The Standard Model:

Current Status & Open Questions

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Fermilab

Interactions . . .

$$\mathcal{L}_{A\text{-}\ell} = \frac{gg'}{\sqrt{g^2 + g'^2}} \bar{e}\gamma^\mu e A_\mu$$

. . . vector interaction; $\Rightarrow A_\mu$ as γ, provided we identify

$$\boxed{gg'/\sqrt{g^2 + g'^2} \equiv e}$$

Define $g' = g \tan\theta_W$ θ_W: weak mixing angle

$$\begin{aligned} g &= e/\sin\theta_W \geq e \\ g' &= e/\cos\theta_W \geq e \end{aligned}$$

$$Z_\mu = b^3_\mu \cos\theta_W - \mathcal{A}_\mu \sin\theta_W \quad A_\mu = \mathcal{A}_\mu \cos\theta_W + b^3_\mu \sin\theta_W$$

$$\mathcal{L}_{Z-\nu} = \frac{-g}{4\cos\theta_W} \bar{\nu}\gamma^\mu(1 - \gamma_5)\nu \, Z_\mu$$

Purely left-handed!

Interactions . . .

$$\mathcal{L}_{Z-e} = \frac{-g}{4\cos\theta_W}\bar{e}\left[L_e\gamma^\mu(1-\gamma_5) + R_e\gamma^\mu(1+\gamma_5)\right] e\, Z_\mu$$

$$\begin{aligned} L_e &= 2\sin^2\theta_W - 1 = 2x_W + \tau_3 \\ R_e &= 2\sin^2\theta_W = 2x_W \end{aligned}$$

Z-decay calculation analogous to $W^\pm$

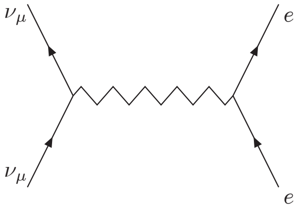
$$\begin{aligned} \Gamma(Z \to \nu\bar{\nu}) &= \frac{G_F M_Z^3}{12\pi\sqrt{2}} \\ \Gamma(Z \to e^+e^-) &= \Gamma(Z \to \nu\bar{\nu})\left[L_e^2 + R_e^2\right] \end{aligned}$$

In the Electroweak Theory . . .

- Electromagnetism is mediated by a massless photon, coupled to the electric charge;
- Mediator of charged-current weak interaction acquires a mass $M_W^2 = \pi\alpha / G_F\sqrt{2}\sin^2\theta_W$,
- Mediator of (new!) neutral-current weak interaction acquires mass $M_Z^2 = M_W^2/\cos^2\theta_W$;
- Massive neutral scalar particle, the Higgs boson, appears, but its mass is not predicted;
- Fermions can acquire mass—values not predicted.

Determine $\sin^2\theta_W$ to predict M_W, M_Z

Neutral-current interactions



New νe reaction, not in V − A

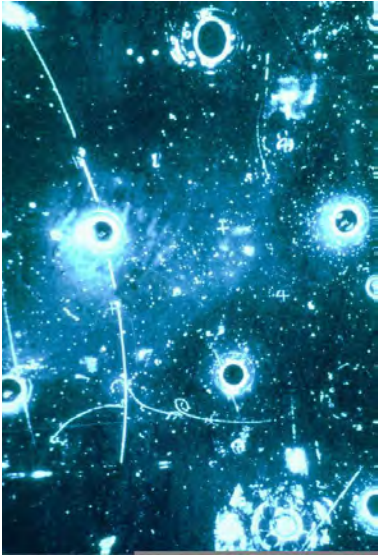
$$\sigma(\nu_\mu e \to \nu_\mu e) = \frac{G_F^2 m_e E_\nu}{2\pi}\left[L_e^2 + R_e^2/3\right]$$

$$\sigma(\bar{\nu}_\mu e \to \bar{\nu}_\mu e) = \frac{G_F^2 m_e E_\nu}{2\pi}\left[L_e^2/3 + R_e^2\right]$$

$$\sigma(\nu_e e \to \nu_e e) = \frac{G_F^2 m_e E_\nu}{2\pi}\left[(L_e + 2)^2 + R_e^2/3\right]$$

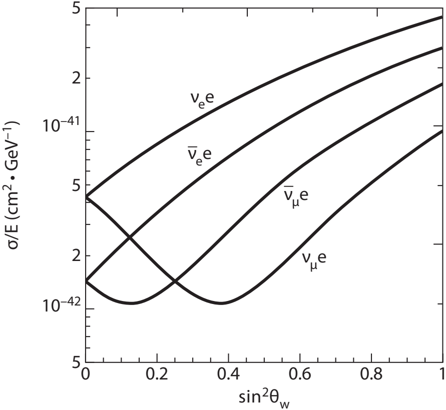
$$\sigma(\bar{\nu}_e e \to \bar{\nu}_e e) = \frac{G_F^2 m_e E_\nu}{2\pi}\left[(L_e + 2)^2/3 + R_e^2\right]$$

Gargamelle $\nu_\mu e$ event (1973)



