Modern lattice QCD part II: *recent progress & future prospects*

Ruth Van de Water
Fermilab

Hadron Collider Physics Summer School
August 19, 2014

Outline

(1) *Recap:* lattice QCD for high-energy physics

(2) Quark-flavor physics

- ❖ The CKM unitary triangle fit
- ❖ Rare decays

(3) New experimental opportunities

- ❖ Muon g-2
- ❖ Dark-matter detection and $\mu \to e$ conversion
- ❖ Neutrino physics
- ❖ Precision Higgs measurements

(4) Summary and outlook

Lattice QCD & high-energy physics

✦ HEP experimental program searching for new physics with two complimentary approaches

(1) Direct production of new particles at colliders

- SUSY, leptoquarks, little Higgs, 4th generation, technicolor, X-dimensions, hidden sector, ...
- New physics may be strongly coupled

(2) Precise measurements of Standard-Model parameters and processes

- Current & future experiments will improve determinations of CKM quark-mixing matrix & PMNS ν-mixing matrix, measurements of decay rates and oscillation frequencies, & observe some rare processes for the first time
- Comparison with Standard-Model predictions & discovery of new physics requires equally precise theoretical predictions

Lattice QCD & high-energy physics

✦ HEP experimental program searching for new physics with two complimentary approaches

(1) Direct production of new particles at colliders

❖ SUSY, leptoquarks, little Higgs, 4th generation, technicolor, X-dimensions, hidden sector, ...

❖ New physics may be strongly coupled

(2) Precise measurements of Standard-Model parameters and processes

❖ Current & future experiments will improve determinations of CKM quark-mixing matrix & PMNS ν-mixing matrix, measurements of decay rates and & observe some

In this talk, primarily focus on role of lattice QCD to interpret precision measurements

❖ Comparison with Standard-Model predictions & discovery of new physics requires equally precise theoretical predictions

Scope of precision measurements

✦ Target processes where new-physics contributions may be observable:

(1) Extremely rare (or even forbidden) in the Standard Model

(2) Predicted to high precision in the Standard Model

neutrino physics

lepton flavor violation

Higgs physics

B & D physics

muon g-2

kaon physics

Scope of precision measurements

- Target processes where new-physics contributions may be observable:
  - **(1)** Extremely rare (or even forbidden) in the Standard Model
  - **(2)** Predicted to high precision in the Standard Model

neutrino physics

lepton flavor violation

Higgs physics

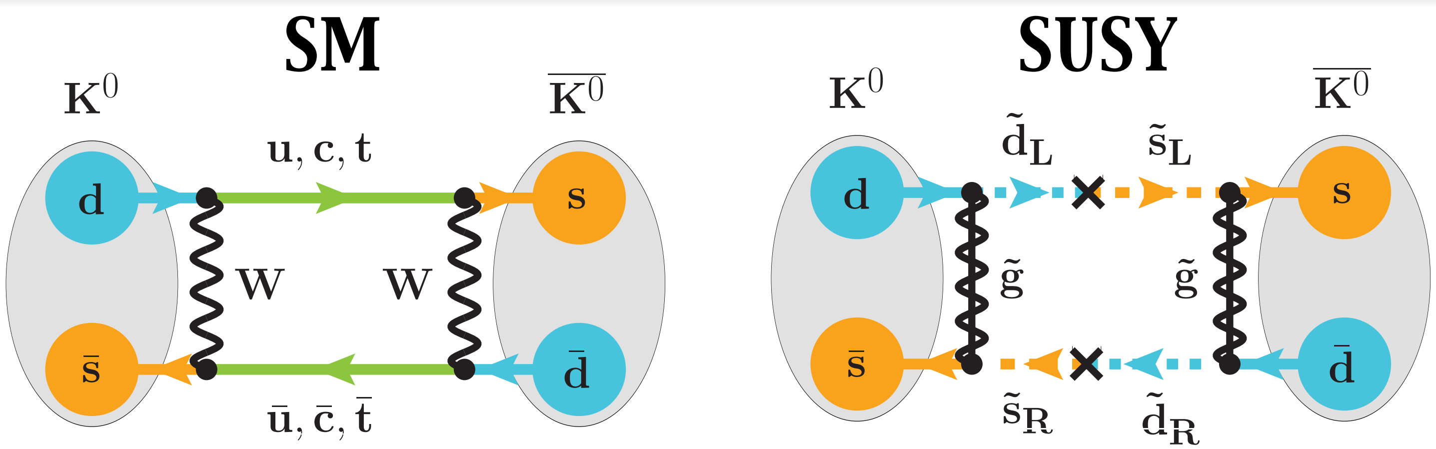
Current & planned experiments cover broad range of topics in particle & nuclear physics

muon g-2

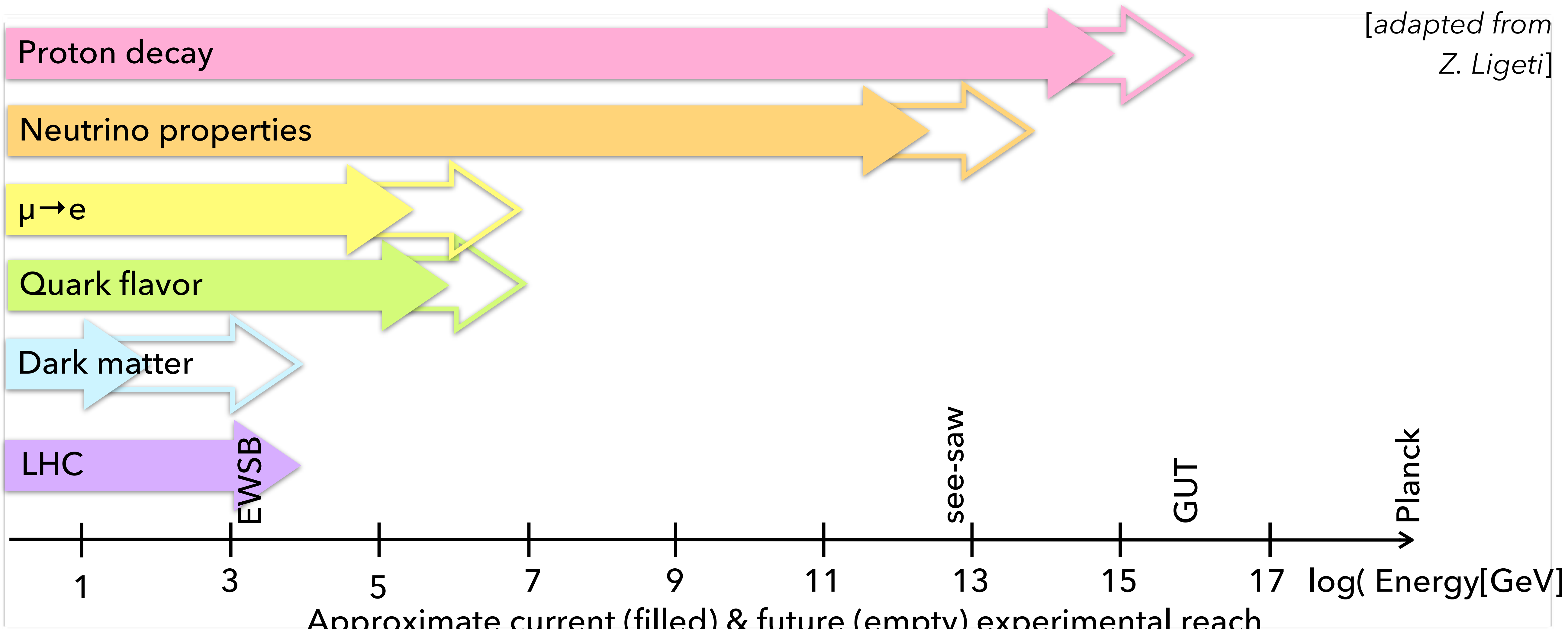
kaon physics

New-physics reach

Quantum-mechanical loops sensitive to new heavy particles above the TeV scale, e.g.:

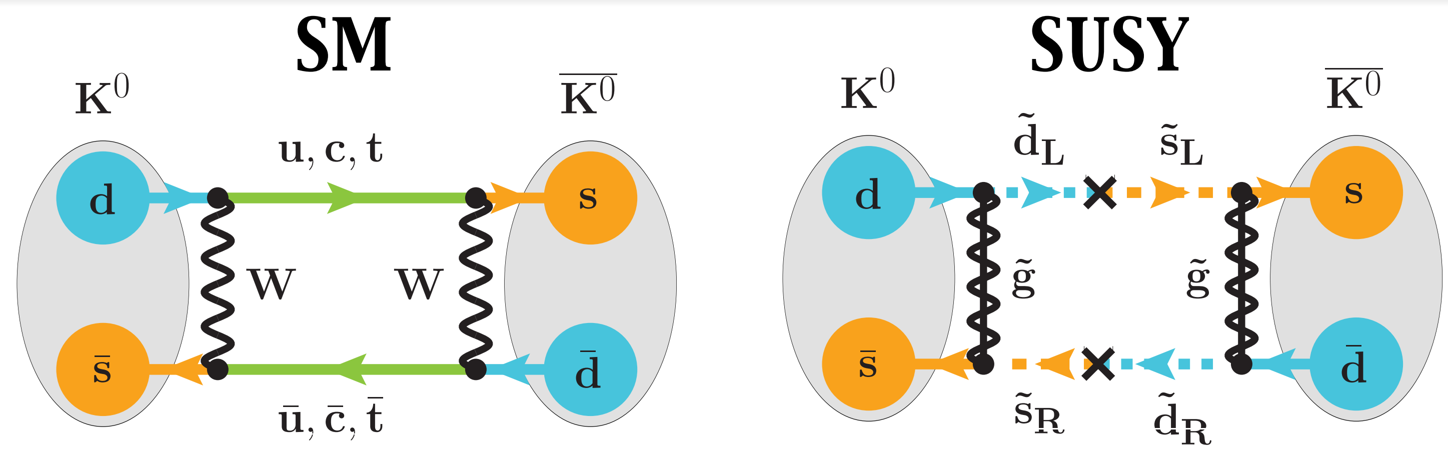


neutral kaon mixing

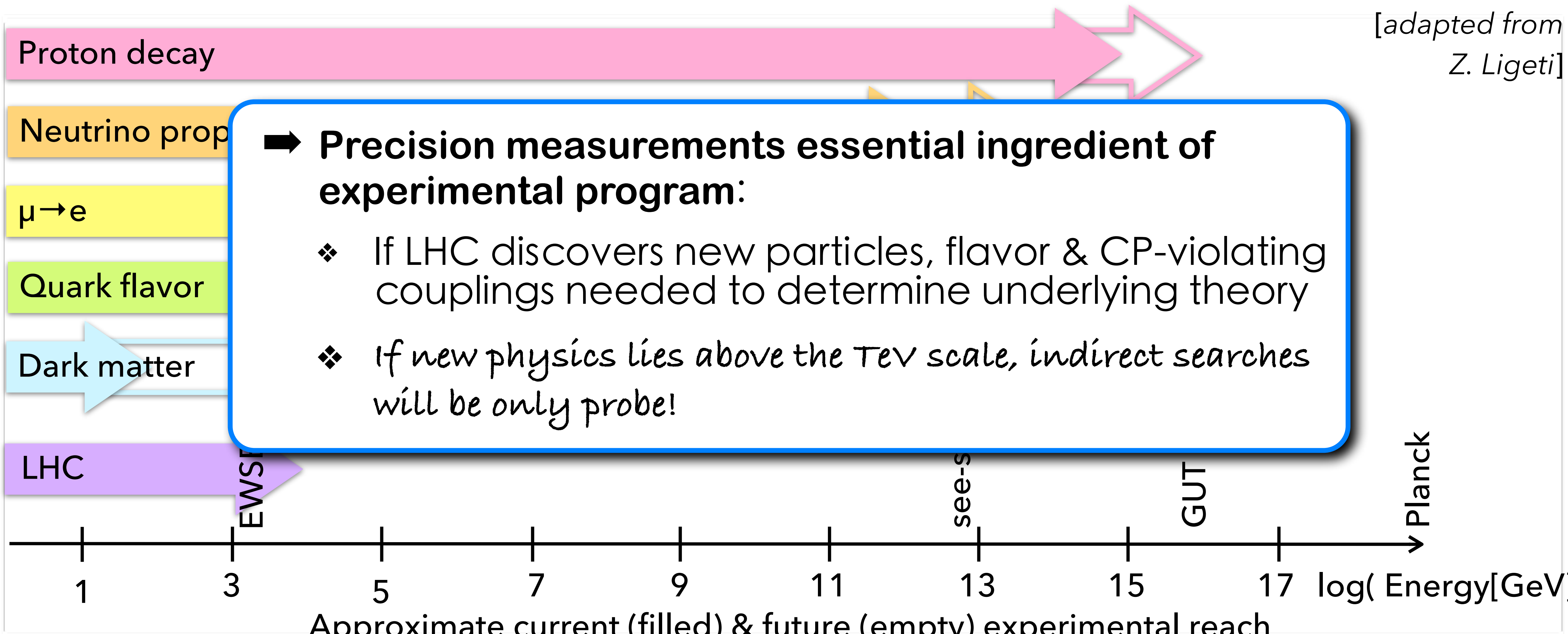


New-physics reach

Quantum-mechanical loops sensitive to new heavy particles above the TeV scale, e.g.:



neutral kaon mixing



➡ **Precision measurements essential ingredient of experimental program**:

- ❖ If LHC discovers new particles, flavor & CP-violating couplings needed to determine underlying theory
- ❖ If new physics lies above the TeV scale, indirect searches will be only probe!

New-physics complementarity

	LHT	RSc	4G	2HDM	RHMFV
$D^0 - \bar{D}^0$ (CPV)	★★★	★★★	★★	★★	
ϵ_K	★★	★★★	★★	★★	★★
$S_{\psi\phi}$	★★★	★★★	★★★	★★★	★★★
$S_{\phi K_S}$	★	★	★★		
$A_{\rm CP}\,(B \to X_s\gamma)$	★		★		
$A_{7,8}(K^*\mu^+\mu^-)$	★★	★	★★		
$B_s \to \mu^+\mu^-$	★	★	★★★	★★★	★★
$K^+ \to \pi^+\nu\bar\nu$	★★★	★★★	★★★		★★
$K_L \to \pi^0\nu\bar\nu$	★★★	★★★	★★★		★★
$\mu \to e\gamma$	★★★	★★★	★★★		
$\tau \to \mu\gamma$	★★★	★★★	★★★		
$\mu + N \to e + N$	★★★	★★★	★★★		
d_n	★	★★★	★	★★★	
d_e	★	★★★	★	★★★	
$(g-2)_\mu$	★	★★	★		

★★★ = sizeable NP effects
★★ = moderate to small NP effects
★ = no visable NP effects

[Buras, Acta Phys.Polon.B41:2487-2561,2010]

- Different processes & observables sensitive to different new-physics scenarios
  - Pattern of measurements can distinguish between models & constrain model parameters

New-physics complementarity

	LHT	RSc	4G	2HDM	RHMFV
$D^0 - \bar{D}^0$ (CPV)	★★★	★★★	★★	★★	
ϵ_K	★★	★★★	★★	★★	★★
$S_{\psi\phi}$	★★★	★★★	★★★	★★★	★★★
$S_{\phi K_S}$	[illegible]	[illegible]	★★		
$A_{\rm CP}(B \to X_s\gamma)$	[illegible]	[illegible]	★		
$A_{7,8}(K^*\mu^+\mu^-)$	[illegible]	[illegible]	★★		
$B_s \to \mu^+\mu^-$	★	★	★★★	★★★	★★
$K^+ \to \pi^+\nu\bar{\nu}$	★★★	★★★	★★★		★★
$K_L \to \pi^0\nu\bar{\nu}$	★★★	★★★	★★★		★★
$\mu \to e\gamma$	★★★	★★★	★★★		
$\tau \to \mu\gamma$	★★★	★★★	★★★		
$\mu + N \to e + N$	★★★	★★★	★★★		
d_n	★	★★★	★	★★★	
d_e	★	★★★	★	★★★	
$(g-2)_\mu$	★	★★	★		

QUARK FLAVOR

★★★ = sizeable NP effects
★★ = moderate to small NP effects
★ = no visable NP effects

[Buras, Acta Phys.Polon.B41:2487-2561,2010]

- Different processes & observables sensitive to different new-physics scenarios
  - Pattern of measurements can distinguish between models & constrain model parameters

New-physics complementarity

	LHT	RSc	4G	2HDM	RHMFV
$D^0 - \bar{D}^0$ (CPV)	★★★	★★★	★★	★★	
ϵ_K	★★	★★★	★★	★★	★★
$S_{\psi\phi}$	★★★	★★★	★★★	★★★	★★★
$S_{\phi K_S}$	[illegible]	[illegible]	★★		
$A_{\rm CP}(B \to X_s\gamma)$	[illegible]	[illegible]	★		
$A_{7,8}(K^*\mu^+\mu^-)$	[illegible]	[illegible]	★★		
$B_s \to \mu^+\mu^-$	★	★	★★★	★★★	★★
$K^+ \to \pi^+\nu\bar{\nu}$	★★★	★★★	★★★		★★
$K_L \to \pi^0\nu\bar{\nu}$	★★★	★★★	★★★		★★
$\mu \to e\gamma$	[illegible]	[illegible]	★★★		
$\tau \to \mu\gamma$	[illegible]	[illegible]	★★★		
$\mu + N \to e + N$	[illegible]	[illegible]	★★★		
d_n	★	★★★	★	★★★	
d_e	★	★★★	★	★★★	
$(g-2)_\mu$	★	★★	★		

QUARK FLAVOR

LEPTON FLAVOR

★★★ = sizeable NP effects
★★ = moderate to small NP effects
★ = no visable NP effects

[Buras, Acta Phys.Polon.B41:2487-2561,2010]

- Different processes & observables sensitive to different new-physics scenarios
  - Pattern of measurements can distinguish between models & constrain model parameters

New-physics complementarity

	LHT	RSc	4G	2HDM	RHMFV
$D^0 - \bar{D}^0$ (CPV)	★★★	★★★	★★	★★	
ϵ_K	★★	★★★	★★	★★	★★
$S_{\psi\phi}$	★★★	★★★	★★★	★★★	★★★
$S_{\phi K_S}$	[illegible]	[illegible]	★★		
$A_{\rm CP}(B \to X_s\gamma)$	[illegible]	[illegible]	★		
$A_{7,8}(K^*\mu^+\mu^-)$	[illegible]	[illegible]	★★		
$B_s \to \mu^+\mu^-$	★	★	★★★	★★★	★★
$K^+ \to \pi^+\nu\bar{\nu}$	★★★	★★★	★★★		★★
$K_L \to \pi^0\nu\bar{\nu}$	★★★	★★★	★★★		★★
$\mu \to e\gamma$	[illegible]	[illegible]	★★★		
$\tau \to \mu\gamma$	[illegible]	[illegible]	★★★		
$\mu + N \to e + N$	[illegible]	[illegible]	★★★		
d_n	[illegible]	[illegible]	★	★★★	
d_e	[illegible]	[illegible]	★	★★★	
$(g-2)_\mu$	★	★★	★		

QUARK FLAVOR

LEPTON FLAVOR

EDMs

★★★ = sizeable NP effects

★★ = moderate to small NP effects

★ = no visable NP effects

[Buras, Acta Phys.Polon.B41:2487-2561,2010]

- Different processes & observables sensitive to different new-physics scenarios
  - Pattern of measurements can distinguish between models & constrain model parameters

New-physics complementarity

	LHT	RSc	4G	2HDM	RHMFV
$D^0 - \bar{D}^0$ (CPV)	★★★	★★★	★★	★★	
ϵ_K	★★	★★★	★★	★★	★★
$S_{\psi\phi}$	★★★	★★★	★★★	★★★	★★★
$S_{\phi K_S}$	[illegible]	[illegible]	★★		
$A_{\mathrm{CP}}(B \to X_s\gamma)$	[illegible]	[illegible]	★		
$A_{7,8}(K^*\mu^+\mu^-)$	[illegible]	[illegible]	★★		
$B_s \to \mu^+\mu^-$	★	★	★★★	★★★	★★
$K^+ \to \pi^+\nu\bar{\nu}$	★★★	★★★	★★★		★★
$K_L \to \pi^0\nu\bar{\nu}$	★★★	★★★	★★★		★★
$\mu \to e\gamma$	[illegible]	[illegible]	★★★		
$\tau \to \mu\gamma$	[illegible]	[illegible]	★★★		
$\mu + N \to e + N$	[illegible]	[illegible]	★★★		
d_n	[illegible]	[illegible]	★	★★★	
d_e	[illegible]	[illegible]	★	★★★	
$(g-2)_\mu$	★	★★	★		

QUARK FLAVOR

LEPTON FLAVOR

EDMs

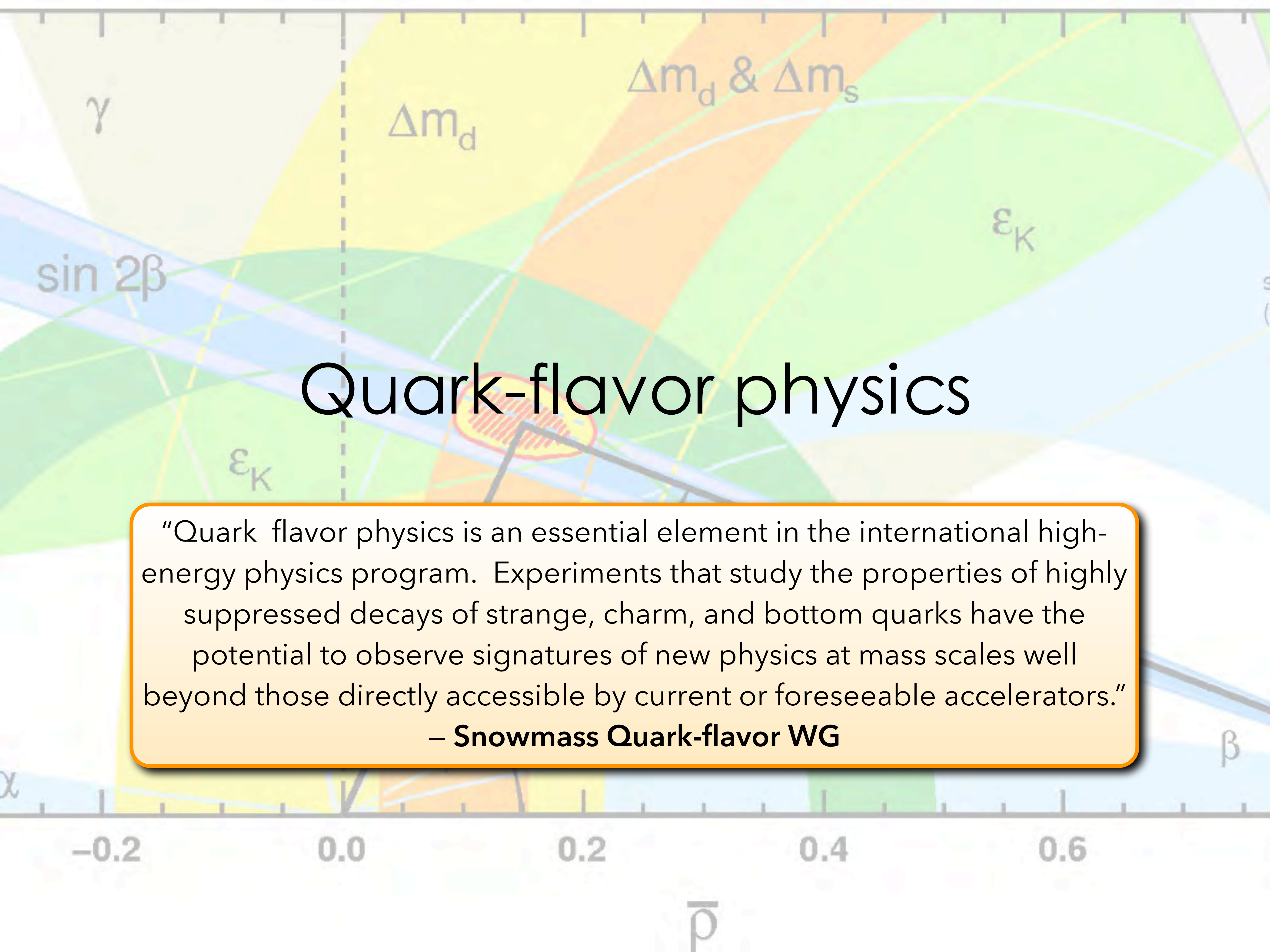
★★★ = sizeable NP effects
★★ = moderate to small NP effects
★ = no visable NP effects

[Buras, Acta Phys.Polon.B41:2487-2561,2010]

- Different processes & observables sensitive to different new-physics scenarios
  - Pattern of measurements can distinguish between models & constrain model parameters

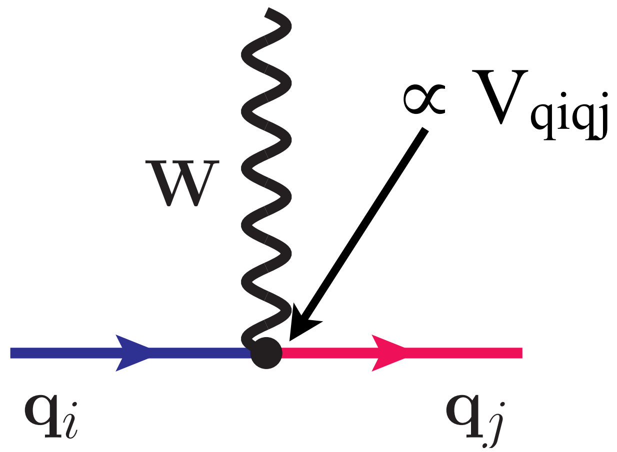
We do not know where the new physics lies ➜ cast a wide net!

Quark-flavor physics

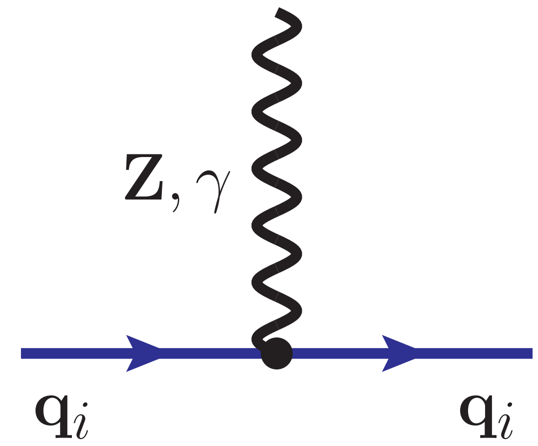


"Quark flavor physics is an essential element in the international high-energy physics program. Experiments that study the properties of highly suppressed decays of strange, charm, and bottom quarks have the potential to observe signatures of new physics at mass scales well beyond those directly accessible by current or foreseeable accelerators."
– **Snowmass Quark-flavor WG**

Flavor structure of the Standard Model



Quarks change flavor via W-boson exchange



No tree-level flavor-changing neutral currents

- Mixing between quark flavors under charged weak interactions parameterized by **Cabibbo-Kobayashi-Maskawa (CKM) matrix:**

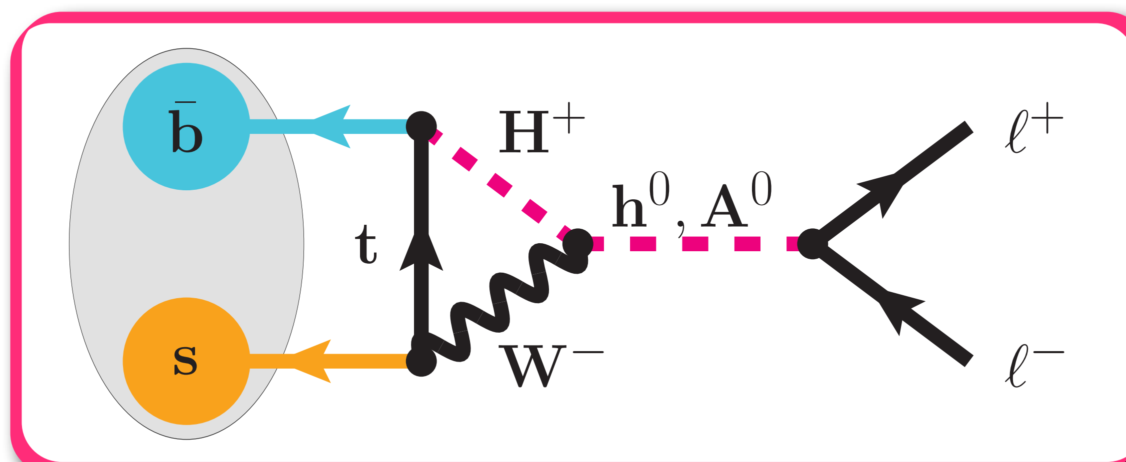
$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{array}{cc} \begin{array}{ccc} \quad d \quad & \quad s \quad & \quad b \quad \end{array} & \\ \begin{pmatrix} 0.9742 & 0.2257 & 3.59 \times 10^{-3} \\ 0.2256 & 0.9733 & 41.5 \times 10^{-3} \\ 8.74 \times 10^{-3} & 40.7 \times 10^{-3} & 0.9991 \end{pmatrix} & \begin{array}{c} u \\ c \\ t \end{array} \end{array}$$

- Largest elements largest along diagonal ➜ mixing most probable within same generation
- Complex phase ➜ CP violation

The Standard-Model CKM framework works!

- Bottom & charm factories established that **CKM paradigm of CP-violation describes experimental observations at the ~10% percent level** ➜ 2008 Nobel Prize for Kobayashi & Maskawa

- ... But *most Standard-Model extensions lead to additional flavor & CP-violation from exchange of new particles*, e.g.:

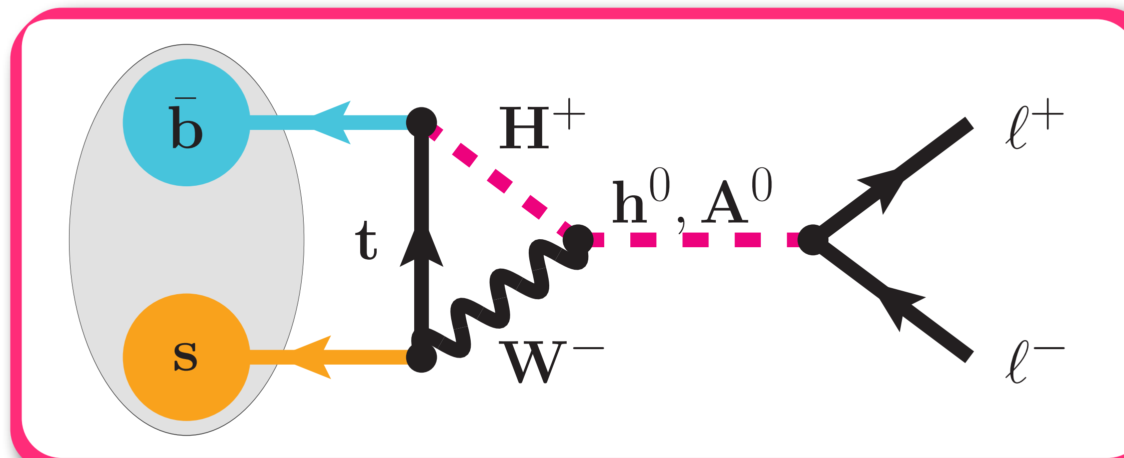


➡ **Goal of quark-flavor physics effort to look for cracks in the CKM framework and evidence for new physics**

The Standard-Model CKM framework works!

- Bottom & charm factories established that **CKM paradigm of CP-violation describes experimental observations at the ~10% percent level** ➜ 2008 Nobel Prize for Kobayashi & Maskawa

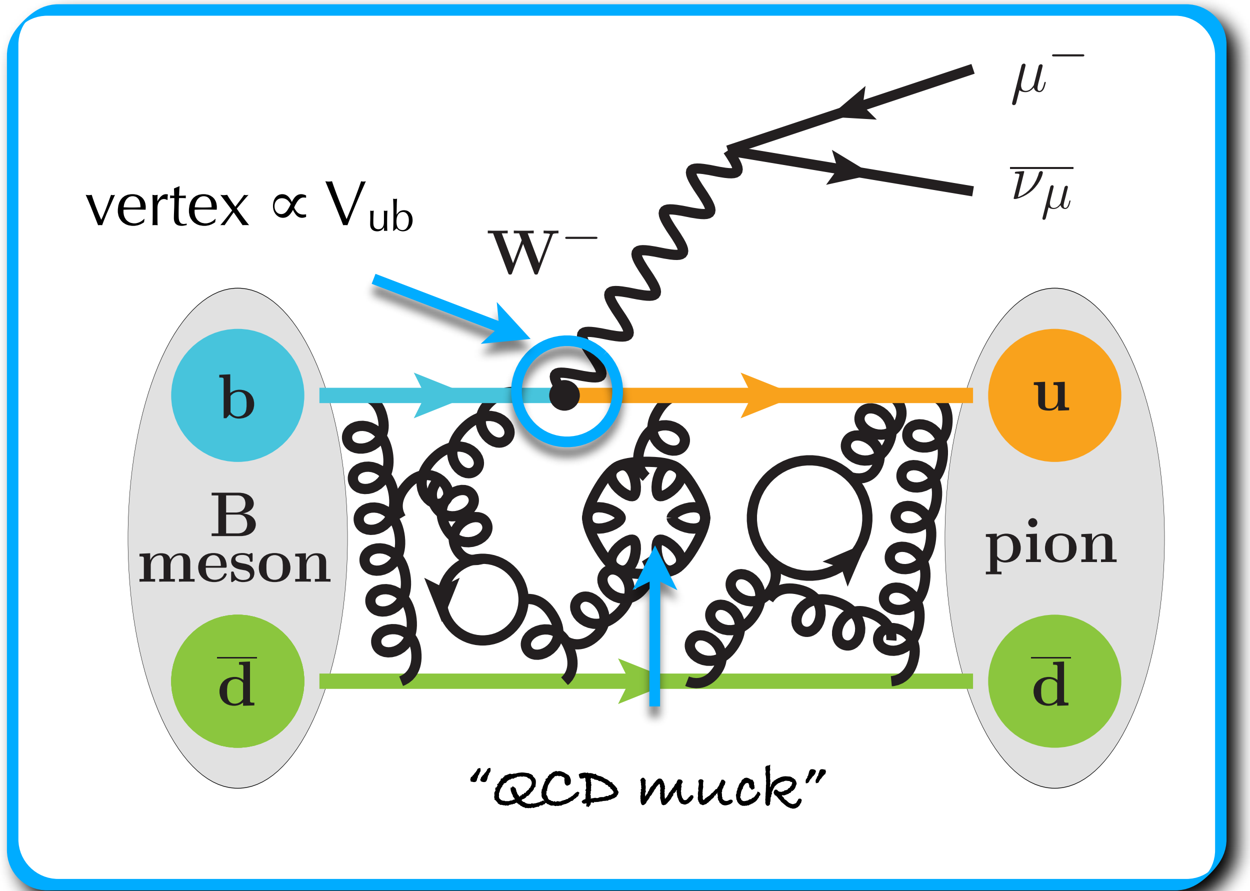
- ... But *most Standard-Model extensions lead to additional flavor & CP-violation from exchange of new particles*, e.g.:



➡ **Goal of quark-flavor physics effort to look for cracks in the CKM framework and evidence for new physics**

"Measuring" the CKM matrix

- CKM matrix elements & phase are fundamental Standard-Model parameters that cannot be calculated from first principles
- Infer their values by comparing experimental measurements of flavor-changing interactions of mesons & baryons with theoretical predictions *assuming the Standard Model*



(Experiment) = (known) x (CKM factors) × (Hadronic Matrix Element)

$\Delta m_{(d,s)}$,

$$\frac{d\Gamma(B \to \pi \ell \nu)}{dq^2}, \frac{d\Gamma(B \to D^{(*)} \ell \nu)}{dw}, \ldots$$

Absorb complex QCD dynamics into hadronic parameters such as decay constants, form factors, and bag-parameters (these are just #s!)

