## 2

## The Standard Model:

Current Status \& Open Questions

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## Parity violation in weak decays

1956 Wu et al.: correlation between spin vector $\vec{J}$ of polarized ${ }^{60} \mathrm{Co}$ and direction $\hat{p}_{e}$ of outgoing $\beta$ particle

## Parity leaves spin (axial vector) unchanged $\mathcal{P}: \vec{J} \rightarrow \vec{J}$

Parity reverses electron direction $\mathcal{P}: \hat{p}_{e} \rightarrow-\hat{p}_{e}$
Correlation $\vec{J} \cdot \hat{p}_{e}$ is parity violating

Parity links left-handed, right-handed $\nu$,

$$
\nu_{L} \stackrel{\mathcal{P}}{\Leftarrow} \stackrel{\nless R}{\rightleftarrows}
$$

$\Rightarrow$ build a manifestly parity-violating theory with only $\nu_{\mathrm{L}}$.

## Parity violation in ${ }^{60} \mathrm{Co} \mathrm{decay}$



## Left-handed Charged-current Interaction

 Polarized $e^{ \pm} p \rightarrow(\bar{\nu}, \nu)+$ anything - no RHCC

## Effective Lagrangian for the Weak Interactions

$$
\mathcal{L}_{\mathrm{V}-\mathrm{A}}=\frac{-G_{\mathrm{F}}}{\sqrt{2}} \bar{\nu} \gamma_{\mu}\left(1-\gamma_{5}\right) e \bar{e} \gamma^{\mu}\left(1-\gamma_{5}\right) \nu
$$

with $G_{F}=1.1663787(6) \times 10^{-5} \mathrm{GeV}^{-2}$. $\bar{\nu}$ e scattering:


$$
\mathcal{M}=-\frac{i G_{F}}{\sqrt{2}} \bar{v}\left(\nu, q_{1}\right) \gamma_{\mu}\left(1-\gamma_{5}\right) u\left(e, p_{1}\right) \bar{u}\left(e, p_{2}\right) \gamma^{\mu}\left(1-\gamma_{5}\right) v\left(\nu, q_{2}\right)
$$

## $\mathcal{L}_{\mathrm{V}-\mathrm{A}}$ Consequences

$$
\begin{aligned}
& \frac{d \sigma_{\mathrm{V}-\mathrm{A}}(\bar{\nu} e \rightarrow \bar{\nu} e)}{d \Omega_{\mathrm{cm}}}=\frac{\overline{|\mathcal{M}|^{2}}}{64 \pi^{2} s}=\frac{G_{\mathrm{F}}^{2} \cdot 2 m E_{\nu}(1-z)^{2}}{16 \pi^{2}} ; \quad z=\cos \theta^{*} \\
& \sigma_{\mathrm{V}-\mathrm{A}}(\bar{\nu} e \rightarrow \bar{\nu} e)=\frac{G_{\mathrm{F}}^{2} \cdot 2 m E_{\nu}}{3 \pi} \approx 0.574 \times 10^{-41} \mathrm{~cm}^{2}\left(\frac{E_{\nu}}{1 \mathrm{GeV}}\right)
\end{aligned}
$$

Repeat for $\nu$ e scattering:

$$
\begin{gathered}
\frac{d \sigma_{\mathrm{V}-\mathrm{A}}(\nu e \rightarrow \nu e)}{d \Omega_{\mathrm{cm}}}=\frac{G_{\mathrm{F}}^{2} \cdot 2 m E_{\nu}}{4 \pi^{2}} \\
\sigma_{\mathrm{V}-\mathrm{A}}(\nu e \rightarrow \nu e)=\frac{G_{\mathrm{F}}^{2} \cdot 2 m E_{\nu}}{\pi} \approx 1.72 \times 10^{-41} \mathrm{~cm}^{2}\left(\frac{E_{\nu}}{1 \mathrm{GeV}}\right)
\end{gathered}
$$

## Problem 6

Trace the origin of the factor-of-three difference between the $\nu e$ and $\bar{\nu} e$ cross sections, which arises from the left-handed nature of the charged-current weak interaction. Analyze the spin configurations for forward and backward scattering for the two cases, and show how angular momentum conservation accounts for the different angular distributions.