$\leftarrow \bar{\nu}_\mu$

“Model-independent” analysis



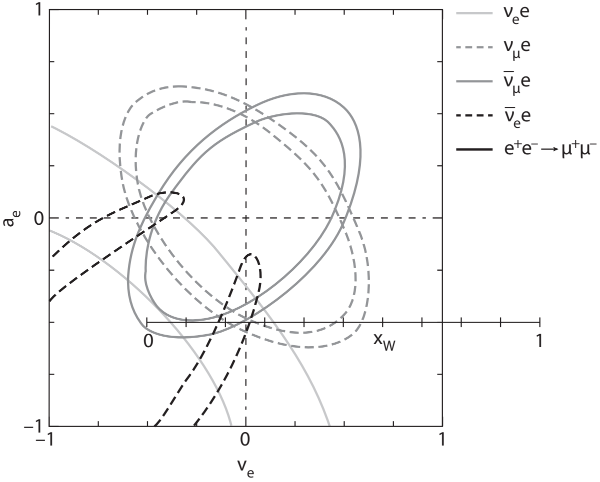
Measure all cross sections to determine chiral couplings L_e and R_e or traditional vector and axial couplings v and a

$$a = \frac{1}{2}(L_e - R_e) \qquad v = \frac{1}{2}(L_e + R_e)$$

$$L_e = v + a \qquad R_e = v - a$$

model-independent in V, A framework

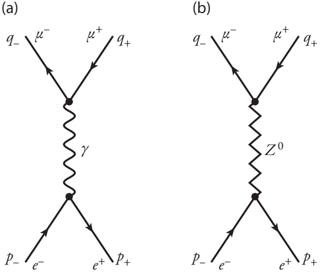
Neutrino–electron scattering



$$x_W \equiv \sin^2\theta_W$$

Twofold ambiguity remains even after measuring all four cross sections: same cross sections result if we interchange $R_e \leftrightarrow -R_e$ $(v \leftrightarrow a)$

Consider $e^+e^- \to \mu^+\mu^-$



$$\begin{aligned}
\mathcal{M} \;=\; & -ie^2\bar{u}(\mu, q_-)\gamma_\lambda Q_\mu v(\mu, q_+)\frac{g^{\lambda\nu}}{s}\bar{v}(e, p_+)\gamma_\nu u(e.p_-) \\
& +\frac{i}{2}\left(\frac{G_F M_Z^2}{\sqrt{2}}\right)\bar{u}(\mu, q_-)\gamma_\lambda[R_\mu(1+\gamma_5)+L_\mu(1-\gamma_5)]v(\mu, q_+) \\
& \times\frac{g^{\lambda\nu}}{s-M_Z^2}\bar{v}(e, p_+)\gamma_\nu[R_e(1+\gamma_5)+L_e(1-\gamma_5)]u(e, p_-)
\end{aligned}$$

muon charge $Q_\mu = -1$

$e^+e^- \to \mu^+\mu^- \ldots$

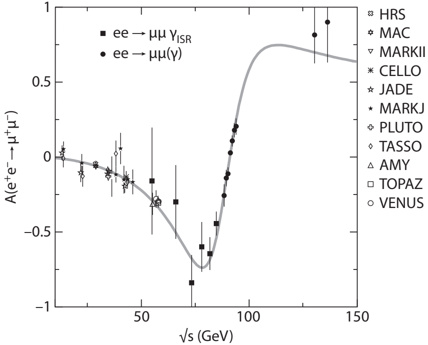
$$
\begin{aligned}
\frac{d\sigma}{dz} = \; & \frac{\pi\alpha^2 Q_\mu^2}{2s}(1+z^2) \\
& - \frac{\alpha Q_\mu G_F M_Z^2 (s - M_Z^2)}{8\sqrt{2}[(s-M_Z^2)^2 + M_Z^2\Gamma^2]} \\
& \times [(R_e + L_e)(R_\mu + L_\mu)(1+z^2) + 2(R_e - L_e)(R_\mu - L_\mu)z] \\
& + \frac{G_F^2 M_Z^4 s}{64\pi[(s-M_Z^2)^2 + M_Z^2\Gamma^2]} \\
& \times [(R_e^2 + L_e^2)(R_\mu^2 + L_\mu^2)(1+z^2) + 2(R_e^2 - L_e^2)(R_\mu^2 - L_\mu^2)z]
\end{aligned}
$$

Measuring F–B asymmetry resolves ambiguity

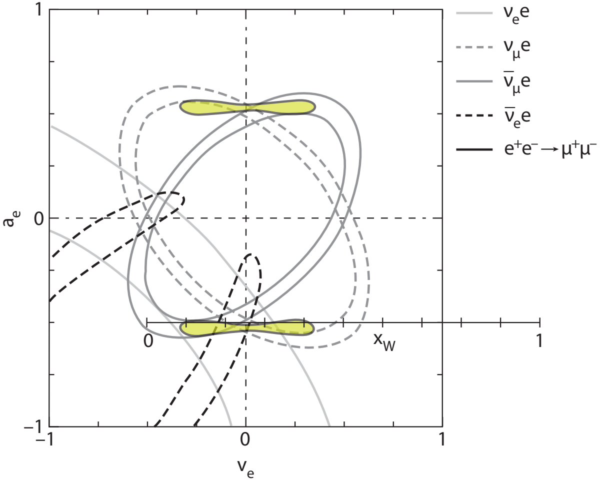
$$A \equiv \left(\int_0^1 dz\, d\sigma/dz - \int_{-1}^0 dz\, d\sigma/dz \right) / \int_{-1}^1 dz\, d\sigma/dz$$

$$\lim_{s/M_Z^2 \ll 1} A = \frac{3 G_F s}{16 \pi \alpha Q_\mu \sqrt{2}} (R_e - L_e)(R_\mu - L_\mu)$$

$$\approx -6.7 \times 10^{-5} \left(\frac{s}{1\ \mathrm{GeV}^2} \right) (2a_e)(2a_\mu) = -\frac{3 G_F s a^2}{4\pi\alpha\sqrt{2}}$$