Lattice QCD & the CKM matrix

- All CKM elements except $|V_{tb}|$ can be obtained from simple "gold-plated" meson decays or mixing processes
  - One hadron in initial state and at most one hadron in final state
  - Hadrons are stable under QCD (or narrow and far from threshold

$$\begin{pmatrix} \mathbf{V_{ud}} & \mathbf{V_{us}} & \mathbf{V_{ub}} \\ \pi \to \ell\nu & K \to \ell\nu & B \to \ell\nu \\ & K \to \pi\ell\nu & B \to \pi\ell\nu \\ \mathbf{V_{cd}} & \mathbf{V_{cs}} & \mathbf{V_{cb}} \\ D \to \ell\nu & D_s \to \ell\nu & B \to D\ell\nu \\ D \to \pi\ell\nu & D \to K\ell\nu & B \to D^*\ell\nu \\ \mathbf{V_{td}} & \mathbf{V_{ts}} & \mathbf{V_{tb}} \\ \langle B_d|\bar{B}_d\rangle & \langle B_s|\bar{B}_s\rangle & \end{pmatrix}$$

*Neutral kaon mixing also gold-plated and can be used to obtain CKM phase (ρ,η)

- **Lattice-QCD community has well-established and successful program to calculate weak matrix elements needed to obtain CKM elements & phase** (see Flavor Lattice Averaging Group (FLAG) review, 1310.8555)

Lattice QCD & the CKM matrix

- **All CKM elements except $|V_{tb}|$ can be obtained from simple "gold-plated" meson decays or mixing processes**
  - One hadron in initial state and at most one hadron in final state
  - Hadrons are stable under QCD (or narrow and far from threshold

$$\begin{pmatrix} \mathbf{V_{ud}} & & \\ \pi \to \ell\nu & K & \\ & K & \\ \mathbf{V_{cd}} & \mathbf{V_{cs}} & \mathbf{V_{cb}} \\ D \to \ell\nu & D_s \to \ell\nu & B \to D\ell\nu \\ D \to \pi\ell\nu & D \to K\ell\nu & B \to D^*\ell\nu \\ \mathbf{V_{td}} & \mathbf{V_{ts}} & \mathbf{V_{tb}} \\ \langle B_d|\bar{B}_d\rangle & \langle B_s|\bar{B}_s\rangle & \end{pmatrix}$$

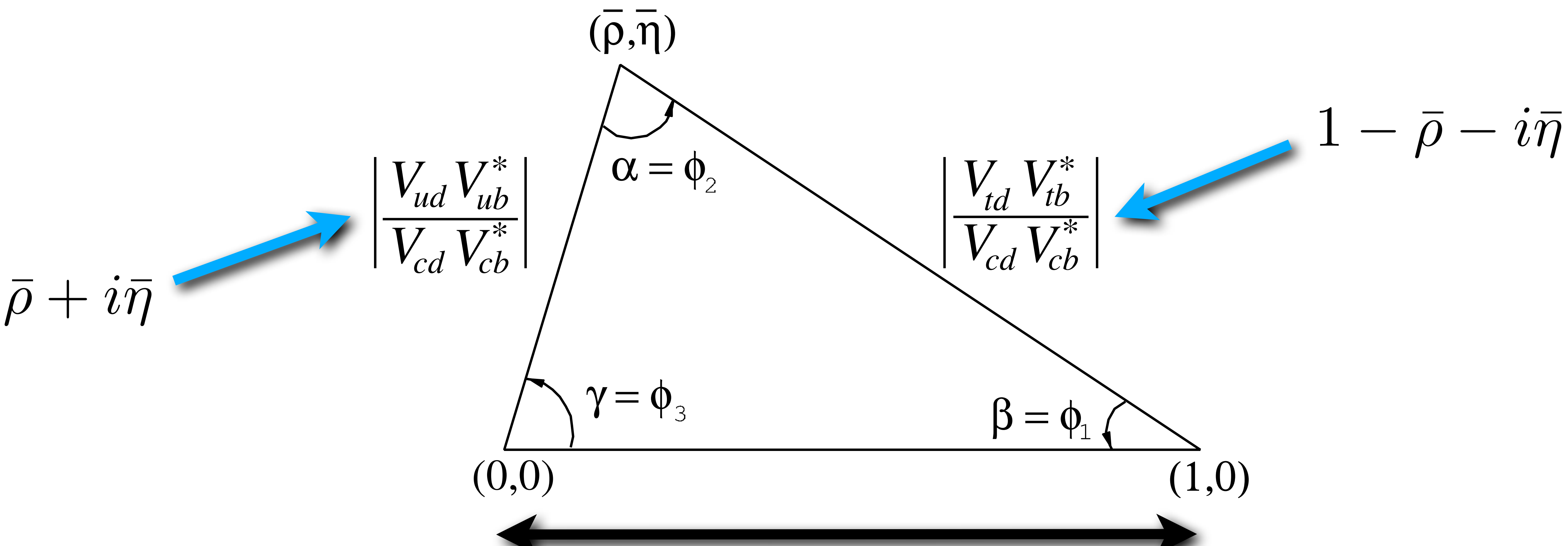
In this talk only present results for **three** (u,d,s) or **four** (u,d,s,c) **dynamical quark flavors**

l kaon mixing also gold-plated and can be used to obtain CKM phase (ρ,η)

- **Lattice-QCD community has well-established and successful program to calculate weak matrix elements needed to obtain CKM elements & phase** (see Flavor Lattice Averaging Group (FLAG) review, 1310.8555)

The CKM unitarity triangle

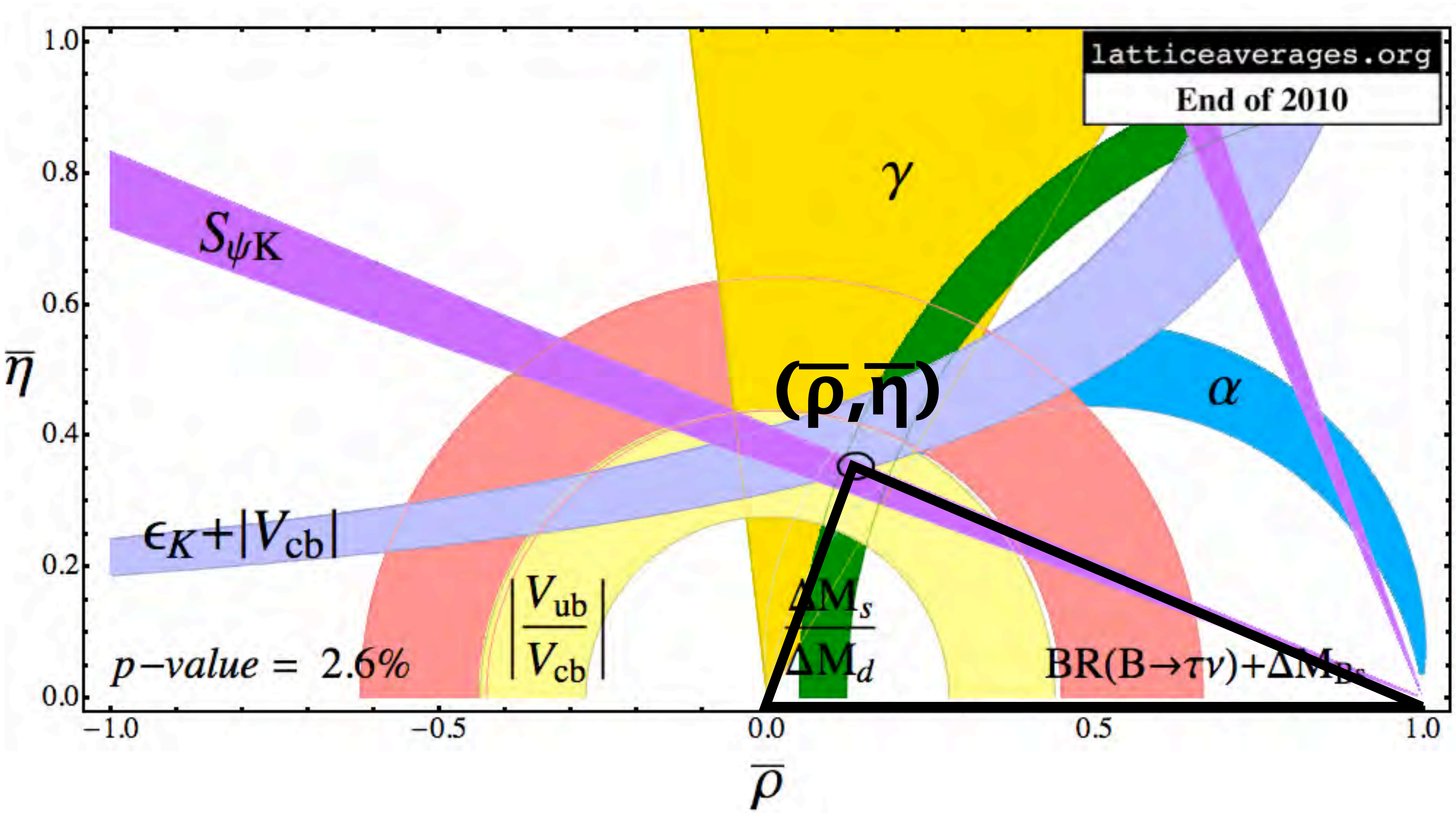
- Standard-Model CKM matrix is unitary ➜ elements are not all independent
- One of the relationships between CKM matrix elements is: $$\boxed{V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0}$$
- **Can express as triangle in complex plane known as the CKM unitarity triangle**



(Rescaled by $|V_{cd}V_{cb}^*|$ so base has unit length)

The global CKM UT fit

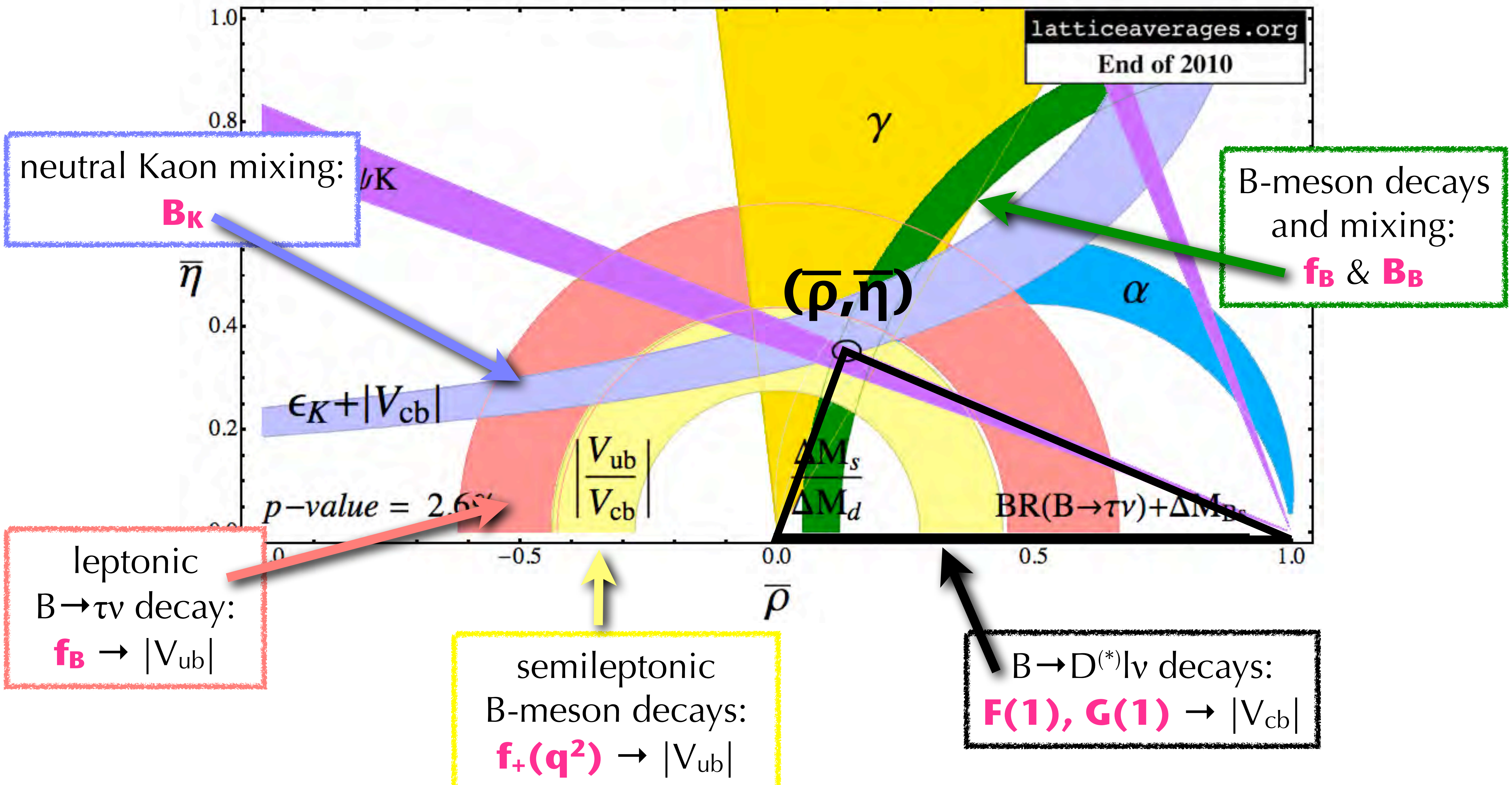
- Can interpret many experimental measurements as determinations of the apex (ρ,η)



- **New quark flavor-changing interactions or CP-violating phases would appear as inconsistencies between measurements of ρ & η that are predicted to be the same within the Standard Model**

Lattice-QCD inputs to the UT fit

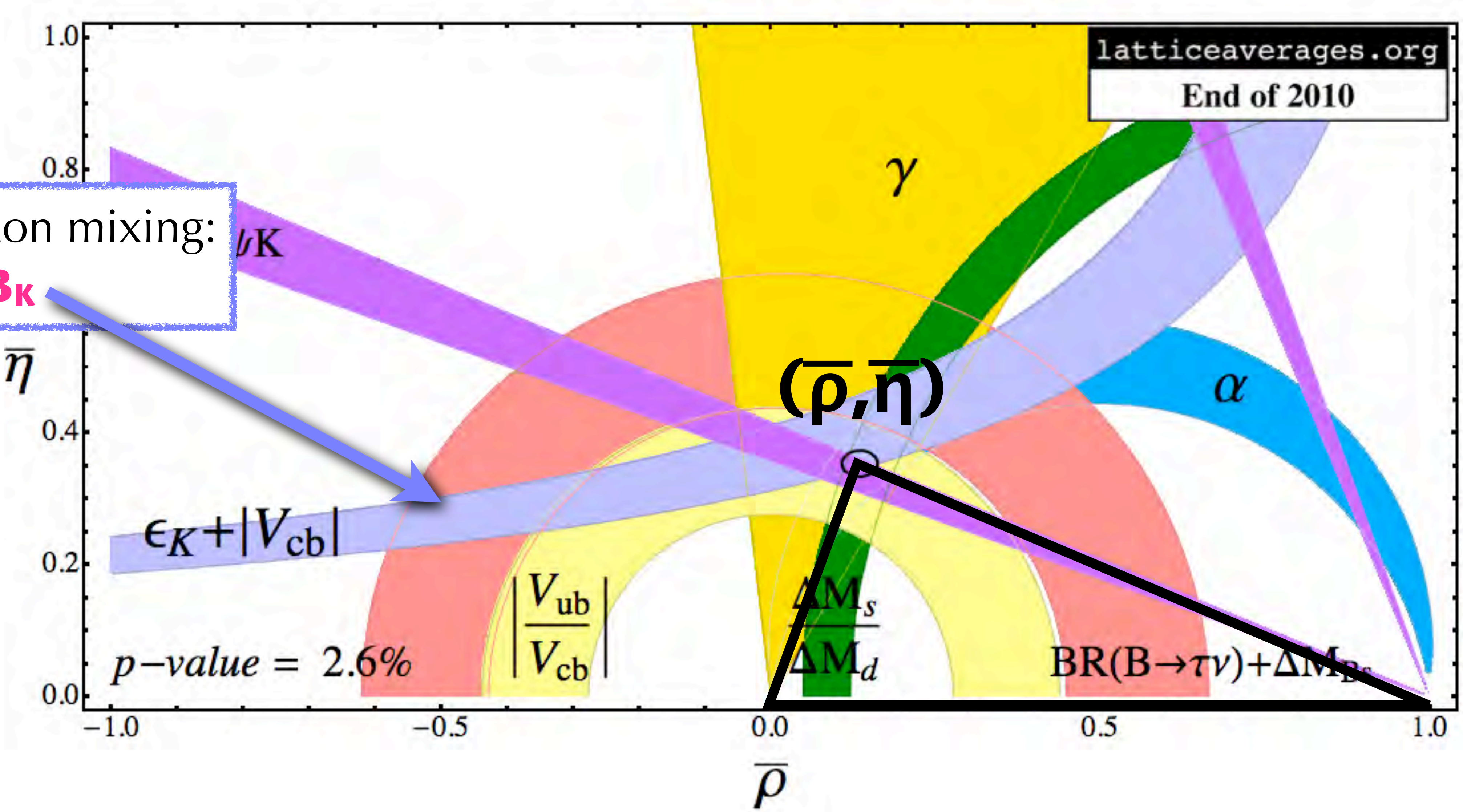
- Many constraints on the unitarity triangle require lattice calculations of hadronic parameters



Neutral kaon mixing

- Many constraints on the unitarity triangle require lattice calculations of hadronic parameters

neutral Kaon mixing:
B_K



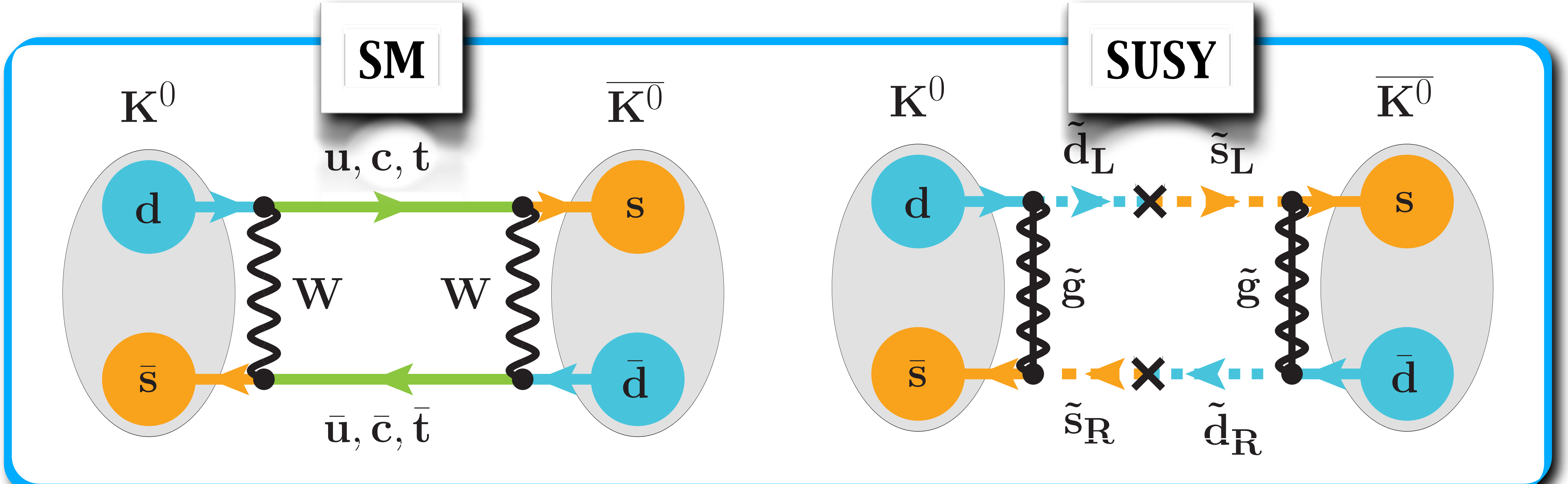
Neutral kaon mixing

- Standard-Model flavor-changing effects suppressed most strongly in kaon sector:

$$\underbrace{V_{ts}^* V_{td}}_{K \text{ system}} \sim 5 \cdot 10^{-4} \ll \underbrace{V_{tb}^* V_{td}}_{B_d \text{ system}} \sim 10^{-2} \ll \underbrace{V_{tb}^* V_{ts}}_{B_s \text{ system}} \sim 4 \cdot 10^{-2}$$

➡ **Kaon system exhibits highest new-physics sensitivity of any neutral meson system**

- Measurements of kaon mixing push scale of new physics with generic O(1) flavor couplings to ≳ 10,000 TeV [Isidori, Nir, Perez, Ann.Rev.Nucl.Part.Sci. 60 (2010) 355]



The kaon mixing parameter B_K

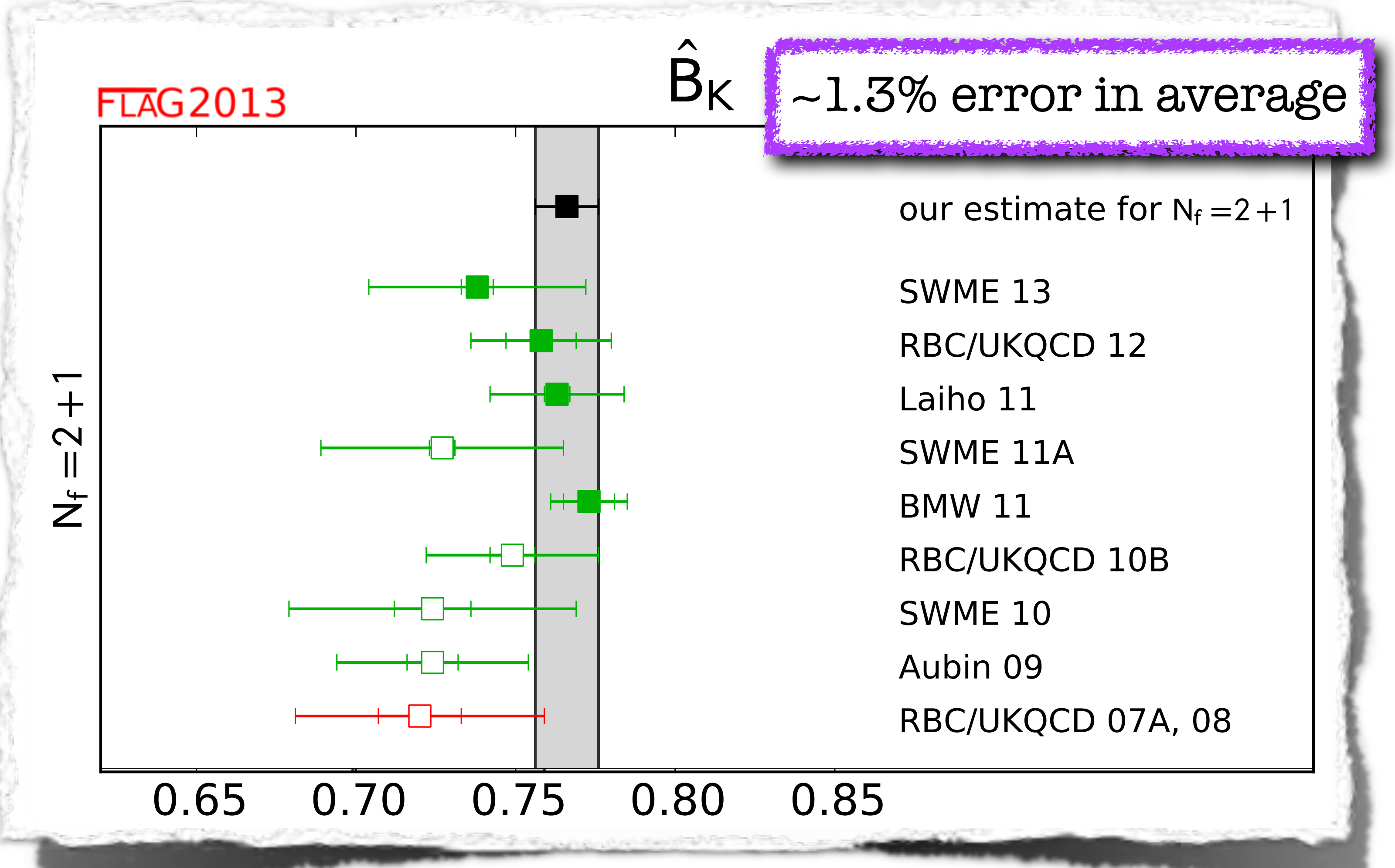
- Indirect CP violation in neutral kaon system (ϵ_K) determines apex (ρ,η) via

$$|\epsilon_K| = C_\epsilon B_K A^2 \bar{\eta}\{-\eta_1 S_0(x_c)(1-\lambda^2/2) + \eta_3 S_0(x_c,x_t) + \eta_2 S_0(x_t) A^2 \lambda^2 (1-\bar{\rho})\}$$

  - C_ε, η_i and S_0 known to NLO (in some cases NNLO) in perturbation theory

- Until recently, ϵ_K constraint limited by ~20% uncertainty in **nonperturbative hadronic matrix element B_K**

- Significant theoretical & computational effort devoted to B_K ➜ **now several independent lattice results in good agreement**



Status of the $|\epsilon_K|$ band

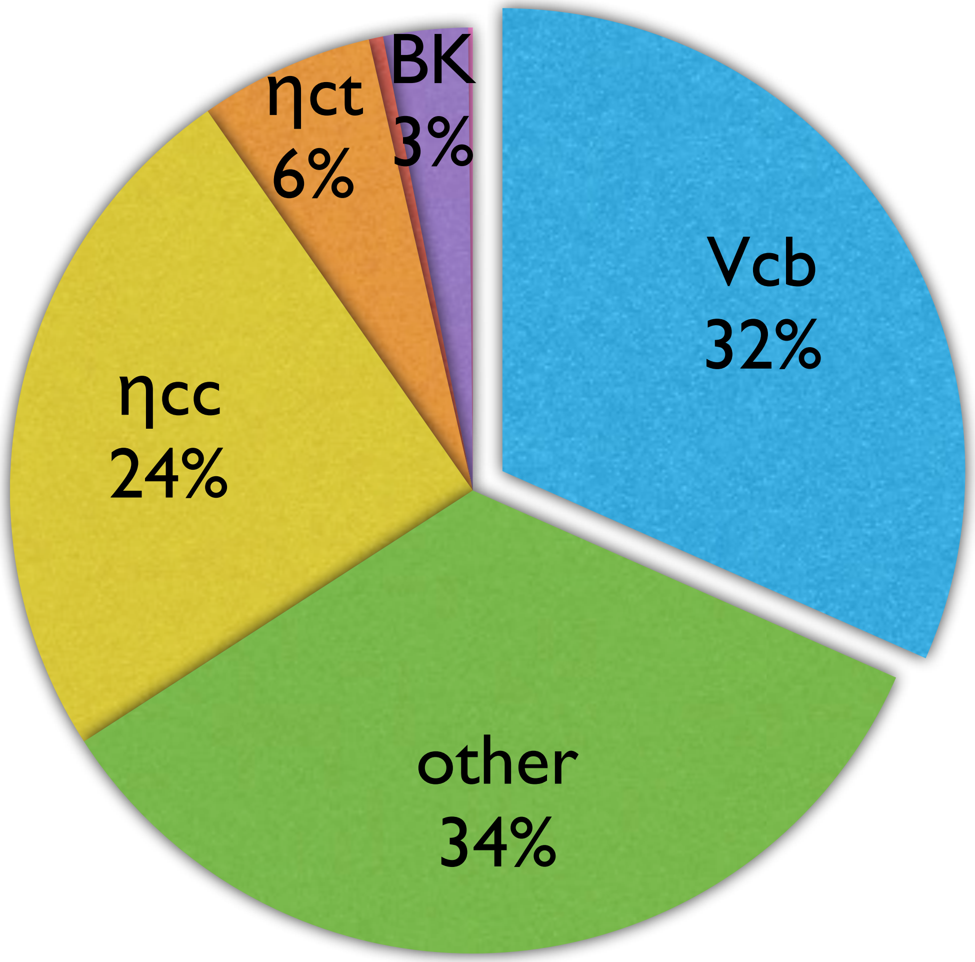
- Brod & Gorbahn [PRL108 (2012) 121801] give following error breakdown for $|\epsilon_K|$ in the Standard Model:

$$|\epsilon_K| = (1.81 \pm 0.14_{\eta_{cc}} \pm 0.02_{\eta_{tt}} \pm 0.07_{\eta_{ct}} \pm 0.05_{\mathrm{LD}} \pm 0.23_{\mathrm{parametric}}) \times 10^{-3}$$

(1) Largest individual uncertainty is from ~10% parametric error in $\propto |V_{cb}|^4$

(2) η_{cc} and η_{ct} are both known to 3-loops (NNLO)

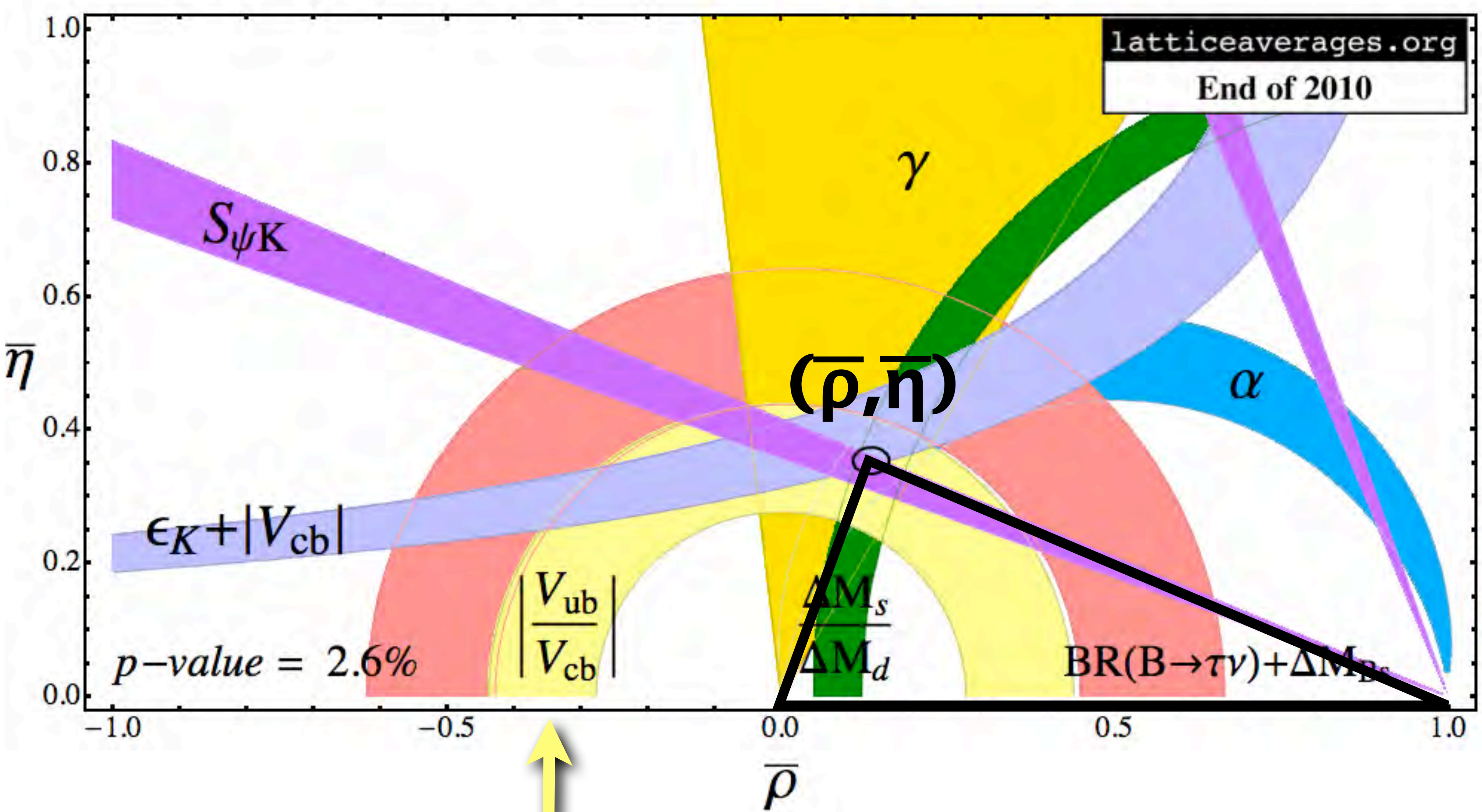
(3) Error from B_K only fourth-largest individual contribution



- Lattice community moving on to other more challenging kaon-physics quantities such as $K\to\pi\pi$ and $\Delta(M_K)$...

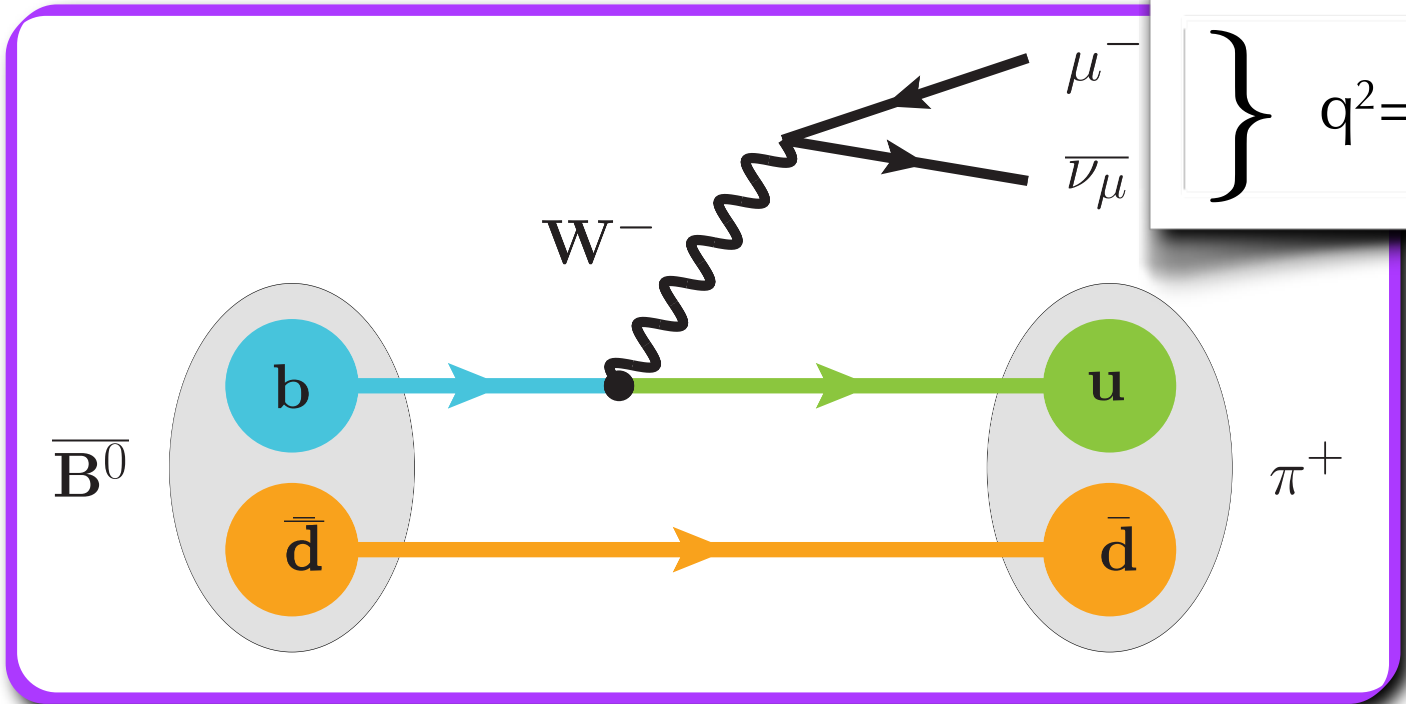
B → $\pi\ell\nu$ semileptonic decay

- Many constraints on the unitarity triangle require lattice calculations of hadronic parameters



semileptonic
B-meson decays:
$f_+(q^2)$ → $|V_{ub}|$

$|V_{ub}|$ from $B \to \pi \ell \nu$ decay



$\}$ $q^2=(m_B{}^2+m_\pi{}^2-2m_B E_\pi)^2$

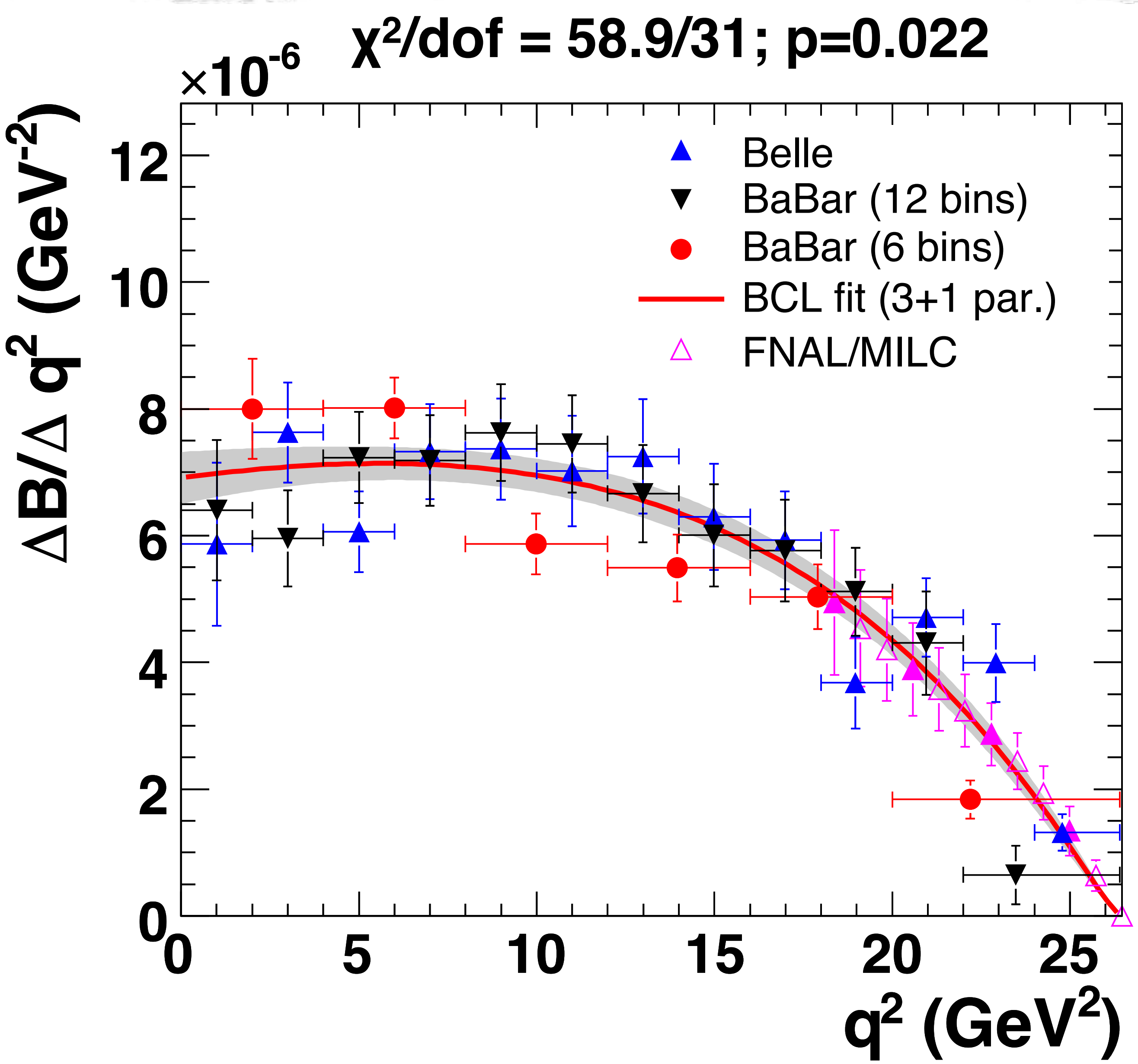
- $B \to \pi \ell \nu$ semileptonic decay enables determination of $|V_{ub}|$ via:

$$\frac{d\Gamma(B^0 \to \pi^- \ell^+ \nu)}{dq^2} = \frac{G_F^2}{192\pi^3 m_B^3}\left[(m_B^2+m_\pi^2-q^2)^2-4m_B^2 m_\pi^2\right]^{3/2} |\mathbf{V_{ub}}|^2 |\mathbf{f_+(q^2)}|^2$$

- **Few percent determination of exclusive $|V_{ub}|$ challenging because:**
  - Lattice statistical & discretization errors grow with increasing pion momentum ➜ determination of **hadronic form factor $f_+(q^2)$** best at large momentum-transfer (q^2)
  - Experimental branching fraction most precise at smallest at low q^2