The Two-Neutrino Experiment
Lederman, Schwartz, Steinberger, 1962

- Make a beam of high-energy $\nu$ from $\pi \rightarrow \mu \nu$
- Observe $\nu N \rightarrow \mu+X$ not $\nu N \rightarrow e+X$
$\Rightarrow$ neutrino produced in $\pi \rightarrow \mu \nu$ decay is $\nu_{\mu}$
Suggests family structure

$$
\binom{\nu_{e}}{e}_{\mathrm{L}} \quad\binom{\nu_{\mu}}{\mu}_{\mathrm{L}}
$$

Generalize the effective Lagrangian:

$$
\mathcal{L}_{\mathrm{V}-\mathrm{A}}^{(e \mu)}=\frac{-G_{\mathrm{F}}}{\sqrt{2}} \bar{\nu}_{\mu} \gamma_{\mu}\left(1-\gamma_{5}\right) \mu \overline{\mathrm{e}} \gamma^{\mu}\left(1-\gamma_{5}\right) \nu_{e}+\text { h.c. }
$$

## (Inverse) Muon Decay

$$
\begin{gathered}
\Gamma\left(\mu \rightarrow e \bar{\nu}_{e} \nu_{\mu}\right)=\frac{G_{F}^{2} m_{\mu}^{5}}{192 \pi^{3}} \\
\sigma\left(\nu_{\mu} e \rightarrow \mu \nu_{e}\right)=\sigma_{\mathrm{V}-\mathrm{A}}\left(\nu_{e} e \rightarrow \nu_{e} e\right)\left[1-\frac{\left(m_{\mu}^{2}-m_{e}^{2}\right)}{2 m_{e} E_{\nu}}\right]^{2}
\end{gathered}
$$

## Partial-wave Unitarity (Probability Conservation)

 PW unitarity constrains the modulus $\left|\mathcal{M}_{J=0}\right|<1$ for an $s$-wave amplitude.Equivalently, the contribution to the cross section is bounded by $\sigma_{0}<\pi / p_{\mathrm{cm}}^{2}$.

$$
\mathcal{M}_{0}=\frac{G_{\mathrm{F}} \cdot 2 m_{e} E_{\nu}}{\pi \sqrt{2}}\left[1-\frac{\left(m_{\mu}^{2}-m_{e}^{2}\right)}{2 m_{e} E_{\nu}}\right]
$$

Satisfies unitarity constraint for

$$
\begin{aligned}
E_{\nu}<\pi / G_{\mathrm{F}} m_{e} \sqrt{2} \approx 3.7 & \times 10^{8} \mathrm{GeV} \\
& \text { or } E_{\mathrm{cm}} \lesssim 300 \mathrm{GeV}
\end{aligned}
$$

Physics must change before $\sqrt{s}=600 \mathrm{GeV}$

## Problem 7

Using the measured lifetimes of the muon and the tau lepton, $\tau_{\ell}=\hbar / \Gamma_{\ell}$, and the branching fractions into the $e \bar{\nu}_{e} \nu_{\ell}$ channel to determine the Fermi couplings for muon and tau interactions, $G_{\mu}$ and $G_{\tau}$. Compare these two values with each other and with the standard value of $G_{F}$.

The equality of $G_{\mu}, G_{\tau}$, and $G_{F}$ and supports the notion that the leptonic (charged-current) weak interactions are of universal strength.

Introduction: J. R. Patterson, "Lepton Universality," SLAC Beam Line (Spring 1995).
Recent BaBar study, Phys. Rev. Lett. 105, 051602 (2008).

## Electroweak theory antecedents

Lessons from experiment and theory

- Parity-violating $V-A$ structure of charged current
- Cabibbo universality of leptonic and semileptonic processes
- Absence of strangeness-changing neutral currents
- Negligible neutrino masses; left-handed neutrinos
- Unitarity: four-fermion description breaks down at $\sqrt{s} \approx 620 \mathrm{GeV} \quad \nu_{\mu} e \rightarrow \mu \nu_{e}$
- $\nu \bar{\nu} \rightarrow W^{+} W^{-}$: divergence problems of ad hoc intermediate vector boson theory


## Formulate electroweak theory

Three crucial clues from experiment:

- Left-handed weak-isospin doublets,

$$
\begin{array}{lll}
\binom{\nu_{e}}{e}_{\mathrm{L}} & \binom{\nu_{\mu}}{\mu}_{\mathrm{L}} & \binom{\nu_{\tau}}{\tau}_{\mathrm{L}} \\
\binom{u}{d^{\prime}}_{\mathrm{L}} & \binom{c}{s^{\prime}}_{\mathrm{L}} & \binom{t}{b^{\prime}}_{\mathrm{L}} ;
\end{array}
$$

- Universal strength of the (charged-current) weak interactions;
- Idealization that neutrinos are massless.