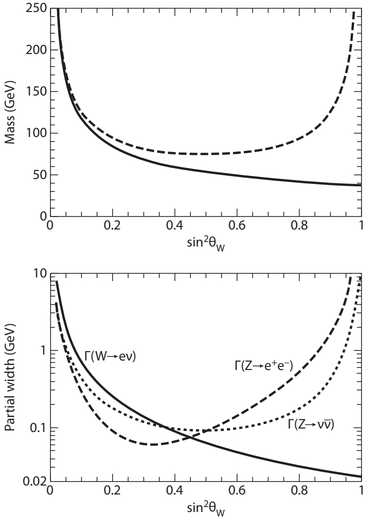
Neutrino–electron scattering + $e^+e^- \to \mu^+\mu^-$



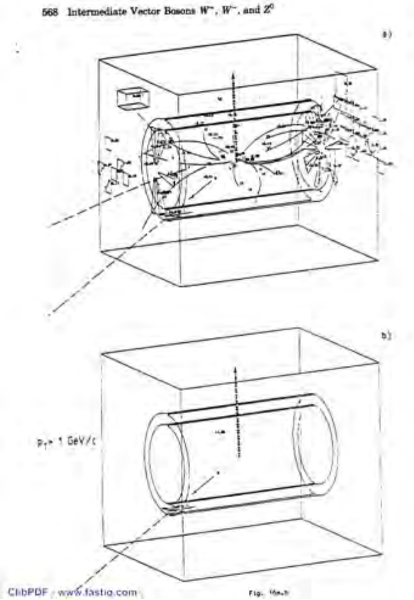
Validate EW theory, measure $\sin^2 \theta_W$

With a measurement of $\sin^2\theta_W = \alpha/\alpha_2$, predict

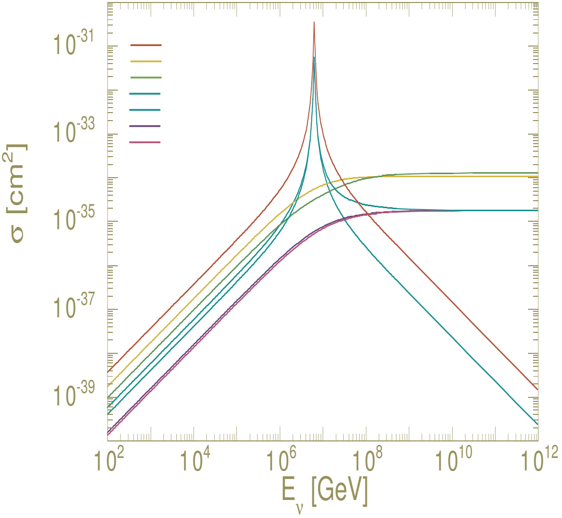
$M_W^2 = \pi\alpha/G_F\sqrt{2}\sin^2\theta_W \approx (37.28\text{ GeV})^2/\sin^2\theta_W \quad M_Z^2 = M_W^2/\cos^2\theta_W$



First Z from UA1



νe cross sections ...



At low energies: $\sigma(\bar{\nu}_e e \to \text{ hadrons}) > \sigma(\nu_\mu e \to \mu\nu_e) > \sigma(\nu_e e \to \nu_e e) >$ $\sigma(\bar{\nu}_e e \to \bar{\nu}_\mu \mu) > \sigma(\bar{\nu}_e e \to \bar{\nu}_e e) > \sigma(\nu_\mu e \to \nu_\mu e) > \sigma(\bar{\nu}_\mu e \to \bar{\nu}_\mu e)$

Electroweak interactions of quarks

- Left-handed doublet

	I_3	Q	$Y = 2(Q - I_3)$
$\mathsf{L}_q = \begin{pmatrix} u \\ d \end{pmatrix}_{\mathsf{L}}$	$\frac{1}{2}$ $-\frac{1}{2}$	$+\frac{2}{3}$ $-\frac{1}{3}$	$\frac{1}{3}$

- two right-handed singlets

	I_3	Q	$Y = 2(Q - I_3)$
$\mathsf{R}_u = u_{\mathsf{R}}$	0	$+\frac{2}{3}$	$+\frac{4}{3}$
$\mathsf{R}_d = d_{\mathsf{R}}$	0	$-\frac{1}{3}$	$-\frac{2}{3}$

▸ Exercises

Electroweak interactions of quarks

- CC interaction

$$\mathcal{L}_{W\text{-}q} = \frac{-g}{2\sqrt{2}} \left[\bar{u}_e \gamma^\mu (1 - \gamma_5) d \, W_\mu^+ + \bar{d} \gamma^\mu (1 - \gamma_5) u \, W_\mu^- \right]$$

identical in form to $\mathcal{L}_{W\text{-}\ell}$: universality $\Leftrightarrow$ weak isospin

- NC interaction

$$\mathcal{L}_{Z\text{-}q} = \frac{-g}{4 \cos\theta_{\mathrm{W}}} \sum_{i=u,d} \bar{q}_i \gamma^\mu \left[L_i (1 - \gamma_5) + R_i (1 + \gamma_5) \right] q_i Z_\mu$$

$$L_i = \tau_3 - 2Q_i \sin^2\theta_{\mathrm{W}} \quad R_i = -2Q_i \sin^2\theta_{\mathrm{W}}$$

equivalent in form (not numbers) to $\mathcal{L}_{Z\text{-}\ell}$

Trouble in Paradise

Universal $u \leftrightarrow d$, $\nu_e \leftrightarrow e$ *not quite right*

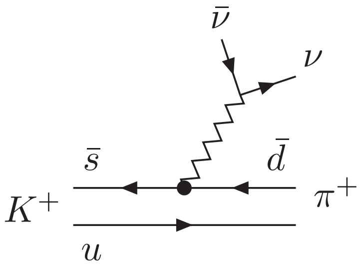
$$\text{Good:} \quad \begin{pmatrix} u \\ d \end{pmatrix}_{\mathsf{L}} \rightarrow \text{Better:} \quad \begin{pmatrix} u \\ d_\theta \end{pmatrix}_{\mathsf{L}}$$

$$d_\theta \equiv d \cos\theta_{\mathsf{C}} + s \sin\theta_{\mathsf{C}} \qquad \cos\theta_{\mathsf{C}} = 0.9736 \pm 0.0010$$