Exclusive determination of |Vub|

[Heavy Flavor Averaging Group for PDG 2012]



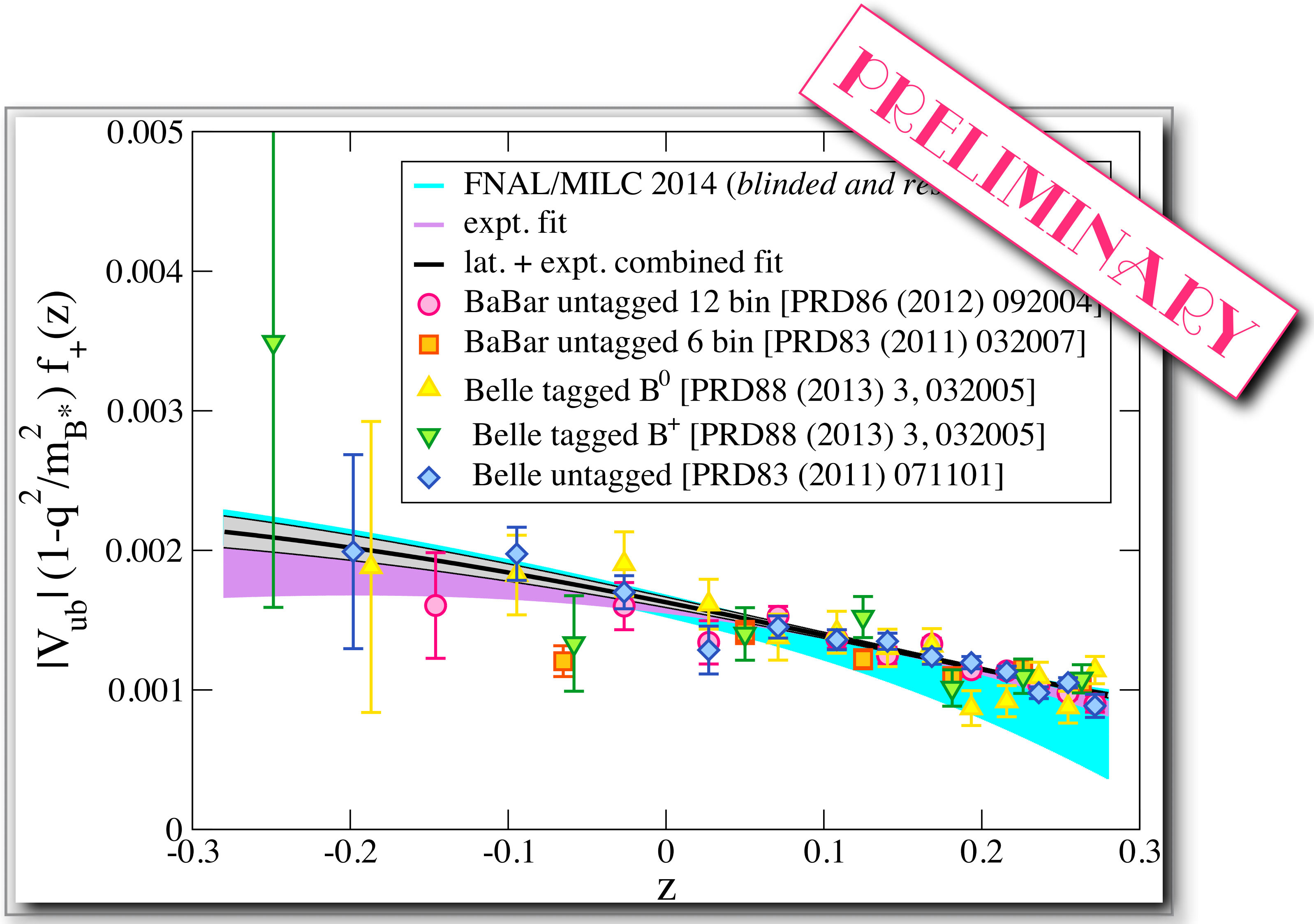
- Fit lattice-QCD and experimental data together to model-independent parameterization based on analyticity, unitarity, and crossing symmetry leaving relative normalization factor ($|V_{ub}|$) as a free parameter [*c.f.* Arnesen *et al.* PRL. 95, 071802 (2005); FNAL/MILC, PRD79 (2009) 054507]

- Combined fit to Belle, BaBar, and FNAL/MILC lattice data gives $|V_{ub}|$ with ~9% uncertainty:

$$|V_{ub}|^{B\to\pi\ell\nu} = (3.23 \pm 0.30) \times 10^{-3}$$

Exclusive determination of |V_{ub}|

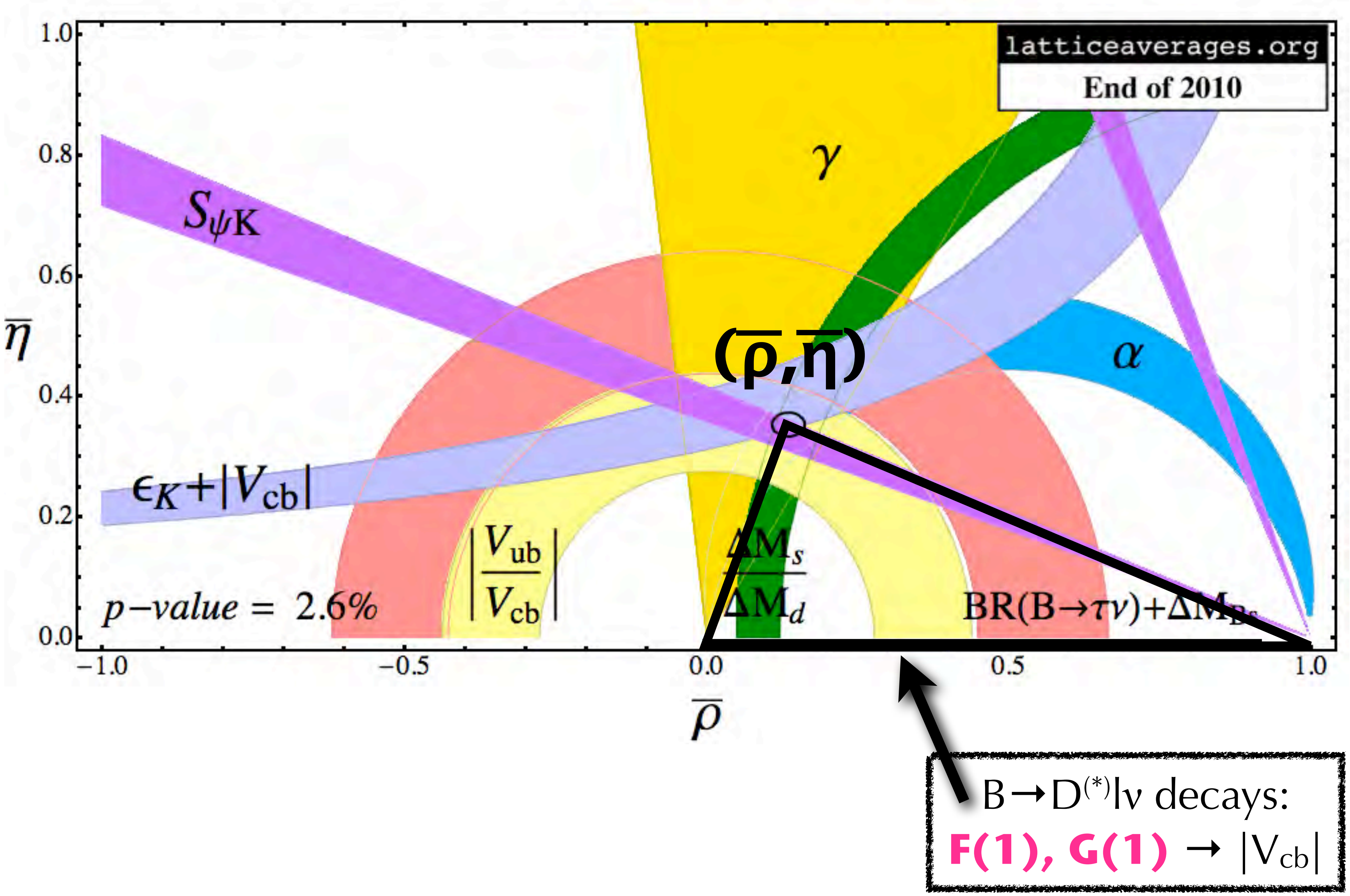
- Anticipate new $B\to\pi\ell\nu$ form-factors results later this year with increased statistics, finer lattice spacings, and different light- and b-quark actions [HPQCD, PoS LATTICE2012 (2012) 118; FNAL/MILC PoS LATTICE2013 (2013) 383 ;RBC/UKQCD, PoS LATTICE2012 (2012) 109]



➜ **Projected error on exclusive |V_{ub}| will decrease from ~9% → ~4%**

B→Dℓν & B→D*ℓν semileptonic decays

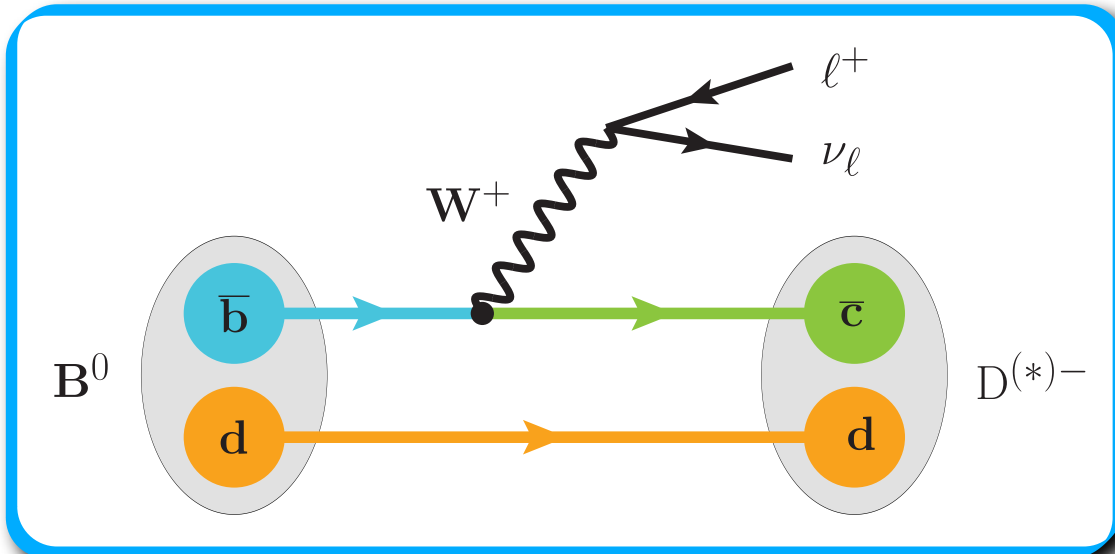
- Many constraints on the unitarity triangle require lattice calculations of hadronic parameters



$|V_{cb}|$ from $B \to D^* \ell\nu$ at zero recoil

[Bailey *et al.* [Fermilab Lattice & MILC Collaborations], arXiv:1403.0635]

- $B \to D^* \ell\nu$ semileptonic decay allows determination of $|V_{cb}|$



$$\boxed{\frac{d\Gamma(B \to D^* l\nu)}{dw} = \frac{G_F^2}{48\pi^3} m_D^3 (m_B + m_D)^2 (w^2 - 1)^{3/2} |\mathbf{V_{cb}}|^2 |\mathcal{F}_{\mathbf{B \to D^*}}(\mathbf{w})|^2} \Big\} \; w \equiv v_B \cdot v_D$$

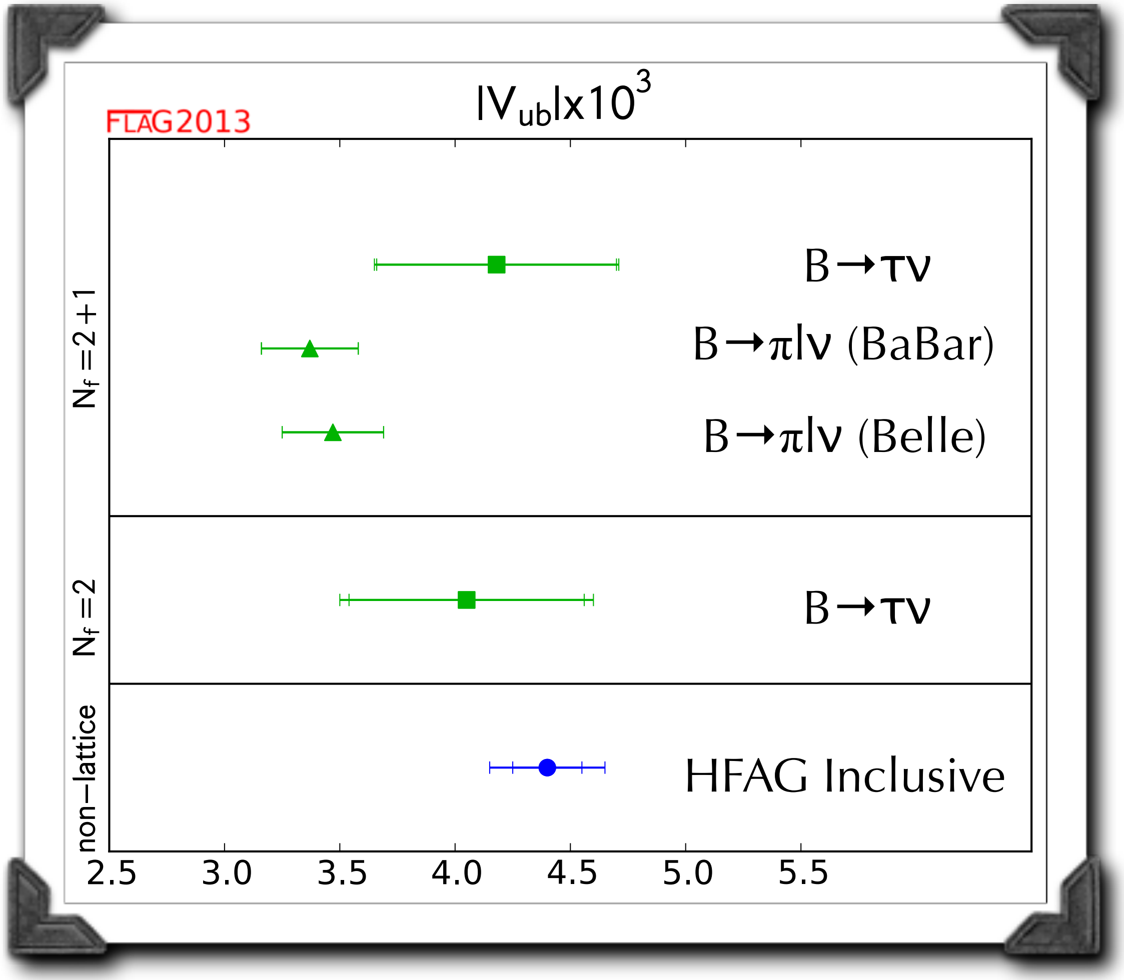
- Need **nonperturbative form factor F(w)** at one normalization point ➜ choose zero recoil (w=1) where it can be computed most precisely
- Fermilab/MILC recently **updated F(1) with increased statistics, lighter quark masses, & finer lattice spacings**, obtaining $|V_{cb}|$ to 1.9% precision
- **QCD error in $|V_{cb}|$ now commensurate with experimental error**

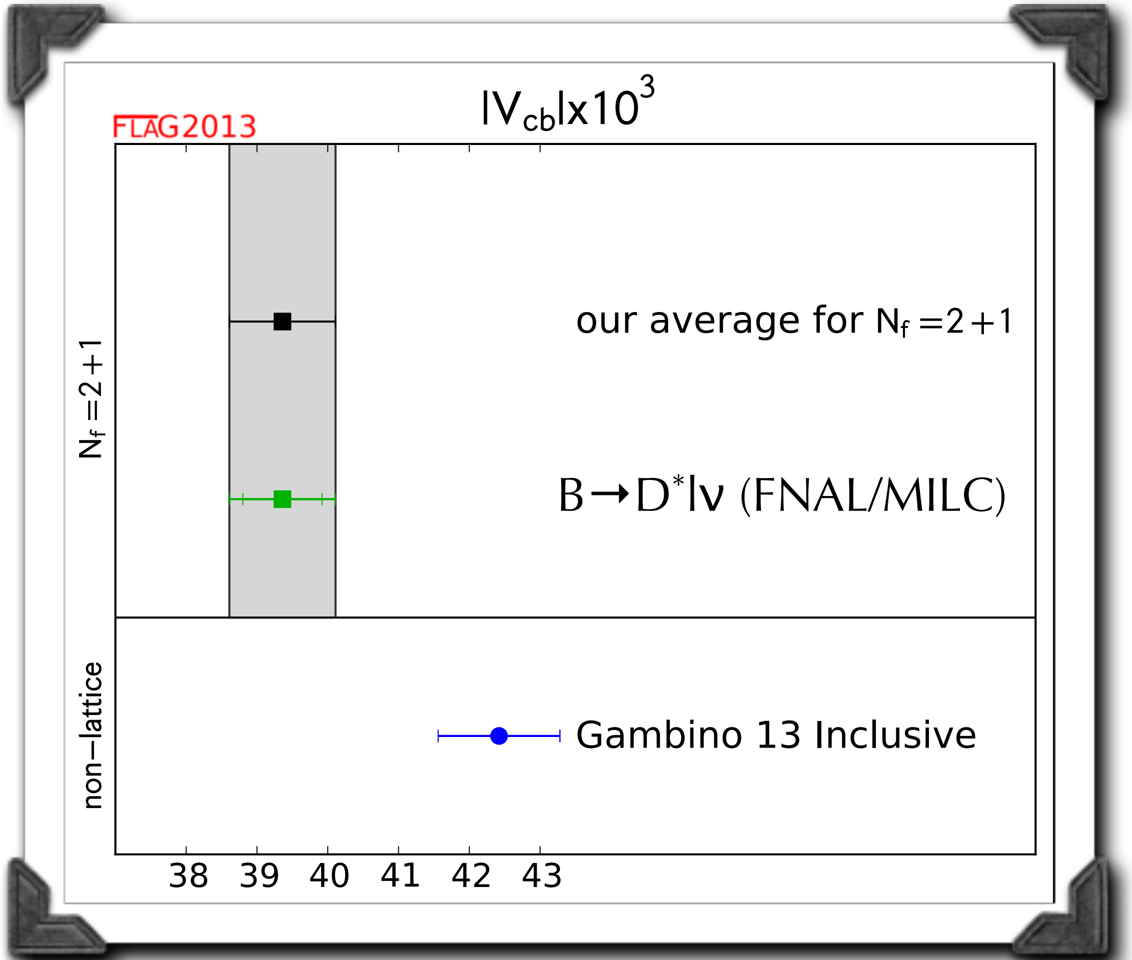
$$F(1) = 0.906(4)_{\text{stat}}(12)_{\text{sys}}$$

$$|V_{cb}| = [39.04(49)_{\text{expt}}(53)_{\text{LQCD}}(19)_{\text{QED}}] \times 10^{-3}$$

The "V_{ub} & V_{cb} puzzles"

- Long-standing ~3σ tensions between determinations of $|V_{ub}|$ and $|V_{cb}|$ from inclusive & exclusive semileptonic B-decays need resolution

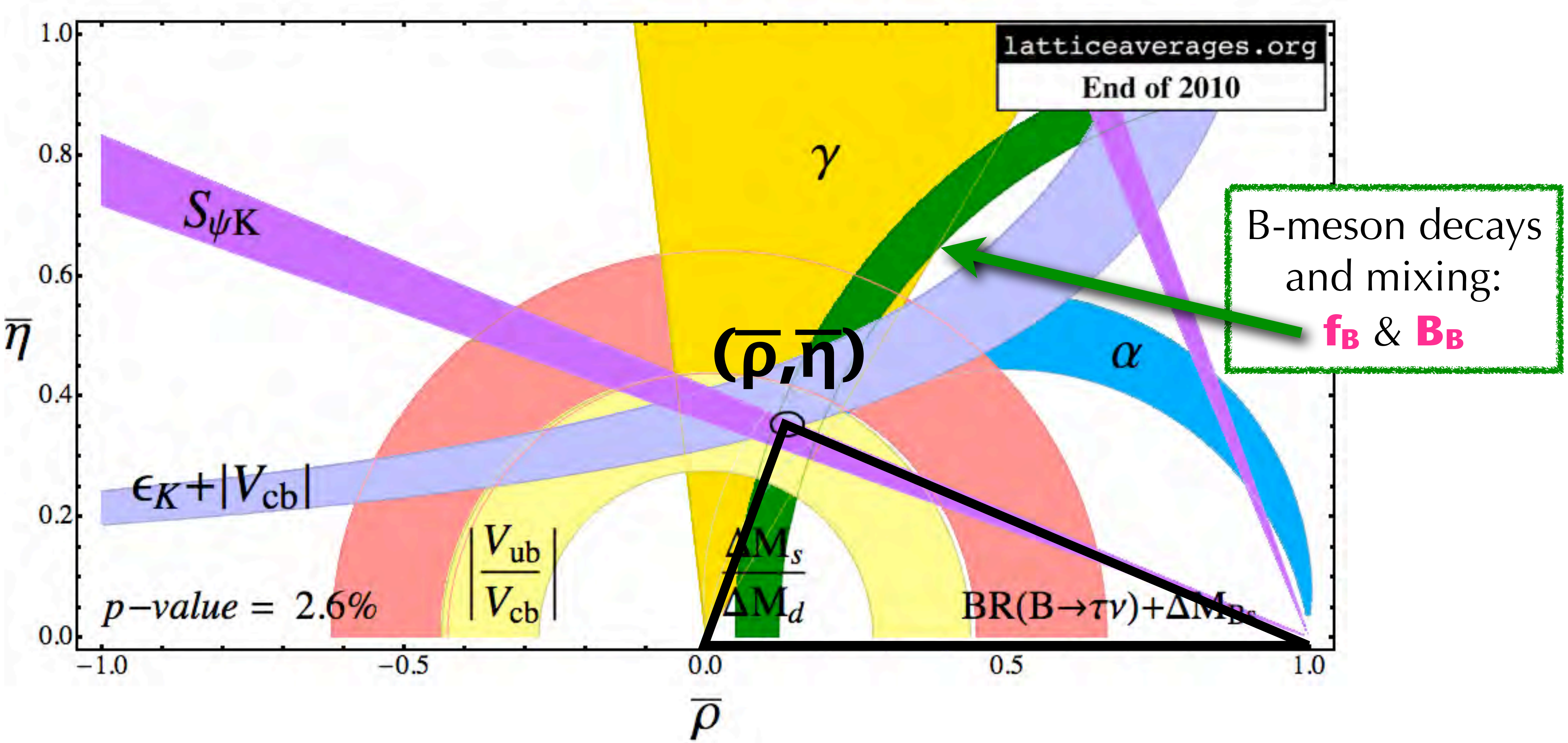




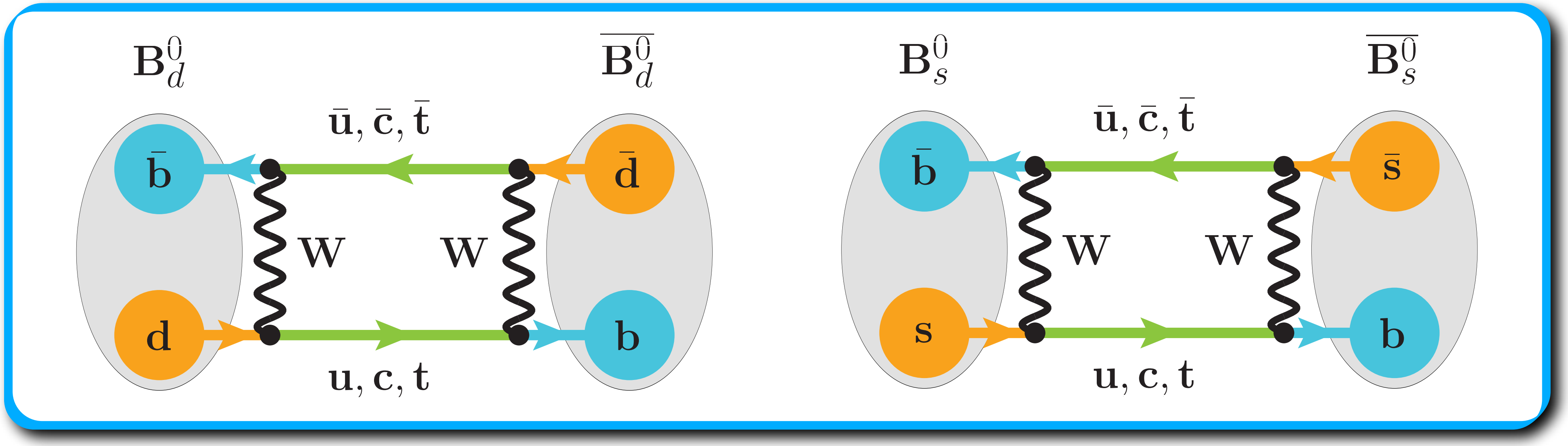
- Experimental measurements and lattice calculations will continue to improve
- **Determinations from other exclusive decays will provide important checks:**
  - Lattice-QCD calculations underway of form factors for $B_s\to K\mu\nu$ to obtain $|V_{ub}|$ (anticipate measurement @ LHCb) and $B\to Dl\nu$ to obtain $|V_{cb}|$ (N_f=2+1 result coming soon)

Neutral B-meson mixing

- Many constraints on the unitarity triangle require lattice calculations of hadronic parameters



Neutral B-meson mixing



- Ratio of B_d to B_s oscillation frequencies (Δm_q) determines apex (ρ,η) via

$$\frac{\Delta m_d}{\Delta m_s} = \left(\frac{f_{B_d}\sqrt{\hat{B}_{B_d}}}{f_{B_s}\sqrt{\hat{B}_{B_s}}}\right)^2 \frac{m_{B_d}}{m_{B_s}} \frac{|V_{td}|^2}{|V_{ts}|^2}$$

$$= \xi^2 \frac{m_{B_d}}{m_{B_s}} \left(\frac{\lambda}{1-\lambda^2/2}\right)^2 \frac{((1-\bar{\rho})^2+\bar{\eta}^2)}{\left(1+\frac{\lambda^2}{1-\lambda^2/2}\bar{\rho}\right)+\lambda^4\bar{\eta}^2}$$

- Δm_q measured to better than 1% ➜ dominant error from uncertainty in **ratio of hadronic matrix elements ξ**

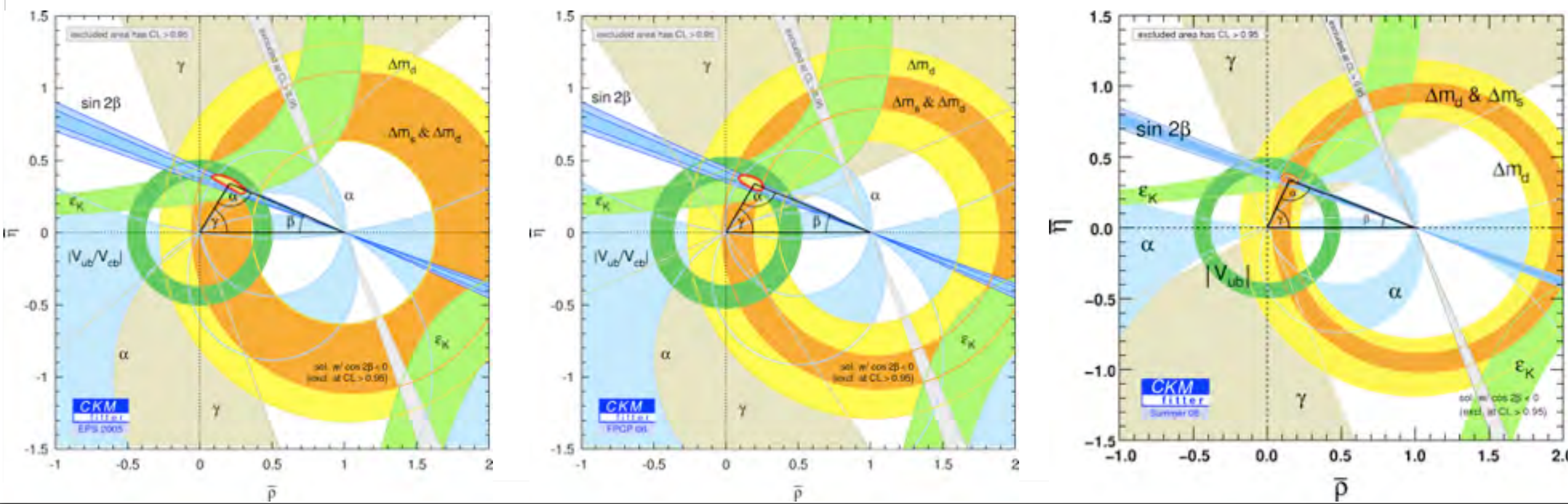
History of B-mixing constraint

- Δm_q measured to better than 1% ➜ dominant error from uncertainty in **ratio of hadronic matrix elements ξ**

2005

2006: D0 and CDF measure Δm_s

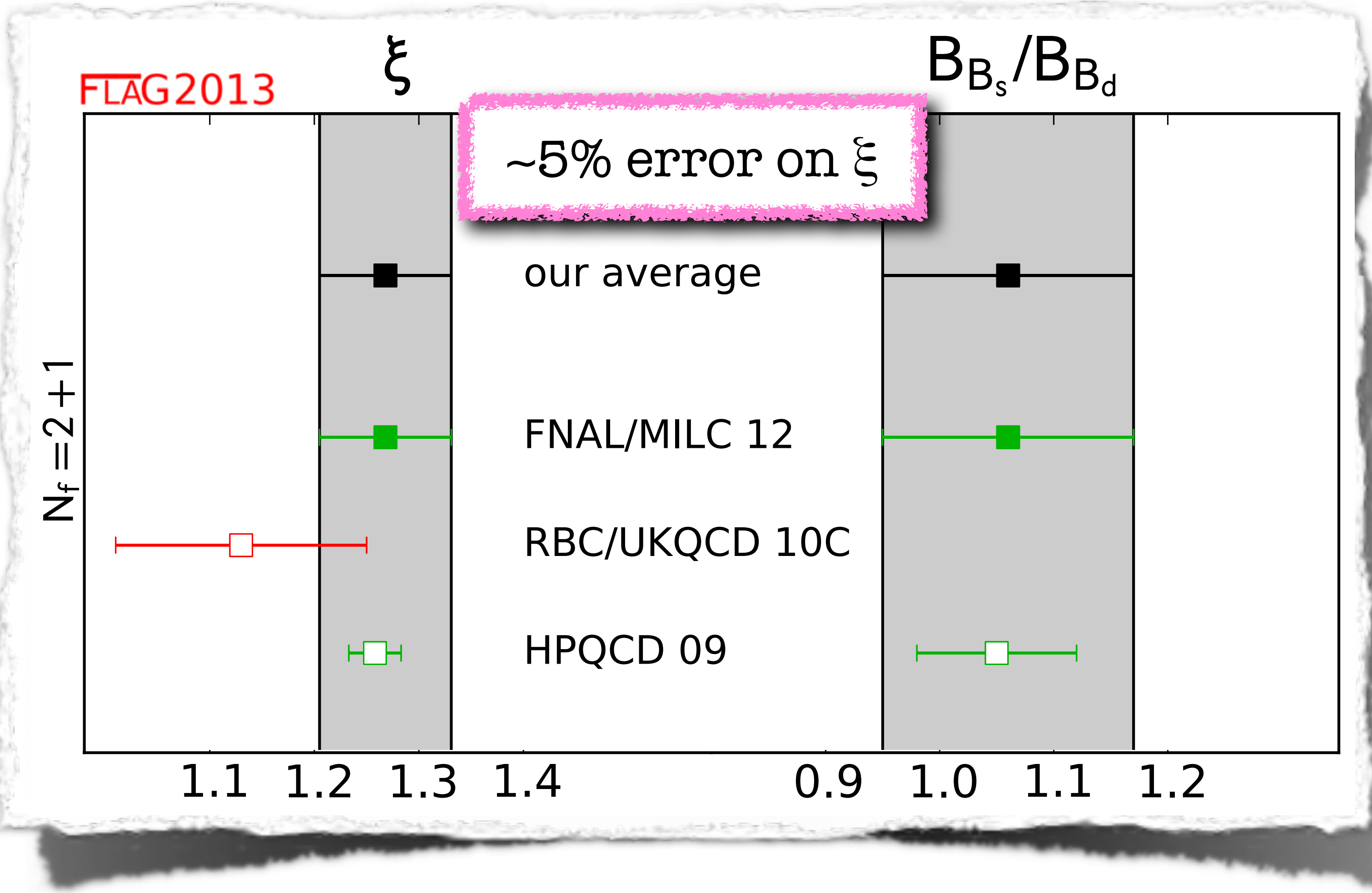
2008: First unquenched lattice calculation of ξ



- Key constraint in global UT fit because approximately perpendicular to narrow constraint from sin(2β)

The SU(3)-breaking ratio ξ

- **Can be computed more precisely than individual B_d and B_s matrix elements because:**
  - Statistical fluctuations highly correlated between numerator & denominator
  - f_{Bd} Sqrt[B_{Bd}]= f_{Bs} Sqrt[B_{Bs}] in SU(3) limit m_s=m_{ud}, so some systematic errors suppressed by $(m_s\text{-}m_{ud})/\Lambda_{QCD}$

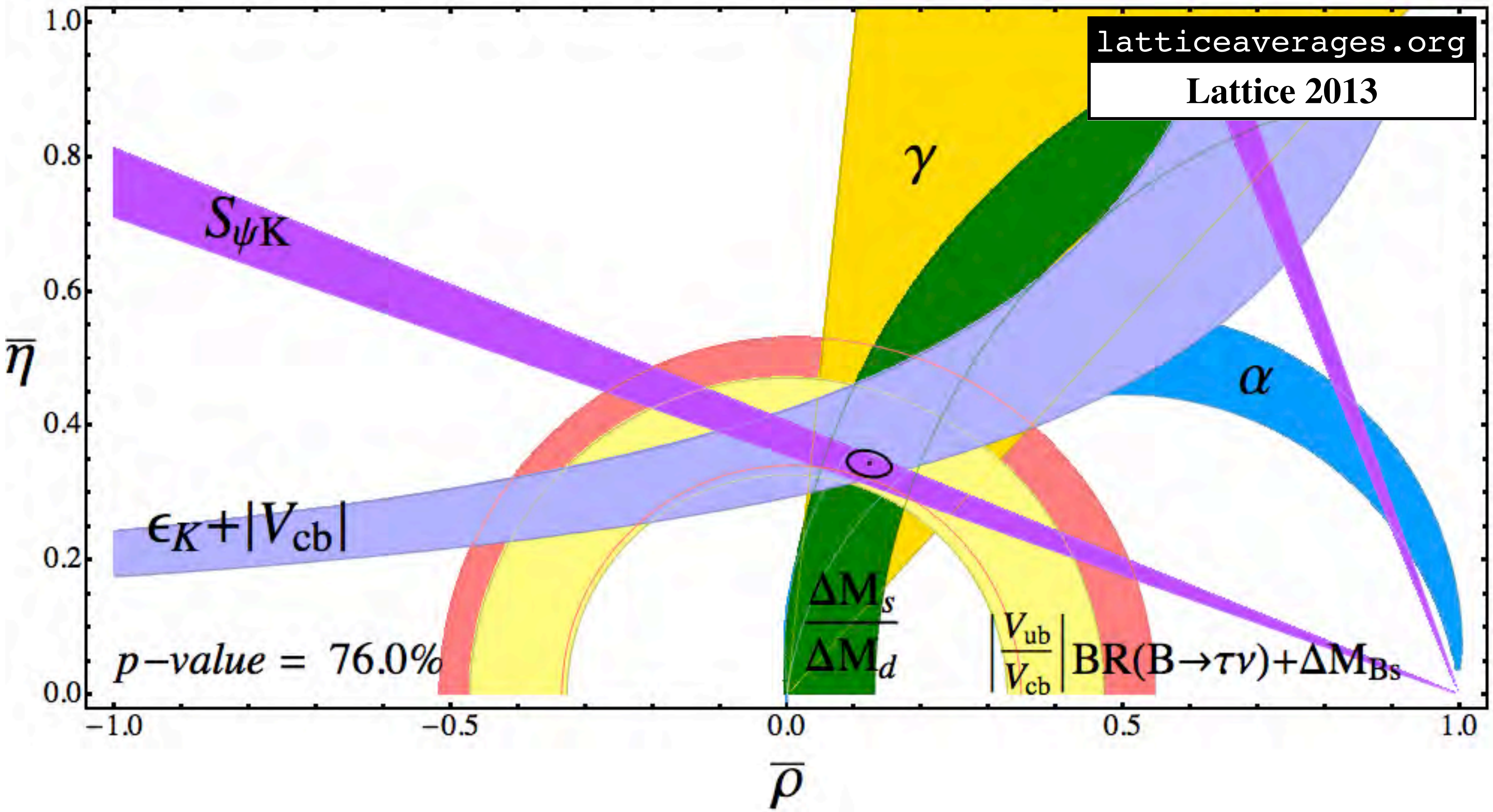


- Calculations in progress with
  - Different lattice light & b-quark actions [RBC/UKQCD, PoS LATTICE2012 (2012) 109]
  - Lighter pions + finer lattice spacings [FNAL/MILC, PoS LATTICE 2013, 477 (2013); C. Davies *for HPQCD*, Lattice 2013]

➡ Expect new N_f=2+1 results within a year with (projected) errors ≲2%

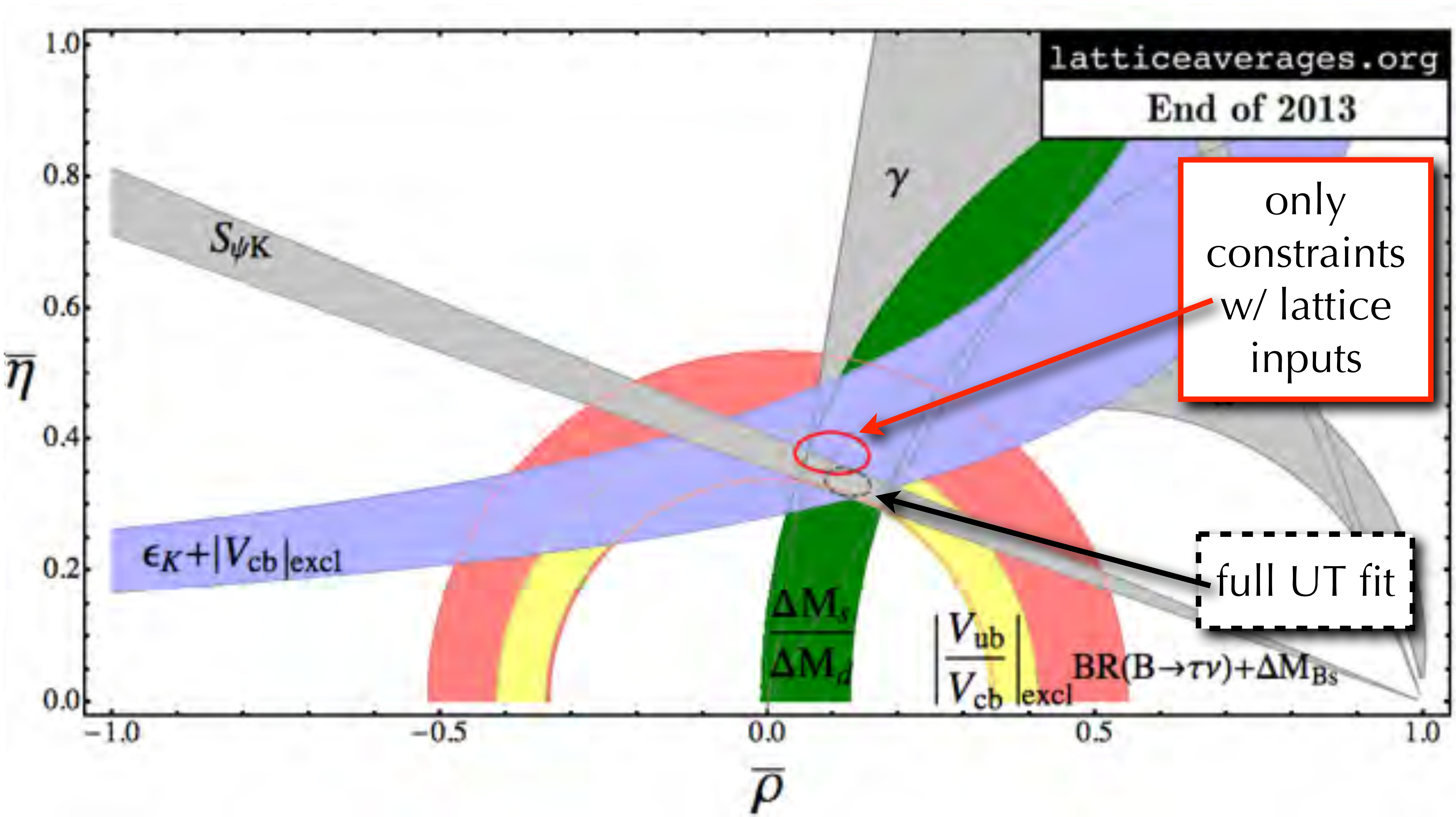
Current status of global CKM UT fit

- (*Disappointingly!*) good agreement with Standard-Model expectations...