First two clues suggest $\operatorname{SU}(2)_{\text {L }}$ gauge symmetry

## A theory of leptons

$$
\mathrm{L}=\binom{\nu_{e}}{e}_{\mathrm{L}} \quad \mathrm{R} \equiv e_{\mathrm{R}}
$$

weak hypercharges $Y_{\mathrm{L}}=-1, Y_{\mathrm{R}}=-2$
Gell-Mann-Nishijima connection, $Q=I_{3}+\frac{1}{2} Y$
$S U(2)_{\mathrm{L}} \otimes U(1)_{Y}$ gauge group $\Rightarrow$ gauge fields:

- weak isovector $\vec{b}_{\mu}$, coupling $g$

$$
b_{\mu}^{\ell}=b_{\mu}^{\ell}-\varepsilon_{j k \ell} \alpha^{j} b_{\mu}^{k}-(1 / g) \partial_{\mu} \alpha^{\ell}
$$

- weak isoscalar $\mathcal{A}_{\mu}$, coupling $g^{\prime} / 2$

$$
\mathcal{A}_{\mu} \rightarrow \mathcal{A}_{\mu}-\partial_{\mu} \alpha
$$

Field-strength tensors

$$
\begin{gathered}
F_{\mu \nu}^{\ell}=\partial_{\nu} b_{\mu}^{\ell}-\partial_{\mu} b_{\nu}^{\ell}+g \varepsilon_{j k \ell} b_{\mu}^{j} b_{\nu}^{k}, \mathrm{SU}(2)_{\mathrm{L}} \\
f_{\mu \nu}=\partial_{\nu} \mathcal{A}_{\mu}-\partial_{\mu} \mathcal{A}_{\nu}, \mathrm{U}(1)_{Y}
\end{gathered}
$$

## Interaction Lagrangian

$$
\begin{gathered}
\mathcal{L}=\mathcal{L}_{\text {gauge }}+\mathcal{L}_{\text {leptons }} \\
\mathcal{L}_{\text {gauge }}=-\frac{1}{4} F_{\mu \nu}^{\ell} F^{\ell \mu \nu}-\frac{1}{4} f_{\mu \nu} f^{\mu \nu} \\
\mathcal{L}_{\text {leptons }}=\overline{\mathrm{R}} i \gamma^{\mu}\left(\partial_{\mu}+i \frac{g^{\prime}}{2} \mathcal{A}_{\mu} Y\right) \mathrm{R} \\
+\overline{\mathrm{L}} i \gamma^{\mu}\left(\partial_{\mu}+i \frac{g^{\prime}}{2} \mathcal{A}_{\mu} Y+i \frac{g}{2} \vec{\tau} \cdot \vec{b}_{\mu}\right) \mathrm{L}
\end{gathered}
$$

Mass term $\mathcal{L}_{e}=-m_{e}\left(\bar{e}_{R} e_{L}+\bar{e}_{L} e_{R}\right)=-m_{e} \bar{e} e$ would violate local gauge invariance

Theory: 4 massless gauge bosons $\left(\begin{array}{llll}\mathcal{A}_{\mu} & b_{\mu}^{1} & b_{\mu}^{2} & b_{\mu}^{3}\end{array}\right)$; Nature: $1(\gamma)$

## Symmetric law need not imply symmetric outcome

 Buckling rod (L. Euler) spontaneous symmetry breaking
radius $R$, moment of inertia $I=\pi R^{2} / 4$, elastic modulus $E$

$$
\text { Critical point: } F_{\mathrm{cr}}=\pi^{2} I E / \ell^{2}
$$

symmetric solution unstable, ground state degenerate

## Symmetric law need not imply symmetric outcome



## Massive Photon? <br> Hiding Symmetry

Recall 2 miracles of superconductivity:

- No resistance ... ... Meissner effect (exclusion of B)

Ginzburg-Landau Phenomenology (not a theory from first principles) normal, resistive charge carriers ... ... + superconducting charge carriers