"Cabibbo-rotated" doublet perfects CC interaction (up to small third-generation effects) but ⇒ serious trouble for NC

$$\begin{aligned}
\mathcal{L}_{Z\text{-}q} \;=\; & \frac{-g}{4\cos\theta_{\mathsf{W}}} Z_\mu \left\{ \bar{u}\gamma^\mu \left[L_u(1-\gamma_5) + R_u(1+\gamma_5) \right] u \right. \\
& + \bar{d}\gamma^\mu \left[L_d(1-\gamma_5) + R_d(1+\gamma_5) \right] d \cos^2\theta_{\mathsf{C}} \\
& + \bar{s}\gamma^\mu \left[L_d(1-\gamma_5) + R_d(1+\gamma_5) \right] s \sin^2\theta_{\mathsf{C}} \\
& + \bar{d}\gamma^\mu \left[L_d(1-\gamma_5) + R_d(1+\gamma_5) \right] s \sin\theta_{\mathsf{C}} \cos\theta_{\mathsf{C}} \\
& \left. + \; \bar{s}\gamma^\mu \left[L_d(1-\gamma_5) + R_d(1+\gamma_5) \right] d \sin\theta_{\mathsf{C}} \cos\theta_{\mathsf{C}} \right\}
\end{aligned}$$

Strangeness-changing NC highly suppressed!



BNL E-787/E-949 has three $K^+ \to \pi^+ \nu\bar{\nu}$ candidates, with $\mathcal{B}(K^+ \to \pi^+ \nu\bar{\nu}) = 1.73^{+1.15}_{-1.05} \times 10^{-10}$
Phys. Rev. Lett. **101**, 191802 (2008)

(SM: $\approx$ 0.85: Brod–Gorbahn, *Phys. Rev. D***78**, 034008 (2008)

Glashow–Iliopoulos–Maiani

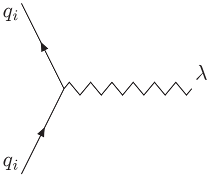
two LH doublets: $\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L \quad \begin{pmatrix} u \\ d_\theta \end{pmatrix}_L \begin{pmatrix} c \\ s_\theta \end{pmatrix}_L$

$$(s_\theta = s \cos\theta_C - d \sin\theta_C)$$

+ right-handed singlets, e_R, μ_R, u_R, d_R, c_R, s_R

Required new charmed quark, c

Cross terms vanish in $\mathcal{L}_{Z\text{-}q}$,



$$\frac{-ig}{4\cos\theta_W}\gamma_\lambda[(1-\gamma_5)L_i + (1+\gamma_5)R_i] \quad ,$$

$$L_i = \tau_3 - 2Q_i \sin^2\theta_W \quad R_i = -2Q_i \sin^2\theta_W$$

flavor-diagonal interaction!

Straightforward generalization to n quark doublets

$$\mathcal{L}_{W\text{-}q} = \frac{-g}{2\sqrt{2}}\left[\bar{\Psi}\gamma^\mu(1-\gamma_5)\mathcal{O}\Psi\, W_\mu^+ + \text{h.c.}\right]$$

composite $\Psi = \begin{pmatrix} u \\ c \\ \vdots \\ d \\ s \\ \vdots \end{pmatrix}$ flavor structure $\mathcal{O} = \begin{pmatrix} 0 & U \\ 0 & 0 \end{pmatrix}$

U: unitary quark mixing matrix

Weak-isospin part: $\mathcal{L}_{Z\text{-}q}^{\text{iso}} = \frac{-g}{4\cos\theta_W}\bar{\Psi}\gamma^\mu(1-\gamma_5)\left[\mathcal{O},\mathcal{O}^\dagger\right]\Psi$

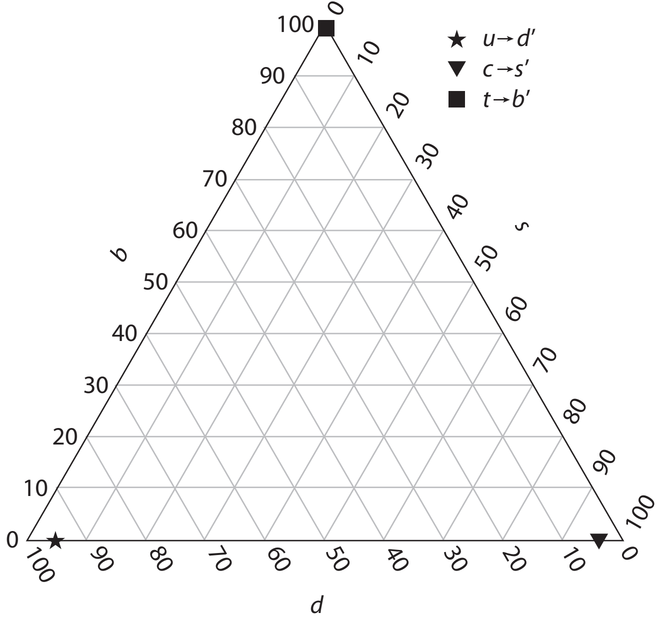
Since $\left[\mathcal{O},\mathcal{O}^\dagger\right] = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} \propto \tau_3$