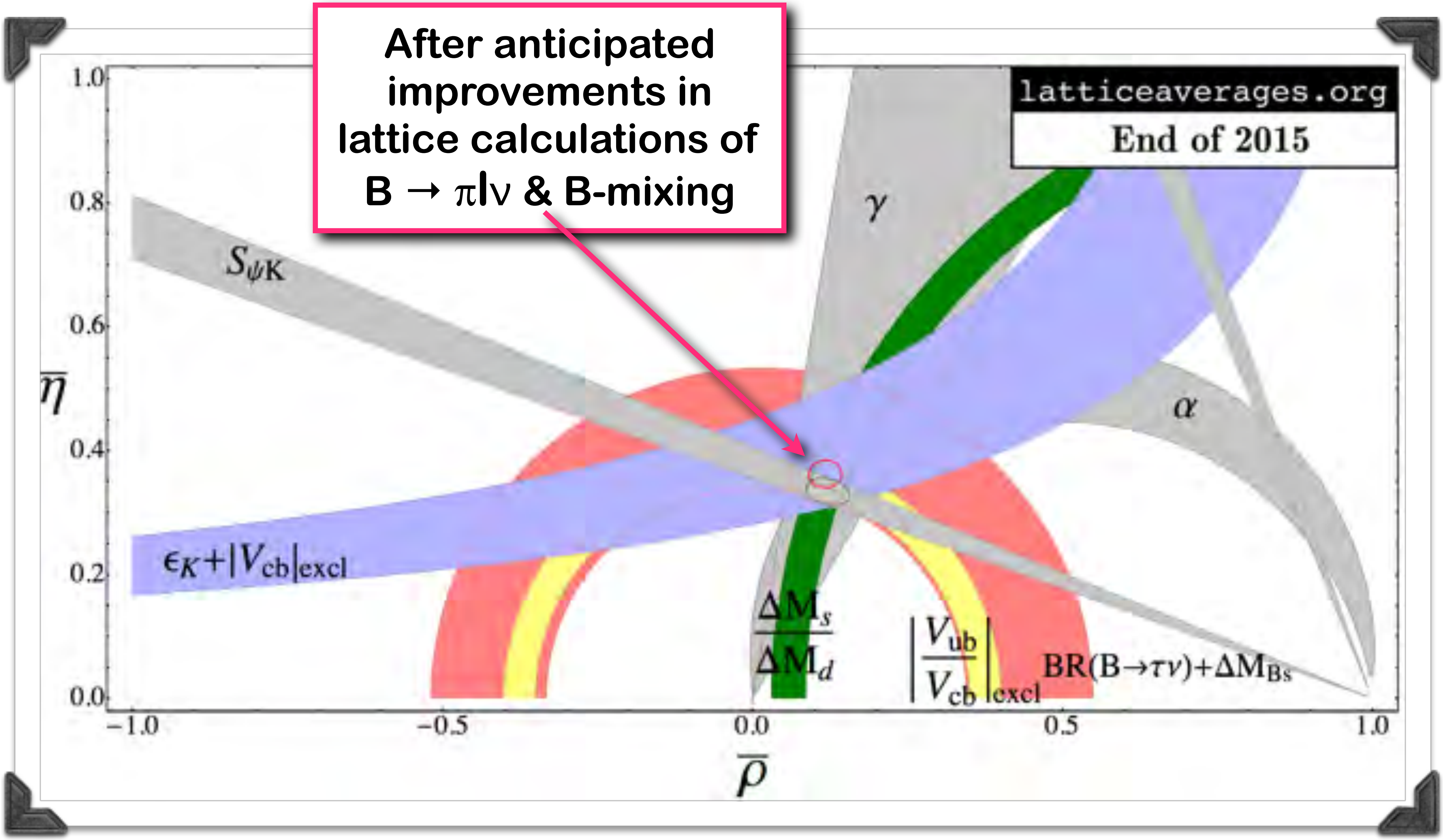
Current status of global CKM UT fit

- *(Disappointingly!)* good agreement with Standard-Model expectations...



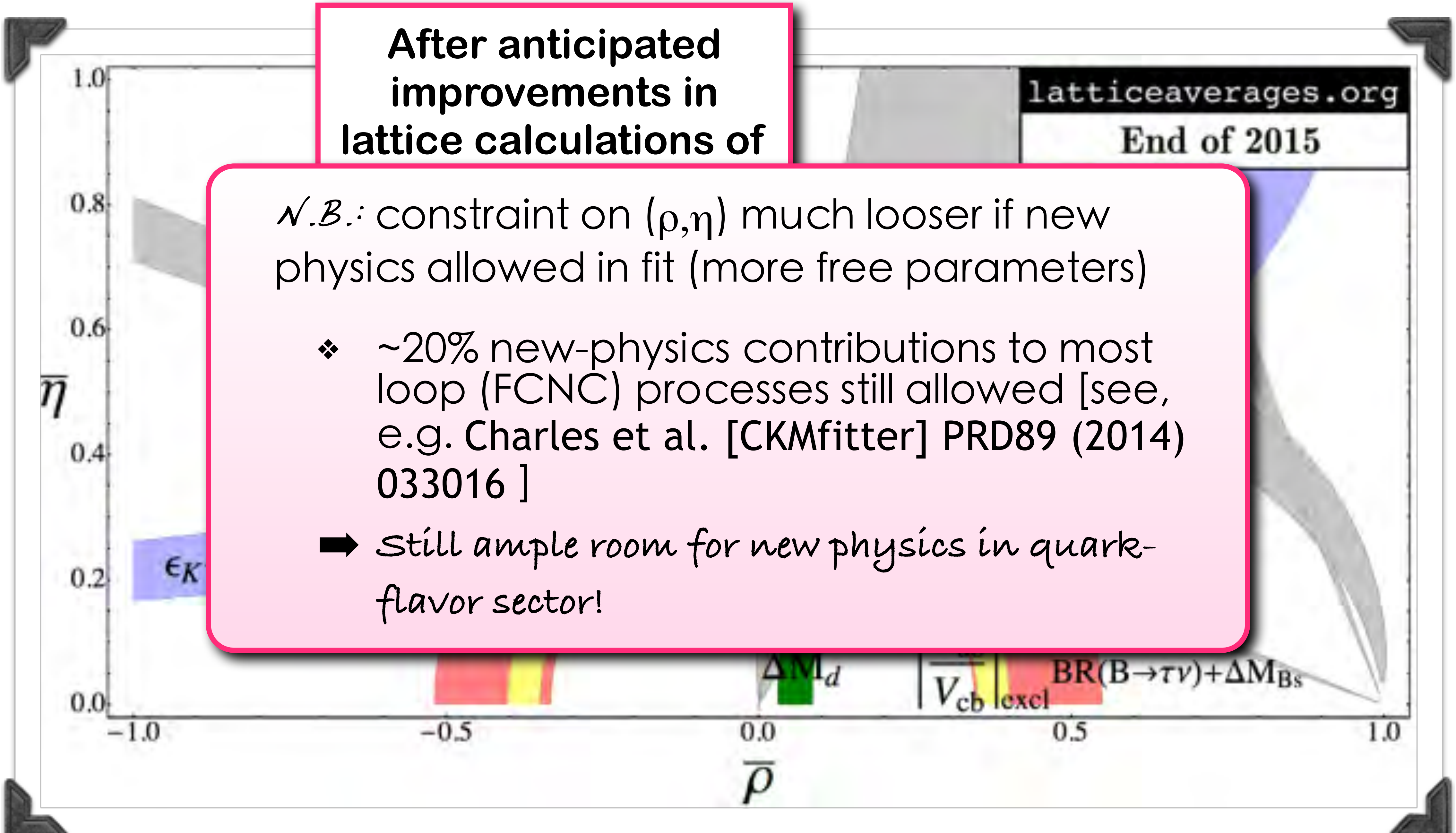
… UT fit at the end of next year?

✦ (*Disappointingly!*) good agreement with Standard-Model expectations…



... UT fit at the end of next year?

✦ *(Disappointingly!)* good agreement with Standard-Model expectations...



After anticipated improvements in lattice calculations of

N.B.: constraint on (ρ,η) much looser if new physics allowed in fit (more free parameters)

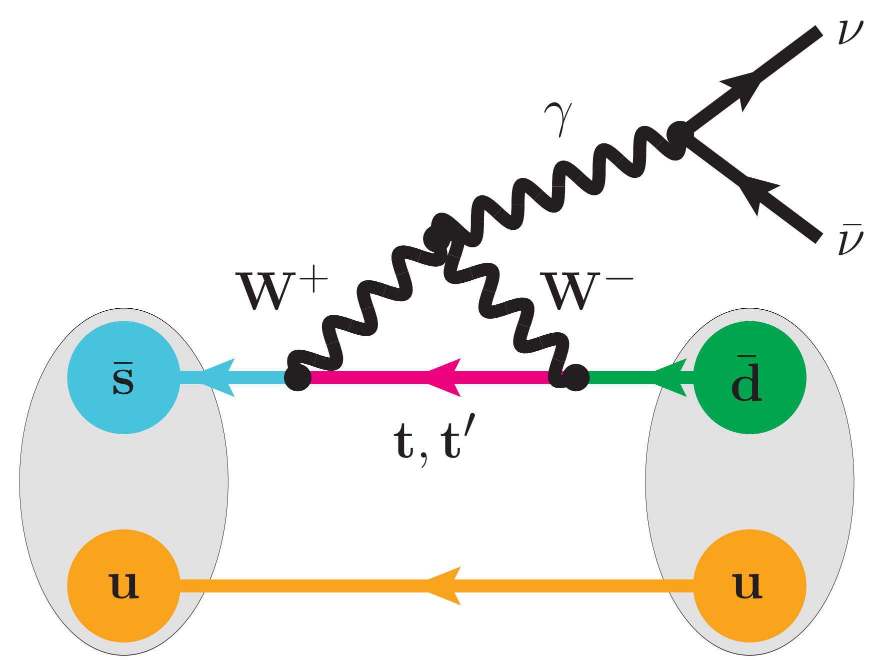
❖ ~20% new-physics contributions to most loop (FCNC) processes still allowed [see, e.g. Charles et al. [CKMfitter] PRD89 (2014) 033016]

➡ *Still ample room for new physics in quark-flavor sector!*

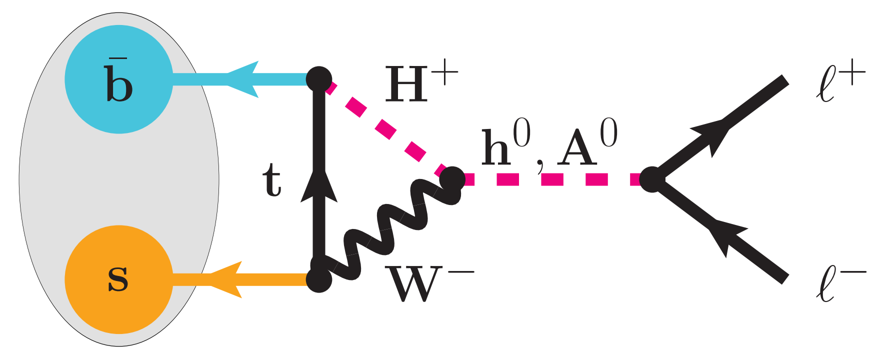
New-physics searches with rare decays

- Can be rare in Standard Model because:
  - **(1)** Loop suppressed (*e.g.* flavor-changing neutral currents)
  - **(2)** Helicity suppressed
  - **(3)** CKM and/or color suppressed
- **Challenging to observe experimentally, but new-physics contributions to decay amplitudes may be easier to observe over suppressed Standard-Model backgrounds**

Measurements of rare kaon & B decays will be at forefront of experimental quark-flavor effort in the coming decade



$\mathbf{K^+ \to \pi^+ \nu\nu}$ & $\mathbf{K_L \to \pi^0 \nu\nu}$ sensitive to 4th generation, warped X-dim, ...



$\mathbf{B_s \to \mu^+ \mu^-}$ sensitive to new b→s FCNCs from 2HDM, ...

$K \to \nu\bar{\nu}$ decays

- **"GOLDEN" MODES:** Standard-Model branching ratios for $K^+ \to \pi^+\nu\bar{\nu}$ and $K_L \to \pi^0\nu\bar{\nu}$ known to precision unmatched by any other quark flavor-changing neutral current processes

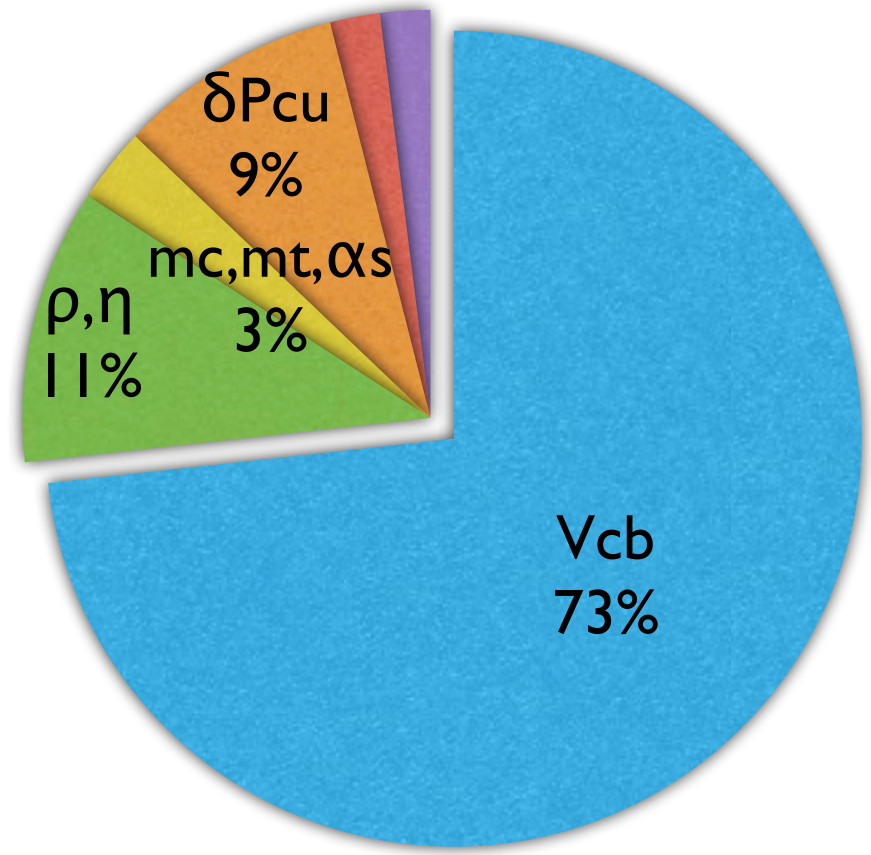
  ➡ Limited by parametric uncertainty from $|V_{cb}|^4$

- Within this decade, **NA62 @ CERN SPS** will measure $\mathcal{O}(100)$ K^+ events (assuming the SM), and **KOTO @ J-PARC** will collect first K^0_L events

- Expect calculations of $B \to D^{(*)} l\nu$ at *nonzero* recoil in next few years to reduce error in $|V_{cb}|$ to ~1.5%, and in Standard-Model branching fractions to ~6%

  ➡ Theory error in Standard-Model predictions will be commensurate with expected experimental error

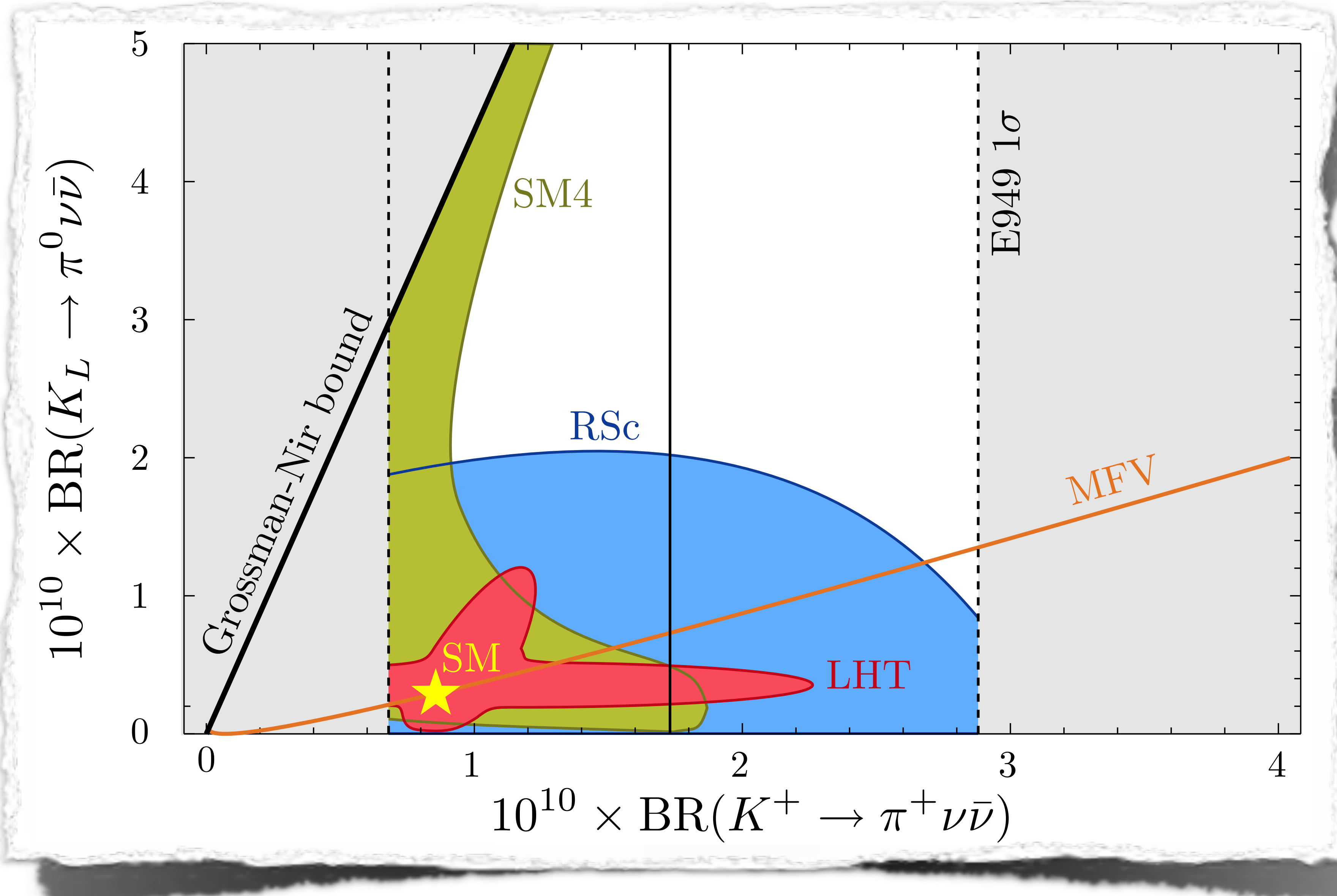
$BR(K^+ \to \pi^+\nu\bar{\nu})$



[Brod & Gorbahn PRD83 (2011) 034030]

Still room for large BSM contributions!

[D. Straub, arXiv:1012.3893 (CKM 2010)]



- Spectacular deviations from Standard Model possible in many new-physics scenarios
- Correlations between the two channels can help distinguish between models

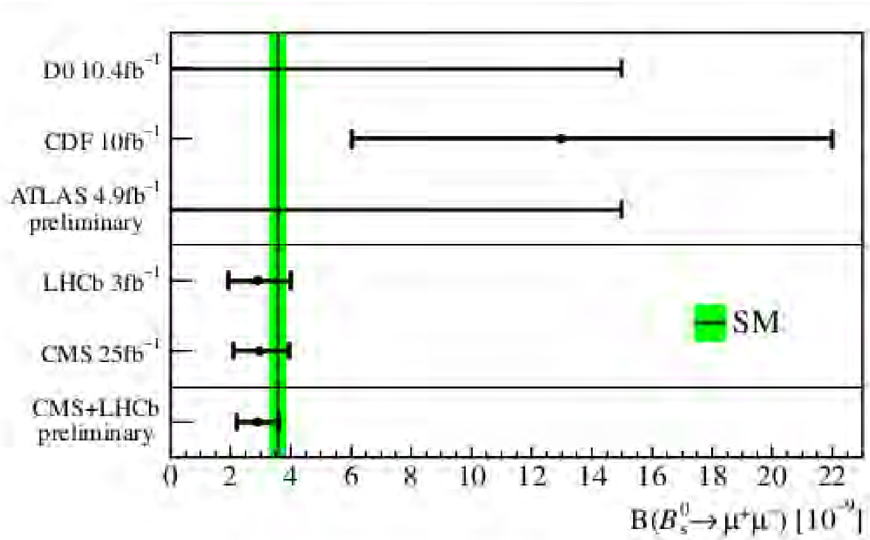
Combined LHCb + CMS Result

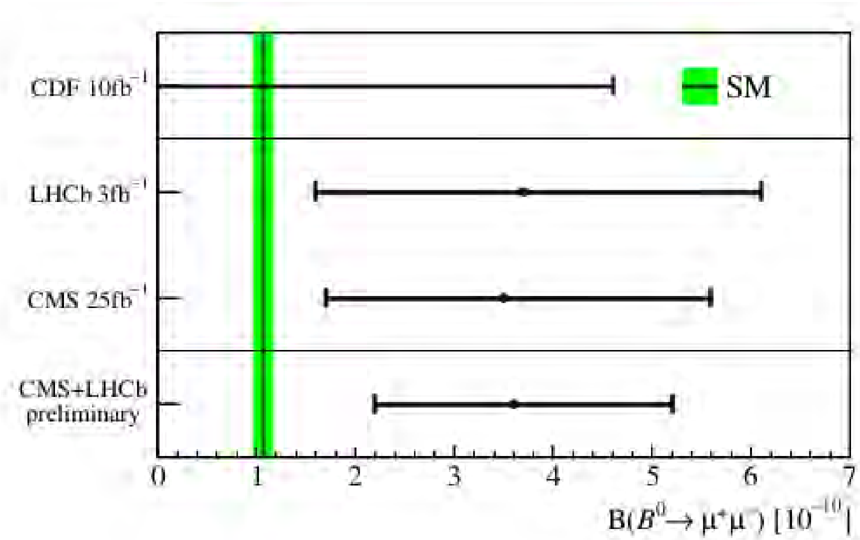
new @ EPS2013

Observation:

BR($B_s \to \mu^+\mu^-$) = $(2.9 \pm 0.7) \times 10^{-9}$

BR($B^0 \to \mu^+\mu^-$) = $3.6^{+1.6}_{-1.4} \times 10^{-10}$





LHCb-CONF-2013-012, CMS-PAS-BPH-13-007

[S. Hansmann-Menzemer, EPS 2013]

$B_s \to \mu^+\mu^-$ decay

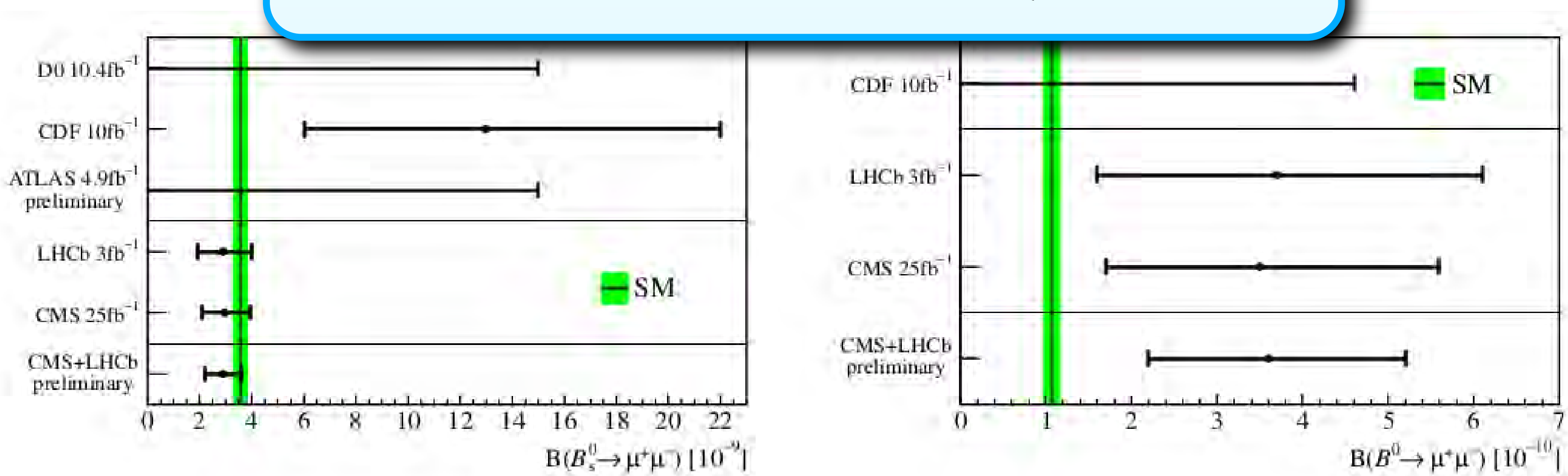
Combined LHCb + CMS Result

new @ EPS2013

Observation:

$$\mathrm{BR}(B_s \to \mu^+\mu^-) = (2.9 \pm 0.7) \times 10^{-9}$$

Compatible with Standard-Model prediction, but experimental & theoretical errors will continue to improve...



LHCb-CONF-2013-012, CMS-PAS-BPH-13-007

[S. Hansmann-Menzemer, EPS 2013]

Standard-Model prediction for $B_s \to \mu^+\mu^-$

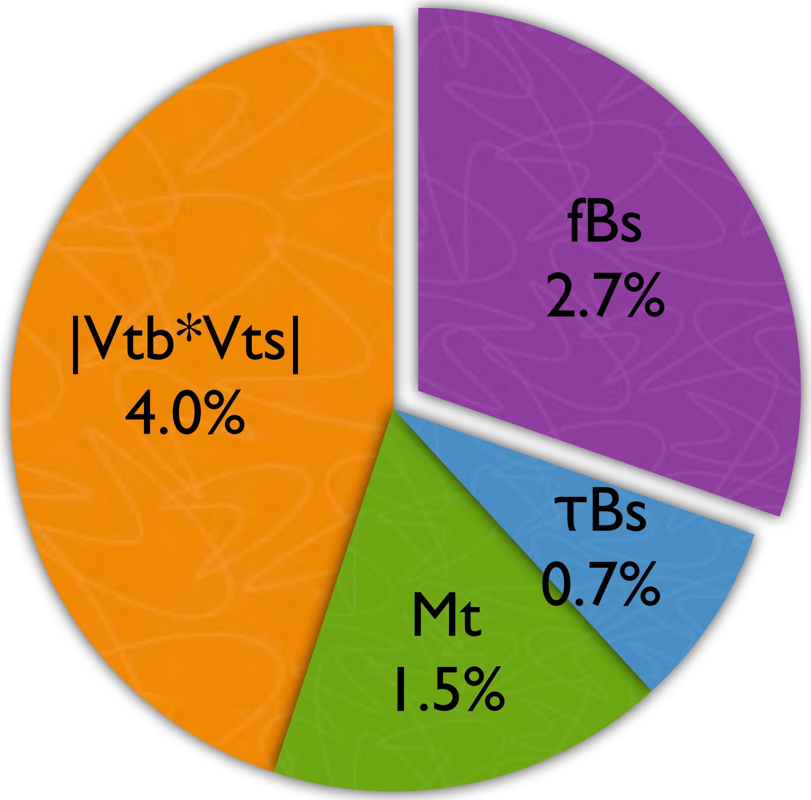
- Standard-Model rate proportional to **pseudoscalar decay constant f_{Bs}**

$$\Gamma(B_s \to \mu^+\mu^-) = \frac{G_F^2}{\pi} \underbrace{Y\left(\frac{\alpha}{4\pi \sin^2 \Theta_W}\right)^2}_{\text{perturbative QCD \& EW corrections}} m_{B_s} \mathbf{f}_{B_s}^2 |V_{tb}^* V_{ts}|^2 m_\mu^2 \sqrt{1 - 4\frac{m_\mu^2}{m_B^2}}$$

- Current **uncertainty in Standard-Model prediction ~5% using 2011 lattice-QCD calculation of f_{Bs}** from HPQCD [McNiele *et al.*, PRD85 (2012) 031503]

- Error in f_{Bs} will continue to improve with analysis of data at even finer lattice spacings ➜ limited by error on $|V_{tb}^*V_{ts}|$ for foreseeable future

BR($B_s \to \mu^+\mu^-$) USING F_{Bs}

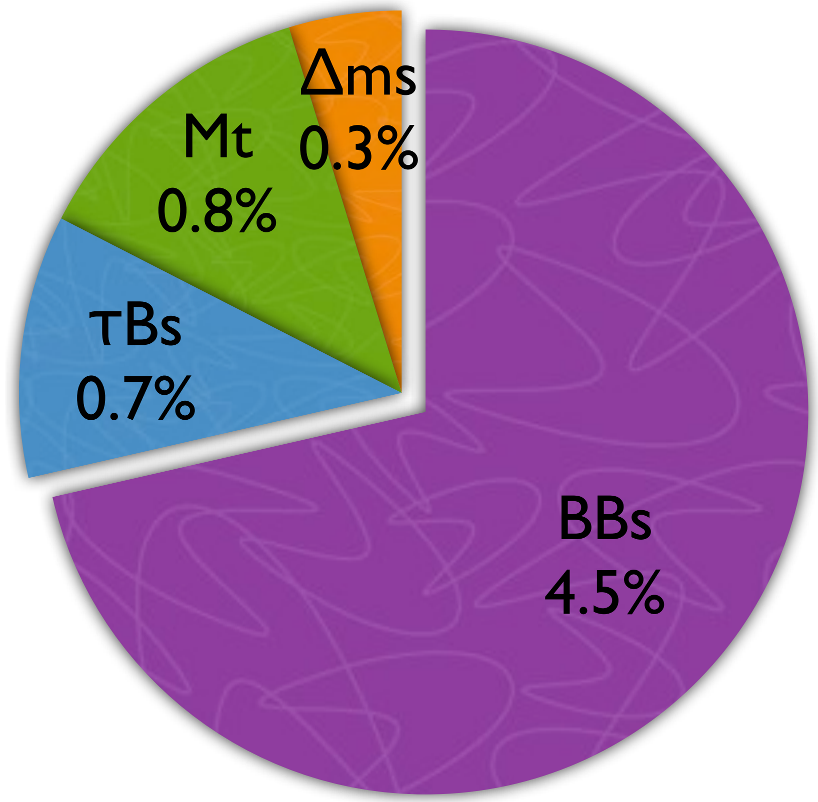


[Buras *et al.*, JHEP 1307 (2013) 77]

Standard-Model prediction for $B_s \to \mu^+\mu^-$

- Alternatively, can express Standard-Model rate in terms of oscillation frequency Δm_s and **hadronic matrix element B_{Bs}**
- Obtain similar precision for **uncertainty in Standard-Model prediction using 2009 lattice-QCD calculation of B_{Bs}** from HPQCD [Gamiz *et al.*, PRD80 (2009) 014503]
- Uncertainty in Standard-Model theory prediction via this approach will shrink with anticipated improvements in B_s-mixing matrix elements

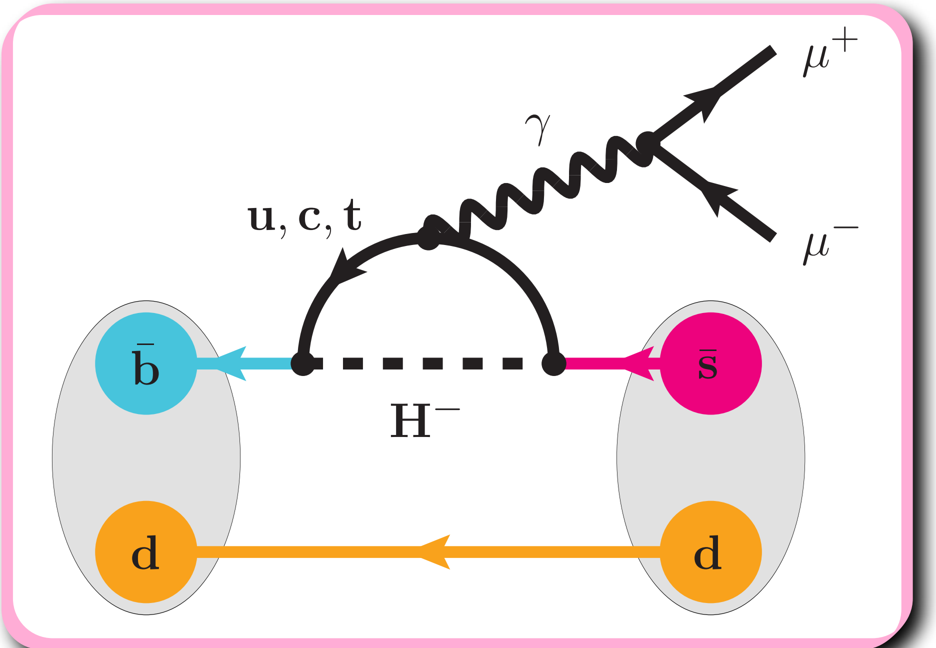
BR($B_s \to \mu^+\mu^-$) USING B_{Bs}



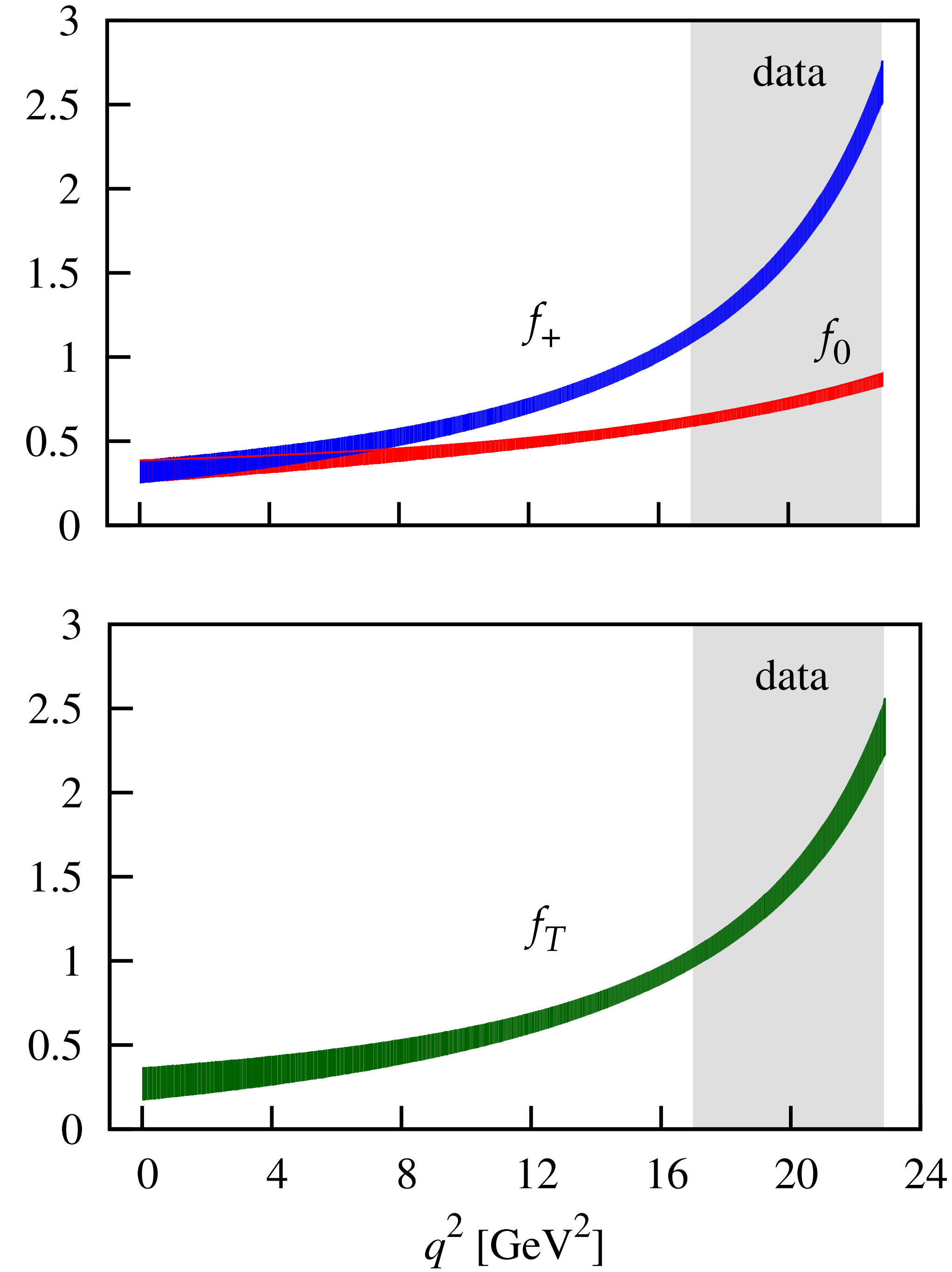
[Buras *et al.*, JHEP 1307 (2013) 77]