$\mathbf{B}=0$ :
$G_{\text {super }}(0)=G_{\text {normal }}(0)+\alpha|\psi|^{2}+\beta|\psi|^{4}$

$$
\left.T>T_{c}: \alpha>\left.0 \quad\langle | \psi\right|^{2}\right\rangle_{0}=0
$$

## Massive Photon? <br> Hiding Symmetry

Recall 2 miracles of superconductivity:

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Ginzburg-Landau Phenomenology (not a theory from first principles) normal, resistive charge carriers ... ... + superconducting charge carriers

$$
\begin{aligned}
& \mathbf{B}=0: \\
& G_{\text {super }}(0)=G_{\text {normal }}(0)+\alpha|\psi|^{2}+\beta|\psi|^{4} \\
& \left.T<T_{c}: \quad \alpha<\left.0 \quad\langle | \psi\right|^{2}\right\rangle_{0} \neq 0
\end{aligned}
$$

## In a nonzero magnetic field ...

$$
\begin{gathered}
G_{\text {super }}(\mathbf{B})=G_{\text {super }}(0)+\frac{\mathbf{B}^{2}}{8 \pi}+\frac{1}{2 m^{*}}\left|-i \hbar \nabla \psi-\frac{e^{*}}{c} \mathbf{A} \psi\right|^{2} \\
\left.\begin{array}{c}
e^{*}=-2 \\
m^{*}
\end{array}\right\} \text { of superconducting carriers }
\end{gathered}
$$

Weak, slowly varying field: $\quad \psi \approx \psi_{0} \neq 0, \nabla \psi \approx 0$
Variational analysis $\leadsto$

$$
\nabla^{2} \mathbf{A}-\frac{4 \pi e^{* 2}}{m^{*} c^{2}}\left|\psi_{0}\right|^{2} \mathbf{A}=0
$$

wave equation of a massive photon
Photon - gauge boson - acquires mass
within superconductor
origin of Meissner effect

## Hiding EW Symmetry

Higgs mechanism: relativistic generalization of Ginzburg-Landau superconducting phase transition

- Introduce a complex doublet of scalar fields

$$
\phi \equiv\binom{\phi^{+}}{\phi^{0}} \quad Y_{\phi}=+1
$$

- Add to $\mathcal{L}$ (gauge-invariant) terms for interaction and propagation of the scalars,

$$
\mathcal{L}_{\text {scalar }}=\left(\mathcal{D}^{\mu} \phi\right)^{\dagger}\left(\mathcal{D}_{\mu} \phi\right)-V\left(\phi^{\dagger} \phi\right)
$$

where $\mathcal{D}_{\mu}=\partial_{\mu}+i \frac{g^{\prime}}{2} \mathcal{A}_{\mu} Y+i \frac{g}{2} \vec{\tau} \cdot \vec{b}_{\mu}$ and

$$
V\left(\phi^{\dagger} \phi\right)=\mu^{2}\left(\phi^{\dagger} \phi\right)+|\lambda|\left(\phi^{\dagger} \phi\right)^{2}
$$

- Add a Yukawa interaction $\mathcal{L}_{\text {Yukawa }}=-\zeta_{e}\left[\overline{\mathrm{R}}\left(\phi^{\dagger} \mathrm{L}\right)+(\overline{\mathrm{L}} \phi) \mathrm{R}\right]$


## Unique and degenerate vacuum states

(a)

(b)


- Arrange self-interactions so vacuum corresponds to a broken-symmetry solution: $\mu^{2}<0$
Choose minimum energy (vacuum) state for vacuum expectation value

$$
\langle\phi\rangle_{0}=\binom{0}{v / \sqrt{2}}, \quad v=\sqrt{-\mu^{2} /|\lambda|}
$$

Hides (breaks) $S U(2)_{L}$ and $U(1)_{Y}$
but preserves $U(1)_{\text {em }}$ invariance
Invariance under $\mathcal{G}$ means $e^{i \alpha \mathcal{G}}\langle\phi\rangle_{0}=\langle\phi\rangle_{0}$, so $\mathcal{G}\langle\phi\rangle_{0}=0$