$\Rightarrow$ NC interaction is flavor-diagonal

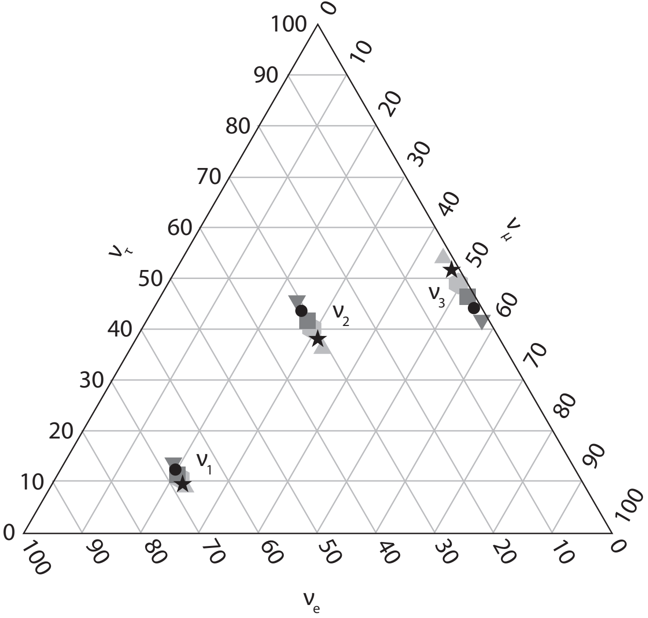
General $n \times n$ mixing matrix U: $n(n-1)/2$ real $\angle$, $(n-1)(n-2)/2$ complex phases

3×3 (Cabibbo–Kobayashi-Maskawa): $3\,\angle + 1$ phase $\Rightarrow$ CP violation

Family patterns among quarks



Family patterns among neutrinos



Problem 9

Refer to the master formula for γ-Z interference given in the expression above for $e^+e^- \to \mu^+\mu^-$.
(a) In the γ-Z interference regime, show that the asymmetries for heavy-quark production are

$$A(c\bar{c}) = \tfrac{3}{2}A(\mu^+\mu^-),$$

$$A(b\bar{b}) = 3A(\mu^+\mu^-).$$

(b) What values would you expect for these asymmetries at $\sqrt{s} = 40$ GeV?

Problem 10

Three observables concerning the b quark are sensitive to different combinations of the chiral couplings: $\Gamma(Z^0 \to b\bar{b})$ is determined by $(L_b^2 + R_b^2)$, $A_{\text{peak}}^{(b\bar{b})}$ is sensitive to $(L_b^2 - R_b^2)/(L_b^2 + R_b^2)$, and the low-energy forward-backward asymmetry $A(b\bar{b})$ is proportional to $(R_b - L_b)$. Generalize the standard $\mathrm{SU(2)_L} \otimes \mathrm{U(1)}_Y$ electroweak theory to include right-handed charged-current interactions of b, so that $L_b = \tau_{3\mathrm{L}} - 2Q_b x_{\mathrm{W}}$ and $R_b = \tau_{3\mathrm{R}} - 2Q_b x_{\mathrm{W}}$. Working to leading order, display allowed regions in the $I_{3\mathrm{L}}$-$I_{3\mathrm{R}}$ plane and determine the weak-isospin quantum numbers of b.

For a thorough analysis and useful compendium of data, see D. Schaile and P. M. Zerwas, *Phys. Rev. D***45**, 3262 (1992).

Successful predictions of $SU(2)_L \otimes U(1)_Y$:

- neutral-current interactions
- necessity of charm
- existence and properties of $W^\pm$ and Z^0

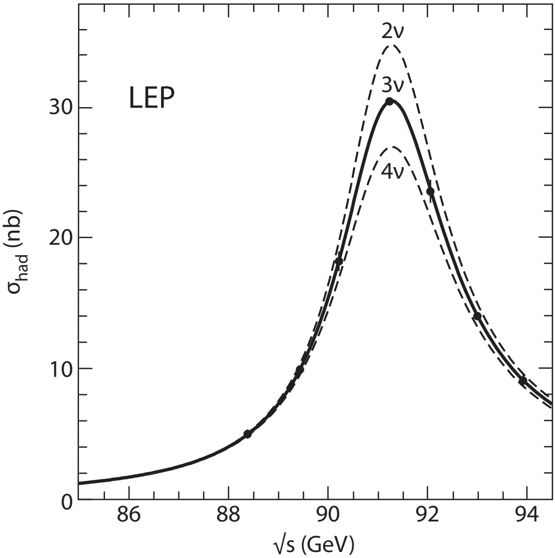
+ a decade of precision EW tests (one-per-mille)

M_Z	$91\,187.6 \pm 2.1$ MeV
Γ_Z	2495.2 ± 2.3 MeV
$\sigma^0_{\text{hadronic}}$	41.540 ± 0.037 nb
Γ_{hadronic}	1744.4 ± 2.0 MeV
Γ_{leptonic}	83.984 ± 0.086 MeV
$\Gamma_{\text{invisible}}$	499.0 ± 1.5 MeV