B→Kℓ⁺ℓ decay

- Sensitive to new b→s flavor-changing interactions



[Bouchard *et al.* PRD88 (2013) 054509]



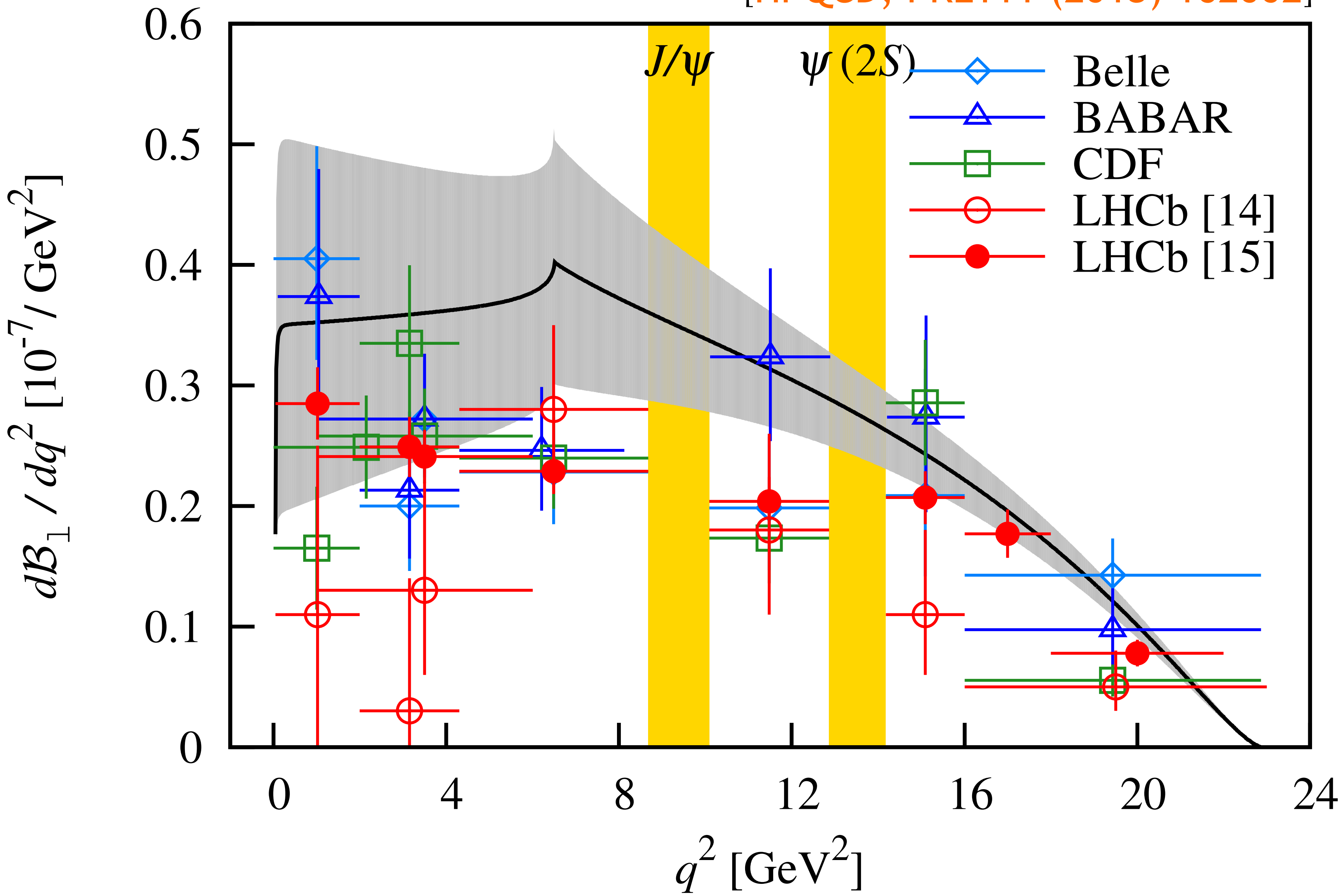
- Standard-Model predictions for rate, forward-backward asymmetry, etc. require hadronic form factors over full q^2 range
- HPQCD Collaboration recently obtained first (2+1)-flavor lattice results for form factors $f_+(q^2)$, $f_0(q^2)$, and $f_T(q^2)$ ➜ sufficient to parameterize $B \to K l^+ l$ in Standard Model & in all possible beyond-the-SM theories

$B \to K \ell^+ \ell^-$ decay

Enabled first model-independent prediction for Standard-Model differential branching fraction ➜ results consistent with experiment

(but both experimental & theory uncertainties will continue to improve...)

[HPQCD, PRL111 (2013) 162002]



Lattice challenges in quark flavor

- **$B \to K^*$ and related form factors**
  - $B \to K^* ll$, $B \to K^* \gamma$, and $B_s \to \varphi\gamma$ have been observed experimentally & rate measurements will continue to improve; comparisons with Standard-Model predictions require form factors over full kinematic range
  - Challenging because final-state K^* and φ are unstable in QCD & widths increase as light-quark masses approach the physical point
  - Initial step recently taken by Prelovsek *et al.*, who completed first lattice study of the $K^{*(}872)$ decay width [Phys.Rev. D88 (2013) 054508]
- **D-meson matrix elements**
  - Important in light of recent experimental evidence for CP-violation in $D \to \pi\pi(KK)$ decays and mixing ➜ *now in the same situation as we've been in for decades with ε′!*
  - Particularly difficult aspect is dealing with intermediate 4π, 6π, etc., states in finite box
  - Progress with generalization of Lüscher formalism to 3π case [Polejaeva & Rusetsky, Briceno & Davoudi, Hansen & Sharpe], but more ideas and hard work are needed

New experimental opportunties

P5 panel identified five "compelling lines of inquiry that show great promise for discovery over the next 10 to 20 years," including:

- Use the Higgs boson as a new tool for discovery
- Pursue the physics associated with neutrino mass
- Explore … new particles, interactions, and physical principles

"Each has the potential to be transformative. Expect surprises."

– **Steve Ritz, March HEPAP meeting**

(Incomplete) HEP experimental horizon

ATLAS/CMS
Δm_s, $B_s \to \mu^+\mu^-$, ...

E14 "KOTO" @ J-PARC
$K^0 \to \pi^0 \nu\bar{\nu}$

Belle II
$\sin(2\beta)$, $B \to \tau(\mu)\nu$,
$B \to \pi(\rho) l\nu$, $B \to D^{(*)} l\nu$,
rare $b \to s\gamma$ & $b \to sll$ decays, ...

LBNF
CP-violation, neutrino mixing & mass hierarchy, proton decay, ...

NOW — **2016** — **2019** — **2020s** — **2030s**

NA62 @ CERN_SPS
$K^+ \to \pi^+ \nu\bar{\nu}$

LHCb
rare $b \to s\gamma$ & $b \to sll$ decays,
$B_s \to \mu^+\mu^-$, D-mixing...

Muon g-2

Mu2e

next high-energy collider
Higgs decay modes

(Incomplete) HEP experimental horizon

ATLAS/CMS
Δm_s, $B_s \to \mu^+\mu^-$, …

E14 "KOTO" @ J-PARC
$K^0 \to \pi^0 \nu\bar{\nu}$

Belle II
$\sin(2\beta)$, $B \to \tau(\mu)\nu$,
$B \to \pi(\rho) l\nu$, $B \to D^{(*)} l\nu$,
rare $b \to s\gamma$ & $b \to sll$ decays, …

LBNF
CP-violation, neutrino mixing & mass hierarchy, proton decay, …

Lattice-QCD calculations needed for ALL of these experiments: here focus on two examples

NOW 20… 2020s 2030s

NA62 @ CERN SPS
$K^+ \to \pi^+ \nu\bar{\nu}$

Muon g-2

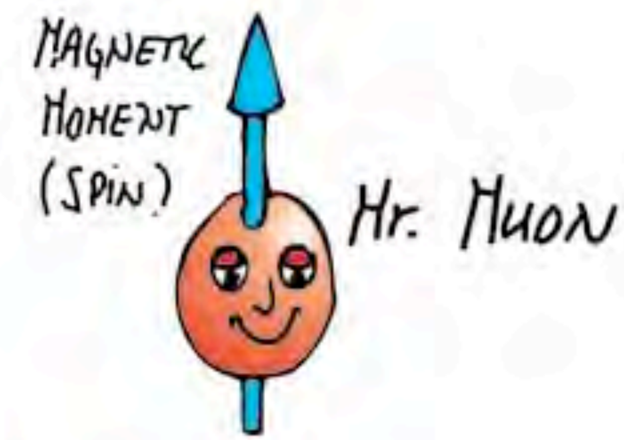
Mu2e

next high-energy collider
Higgs decay modes

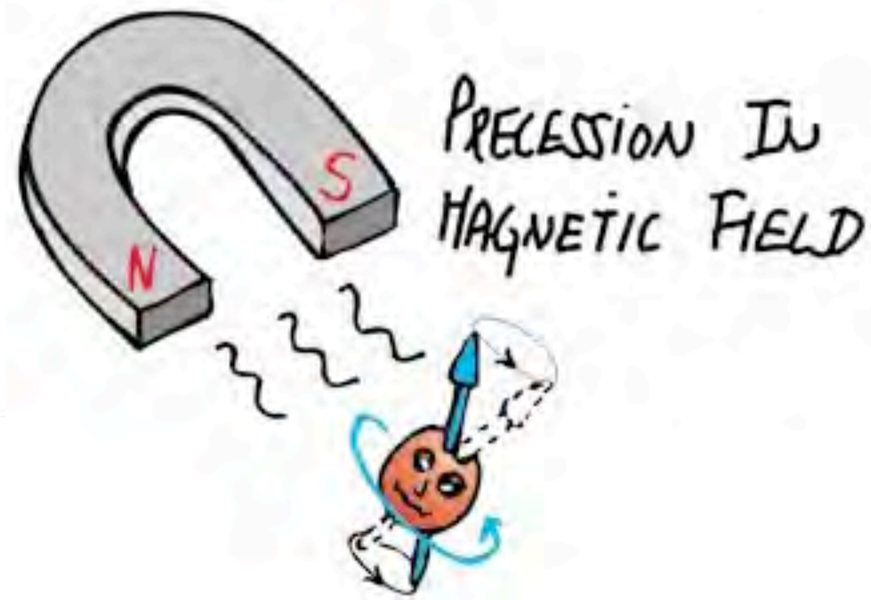
LHCb
rare $b \to s\gamma$ & $b \to sll$ decays,
$B_s \to \mu^+\mu^-$, D-mixing…

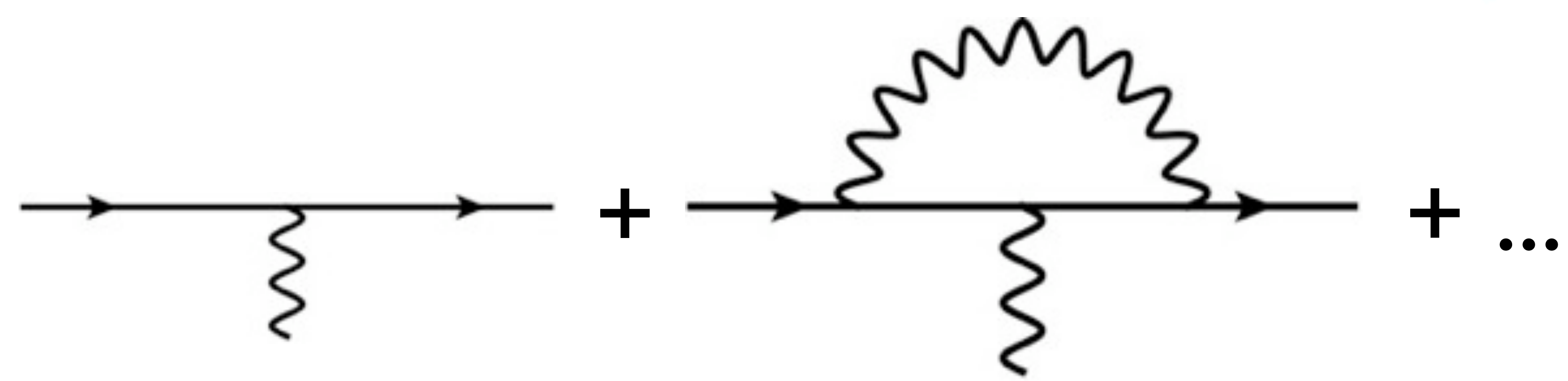
Muon anomalous magnetic moment (g-2)

- Intrinsic magnetic moment of the muon is given by $$\vec{\mu}_\mu = g_\mu \frac{e}{2m_\mu}\vec{S}$$



- The g-factor **g_μ=2 in the free Dirac theory, but corrections arise due to quantum fluctuations**
- Difference from the free case is called the "anomaly": $a_\mu \equiv (g_\mu-2)/2$
  - Schwinger calculated leading QED contribution to a_μ in 1948!

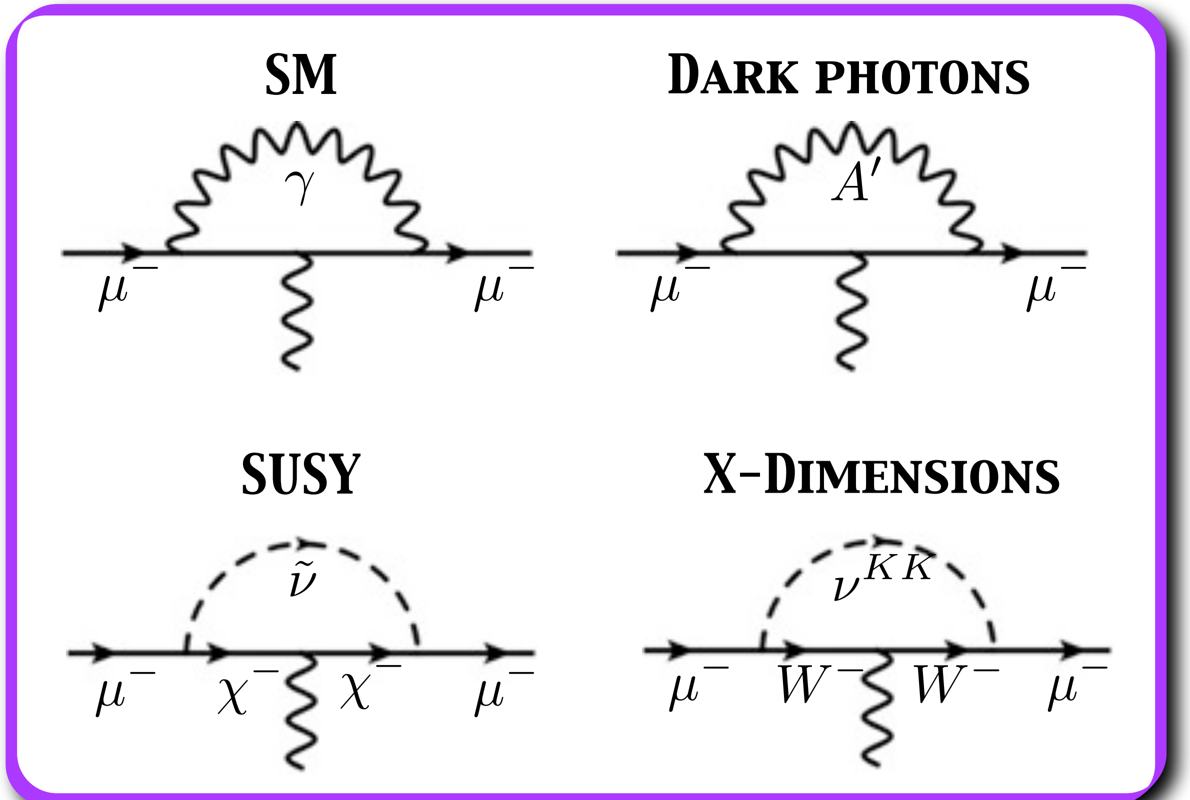


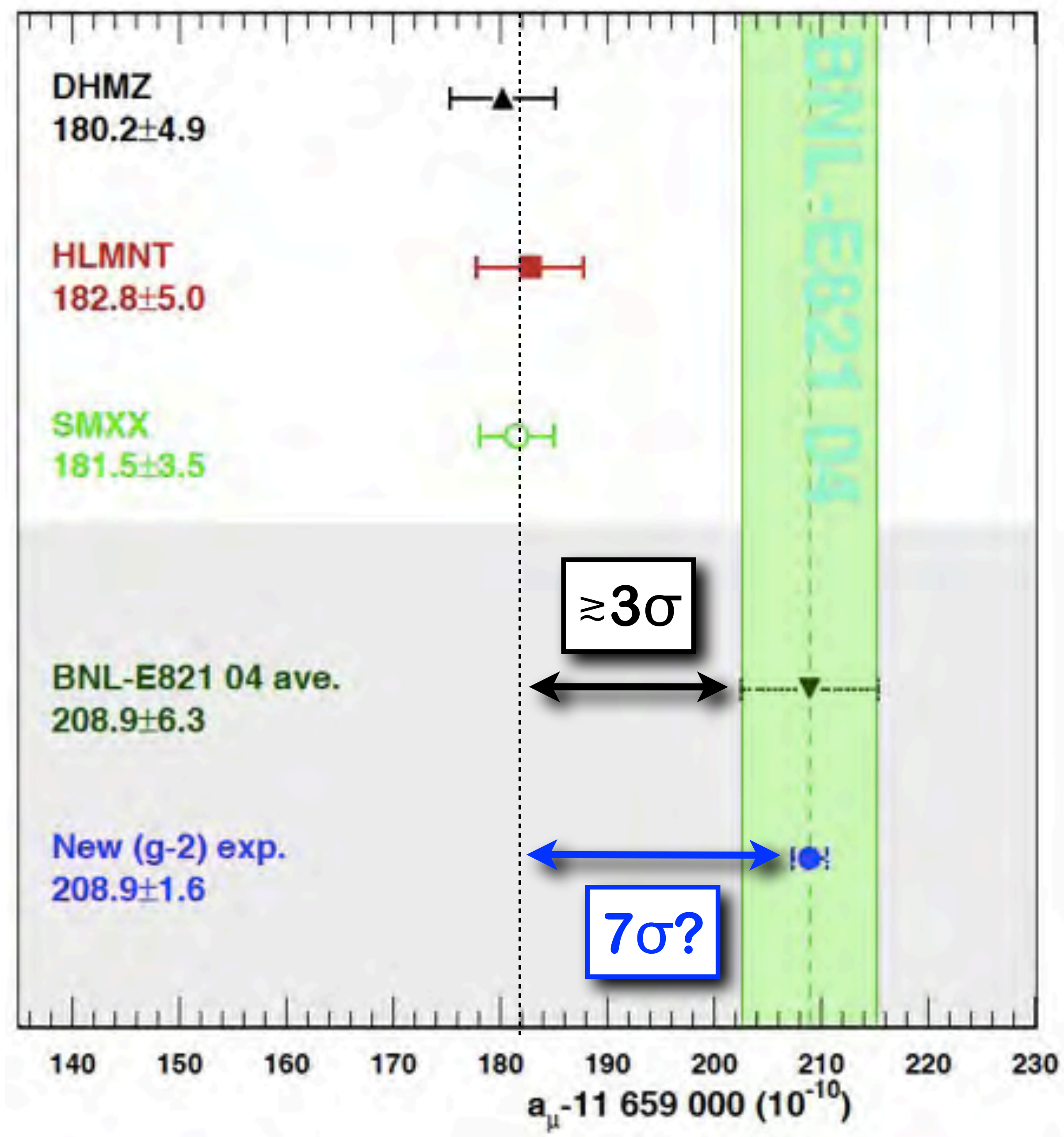


g_μ = 2 + α/π + ...

Experimental status of g-2

- Muon anomalous magnetic moment among best-measured quantities (0.54 ppm!) ➜ provides some of most precise constraints on extensions of Standard Model





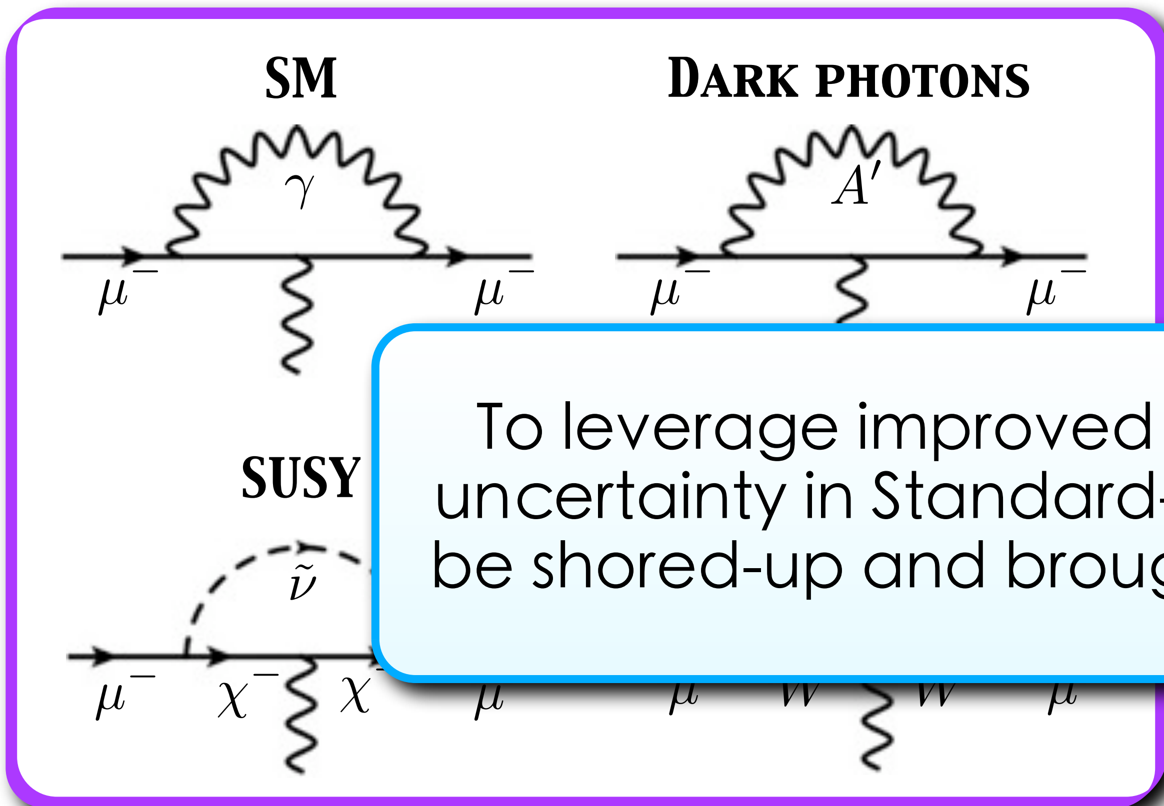
[Blum *et al.*, arXiv:1311.2198]

- **BNL measurement** disagrees with Standard Model by ≳3σ, while

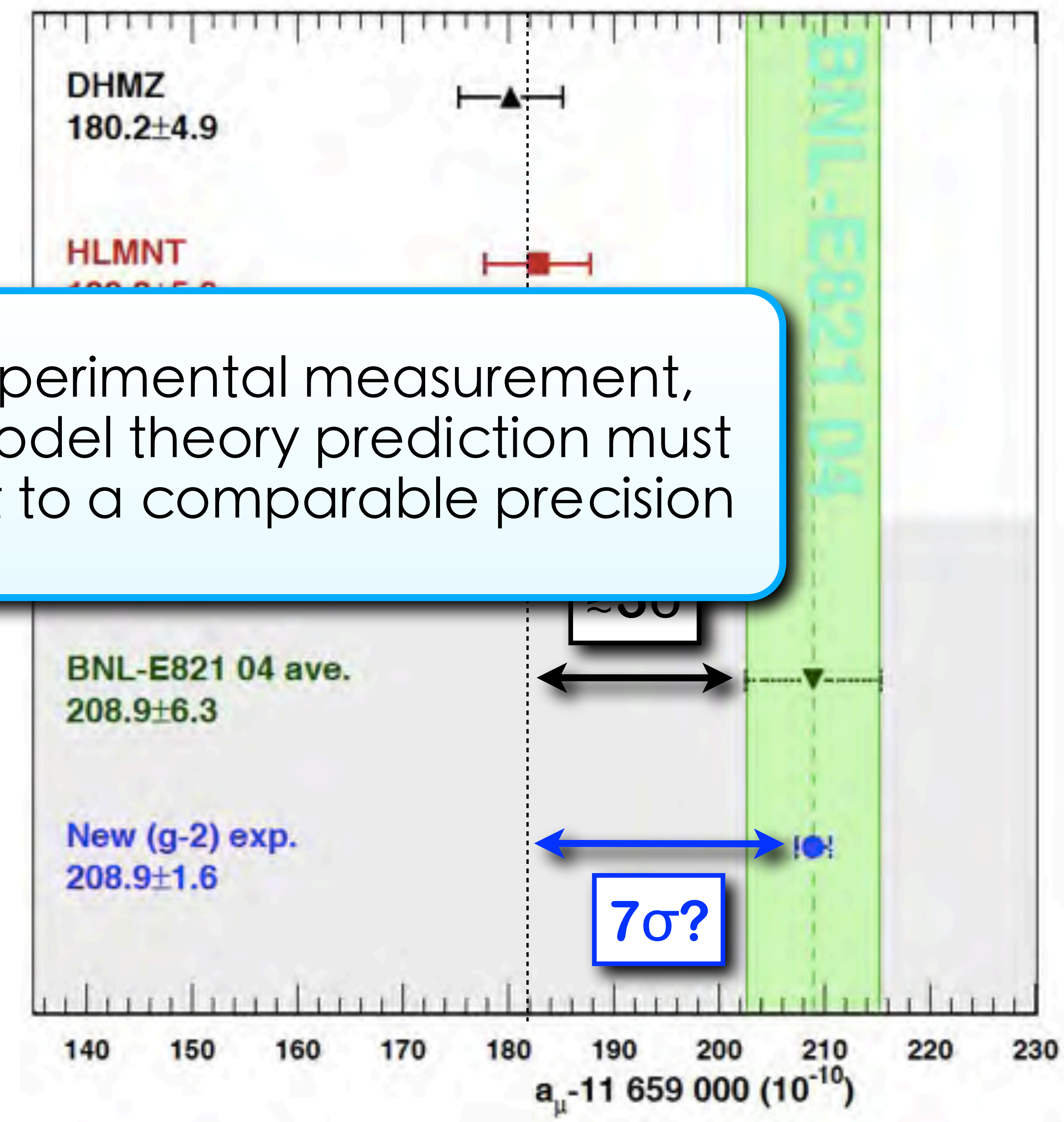
  Fermilab Muon g-2 Experiment aims to reduce error by factor of four

Experimental status of g-2

- Muon anomalous magnetic moment among best-measured quantities (*0.54 ppm!*) ➜ **provides some of most precise constraints on extensions of Standard Model**



To leverage improved experimental measurement, uncertainty in Standard-Model theory prediction must be shored-up and brought to a comparable precision

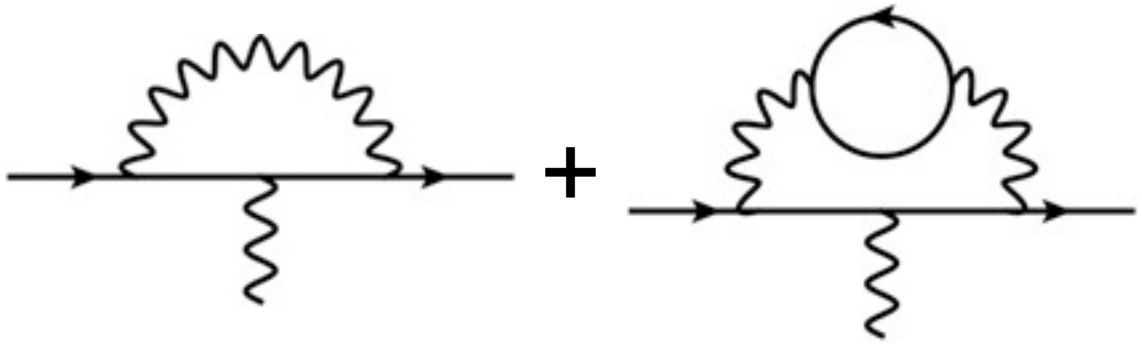


[Blum *et al.*, arXiv:1311.2198]

- **BNL measurement** disagrees with Standard Model by ≳3σ, while

  Fermilab Muon g-2 Experiment aims to reduce error by factor of four

Theoretical status of g-2 (in Standard Model)



QED (4 loops) & EW (2 loops)

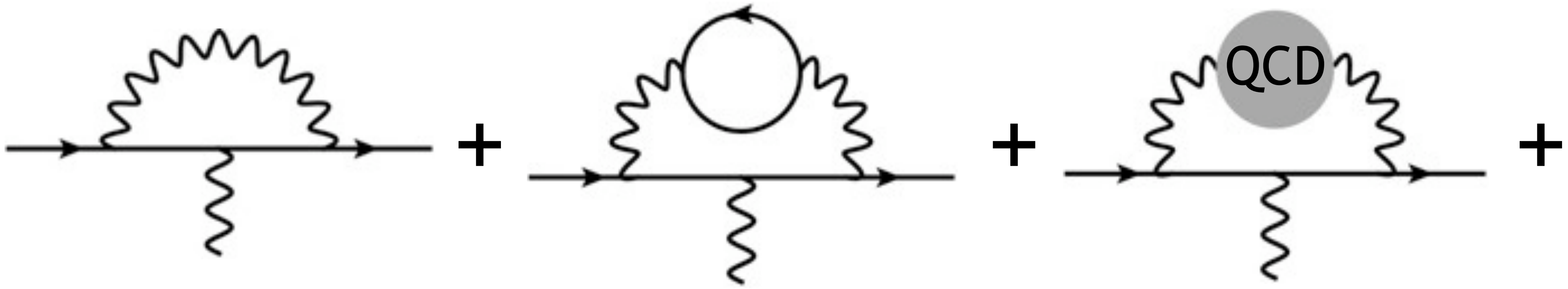
Contribution	Result ($\times 10^{11}$)			Error
QED (leptons)	116 584 718	$\pm$ 0.14	$\pm 0.04_{\alpha}$	0.00 ppm
HVP(lo) [1]	6 923	$\pm$ 42		0.36 ppm
HVP(ho)	-98	$\pm 0.9_{\text{exp}}$	$\pm 0.3_{\text{rad}}$	0.01 ppm
HLbL [2]	105	$\pm$ 26		0.22 ppm
EW	154	$\pm$ 2	$\pm$ 1	0.02 ppm
Total SM	116 591 802	$\pm$ 49		0.42 ppm

[1] Davier, Hoecker, Malaescu, Zhang, Eur.Phys.J. C71 (2011) 1515

[2] Prades, de Rafael, Vainshtein, arXiv:0901.030

Theoretical status of g-2 (in Standard Model)

Hadronic vacuum polarization (HVP):



QED (4 loops) & EW (2 loops)

from experimental result for $e^+e^- \to$ hadrons plus dispersion relation

Contribution	Result ($\times 10^{11}$)		Error
QED (leptons)	116 584 718 $\pm$ 0.14	$\pm\ 0.04_\alpha$	0.00 ppm
HVP(lo) [1]	6 923 $\pm$ 42		0.36 ppm
HVP(ho)	-98 $\pm\ 0.9_{\mathrm{exp}}$	$\pm\ 0.3_{\mathrm{rad}}$	0.01 ppm
HLbL [2]	105 $\pm$ 26		0.22 ppm
EW	154 $\pm$ 2	$\pm$ 1	0.02 ppm
Total SM	116 591 802 $\pm$ 49		0.42 ppm

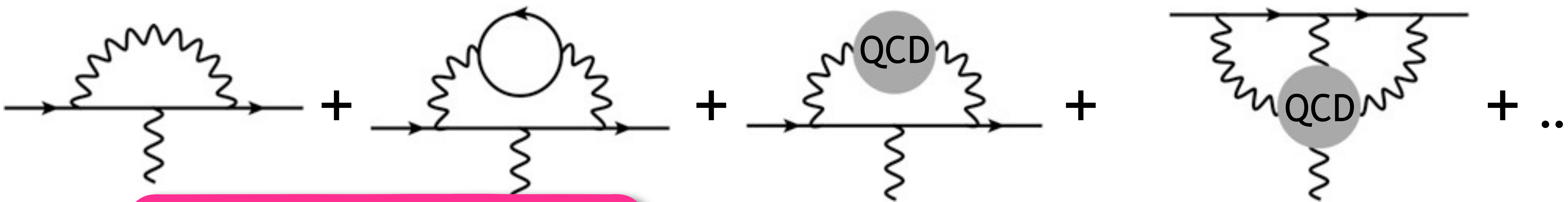
[1] Davier, Hoecker, Malaescu, Zhang, Eur.Phys.J. C71 (2011) 1515

[2] Prades, de Rafael, Vainshtein, arXiv:0901.030

Theoretical status of g-2 (in Standard Model)

Hadronic vacuum polarization (HVP):

Hadronic light-by-light (HLbL):



QE

HLbL error more subjective than for other contributions, and somewhat controversial

from experimental result for $e^+e^- \to$ hadrons plus dispersion relation

estimated from models such as large N_c, χPT, vector meson dominance, etc...

Contribution	Result ($\times 10^{11}$)		Error
QED (leptons)	116 584 718 $\pm$ 0.14	$\pm$ 0.04$_\alpha$	0.00 ppm
HVP(lo) [1]	6 923 $\pm$ 42		0.36 ppm
HVP(ho)	-98 $\pm$ 0.9$_{\text{exp}}$	$\pm$ 0.3$_{\text{rad}}$	0.01 ppm
HLbL [2]	105 $\pm$ 26		0.22 ppm
EW	154 $\pm$ 2	$\pm$ 1	0.02 ppm
Total SM	116 591 802 $\pm$ 49		0.42 ppm

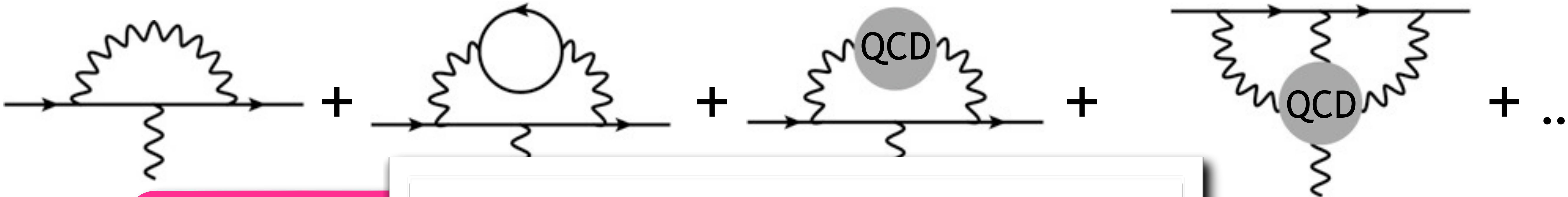
[1] Davier, Hoecker, Malaescu, Zhang, Eur.Phys.J. C71 (2011) 1515

[2] Prades, de Rafael, Vainshtein, arXiv:0901.030

Theoretical status of g-2 (in Standard Model)

Hadronic vacuum polarization (HVP):

Hadronic light-by-light (HLbL):



QE

HLbL error more
for other cont
somewhat c

Precision goals for hadronic contributions set by Muon g-2 Experiment are:

$\delta(a_\mu^{HVP})$~0.2%, $\delta(a_\mu^{HLbL})$~15%

nated from models
as large N_c, χPT,
vector meson
minance, etc...