$$
\begin{aligned}
& \tau_{1}\langle\phi\rangle_{0}=\left(\begin{array}{ll}
0 & 1 \\
1 & 0
\end{array}\right)\binom{0}{v / \sqrt{2}}=\binom{v / \sqrt{2}}{0} \neq 0 \quad \text { broken! } \\
& \tau_{2}\langle\phi\rangle_{0}=\left(\begin{array}{cc}
0 & -i \\
i & 0
\end{array}\right)\binom{0}{v / \sqrt{2}}=\binom{-i v / \sqrt{2}}{0} \neq 0 \text { broken! } \\
& \tau_{3}\langle\phi\rangle_{0}=\left(\begin{array}{cc}
1 & 0 \\
0 & -1
\end{array}\right)\binom{0}{v / \sqrt{2}}=\binom{0}{-v / \sqrt{2}} \neq 0 \text { broken! } \\
& Y\langle\phi\rangle_{0}=Y_{\phi}\langle\phi\rangle_{0}=+1\langle\phi\rangle_{0}=\quad\binom{0}{v / \sqrt{2}} \neq 0 \quad \text { broken! }
\end{aligned}
$$

Examine electric charge operator $Q$ on the (neutral) vacuum

$$
\begin{aligned}
Q\langle\phi\rangle_{0} & =\frac{1}{2}\left(\tau_{3}+Y\right)\langle\phi\rangle_{0} \\
& =\frac{1}{2}\left(\begin{array}{cc}
Y_{\phi}+1 & 0 \\
0 & Y_{\phi}-1
\end{array}\right)\langle\phi\rangle_{0} \\
& =\left(\begin{array}{ll}
1 & 0 \\
0 & 0
\end{array}\right)\binom{0}{v / \sqrt{2}} \\
& =\binom{0}{0} \text { unbroken! }
\end{aligned}
$$

Four original generators are broken, electric charge is not

- $\operatorname{SU}(2)_{\mathrm{L}} \otimes \mathrm{U}(1)_{Y} \rightarrow \mathrm{U}(1)_{\mathrm{em}}$ (will verify)
- Expect massless photon
- Expect gauge bosons corresponding to

$$
\tau_{1}, \tau_{2}, \frac{1}{2}\left(\tau_{3}-Y\right) \equiv K \quad \text { to acquire masses }
$$

## Expand about the vacuum state

$$
\begin{aligned}
& \text { Let } \phi=\binom{0}{(v+\eta) / \sqrt{2}} ; \text { in unitary gauge } \\
& \qquad \begin{aligned}
\mathcal{L}_{\text {scalar }}= & \frac{1}{2}\left(\partial^{\mu} \eta\right)\left(\partial_{\mu} \eta\right)-\mu^{2} \eta^{2} \\
& +\frac{v^{2}}{8}\left[g^{2}\left|b_{\mu}^{1}-i b_{\mu}^{2}\right|^{2}+\left(g^{\prime} \mathcal{A}_{\mu}-g b_{\mu}^{3}\right)^{2}\right] \\
& + \text { interaction terms }
\end{aligned}
\end{aligned}
$$

"Higgs boson" $\eta$ has acquired (mass) $)^{2} M_{H}^{2}=-2 \mu^{2}>0$

$$
\begin{gathered}
\text { Define } W_{\mu}^{ \pm}=\frac{b_{\mu}^{1} \mp i b_{\mu}^{2}}{\sqrt{2}} \\
\frac{g^{2} v^{2}}{8}\left(\left|W_{\mu}^{+}\right|^{2}+\left|W_{\mu}^{-}\right|^{2}\right) \Longleftrightarrow M_{W^{ \pm}}=g v / 2
\end{gathered}
$$

$\left(v^{2} / 8\right)\left(g^{\prime} \mathcal{A}_{\mu}-g b_{\mu}^{3}\right)^{2} \ldots$

Now define orthogonal combinations

$$
\begin{aligned}
& Z_{\mu}=\frac{-g^{\prime} \mathcal{A}_{\mu}+g b_{\mu}^{3}}{\sqrt{g^{2}+g^{\prime 2}}} \quad A_{\mu}=\frac{g \mathcal{A}_{\mu}+g^{\prime} b_{\mu}^{3}}{\sqrt{g^{2}+g^{\prime 2}}} \\
& M_{Z^{0}}=\sqrt{g^{2}+g^{\prime 2}} \quad v / 2=M_{W} \sqrt{1+g^{\prime 2} / g^{2}}
\end{aligned}
$$

$A_{\mu}$ remains massless

$$
\begin{aligned}
& \mathcal{L}_{\text {Yukawa }}=-\zeta_{e} \frac{(v+\eta)}{\sqrt{2}}\left(\bar{e}_{R} e_{L}+\bar{e}_{L} e_{R}\right) \\
&=-\frac{\zeta_{e} v}{\sqrt{2}} \bar{e} e-\frac{\zeta_{e} \eta}{\sqrt{2}} \bar{e} e \\
& \text { electron acquires } m_{e}=\zeta_{e} v / \sqrt{2}
\end{aligned}
$$

Higgs-boson coupling to electrons: $m_{e} / v \quad$ ( $\propto$ mass)
Desired particle content ... plus a Higgs scalar
Values of couplings, electroweak scale $v$ ?
What about interactions?