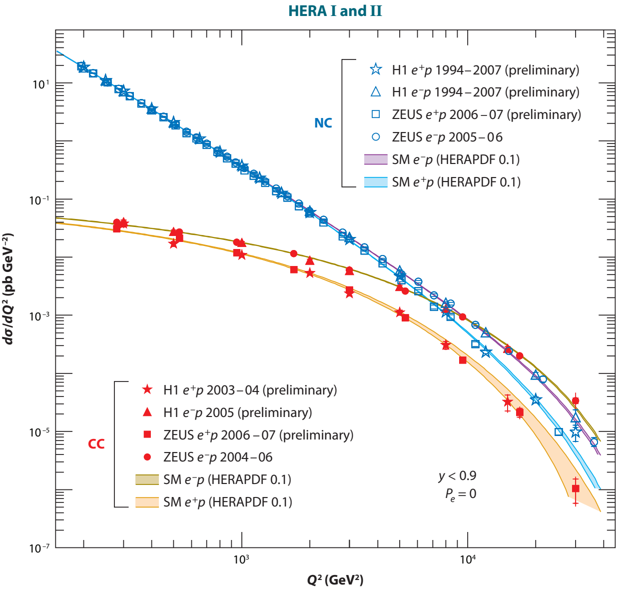
$$\Gamma_{\text{invisible}} \equiv \Gamma_Z - \Gamma_{\text{hadronic}} - 3\Gamma_{\text{leptonic}}$$

light ν : $N_\nu = \Gamma_{\text{invisible}}/\Gamma^{\text{SM}}(Z \to \nu_i \bar{\nu}_i) = 2.92 \pm 0.05 \quad (\nu_e, \nu_\mu, \nu_\tau)$

Three light neutrinos

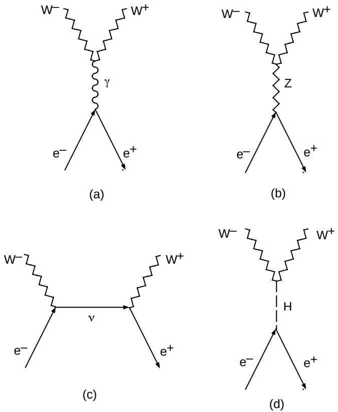


Electroweak theory tests: tree level



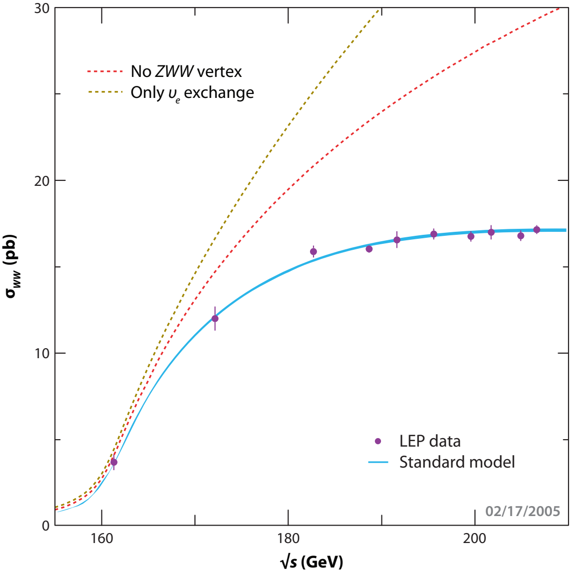
Electroweak theory tests: tree level

S-matrix analysis of $e^+e^- \to W^+W^-$



Individual $J = 1$ partial-wave amplitudes $\mathcal{M}_\gamma^{(1)}$, $\mathcal{M}_Z^{(1)}$, $\mathcal{M}_\nu^{(1)}$ have unacceptable high-energy behavior ($\propto s$)

Electroweak theory tests: tree level



. . . sum is well-behaved; gauge symmetry!

Why a Higgs boson "must" exist

$J = 0$ amplitude exists because electrons have mass, and can be found in "wrong" helicity state

$$\mathcal{M}_{\nu}^{(0)} \propto s^{\frac{1}{2}} \quad \text{: unacceptable HE behavior}$$

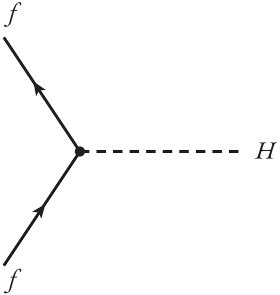
saturate p.w. unitarity at

$$\sqrt{s_e} \simeq \frac{4\pi\sqrt{2}}{\sqrt{3}\, G_{\mathrm{F}}\, m_e} \approx 1.7 \times 10^9 \text{ GeV}$$

Divergence canceled by Higgs-boson contribution

$\Rightarrow He\bar{e}$ coupling must be $\propto m_e$,

because "wrong-helicity" amplitudes $\propto m_e$



$$\frac{-i\zeta_f}{\sqrt{2}} = \frac{-im_f}{v} = \frac{-igm_f}{2M_W} = -im_f(G_F\sqrt{2})^{1/2}$$

If the Higgs boson did not exist, something else would have to cure divergent behavior

If gauge symmetry were unbroken ...

- no Higgs boson; no longitudinal gauge bosons
- no extreme divergences; no wrong-helicity amplitudes

...and no viable low-energy phenomenology

In spontaneously broken theory ...

- gauge structure of couplings eliminates the most severe divergences
- lesser—but potentially fatal—divergence arises because the electron has mass ...due to SSB
- SSB provides its own cure—the Higgs boson

Similar interplay and compensation *must exist* in any acceptable theory

Anticipating m_t

$$\rho \equiv \frac{M_W^2}{M_Z^2 \cos^2\theta_W}$$

measures relative strength of NC, CC at low energies

$$\sigma(\nu_\mu e \to \nu_\mu e) = \frac{G_F^2 mE}{2\pi}\rho^2\left[(2x_W - 1)^2 + \frac{4x_W^2}{3}\right]$$

$$\sigma(\bar{\nu}_\mu e \to \bar{\nu}_\mu e) = \frac{G_F^2 mE}{2\pi}\rho^2\left[\frac{(2x_W - 1)^2}{3} + 4x_W^2\right]$$