Contribution	Result ($\times 10^{11}$)		Error
QED (leptons)	116 584 718 $\pm$ 0.14	$\pm$ 0.04$_\alpha$	0.00 ppm
HVP(lo) [1]	6 923 $\pm$ 42		0.36 ppm
HVP(ho)	-98 $\pm$ 0.9$_{\mathrm{exp}}$	$\pm$ 0.3$_{\mathrm{rad}}$	0.01 ppm
HLbL [2]	105 $\pm$ 26		0.22 ppm
EW	154 $\pm$ 2	$\pm$ 1	0.02 ppm
Total SM	116 591 802 $\pm$ 49		0.42 ppm

[1] Davier, Hoecker, Malaescu, Zhang, Eur.Phys.J. C71 (2011) 1515
[2] Prades, de Rafael, Vainshtein, arXiv:0901.030

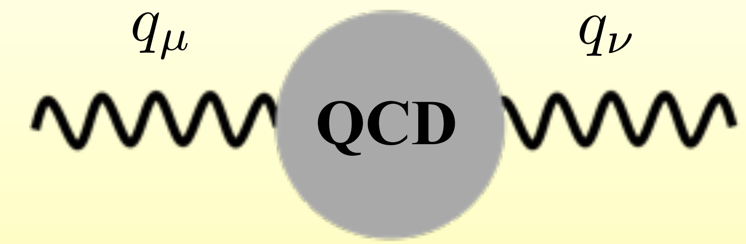
Standard lattice method for a_μ^{HVP}

[Blum, Phys.Rev.Lett. 91 (2003) 052001]

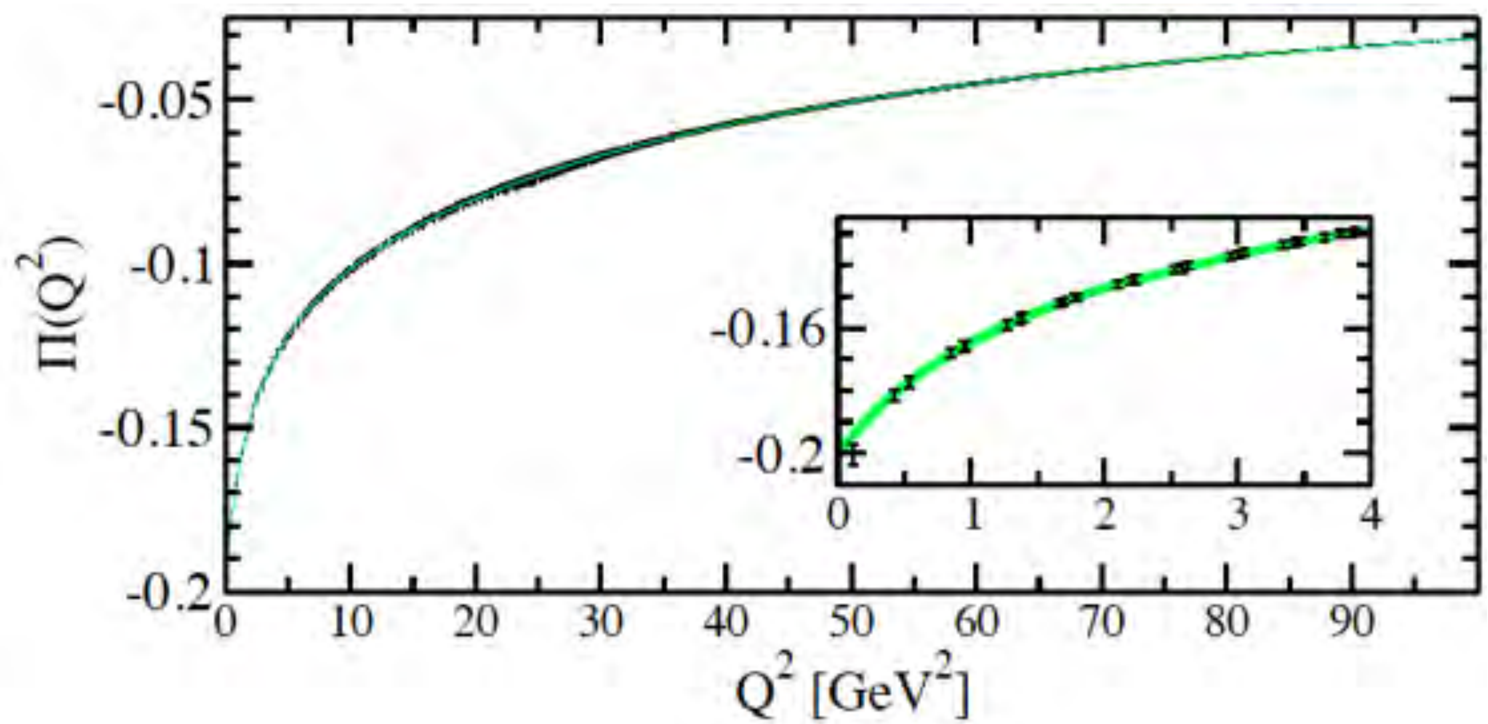
- Calculate a_μ^{HVP} directly from Euclidean space vacuum polarization function
- $\Pi(Q^2)$ a simple correlation function of two electromagnetic currents
- In Euclidean space, $\Pi(Q^2)$ has a **smooth Q^2 dependence with no resonance structure**

$$a_\mu^{\mathrm{HVP(LO)}} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dQ^2 f(Q^2) \left[\Pi(Q^2) - \Pi(0)\right]$$

$$i\Pi_{\mu\nu}(q^2) =$$



[plot from Dru Renner]



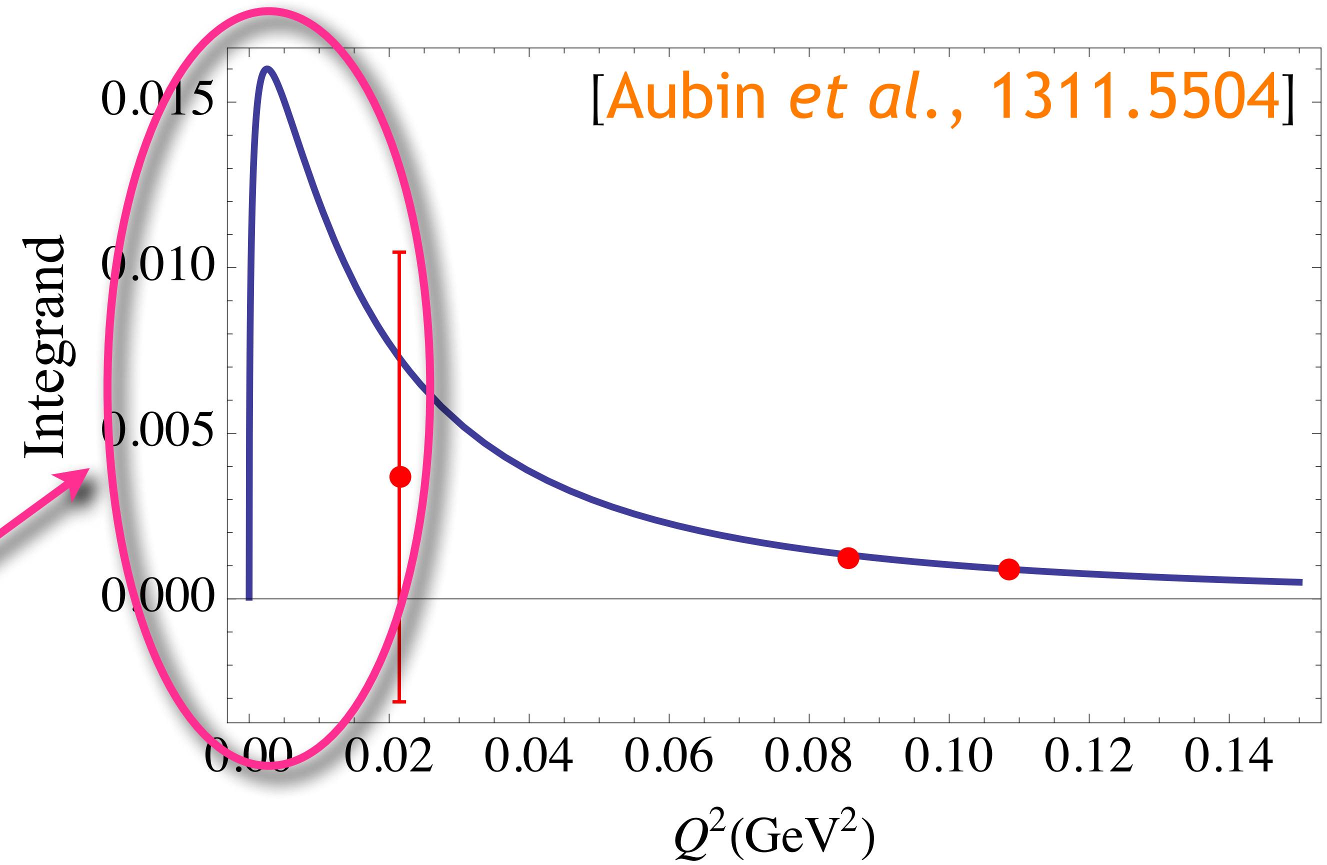
Standard lattice method for a_μ^{HVP}

[Blum, Phys.Rev.Lett. 91 (2003) 052001]

- Calculate a_μ^{HVP} directly from Euclidean space vacuum polarization function
- $\Pi(Q^2)$ a simple correlation function of two electromagnetic currents
- In Euclidean space, $\Pi(Q^2)$ has a **smooth Q^2 dependence with no resonance structure**
- **Integrand $f(Q^2)[\Pi(Q^2)\text{-}\Pi(0)]$, however, peaks around $Q^2 \approx (m_\mu/2)^2$, where lattice data is sparse and noisy** ➜ need precise determination of $\Pi(Q^2)$ in this region to obtain precise result for a_μ^{HVP}

$$a_\mu^{\mathrm{HVP(LO)}} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dQ^2 f(Q^2)\left[\Pi(Q^2) - \Pi(0)\right]$$

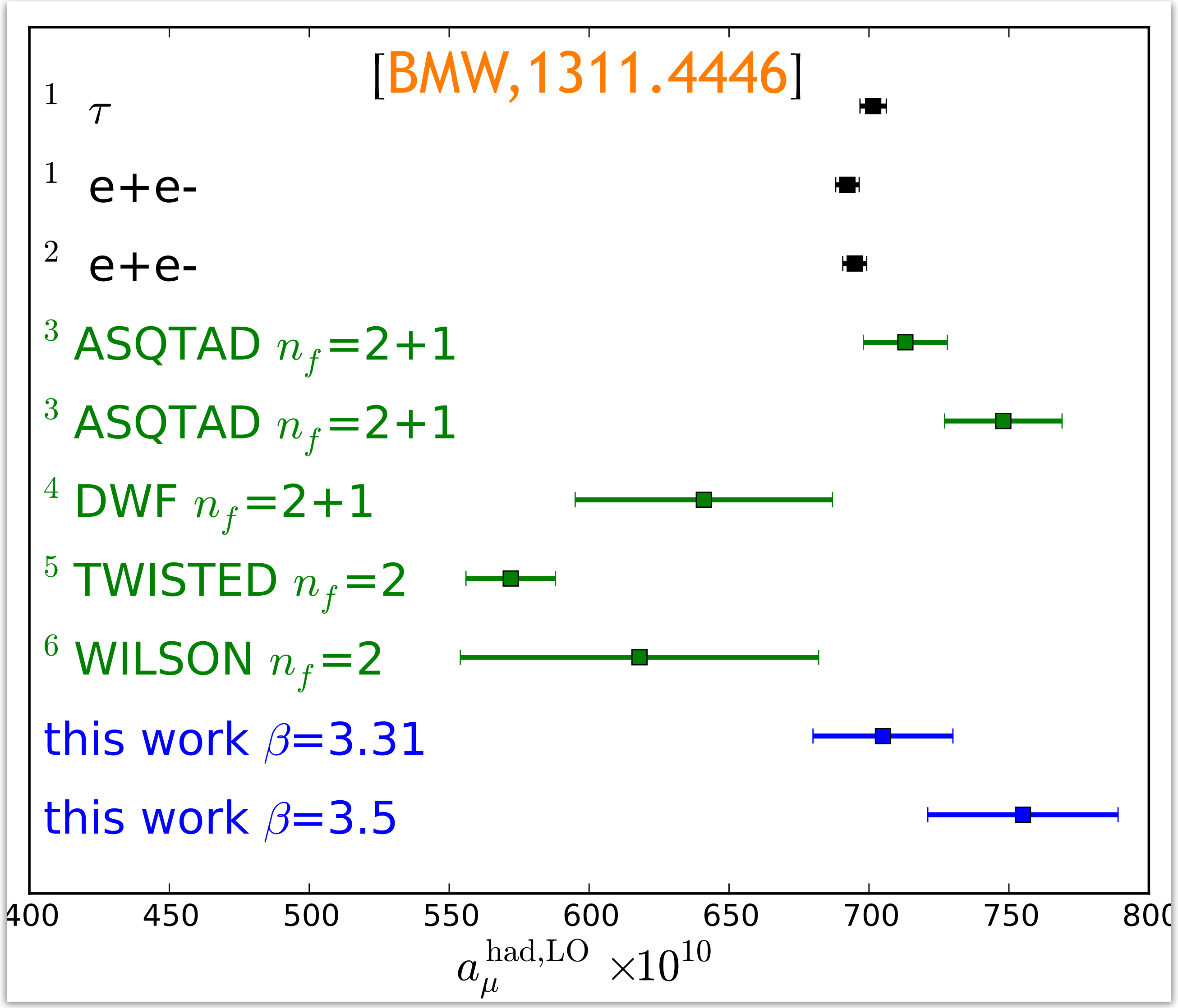
$$i\Pi_{\mu\nu}(q^2) =$$



Standard lattice method for a_μ^{HVP}

[Blum, Phys.Rev.Lett. 91 (2003) 052001]

Several independent ongoing efforts using this approach

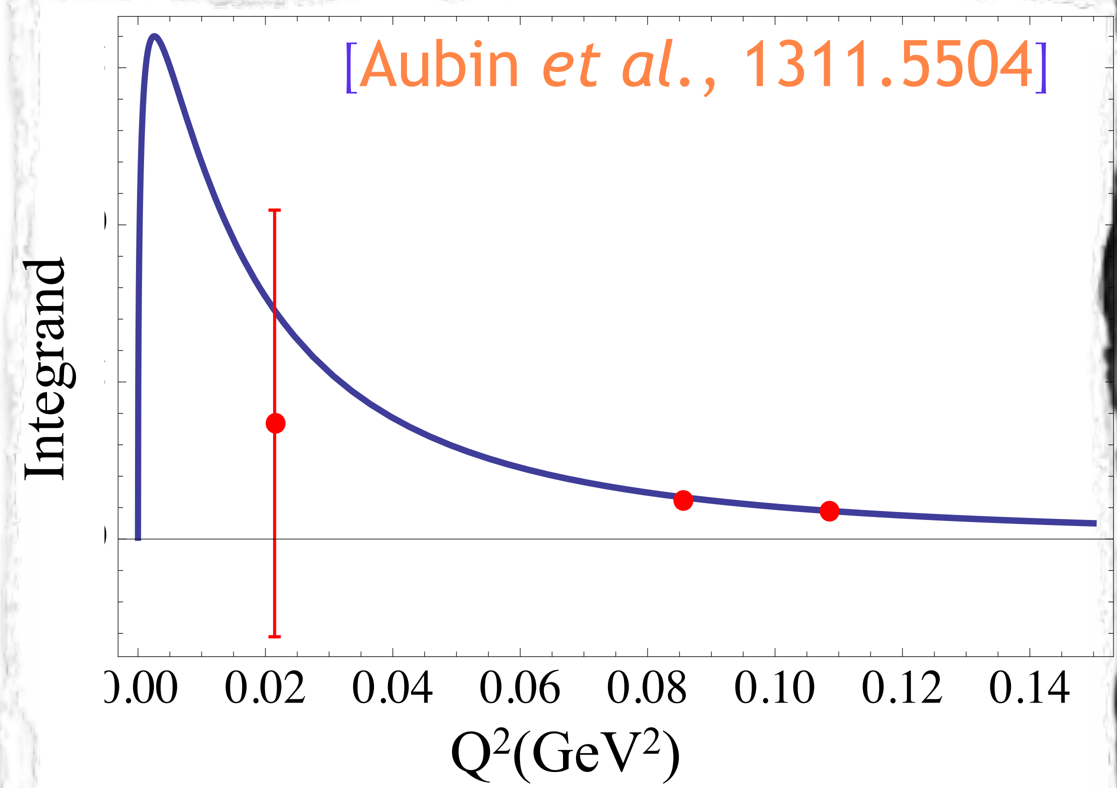


Present errors in ~5-10% percent range (and mostly neglect quark-disconnected contributions)

(Select) Recent progress on a_μ^{HVP}

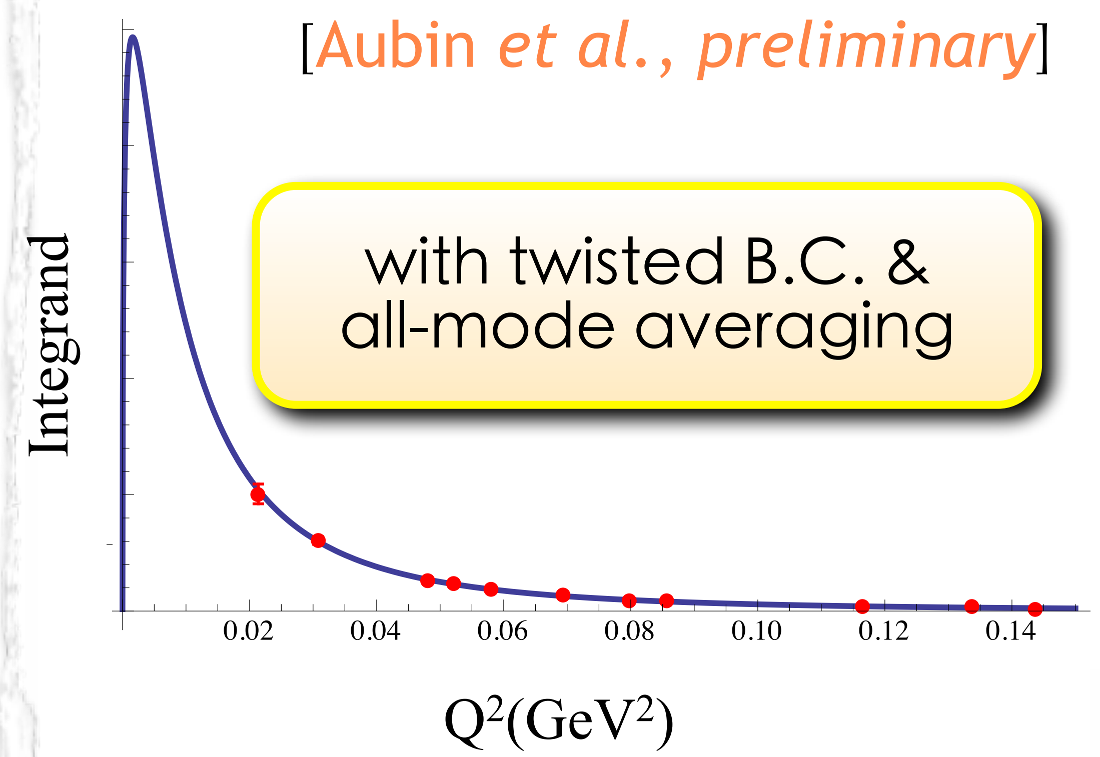
- **Twisted boundary conditions** for fermion fields to access momentum values below the minimum discrete lattice momentum ($2\pi/L$) [spatial lattice volume=L^3] [Della Morte *et al.*, JHEP 1203 (2012) 055; Aubin *et al.*, PRD88 (2013) 7, 074505]

- **All-mode averaging** to reduce statistical errors [Blum *et al*, PRD 88 (2013) 094503]



(Select) Recent progress on a_μ^{HVP}

- **Twisted boundary conditions** for fermion fields to access momentum values below the minimum discrete lattice momentum ($2\pi/L$) [spatial lattice volume=L^3] [Della Morte *et al.*, JHEP 1203 (2012) 055; Aubin *et al.*, PRD88 (2013) 7, 074505]
- **All-mode averaging** to reduce statistical errors [Blum *et al*, PRD 88 (2013) 094503]

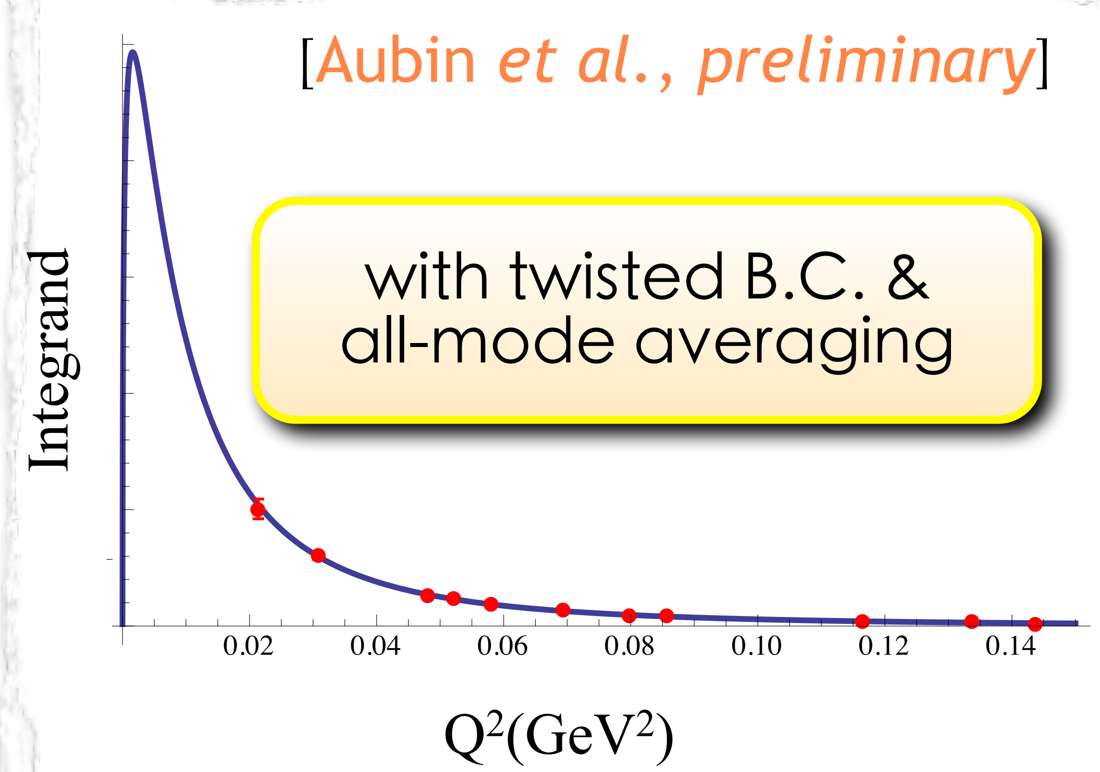


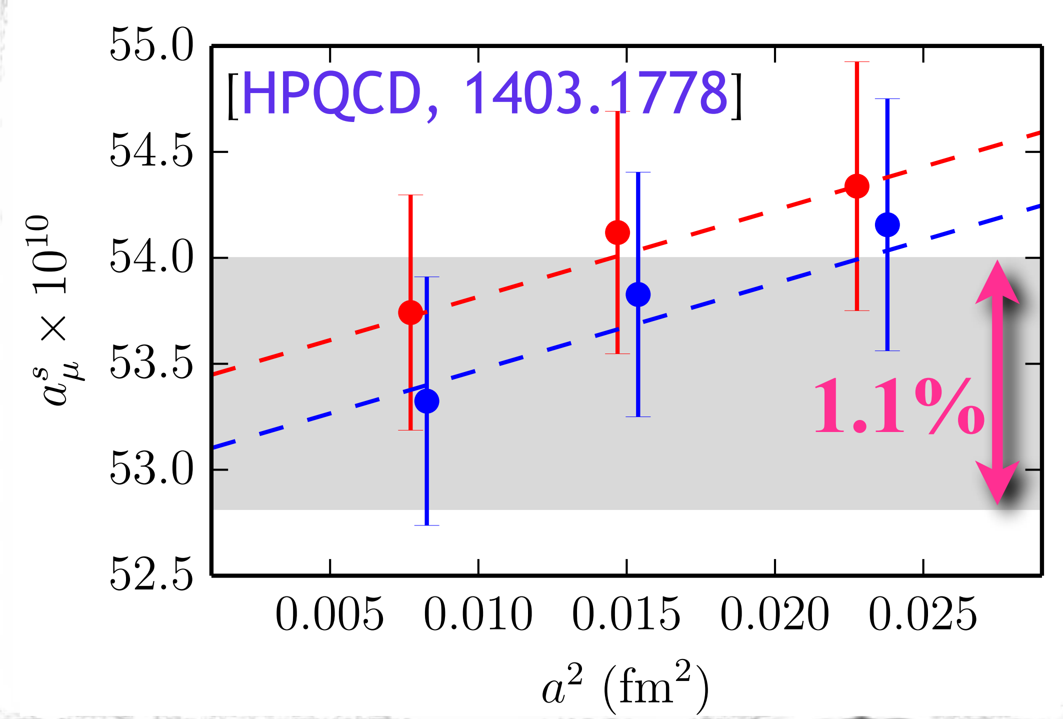
(Select) Recent progress on a_μ^{HVP}

- **Twisted boundary conditions** for fermion fields to access momentum values below the minimum discrete lattice momentum ($2\pi/L$) [spatial lattice volume=L^3] [Della Morte *et al.*, JHEP 1203 (2012) 055; Aubin *et al.*, PRD88 (2013) 7, 074505]

- **All-mode averaging** to reduce statistical errors [Blum *et al*, PRD 88 (2013) 094503]

- **New HPQCD method sidesteps $q^2 \to 0$ extrapolation** by expressing a_μ^{HVP} in terms of derivatives of vacuum polarization function $\Pi(q^2)$ at $q^2=0$ [Chakraborty *et al.*, 1403.1778]
  - Illustrate with strange and charm contributions and obtain a_μ^s to ~1%
  - Correlator noisier for light quarks, but estimate that similar precision can be obtained for $a_\mu^{u,d}$ with 10× larger gauge-field ensembles





(Select) Recent progress on a_μ^{HVP}

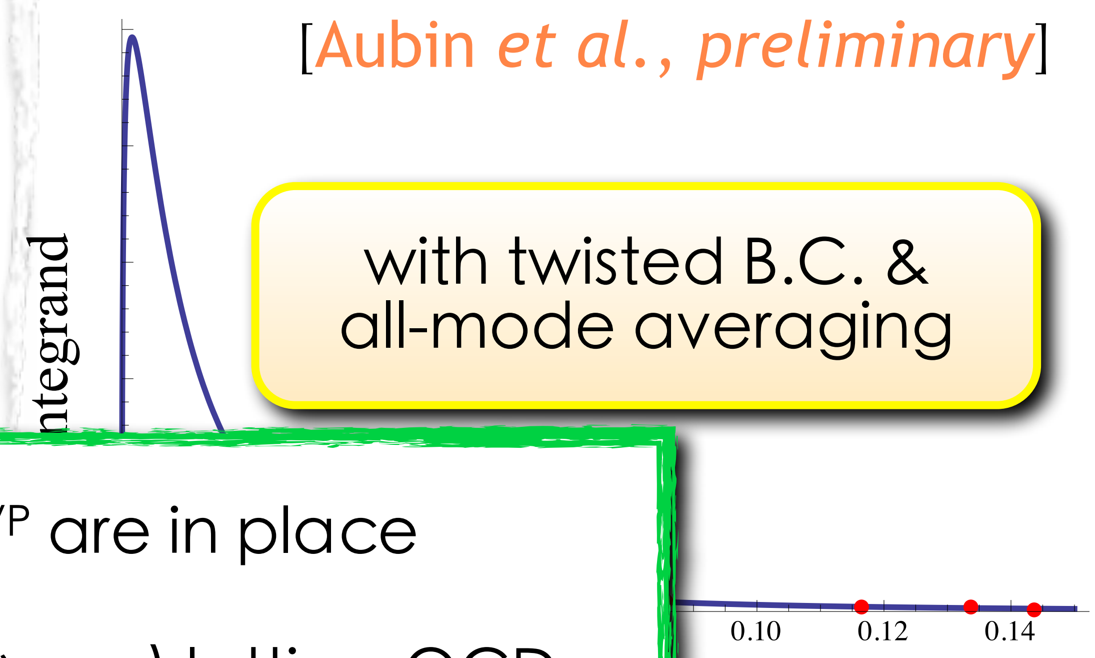
- **Twisted boundary conditions** for fermion fields to access momentum values below the minimum discrete lattice momentum ($2\pi/L$) [spatial lattice volume=L^3] [Della Morte *et al.*, JHEP 1203 (2012) 055; Aubin *et al.*, PRD88 (2013) 7, 074505]

- **All-mode a**
  [Blum *et al*

- **New HPQC extrapolati**
  derivatives o
  at $q^2=0$ [Chakraborty *et al.*, 1403.1778]

  - Illustrate with strange and charm contributions and obtain a_μ^s to ~1%

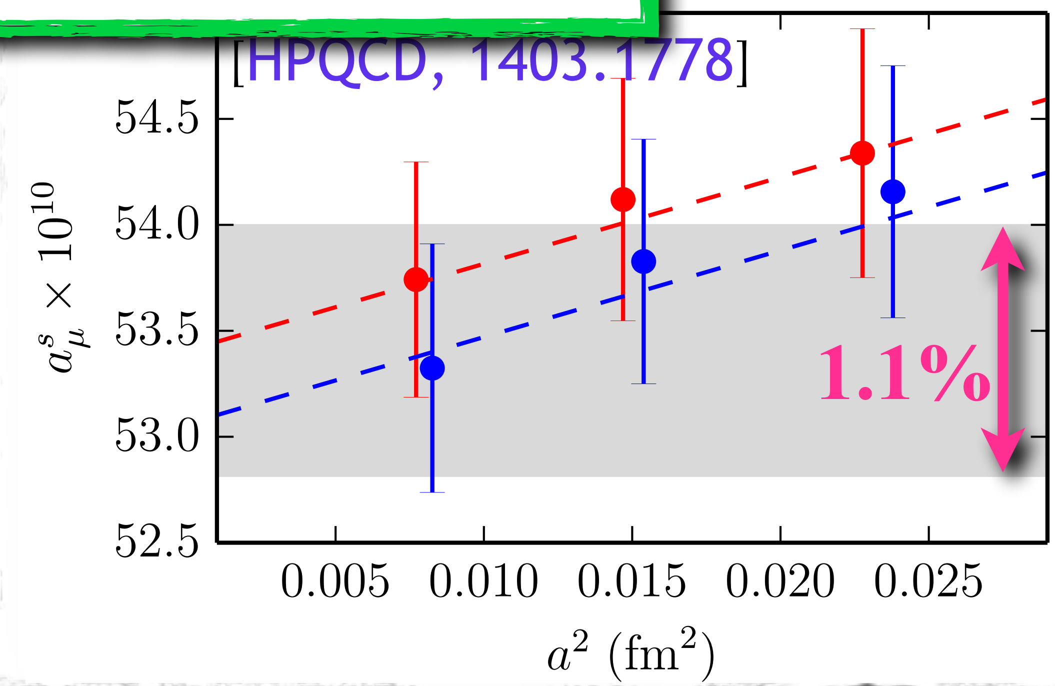
  - Correlator noisier for light quarks, but estimate that similar precision can be obtained for $a_\mu^{u,d}$ with 10× larger gauge-field ensembles



Theoretical methods for a_μ^{HVP} are in place

➜

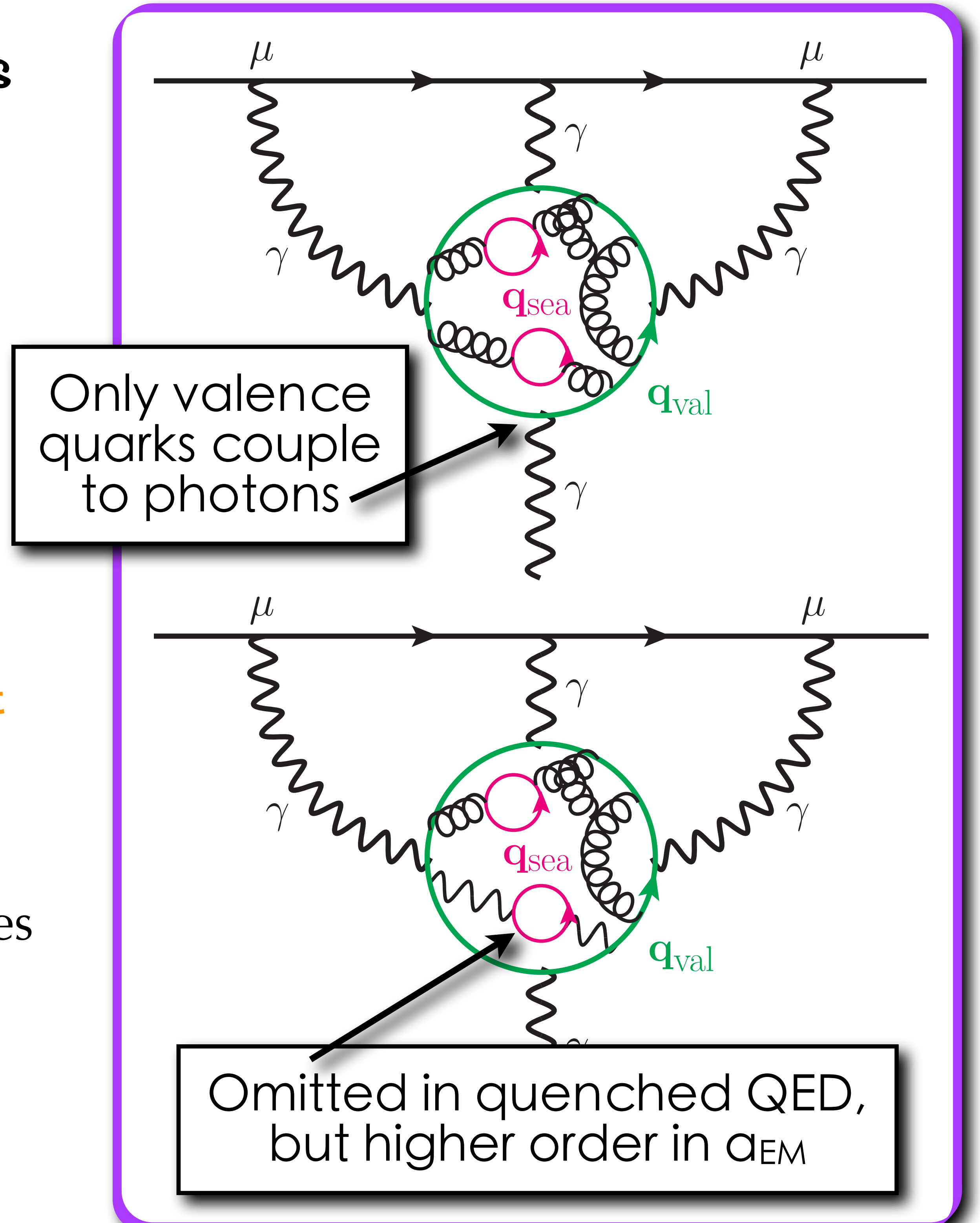
should enable a few percent (or better) lattice-QCD calculation on the timescale of the coming experiment



Lattice calculation of a_μ^{HLbL}

[Hayakawa et al., PoS LAT2005 (2006) 353]

- **Requires inclusion of QED in lattice simulations**
- Promising approach to **compute full hadronic amplitude nonperturbatively including photon field along with gluon field in gauge link** ➜ simulation & analysis follow conventional lattice-QCD calculation
- Thus far only pilot study in quenched QED (sea quarks do not carry EM charge), but early results in ballpark of model estimates [Blum *et al.*, PoS LATTICE2012 (2012) 022 & arXiv:1407.2923]
- Still much to do for realistic result with controlled uncertainty: larger spatial volumes (*massless photon!*), extrapolation to physical pion mass & continuum, momentum extrapolation $Q^2 \to 0$, quark-disconnected contributions, ...

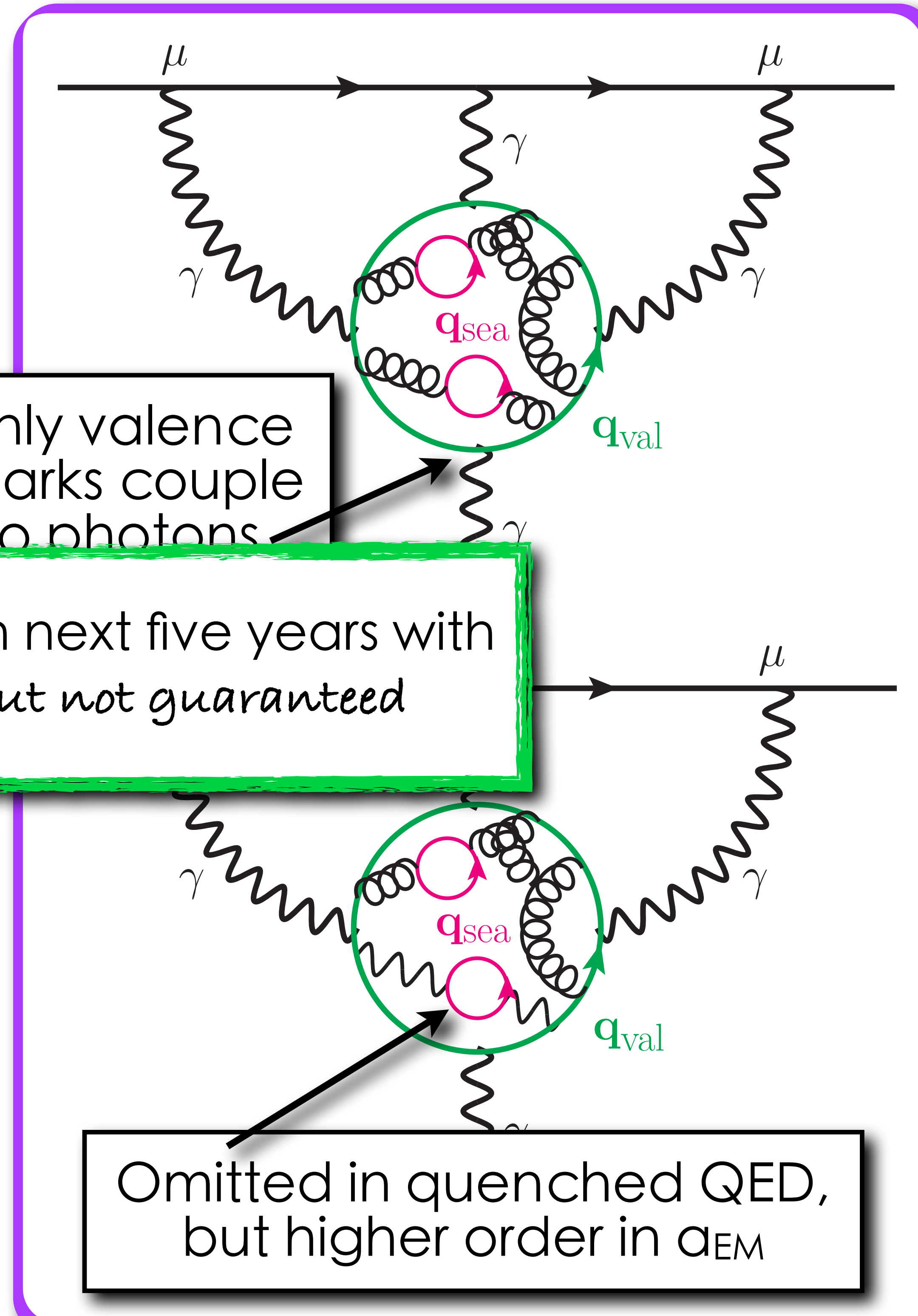


Lattice calculation of a_μ^{HLbL}

[Hayakawa et al., PoS LAT2005 (2006) 353]

- ***Requires inclusion of QED in lattice simulations***
- Promising approach to **compute full hadronic amplitude nonperturbatively including photon field along with gluon field in gauge link** ➜ simulation & analysis follow conventional lattice-QCD calculation
- Thus far only pilot study in quenched QED (sea quarks do… early results in … [Blum *et al.*, P… arXiv:1407.2923]
- Still much to do for realistic result with controlled uncertainty: larger spatial volumes (*massless photon!*), extrapolation to physical pion mass & continuum, momentum extrapolation $Q^2 \rightarrow 0$, quark-disconnected contributions, ...