## Interactions . . .

$$
\mathcal{L}_{W-\ell}=-\frac{g}{2 \sqrt{2}}\left[\bar{\nu} \gamma^{\mu}\left(1-\gamma_{5}\right) e W_{\mu}^{+}+\bar{e} \gamma^{\mu}\left(1-\gamma_{5}\right) \nu W_{\mu}^{-}\right]
$$

+ similar terms for $\mu$ and $\tau$


W

$$
\sim=\frac{-i\left(g_{\mu \nu}-k_{\mu} k_{\nu} / M_{W}^{2}\right)}{k^{2}-M_{W}^{2}}
$$

## Compute $\nu_{\mu} e \rightarrow \mu \nu_{e}$

$$
\sigma\left(\nu_{\mu} e \rightarrow \mu \nu_{e}\right)=\frac{g^{4} m_{e} E_{\nu}}{16 \pi M_{W}^{4}} \frac{\left[1-\left(m_{\mu}^{2}-m_{e}^{2}\right) / 2 m_{e} E_{\nu}\right]^{2}}{\left(1+2 m_{e} E_{\nu} / M_{W}^{2}\right)}
$$

Reproduces 4 -fermion result at low energies if

$$
\frac{g^{4}}{16 M_{W}^{4}}=2 G_{F}^{2} \Rightarrow \frac{g}{2 \sqrt{2}}=\left(\frac{G_{F} M_{W}^{2}}{\sqrt{2}}\right)^{\frac{1}{2}}
$$

Using $M_{W}=g v / 2$, determine the electroweak scale

$$
\begin{aligned}
& v=\left(G_{F} \sqrt{2}\right)^{-\frac{1}{2}} \approx 246 \mathrm{GeV} \\
\Rightarrow & \left\langle\phi^{0}\right\rangle_{0}=\left(G_{F} \sqrt{8}\right)^{-\frac{1}{2}} \approx 174 \mathrm{GeV}
\end{aligned}
$$

## W-propagator modifies HE behavior

$$
\begin{aligned}
\sigma\left(\nu_{\mu} e \rightarrow \mu \nu_{e}\right)= & \frac{g^{4} m_{e} E_{\nu}}{16 \pi M_{W}^{4}} \frac{\left[1-\left(m_{\mu}^{2}-m_{e}^{2}\right) / 2 m_{e} E_{\nu}\right]^{2}}{\left(1+2 m_{e} E_{\nu} / M_{W}^{2}\right)} \\
\lim _{E_{\nu} \rightarrow \infty} \sigma\left(\nu_{\mu} e \rightarrow \mu \nu_{e}\right)= & \frac{g^{4}}{32 \pi M_{W}^{2}}=\frac{G_{F}^{2} M_{W}^{2}}{\sqrt{2}} \\
& \text { no asymptotic growth with energy! }
\end{aligned}
$$

Partial-wave unitarity respected for

$$
s<M_{W}^{2}\left[\exp \left(\pi \sqrt{2} / G_{F} M_{W}^{2}\right)-1\right]
$$

## Problem 8

(a) Write the cross section given on the preceding page for $\sigma\left(\nu_{\mu} e \rightarrow \mu \nu_{e}\right)$ in terms of $G_{F}$, rather than $g$.
(b) Now put aside the threshold factor that results from the muon-electron mass difference. Plot the cross section for $1 \mathrm{GeV} \leq E_{\nu} \leq 10 \mathrm{TeV}$. Also plot the point-coupling expression over the same range.
(c) At what value of $E_{\nu}$ does the $W$-boson propagator begin to have a perceptible effect on the cross section? (d) How well would you have to determine the cross section to derive a useful estimate of $M_{W}$ ?
A. Aktas, et al. (H1 Collaboration), Phys. Lett. B 632, 35 (2006); Z. Zhang, Nucl. Phys. Proc. Suppl. 191, 271 (2009).