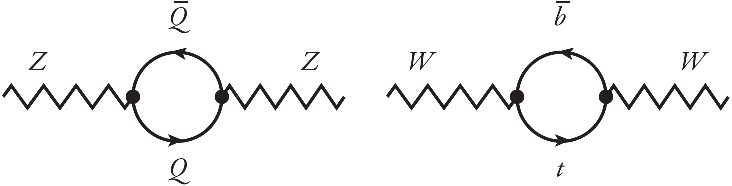
$$\sigma(\nu_\mu e \to \mu\nu_e) = \frac{2G_F^2 mE}{\pi}\left[1 - (\mu^2 - m^2)/2mE\right]^2$$

Anticipating m_t

$\sigma(\nu_\mu e \to \nu_\mu e)/\sigma(\nu_\mu e \to \mu\nu_e)$ determines x_W
comparison of NC and CC cross sections yields ρ.
CHARM-II Collaboration:

$$\begin{aligned} \sin^2\theta_W(\nu_\mu e) &= 0.237 \pm 0.007 \text{ (stat.)} \pm 0.007 \text{ (sys.)} \\ \rho(\nu_\mu e) &= 1.006 \pm 0.014 \text{ (stat.)} \pm 0.033 \text{ (sys.)}. \end{aligned}$$

SM with a single Higgs doublet, $\rho = 1$ at tree level.
Quantum corrections:



Anticipating m_t

With $\rho = 1 + \Delta\rho$,

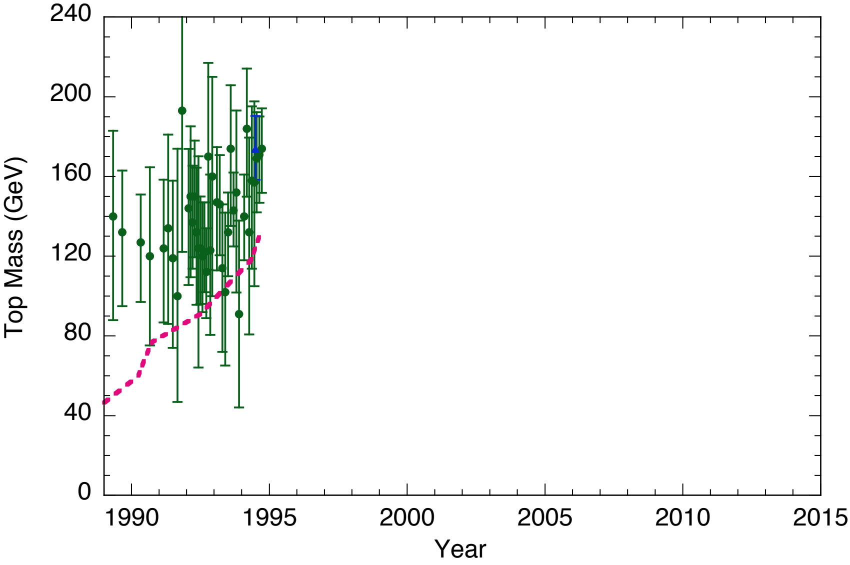
$$\Delta\rho = \frac{3G_F}{8\pi^2\sqrt{2}}\left[m_b^2 + m_t^2 + \frac{2m_t^2 m_b^2}{(m_t^2 - m_b^2)}\ln\left(\frac{m_b^2}{m_t^2}\right)\right].$$

(i) In limit $m_b \to m_t$, $\Delta\rho \to 0$ independent of m_t;
(ii) In limit $m_b \ll m_t$

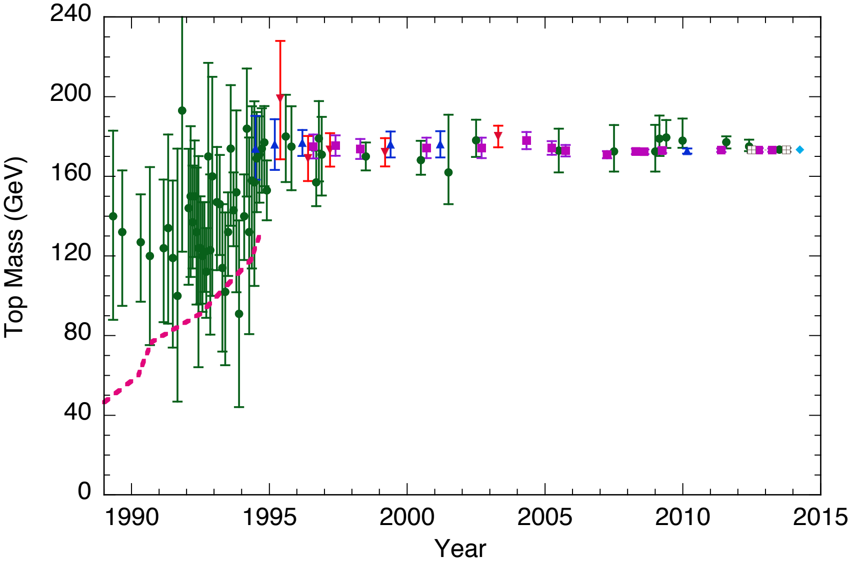
$$\Delta\rho \to \frac{3G_F m_t^2}{8\pi^2\sqrt{2}}.$$

Acute probe of m_t!

Electroweak theory tests: loop level



Electroweak theory tests: loop level



The importance of the 1-TeV scale . . .