Lattice calculation of a_μ^{HLbL} in next five years with ~15% precision is possible, *but not guaranteed*



Precision Higgs measurements

- Now that the Higgs mass is known, **can predict all Higgs-boson couplings and properties within the Standard Model** and look for deviations
- Future high-energy/luminosity colliders will measure Higgs partial widths to sub-percent precision, but commensurate theoretical uncertainties on Standard-Model predictions needed to fully exploit measurements
- Parametric errors from m_c, m_b, and α_s are largest sources of uncertainty in SM width predictions for the dominant Higgs decay mode H→bb, many other Higgs decay channels, and the Higgs total width [LHC Higgs X-Section WG, EPJ C71 (2011) 1753]

Channel	$\Delta\alpha_s$	Δm_b	Δm_c	Theory Uncertainty	Total Uncertainty
$H \to \gamma\gamma$	0%	0%	0%	±1%	±1%
$H \to b\bar{b}$	$\mp 2.3\%$	$^{+3.3\%}_{-3.2\%}$	0%	±2%	±6%
$H \to c\bar{c}$	$^{-7.1\%}_{+7.0\%}$	$\mp 0.1\%$	$^{+6.2\%}_{-6.1\%}$	±2%	±11%
$H \to gg$	$^{+4.2\%}_{-4.1\%}$	$\mp 0.1\%$	0%	±3%	±7%
$H \to \tau^+\tau^-$	0%	0%	0%	±2%	±2%
$H \to WW^*$	0%	0%	0%	±0.5%	±0.5%
$H \to ZZ^*$	0%	0%	0%	±0.5%	±0.5%

[Snowmass Higgs WG Report, 1310.8361]

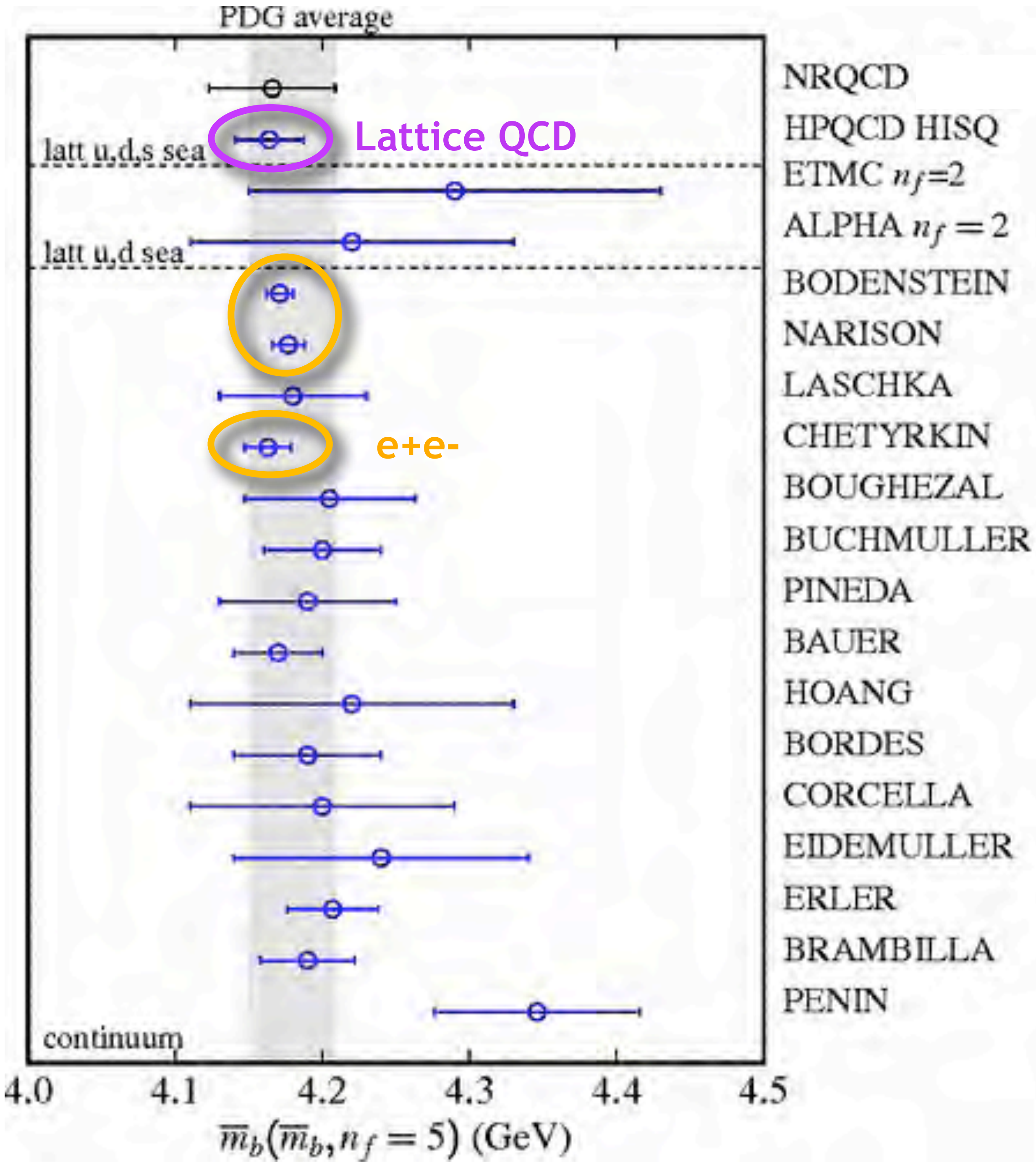
Heavy-quark masses from lattice QCD

[McNeile et al., PRD82 (2010) 034512]

- Most precise m_c and m_b obtained by fitting moments of correlation functions of the quarks' electromagnetic current to $O(\alpha_S^3)$ perturbative expressions

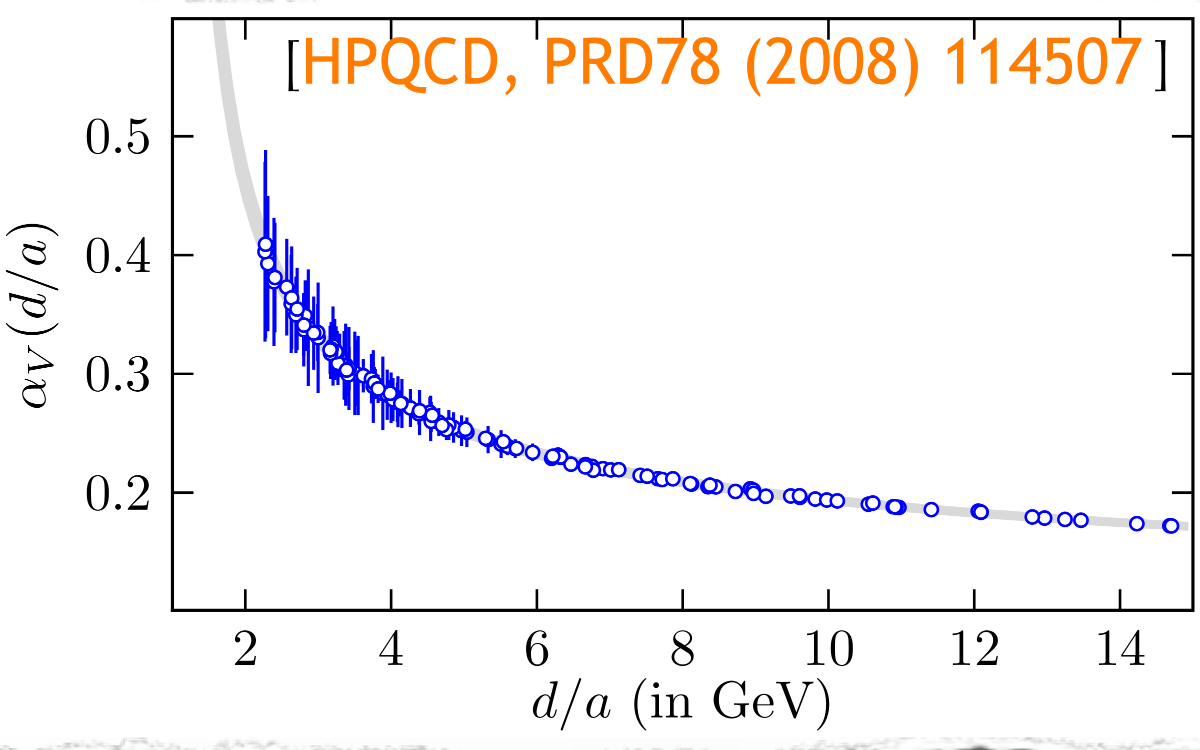
- Moments can be obtained from experimental e+e- annihilation data, and also computed numerically with lattice-QCD simulations
  - Lattice moments have negligible statistical uncertainties, so cleaner than e+e- data
  - Can vary lattice quark-mass between m_c and m_b to control and estimate errors

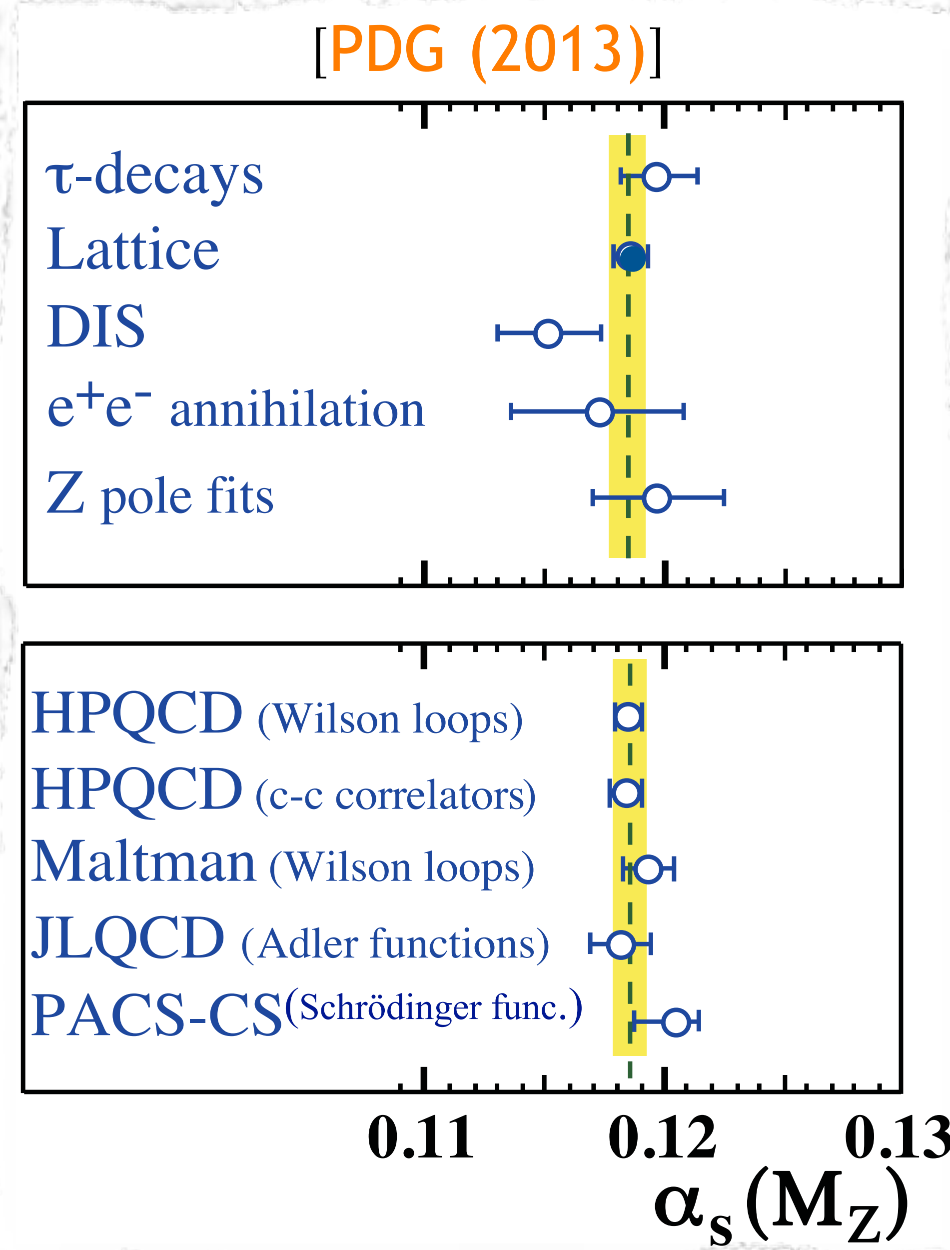
- **HPQCD obtains m_c and m_b to about a half percent precision** and finds good agreement with non-lattice determinations
  - m_c will only improve modestly without higher-order PT calculation, but m_b will improve significantly with simulations using finer lattice spacings



Strong coupling from lattice QCD

- **There are several good independent lattice methods available to obtain α_s**
  - Results consistent, and each is more precise than from non-lattice methods
  - Most precise determination from fitting NNLO QCD β-function to 22 short-distance lattice quantities built from Wilson loops





- In the coming years, **anticipate improvements in individual calculations & increased corroboration from new ones**

Expected precision of SM Higgs couplings

- Uncertainties in m_c, m_b, and α_s have led some to conclude that (sub)percent measurements of Higgs properties may never be useful [Almeida *et al.*, PRD89 (2014) 033006]
- In fact, however, lattice calculations have already determined m_c, m_b, & α_s more precisely than is currently being assumed in discussions of Higgs decay channels

	Higgs X-section Working Group	PDG	non-lattice	**Lattice (2013)**	Lattice (2018)
$\Delta\alpha_s$	0.002	0.0007	0.0012	**0.0006**	0.0004
Δm_c (GeV)	0.03	0.025	0.013	**0.006**	0.004
Δm_b (GeV)	0.06	0.03	0.016 [21]	**0.023**	0.011

[Snowmass Higgs WG Report, 1310.8361]

- Lepage, Mackenzie, & Peskin [1404.0319] use toy Monte-Carlo calculations to estimate how much the uncertainties in m_c, m_b, and α_s from lattice QCD could be decreased over the next decade given the anticipated ~100x growth in computing resources + current analysis methods
  - ➡ Show that lattice QCD will comfortably be able to achieve m_c, m_b, & α_s to precisions needed by a high-luminosity ILC

Summary & outlook

"[An] area of striking progress has been lattice gauge theory. ... It is now possible to compute the spectrum of hadrons with high accuracy, and lattice computations have been crucial in the measurement of the properties of heavy quarks. Continuing improvements in calculational methods are anticipated in coming years."

– **Snowmass Executive Summary**

Forecasts and plans

- **Petascale computing resources** enabling simulations with lighter pions, finer lattice spacings, and larger volumes ➜ will continue to increase precision in parameters of QCD Lagrangian & simplest quark flavor-changing matrix elements

[Snowmass Quark-flavor WG report, 1311.1076]

Quantity	CKM element	Present expt. error	2007 forecast lattice error	Present lattice error	2018 lattice error
f_K/f_π	$\|V_{us}\|$	0.2%	0.5%	0.4%	0.15%
$f_+^{K\pi}(0)$	$\|V_{us}\|$	0.2%	–	0.4%	0.2%
$D \to \pi\ell\nu$	$\|V_{cd}\|$	2.6%	–	4.4%	2%
$D \to K\ell\nu$	$\|V_{cs}\|$	1.1%	–	2.5%	1%
$B \to D^*\ell\nu$	$\|V_{cb}\|$	1.3%	–	1.8%	$< 1\%$
$B \to \pi\ell\nu$	$\|V_{ub}\|$	4.1%	–	8.7%	2%
f_B	$\|V_{ub}\|$	9%	–	2.5%	$< 1\%$
ξ	$\|V_{ts}/V_{td}\|$	0.4%	2-4%	4%	$< 1\%$
B_K	$\mathrm{Im}(V_{td}^2)$	0.5%	3.5–6%	1.3%	$< 1\%$

Forecasts and plans

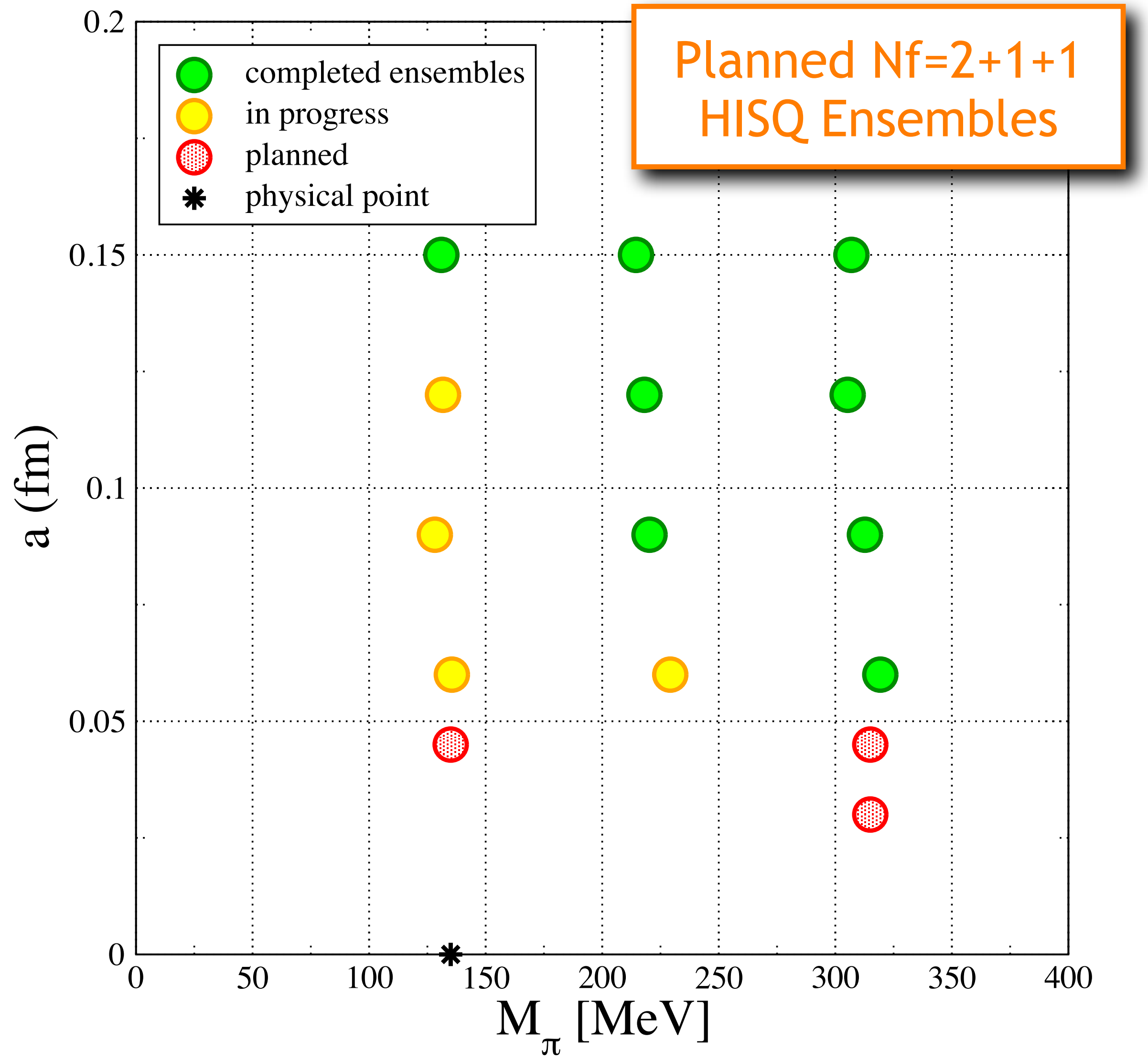
- **Petascale computing resources** enabling simulations with lighter pions, finer lattice spacings, and larger volumes ➜ will continue to increase precision in parameters of QCD Lagrangian & simplest quark flavor-changing matrix elements

- The following improvements will become widespread over the next five years

  (1) Physical-mass pions

  (2) Inclusion of isospin-breaking and EM

  (3) Dynamical charm quarks



Forecasts and plans

- **Petascale computing resources** enabling simulations with lighter pions, finer lattice spacings, and larger volumes ➜ will continue to increase precision in parameters of QCD Lagrangian & simplest quark flavor-changing matrix elements
- The following improvements will become widespread over the next five years

  (1) Physical-mass pions

  (2) Inclusion of isospin-breaking and EM

  (3) Dynamical charm quarks
- Given success with simplest quantities, expanding lattice program to meet needs of current & upcoming experiments, *e.g.*:
  - $K \rightarrow \pi\pi$ decays ($\Delta I = 1/2$ rule and ϵ'/ϵ)
  - Rare-decay form factors
  - Hadronic contributions to muon g-2
  - Nucleon couplings & form factors
  - ...

Forecasts and plans

- **Petascale computing resources** enabling simulations with lighter pions, finer lattice spacings, and larger volumes ➜ will continue to increase precision in parameters of QCD Lagrangian & simplest quark flavor-changing matrix elements
- The following improvements will become widespread over the next five years
  (1) Physical-mass pions
  (2) Inclusion of isospin-breaking and EM
  (3) Dynamical charm quarks
- Given success with simplest quantities, expanding lattice program to meet needs of current & upcoming experiments, e.g.:
  - $K \to \pi\pi$ decays ($\Delta I = 1/2$ rule and ϵ'/ϵ)
  - Rare-decay form factors
  - Hadronic contributions to muon g-2
  - Nucleon couplings & form factors
  - ...

Improved algorithms and analysis methods also being pursued, but difficult to predict

(Lattice) QCD is everywhere!

- **Lattice-QCD calculations are needed throughout high-energy physics experiments:**
  - Rare decay form factors (Belle II, LHCb, KOTO, NA62)
  - Hadronic contributions to g-2
  - Neutrino-nucleon cross-sections (NOvA, miniBoone, ...)
  - Muon-nucleon cross sections (Mu2e)
  - Dark-matter-nucleon cross-sections (super-CDMS, LZ,..)
  - Quark masses & strong coupling for SM Higgs predictions (ATLAS & CMS, ILC...)
- Success of future experimental high-energy physics program hinges on reliable theoretical predictions on same time scale as experiments and with commensurate uncertainties

(Lattice) QCD is everywhere!

- **Lattice-QCD calculations are needed throughout high-energy physics experiments:**
  - Rare decay form factors (Belle II, LHCb, KOTO, NA62)
  - Hadronic contributions to g-2
  - Neutrino-nucleon cross-sections (NOvA, miniBoone, ...)
  - Muon-nucleon cross sections (Mu2e)
  - Dark-matter-nucleon cross-sections (super-CDMS, LZ,..)
  - Quark masses & strong coupling for SM Higgs predictions (ATLAS & CMS, ILC...)

Ask me later!

- **Success of future experimental high-energy physics program hinges on reliable theoretical predictions on same time scale as experiments and with commensurate uncertainties**

(Lattice) QCD is everywhere!

- **Lattice-QCD calculations are needed throughout high-energy physics experiments:**
    - Rare decay form factors (Belle II, LHCb, KOTO, NA62)
    - Hadronic contributions to g-2
    - N
    - N
    - Dark-matter-nucleon cross-sections (super-CDMS, LZ,..)
    - Quark masses & strong coupling for SM Higgs predictions (ATLAS & CMS, ILC...)

> Continued support for lattice-QCD hardware and software is essential to achieve scientific goals and fully capitalize on enormous investments in the HEP (and NP) experimental programs

- **Success of future experimental high-energy physics program hinges on reliable theoretical predictions on same time scale as experiments and with commensurate uncertainties**

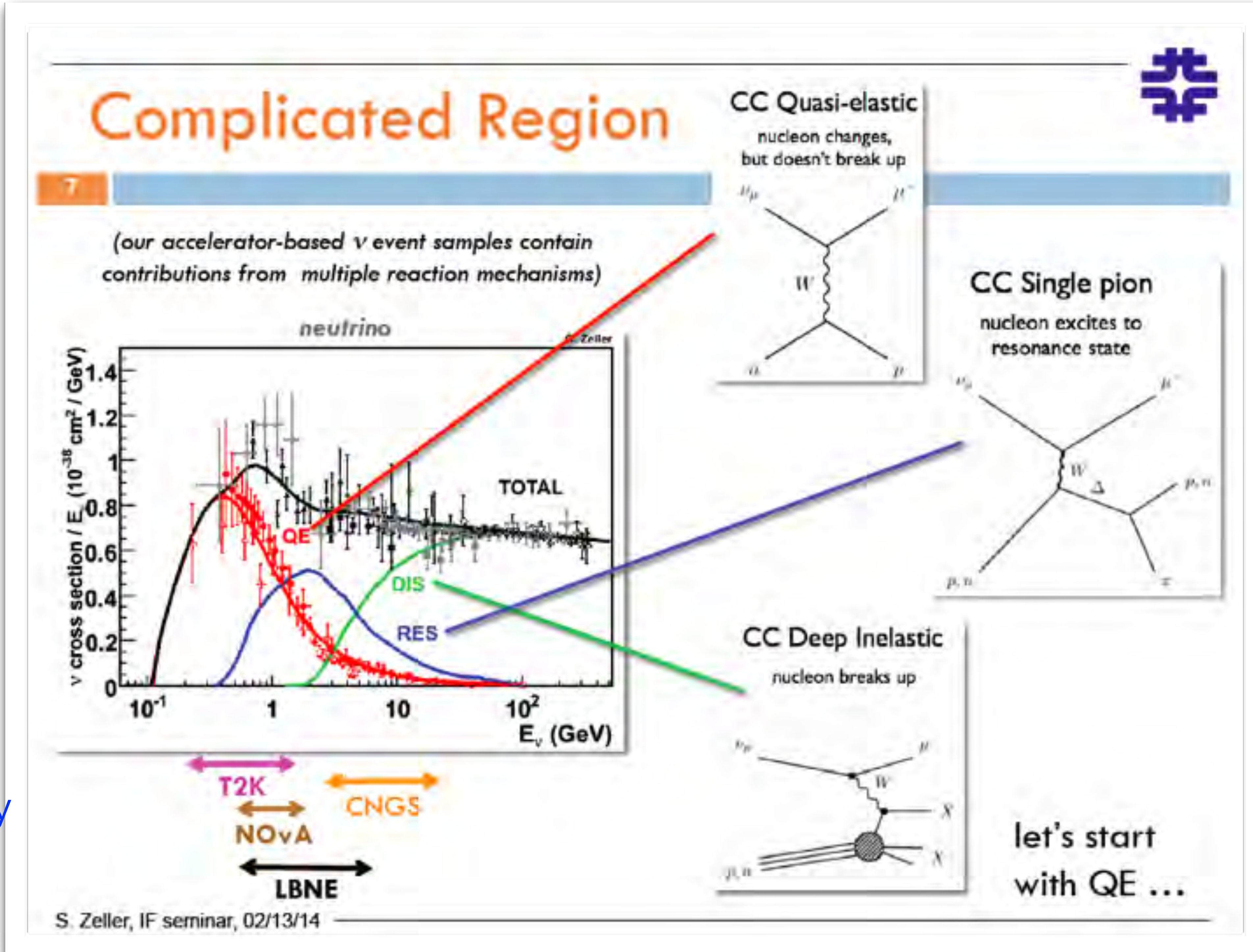
References (with hyperlinks)

- 2013 USQCD white paper "Lattice QCD at the Intensity Frontier"
- Snowmass reports:
  - "Charged Leptons"
  - "Higgs Working Group Report of the Snomass 2013 Community Planning Study"
  - "Lattice field theory for the energy and intensity frontiers: Scientific goals and computing needs"
  - "Neutrinos"
  - "Report of the Snowmass 2013 Energy Frontier QCD Working Group"
  - "Report of the Quark Flavor Physics Working Group"
- 2013 Argonne Intensity Frontier Workshop
- 2012 Project X workshop report "Project X: Physics Opportunities"
- 2011 Rockville workshop report "Fundamental Physics at the Intensity Frontier"

Extras

Neutrino physics

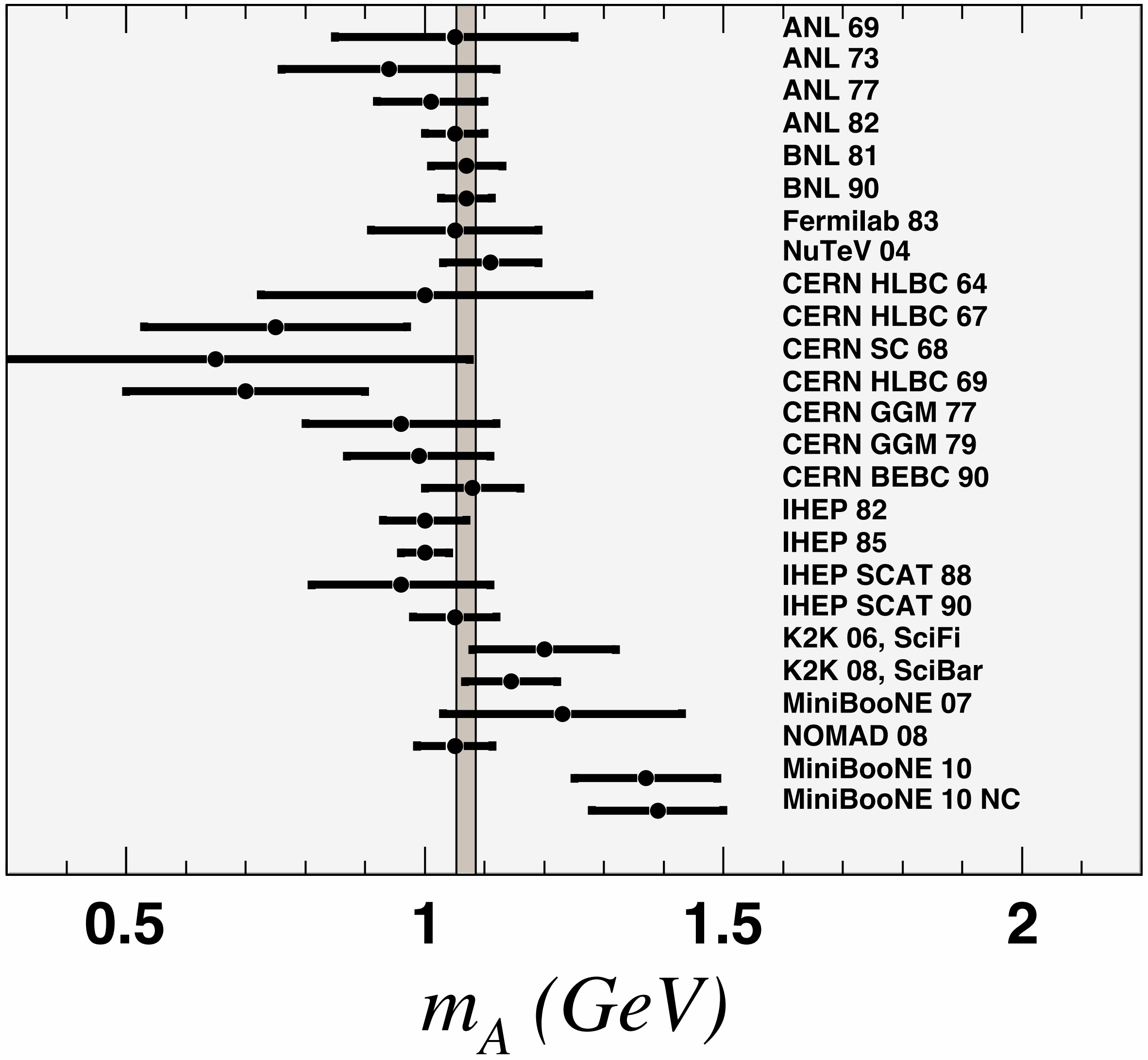
- Accelerator-based ν experiments in low-energy regime complicated by nuclear environment
- Largest contribution to signal sample in most oscillation experiments from charged-current quasielastic (CCQE) scattering on bound neutron
- Measurement of ν oscillation parameters and possible discovery of new ν states limited by understanding of CCQE cross section



CCQE and the axial form factor

- **CCQE described by axial-vector form-factor of nucleon $F_A(q^2)$**
- Typically q^2 dependence modeled by dipole form, $F_A(q^2) = \dfrac{g_A}{(1+q^2/m_A^2)^2}$, with g_A taken from neutron decay:
- Fits to dipole form over different q^2 ranges and by different experiments lead to inconsistent determinations of axial mass m_A
  - Difference may stem from nuclear effects, inadequate model parameterization, or both
- Shape of $F_A(q^2)$ can be calculated from first principles by merging constraints from analyticity [Bhattacharya *et al.*, PRD84 (2011) 073006] with lattice QCD
- Axial-vector form factor also enters Standard-Model prediction for neutrinoless double β-decay [see, *e.g.*, Barea, Kotila, Iachelo, PRC87 (2013) 014315]

[Hill, "Lattice Meets Experiment" 2014]



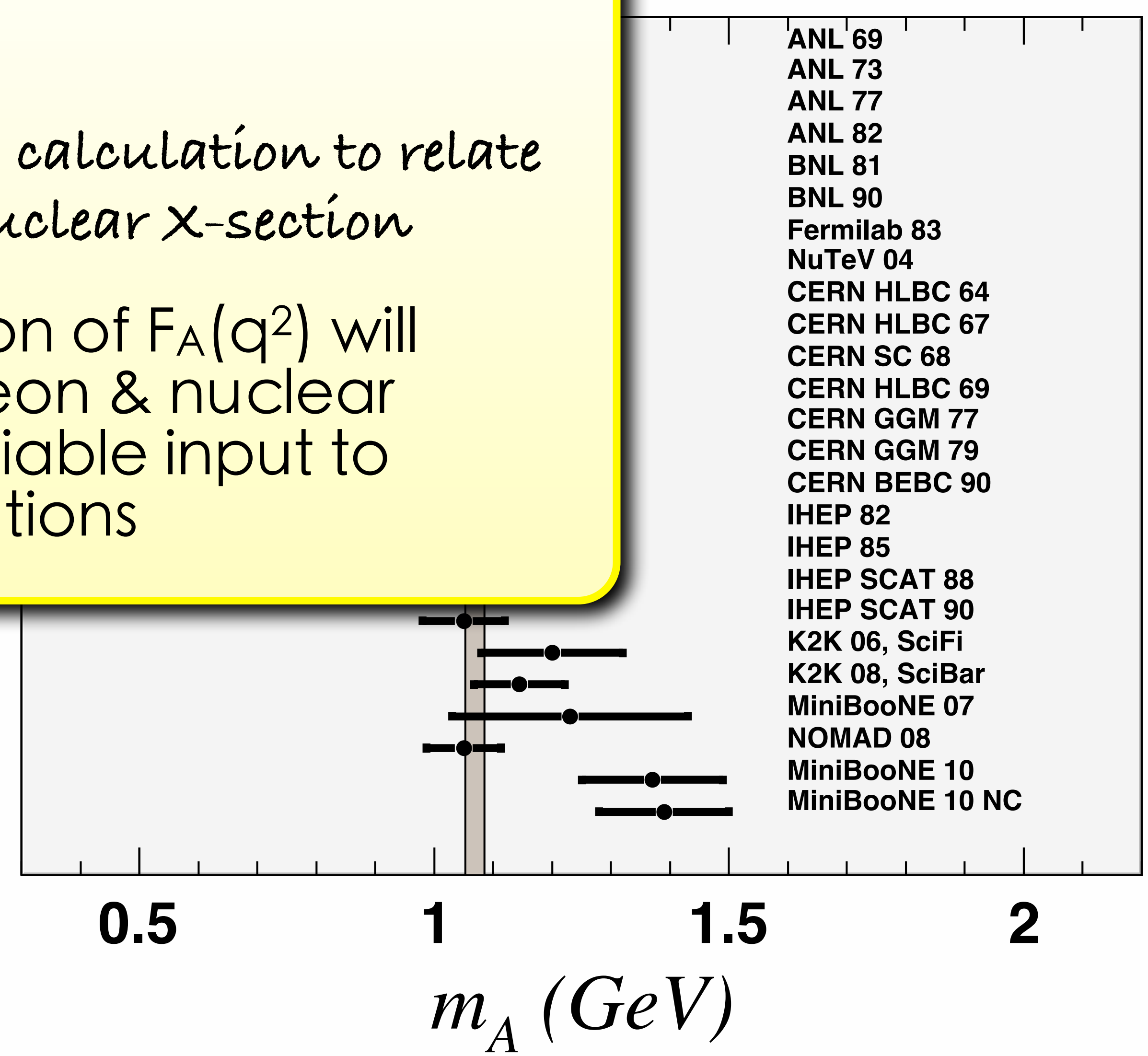
CCQE and the axial form factor

- **CCQE described by axial-vector form-factor of nucleon $F_A(q^2)$**
- Typically q^2 dependence modeled by dipole form, $F_A(q^2) = \dfrac{g_A}{(1+q^2/m_A^2)^2}$, with g_A taken from neutron decay:
- Fits to dipole fo… Experiment" 2014] and by differen… inconsistent de…
  - Difference r… inadequate…
- Shape of $F_A(q^2$… first principles… analyticity [Bha… PRD84 (2011) 073006] with lattice QCD
- Axial-vector form factor also enters Standard-Model prediction for neutrinoless double β-decay [see, *e.g.*, Barea, Kotila, Iachelo, PRC87 (2013) 014315]