## $W$-boson properties

No prediction yet for $M_{W}$ (haven't determined $g$ )
Leptonic decay $W^{-} \rightarrow e^{-} \nu_{e}$

$$
W^{e(p) \quad p \approx\left(\frac{M_{W}}{2} ; \frac{M_{W} \sin \theta}{2}, 0, \frac{M_{W} \cos \theta}{2}\right)} \begin{aligned}
& \bar{\nu}_{e}(q) \quad q \approx\left(\frac{M_{W}}{2} ;-\frac{M_{W} \sin \theta}{2}, 0,-\frac{M_{W} \cos \theta}{2}\right)
\end{aligned}
$$

$$
\mathcal{M}=-i\left(\frac{G_{F} M_{W}^{2}}{\sqrt{2}}\right)^{\frac{1}{2}} \bar{u}(e, p) \gamma_{\mu}\left(1-\gamma_{5}\right) v(\nu, q) \varepsilon^{\mu}
$$

$\varepsilon^{\mu}=(0 ; \hat{\varepsilon}): W$ polarization vector in its rest frame

$$
\begin{aligned}
|\mathcal{M}|^{2} & =\frac{G_{F} M_{W}^{2}}{\sqrt{2}} \operatorname{tr}\left[\notin\left(1-\gamma_{5}\right) \phi\left(1+\gamma_{5}\right) \not^{* *} p\right] ; \\
\operatorname{tr}[\cdots] & =\left[\varepsilon \cdot q \varepsilon^{*} \cdot p-\varepsilon \cdot \varepsilon^{*} q \cdot p+\varepsilon \cdot p \varepsilon^{*} \cdot q+i \epsilon_{\mu \nu \rho \sigma} \varepsilon^{\mu} q^{\nu} \varepsilon^{* \rho} p^{\sigma}\right]
\end{aligned}
$$

$$
\operatorname{tr}[\cdots]=\left[\varepsilon \cdot q \varepsilon^{*} \cdot p-\varepsilon \cdot \varepsilon^{*} q \cdot p+\varepsilon \cdot p \varepsilon^{*} \cdot q+i \epsilon_{\mu \nu \rho \sigma} \varepsilon^{\mu} q^{\nu} \varepsilon^{* \rho} p^{\sigma}\right]
$$

decay rate is independent of $W$ polarization; look first at longitudinal pol. $\varepsilon^{\mu}=(0 ; 0,0,1)=\varepsilon^{* \mu}$, eliminate $\epsilon_{\mu \nu \rho \sigma}$

$$
\begin{gathered}
|\mathcal{M}|^{2}=\frac{4 G_{F} M_{W}^{4}}{\sqrt{2}} \sin ^{2} \theta \\
\frac{d \Gamma_{0}}{d \Omega}=\frac{|\mathcal{M}|^{2}}{64 \pi^{2}} \frac{\mathcal{S}_{12}}{M_{W}^{3}} \\
\mathcal{S}_{12}=\sqrt{\left[M_{W}^{2}-\left(m_{e}+m_{\nu}\right)^{2}\right]\left[M_{W}^{2}-\left(m_{e}-m_{\nu}\right)^{2}\right]}=M_{W}^{2} \\
\frac{d \Gamma_{0}}{d \Omega}=\frac{G_{F} M_{W}^{3}}{16 \pi^{2} \sqrt{2}} \sin ^{2} \theta \quad \Gamma(W \rightarrow e \nu)=\frac{G_{F} M_{W}^{3}}{6 \pi \sqrt{2}}
\end{gathered}
$$

Other helicities: $\varepsilon_{ \pm 1}^{\mu}=(0 ;-1, \mp i, 0) / \sqrt{2}$

$$
\frac{d \Gamma_{ \pm 1}}{d \Omega}=\frac{G_{F} M_{W}^{3}}{32 \pi^{2} \sqrt{2}}(1 \mp \cos \theta)^{2}
$$

Extinctions at $\cos \theta= \pm 1$ are consequences of angular momentum conservation:

(situation reversed for $W^{+} \rightarrow e^{+} \nu_{e}$ )
$e^{+}$follows polarization direction of $W^{+}$
$e^{-}$avoids polarization direction of $W^{-}$ important for discovery of $W^{ \pm}$in $\bar{p} p(\bar{q} q) \quad \mathrm{C}$ violation

## UA1 $W \rightarrow e \nu$ decay angular distribution