EW theory does not predict Higgs-boson mass

▷ Conditional *upper bound* from Unitarity

Compute amplitudes $\mathcal{M}$ for gauge boson scattering at high energies, make a partial-wave decomposition

$$\mathcal{M}(s,t) = 16\pi \sum_J (2J+1) a_J(s) P_J(\cos\theta)$$

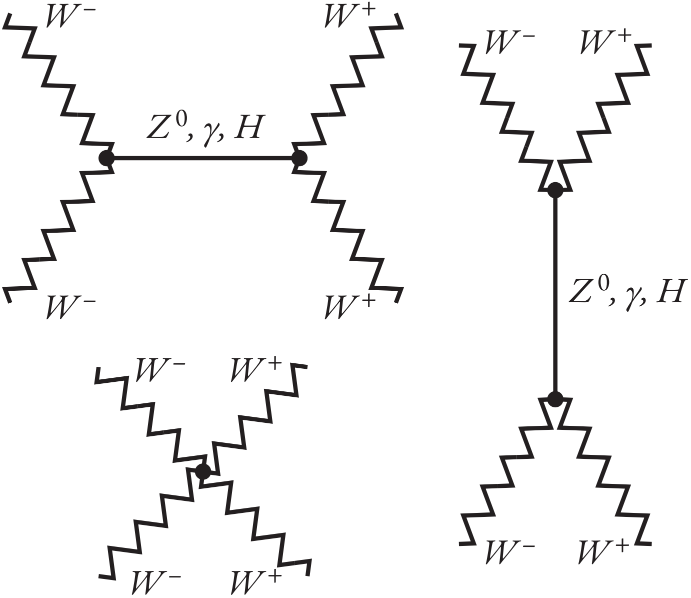
Most channels decouple – pw amplitudes are small at "all" energies – $\forall M_H$.

Four interesting channels:

$$W_L^+ W_L^- \quad Z_L^0 Z_L^0/\sqrt{2} \quad HH/\sqrt{2} \quad HZ_L^0$$

L: longitudinal, $1/\sqrt{2}$ for identical particles

The importance of the 1-TeV scale . . .



The importance of the 1-TeV scale . . .

In HE limit, s-wave amplitudes $\propto G_F M_H^2$

$$\lim_{s \gg M_H^2} (a_0) \to \frac{-G_F M_H^2}{4\pi\sqrt{2}} \cdot \begin{bmatrix} 1 & 1/\sqrt{8} & 1/\sqrt{8} & 0 \\ 1/\sqrt{8} & 3/4 & 1/4 & 0 \\ 1/\sqrt{8} & 1/4 & 3/4 & 0 \\ 0 & 0 & 0 & 1/2 \end{bmatrix}$$

Require that largest eigenvalue respect partial-wave unitarity condition $|a_0| \leq 1$

$$\Longrightarrow \quad M_H \leq \left(\frac{8\pi\sqrt{2}}{3G_F}\right)^{1/2} \approx 1 \text{ TeV}$$

condition for perturbative unitarity

The importance of the 1-TeV scale . . .

If the bound is respected

- weak interactions remain weak at all energies
- perturbation theory is everywhere reliable

If the bound is violated

- perturbation theory breaks down
- weak interactions among $W^{\pm}$, Z, H become strong on 1-TeV scale

New phenomena are to be found in the EW interactions at energies not much larger than 1 TeV

Divergence cancellation in WW scattering

In general, the diagrams for ▸ WW scattering contribute

$$a_J = A(q/M_W)^4 + B(q/M_W)^2 + C$$

A terms cancelled by gauge symmetry

Residual B terms cancelled by Higgs exchange

C term scale set by M_H

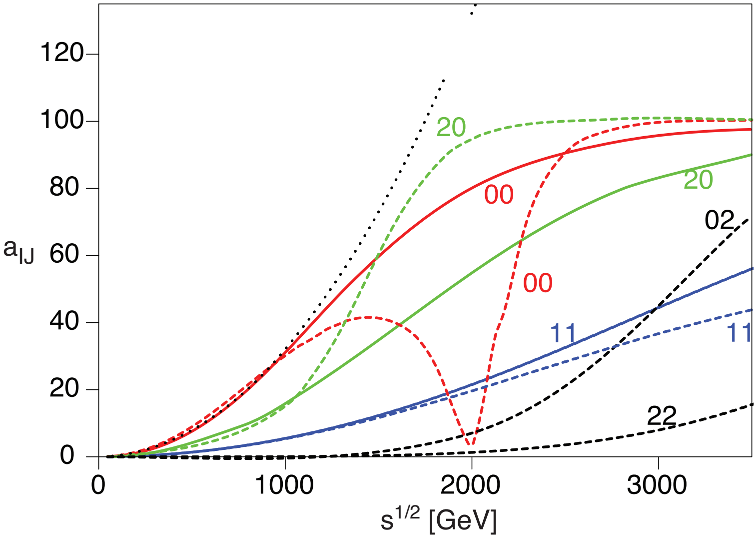
Higgs boson's key role in high-energy behavior

$a_{I=0\,J=0}$ and a_{11} attractive; a_{20} repulsive

Motivates study of WW scattering at high energies

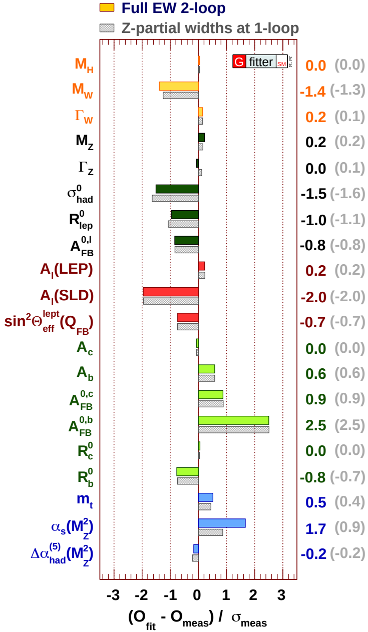
ATLAS $W^{\pm}W^{\pm}$ study

High-energy *WW* scattering (*K*-matrix unitarization)



$\cdots$ *un*unitarized a_{00}; — LO; - - NLO

Electroweak theory tests: loop level (Gfitter *now*)



Electroweak theory tests: Higgs influence *before*