine tuning.

Classical scale invariance is not relevant for the question of fine tuning. Only symmetries of the quantum theory matter.

Dimensional regularization with minimal subtraction assumes scale invariance of the UV theory by being insensitive to heavy scales.

Classical scale invariance is not relevant for the question of fine tuning. Only symmetries of the quantum theory matter.

Dimensional regularization with minimal subtraction assumes scale invariance of the UV theory by being insensitive to heavy scales.
Choice of counter terms is supposed to render the theory finite and preserve the symmetries.

$$\delta m_h^2 = \frac{N_c \lambda_t^2}{4\pi^2} m_t^2 \left(\frac{1}{\epsilon} - \gamma - \log \frac{m_t^2}{4\pi\mu^2} + \frac{1}{2} \right)$$

Choice of counter terms is supposed to render the theory finite and preserve the symmetries.

$$\delta m_h^2 = \frac{N_c \lambda_t^2}{4\pi^2} m_t^2 \left(\frac{1}{\epsilon} - \gamma - \log \frac{m_t^2}{4\pi\mu^2} + \frac{1}{2} \right)$$

Choice of counter terms is supposed to render the theory finite and preserve the symmetries.

$$\mathcal{L}_{c.t.} = H^\dagger H \, \frac{N_c \lambda_t^2}{4\pi^2} m_t^2 \frac{1}{\epsilon}$$

$$\delta m_h^2 = \frac{N_c \lambda_t^2}{4\pi^2} m_t^2 \left(\frac{1}{\epsilon} - \gamma - \log \frac{m_t^2}{4\pi\mu^2} + \frac{1}{2} \right)$$

Choice of counter terms is supposed to render the theory finite and preserve the symmetries.

$$\mathcal{L}_{c.t.} = H^\dagger H \, \frac{N_c \lambda_t^2}{4\pi^2} m_t^2 \frac{1}{\epsilon}$$

$$\mathcal{L}_{c.t.} = H^\dagger H \, \frac{N_c \lambda_t^2}{4\pi^2} m_t^2 \left(\frac{1}{\epsilon} - \gamma + \log(4\pi) \right)$$

$$\delta m_h^2 = \frac{N_c \lambda_t^2}{4\pi^2} m_t^2 \left(\frac{1}{\epsilon} - \gamma - \log \frac{m_t^2}{4\pi\mu^2} + \frac{1}{2} \right)$$

Choice of counter terms is supposed to render the theory finite and preserve the symmetries.

$$\mathcal{L}_{c.t.} = H^\dagger H \, \frac{N_c \lambda_t^2}{4\pi^2} m_t^2 \frac{1}{\epsilon}$$

$$\mathcal{L}_{c.t.} = H^\dagger H \, \frac{N_c \lambda_t^2}{4\pi^2} m_t^2 \left(\frac{1}{\epsilon} - \gamma + \log(4\pi) \right)$$

$$\mathcal{L}_{c.t.} = H^\dagger H \, \frac{N_c \lambda_t^2}{4\pi^2} m_t^2 \left(\frac{1}{\epsilon} + 2014 \cdot 8^{23} \right)$$

Dimensional regularization with MS seems to imply that contributions to scalar mass arise only from massive particles because it is a mass independent regulator.

Using Wilsonian renormalization

$$e^{i\int \mathcal{L}_W(H,\ldots;\Lambda')} = \int_{\Lambda'<k<\Lambda} \mathcal{D}\varphi \; e^{i\int \mathcal{L}_W(H,\ldots;\Lambda)}$$

makes it apparent that scalar mass contribution arise from each momentum shell.

For example, in supersymmetric theories contributions to the scalar mass cancel for each momentum shell between the fermionic and bosonic fields.

Scale invariance at the highest scales

It is proposed that SM and gravity are described by a field theory at energies above the Planck scale. The theory approaches a UV fixed point and therefore might be valid up to arbitrarily high scales.

Scale invariance at the highest scales

It is proposed that SM and gravity are described by a field theory at energies above the Planck scale. The theory approaches a UV fixed point and therefore might be valid up to arbitrarily high scales.

Normally we don't worry about logarithmic divergencies since log(M_Planck/M_weak) is not huge. For an ultimate theory any leftover divergence that cannot be attributed to a CFT behavior implies (infinite) fine tuning. [Even without a divergence there could be large contributions to the Higgs mass from high scales.]

The fixed point needs to be approached sufficiently fast.

Asymptotically free theories present a puzzle:

At short distances, one can use the OPE

$$\langle 0|T\,\mathcal{O}^\dagger(x)\mathcal{O}(0)|0\rangle \propto \left(\frac{1}{|x|^2}\right)^3 \left(\log\frac{1}{|x|^2 M^2}\right)^{-a/b_0}$$

$$\delta m_h^2 \propto \int_{z_{IR}}^{\infty} dz \left(\frac{1}{\log(z/M^2)}\right)^{a/b_0} \quad [\text{where } z = \frac{1}{x^2}]$$

Asymptotically free theories present a puzzle:

At short distances, one can use the OPE

$$\langle 0|T\,\mathcal{O}^\dagger(x)\mathcal{O}(0)|0\rangle \propto \left(\frac{1}{|x|^2}\right)^3 \left(\log\frac{1}{|x|^2 M^2}\right)^{-a/b_0}$$

$$\delta m_h^2 \propto \int_{z_{IR}}^{\infty} dz \left(\frac{1}{\log(z/M^2)}\right)^{a/b_0} \quad [\text{where } z = \frac{1}{x^2}]$$

- dim reg: d<2 and analytic continuation yields multi-valued function
- subtracting a CFT power law does not give finite result
- could not find a meaningful way to regulate this integral

Summary

- Even if scale invariance solves the hierarchy problem in the UV, SM needs to be modified at the TeV scale due to threshold contributions
- Scale invariance of the quantum theory could ameliorate the hierarchy problem, but classical scale invariance does not cut it.
- Asymptotic safety faces additional challenges from naturalness.

The end.