N.B.: neutrino target really atomic nucleus, rather than nucleon

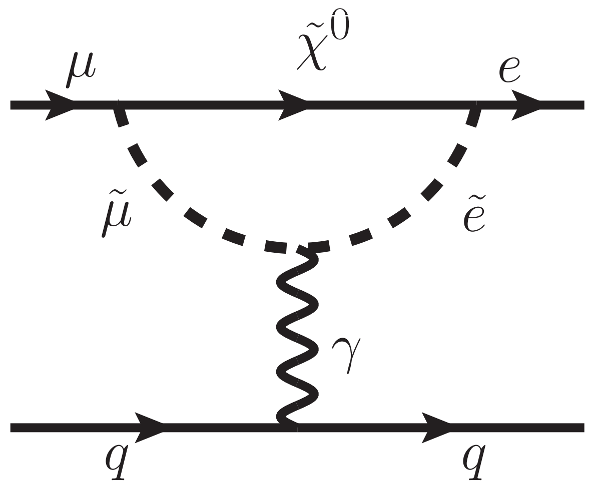
➡ *Still need nuclear theory calculation to relate nucleon form factor to nuclear X-section*

- Lattice-QCD calculation of $F_A(q^2)$ will cleanly separate nucleon & nuclear effects and provide reliable input to nuclear theory calculations

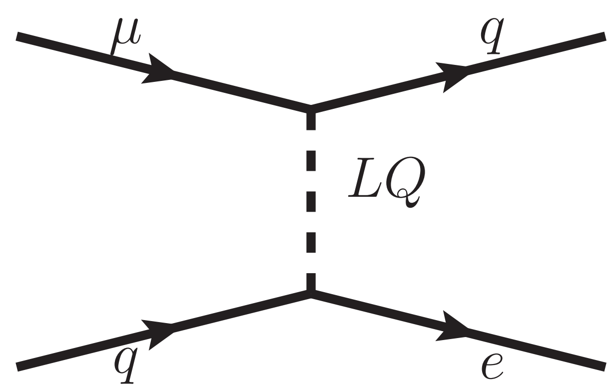


Muon-to-electron conversion

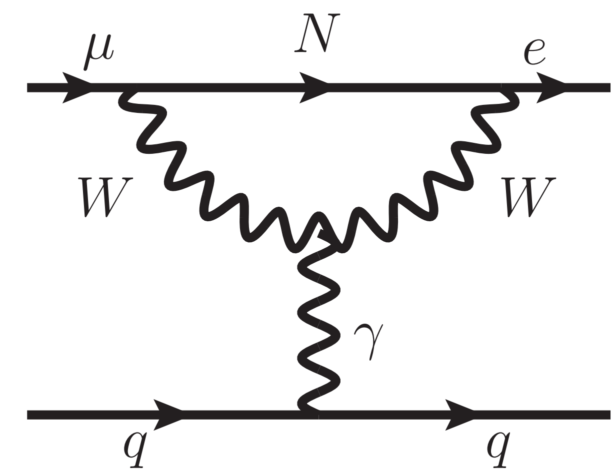
SUSY



LEPTOQUARKS



TYPE-I SEESAW



- Charged-lepton flavor violation so highly suppressed in the Standard Model that any observation would be unambiguous evidence of new physics

- Several experiments searching for μ → e conversion running or on the horizon, motivated in part by new-physics models that predict measurable rates close to current limits:
  - **MEG@PSI** searching for μ → eɣ, while **Mu3e** proposes improved search for μ → eee
  - **Mu2e @ Fermilab** aims to search for μN → eN (where N is a nucleus) with a sensitivity four orders of magnitude below the current best limit

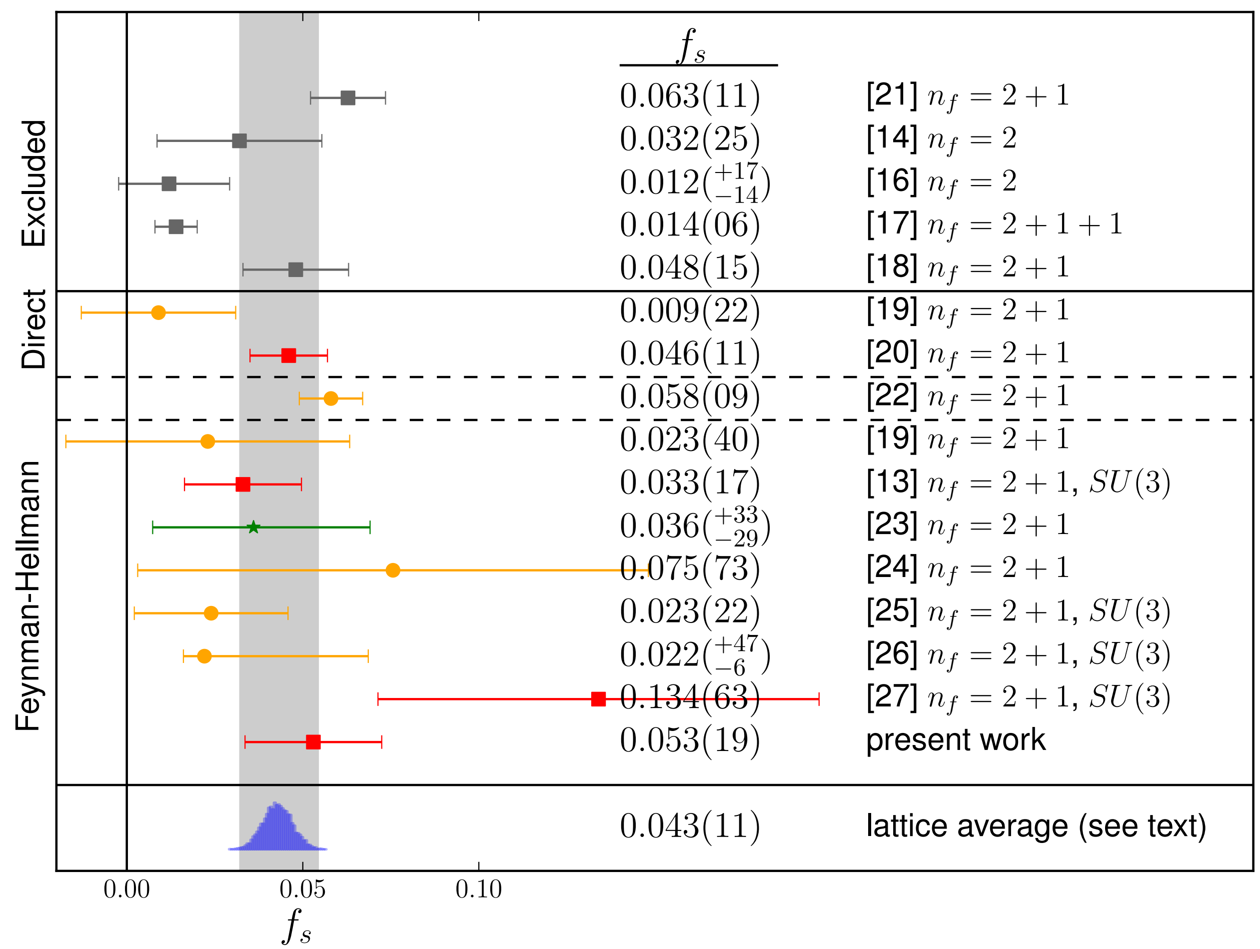
Model discrimination in CLFV

- If observed, combining measured μ → eγ and μ → e conversion rates on different target nuclei can distinguish between models and reveal information on the underlying theory
    - ❖ **Model predictions depend upon nucleon light- and strange-quark contents**

- Lattice calculations of $\sigma_{\pi N}$ and $f_s = m_s\langle N|ss|N\rangle/m_N$ have improved significantly in recent years, and already rule out large f_s favored by early non-lattice estimates

- Present lattice uncertainty in $y=2\langle p|ss|p\rangle/\langle p|uu+dd|p\rangle$ sufficient to test models in which a single operator dominates, but improvement is needed to test two-operator models [Cirigliano *et al.*, PRD80 (2009) 013002]
    - ❖ Pinning down values with ~10-20% errors in the next five years is both realistic and sufficient

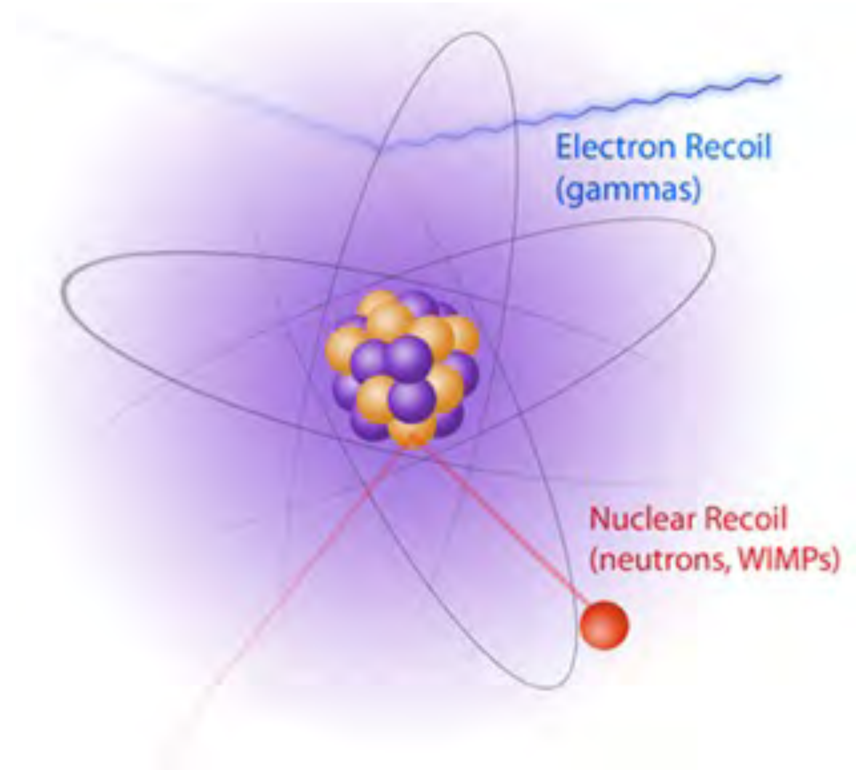
[Junnarkar & Walker-Loud, PRD87 (2013) 11, 114510]



Proton decay & other new interations

- Interpretation of many experimental measurements as constraints on TeV-scale or GUT-scale new physics requires knowledge of nucleon matrix elements

  - **PROTON DECAY:** large underground detectors for neutrino physics also sensitive to proton decay; GUT model predictions for proton lifetime depend upon expectation values $\langle \pi, K, \eta, \ldots \mid \mathcal{O}_{NP} \mid p \rangle$ of new-physics operators



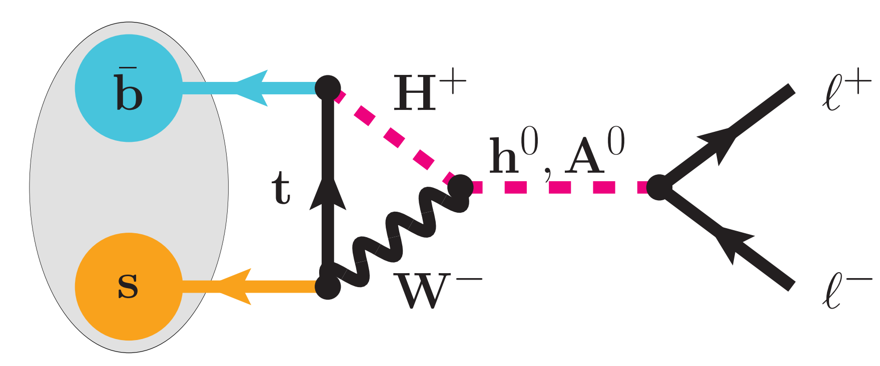
  - **DARK-MATTER DETECTION:** for spin-independent dark matter (*e.g.* mediated by Higgs exchange), cross-section for DM-nucleon scattering depends upon the light- and strange-quark contents of the nucleon (same matrix elements as for $\mu \to e$)

  - **NEUTRON BETA DECAY:** constraints on new TeV-scale interactions depend on the neutron scalar and tensor charges g_S and g_T

- For all of these matrix elements, **lattice calculations with 10–20% precision are sufficient for the time being** and can be achieved in the next five years

The "New-physics flavor puzzle"

- **Most Standard-Model extensions lead to additional flavor & CP violation from exchange of new particles**
- Absence of large signals in flavor-changing neutral currents strongly constrains TeV-scale new physics



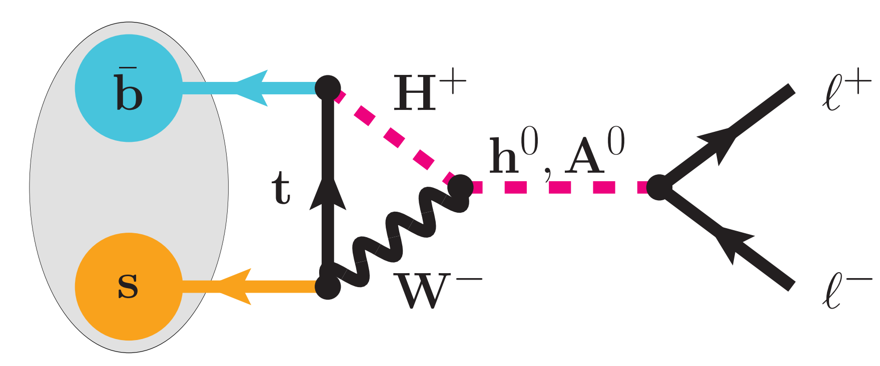
$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\rm SM} + \frac{C_{\rm NP}}{\Lambda^2_{\rm NP}} \mathcal{O}_{\Delta F=2}$$

Operator	Bounds on Λ in TeV ($c_{ij}=1$) Re	Bounds on Λ in TeV ($c_{ij}=1$) Im	Bounds on c_{ij} ($\Lambda = 1$ TeV) Re	Bounds on c_{ij} ($\Lambda = 1$ TeV) Im	Observables
$(\bar s_L \gamma^\mu d_L)^2$	9.8×10^2	1.6×10^4	9.0×10^{-7}	3.4×10^{-9}	Δm_K; ϵ_K
$(\bar s_R\, d_L)(\bar s_L d_R)$	1.8×10^4	3.2×10^5	6.9×10^{-9}	2.6×10^{-11}	Δm_K; ϵ_K
$(\bar c_L \gamma^\mu u_L)^2$	1.2×10^3	2.9×10^3	5.6×10^{-7}	1.0×10^{-7}	Δm_D; $\lvert q/p\rvert, \phi_D$
$(\bar c_R\, u_L)(\bar c_L u_R)$	6.2×10^3	1.5×10^4	5.7×10^{-8}	1.1×10^{-8}	Δm_D; $\lvert q/p\rvert, \phi_D$
$(\bar b_L \gamma^\mu d_L)^2$	5.1×10^2	9.3×10^2	3.3×10^{-6}	1.0×10^{-6}	Δm_{B_d}; $S_{\psi K_S}$
$(\bar b_R\, d_L)(\bar b_L d_R)$	1.9×10^3	3.6×10^3	5.6×10^{-7}	1.7×10^{-7}	Δm_{B_d}; $S_{\psi K_S}$
$(\bar b_L \gamma^\mu s_L)^2$	1.1×10^2		7.6×10^{-5}		Δm_{B_s}
$(\bar b_R\, s_L)(\bar b_L s_R)$	3.7×10^2		1.3×10^{-5}		Δm_{B_s}

[Isidori, Nir, Perez, Ann.Rev.Nucl.Part.Sci. 60 (2010) 355]

The "New-physics flavor puzzle"

- **Most Standard-Model extensions lead to additional flavor & CP violation from exchange of new particles**
- Absence of large signals in flavor-changing neutral currents strongly constrains TeV-scale new physics



$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\rm SM} + \frac{C_{\rm NP}}{\Lambda^2_{\rm NP}} \mathcal{O}_{\Delta F=2}$$

Operator	Bounds on Λ in TeV (c … $= 1$)	Bounds on c … ($\Lambda = 1$ TeV)	Observables
$(\bar{s}_L \gamma^\mu d_L)^2$	[illegible]	[illegible]	$\Delta m_K;\ \epsilon_K$
$(\bar{s}_R\, d_L)(\bar{s}_L d_R)$	[illegible]	[illegible]	$\Delta m_K;\ \epsilon_K$
$(\bar{c}_L \gamma^\mu u_L)^2$	[illegible]	[illegible]	$\Delta m_D;\ \lvert q/p\rvert, \phi_D$
$(\bar{c}_R\, u_L)(\bar{c}_L u_R)$	[illegible]	[illegible]	$\Delta m_D;\ \lvert q/p\rvert, \phi_D$
$(\bar{b}_L \gamma^\mu d_L)^2$	[illegible]	[illegible]	$\Delta m_{B_d};\ S_{\psi K_S}$
$(\bar{b}_R\, d_L)(\bar{b}_L d_R)$	[illegible]	[illegible]	$\Delta m_{B_d};\ S_{\psi K_S}$
$(\bar{b}_L \gamma^\mu s_L)^2$	[illegible]	[illegible]	Δm_{B_s}
$(\bar{b}_R\, s_L)(\bar{b}_L s_R)$	3.7×10[illegible]	1.3×10[illegible]	Δm_{B_s}

If there is new physics at the TeV scale, its flavor structure is unnatural:

- Couplings are tiny (<<1)
- Masses are degenerate ($\Delta m_{ij} << m_i$)
- Phases are aligned with Standard Model

[Isidori, Nir, Perez, Ann.Rev.Nucl.Part.Sci. 60 (2010) 355]

The Wolfenstein parameterization

- Standard-Model CKM matrix is unitary ➜ **elements are not all independent**
- Wolfenstein parameterization expresses CKM matrix as expansion in $\lambda=|V_{us}|\sim 0.22$

$$V_{\rm CKM} \approx \begin{array}{c} \begin{array}{ccc} d & s & b \end{array} \\ \left(\begin{array}{ccc} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{array} \right) \end{array} \begin{array}{c} u \\ c \\ t \end{array}$$

$$\lambda \equiv |V_{us}| \approx 0.22 \qquad A \equiv \frac{|V_{cb}|}{\lambda^2} \qquad \rho \equiv \frac{\mathrm{Re}(V_{ub})}{A\lambda^3} \qquad \eta \equiv -\frac{\mathrm{Im}(V_{ub})}{A\lambda^3}$$

- Makes relationships between matrix elements manifest:
  - **Only four independent parameters** (two real amplitudes & one complex phase)
  - Provides tests of Standard-Model CKM framework!

Recent highlight:

K→πℓν form factor at the physical pion mass

- K→πℓν form factor enables determination of $|V_{us}|$ in the Standard Model via:

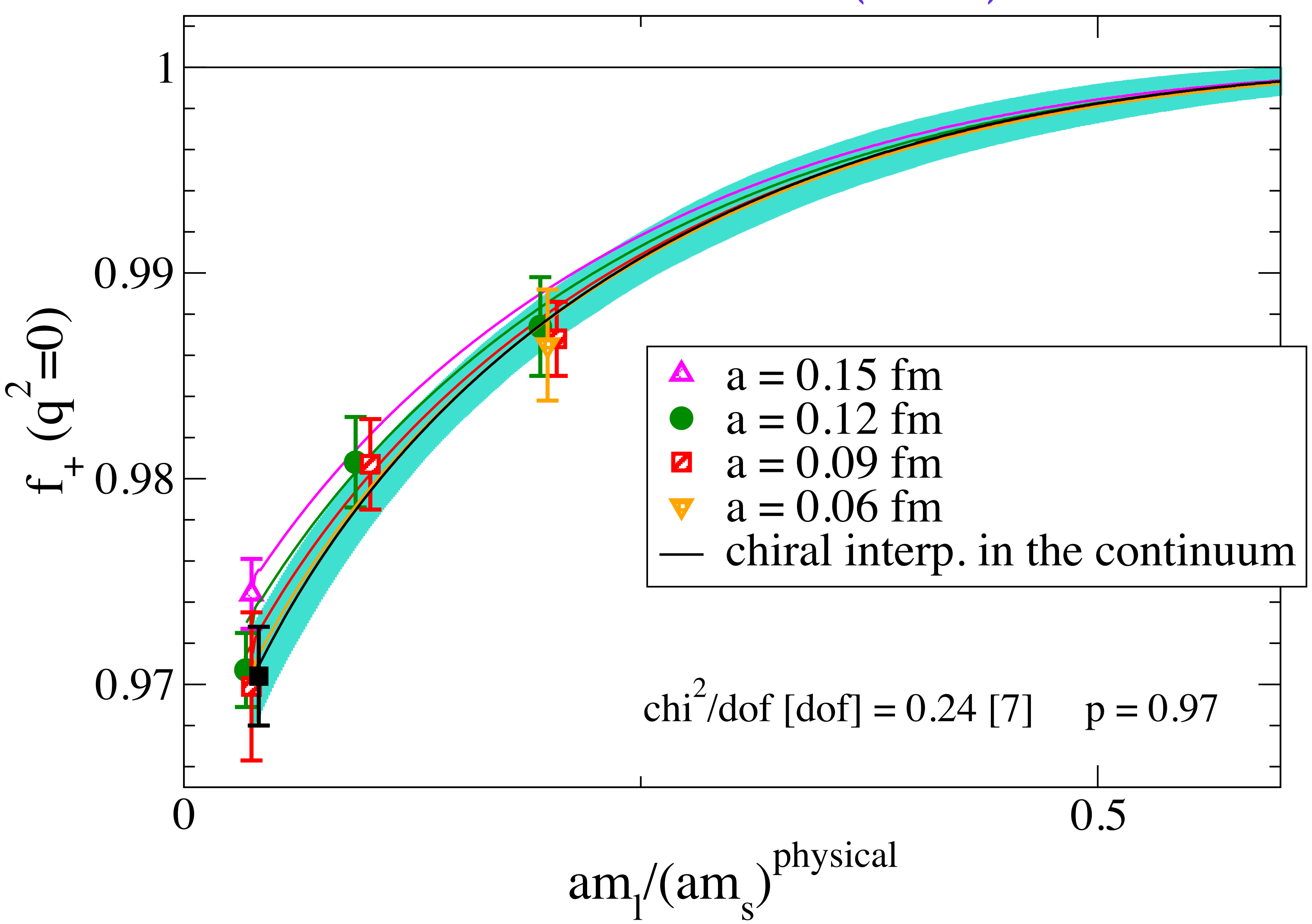
$$\Gamma(K \to \pi\ell\nu) = \frac{G_F^2 m_K^5}{192\pi^3} C_K^2 S_{\rm EW} |V_{us}|^2 |f_+^{K^0\pi^-}(0)|^2 I_{K\ell} \left(1 + \delta_{\rm EM}^{K\ell} + \delta_{SU(2)}^{K\pi}\right)^2$$

[Bazavov *et al.* PRL 112 (2014) 112001]

- Fermilab/MILC recently obtained **first result for $f_+^{K\pi}(q^2=0)$ at the physical pion mass, removing previously dominant uncertainty from chiral extrapolation**

- Single most precise result for $f_+(0)$ enables 0.4% determination of $|V_{us}|$:

$$f_+^{K\pi}(0) = 0.9704(24)_{\rm stat}(22)_{\rm sys}$$
$$|V_{us}| = 0.22290(74)_{\rm theo}(52)_{\rm exp}$$

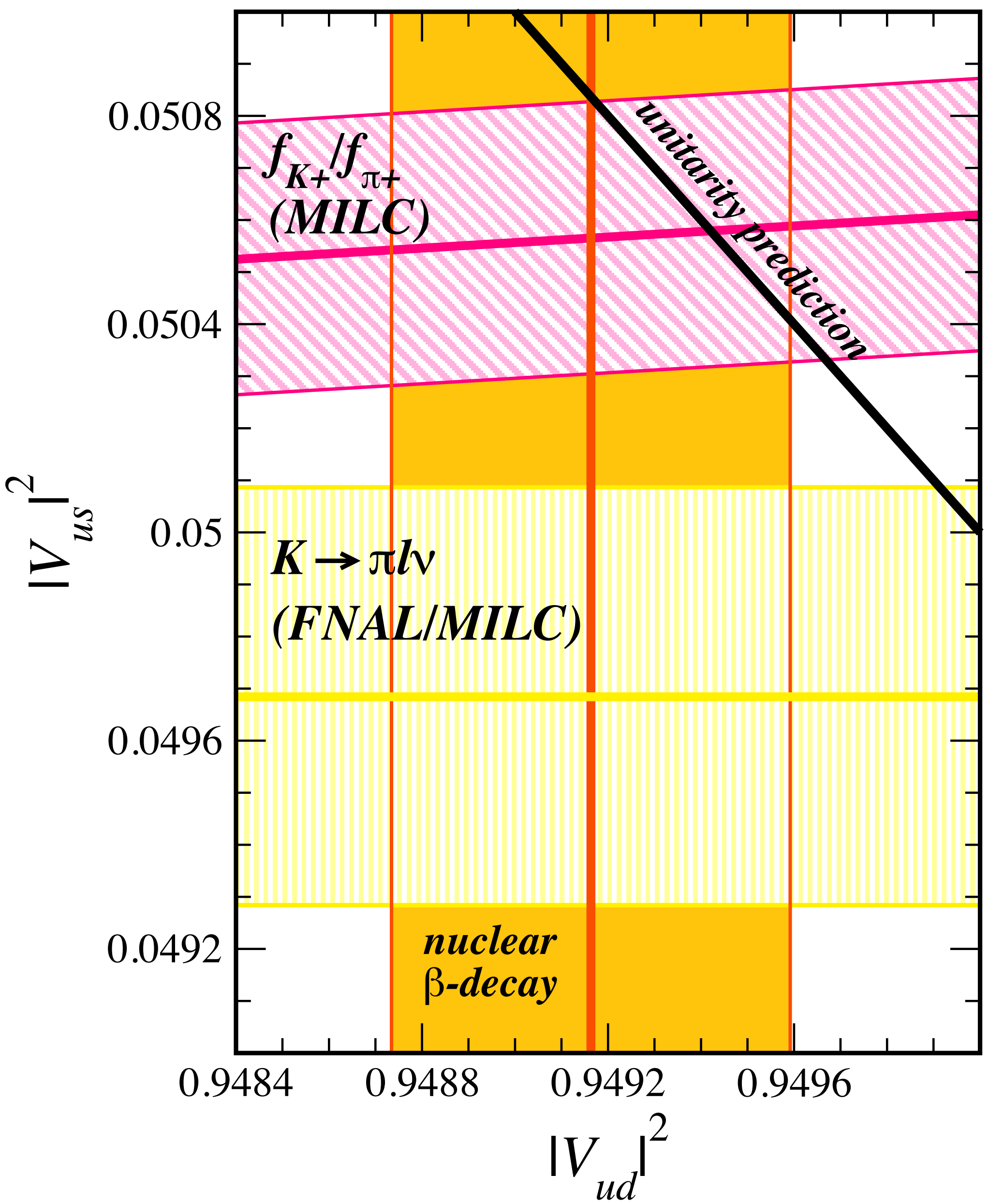


Recent highlight:

K→πℓν form factor at the physical pion mass

In test of first-row unitarity, error from $|V_{us}|$ now smaller than that from $|V_{ud}|$:

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 - 1 = -0.00115(40)_{V_{us}}(43)_{V_{ud}}$$



➜ Motivates revisiting error on $|V_{ud}|$ from nuclear β decays

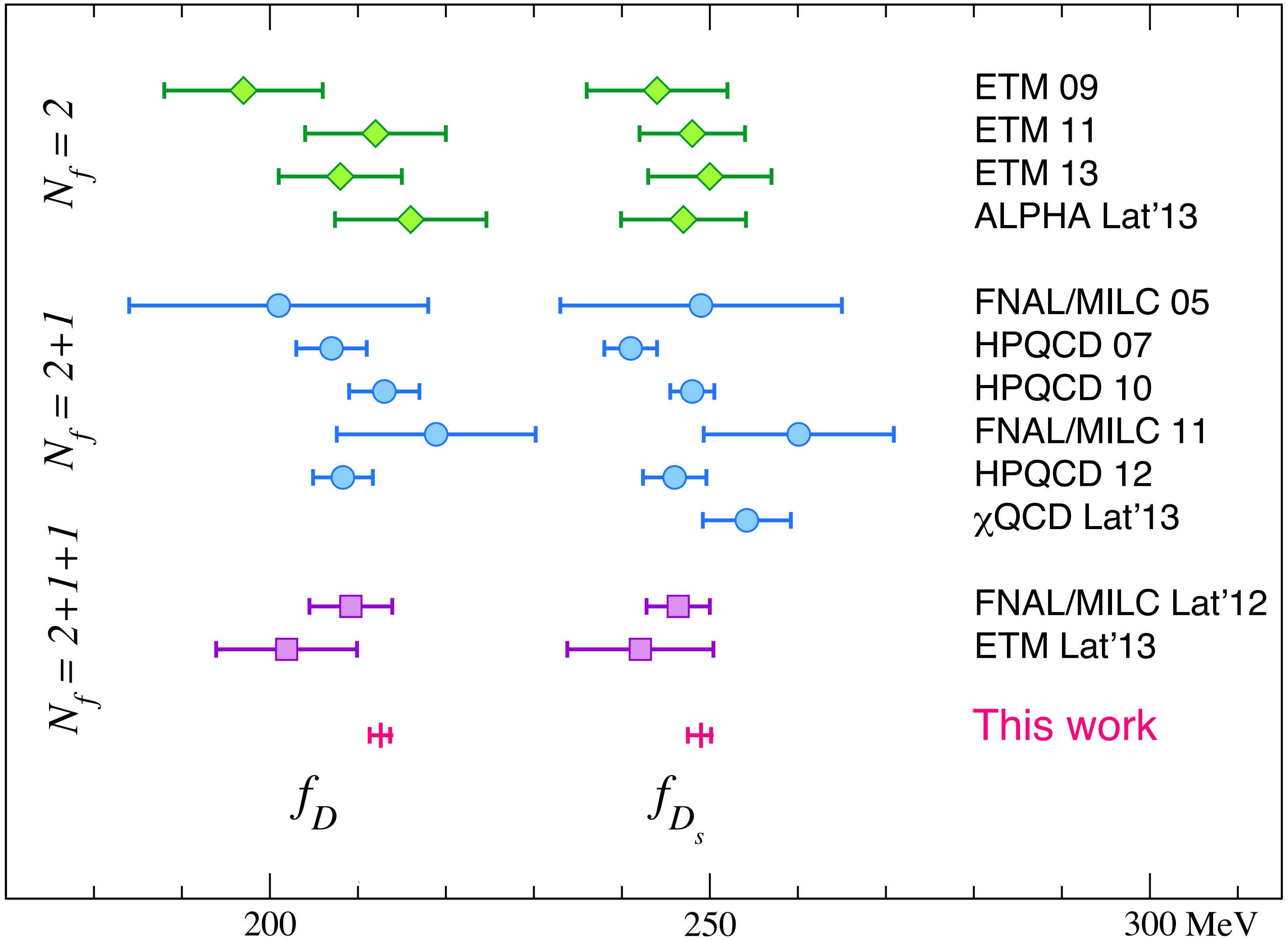
Recent highlight: f_D & f_{D_s} at the physical pion mass

- f_D & f_{D_s} can be used to obtain $|V_{cd}|$ and $|V_{cs}|$ via:

$$\Gamma(D_{(s)} \to \ell\nu) = \frac{G_F^2}{8\pi} \mathbf{f}^2_{\mathbf{D}_{(s)}} M_{D_{(s)}} \left(1 - \frac{m_\ell^2}{M^2_{D_{(s)}}}\right)^2 |\mathbf{V}_{\mathbf{cd(s)}}|^2$$

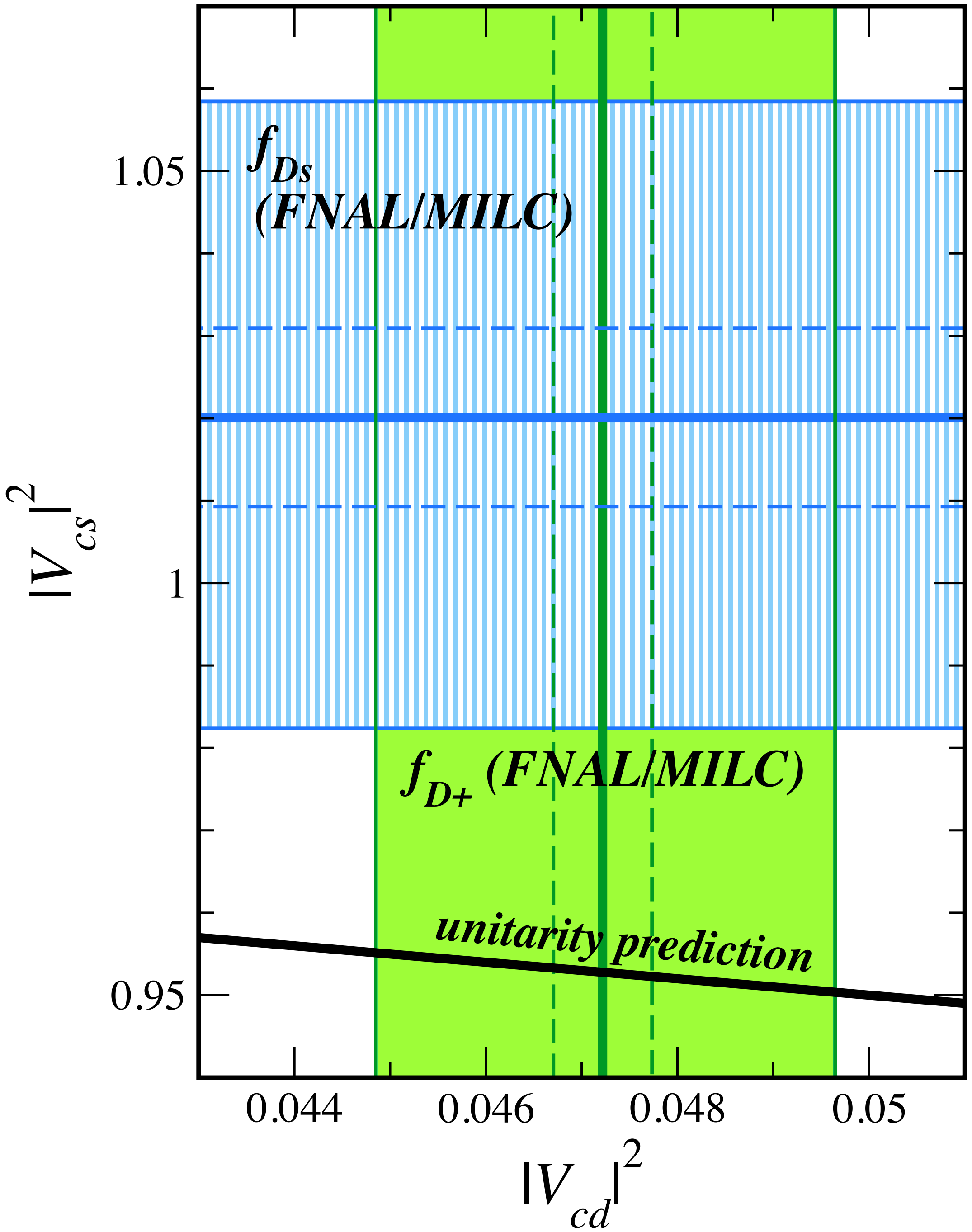
[arXiv:1407.3772]

- Fermilab/MILC recently obtained first four-flavor results for f_D & f_{D_s} with physical pions
- HISQ action for u,d,s, and c quarks and fine lattice spacings eliminates renormalization error and leads to small discretization errors
- 0.5% errors on f_D & f_{D_s} and 0.3% error on f_{D_s}/f_D 2–4× more precise than previous best results



Recent highlight:

f_D & f_{Ds} at the physical pion mass



- Errors on $|V_{cd}|$ & $|V_{cs}|$ limited by experimental branching fractions

$$|V_{cd}| = 0.217(1)_{LQCD}(5)_{expt}(1)_{EM}$$
$$|V_{cs}| = 1.010(5)_{LQCD}(18)_{expt}(6)_{EM}$$

➡ As measurements improve, need more reliable estimate of structure-dependent EM contributions

- $|V_{cd}|$ & $|V_{cs}|$ agree with determinations from semileptonic D decays, but **some tension with CKM unitarity**

$$|V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 - 1 = -0.07(4)$$

In progress: $K \to \pi\pi$ decays

- Direct CP-violation in $K \to \pi\,\pi$ decays (ϵ'_K/ϵ_K) measured experimentally to <10% precision more than a decade ago [NA48, KTeV], but utility for testing Standard Model handicapped by large uncertainty in corresponding weak matrix elements

- RBC/UKQCD presently attacking $K \to \pi\pi$ amplitudes via "direct" Lellouch-Lüscher approach

  - Computed $\Delta I = 3/2$ matrix elements with physical pion and kaon masses, obtaining $\mathrm{Re}(A_2)$ & $\mathrm{Im}(A_2)$ with ~20% errors [PRL108 (2012) 141601], and now analyzing data at a second lattice spacing for a continuum limit [arXiv:1311.3844]

  - Performed successful pilot study of $\Delta I = 1/2$ matrix elements with ~330 MeV pions [PRD84 (2011) 114503; Q. Liu Ph.D. thesis (2012)], and now beginning large-scale calculation with physical pions and kaons

  - **Should yield first ab initio QCD calculation of $\Delta I=1/2$ rule and calculation of $\varepsilon'_K/\varepsilon_K$ with ~20-30% precision in the next one or two years**

- Methods for long-distance matrix elements needed for rare kaon decays also being studied, with approach worked out for simplest quantities $\Delta(M_K)$ and ϵ_K in [Christ, PoS (Lattice2010) 300, LATTICE2011 277], but too soon to predict time needed to obtain controlled results

Coming soon: B→Dℓυ at nonzero recoil

[Qiu *et al.* [Fermilab Lattice & MILC Collaborations], PoS LATTICE 2013, 385 (2013)]

$$\frac{d\Gamma(B \to Dl\nu)}{dw} = \frac{G_F^2}{48\pi^3} m_D^3 (m_B + m_D)^2 (w^2 - 1)^{3/2} |\mathbf{V_{cb}}|^2 |\mathcal{G}_{\mathbf{B\to D}}(\mathbf{w})|^2$$

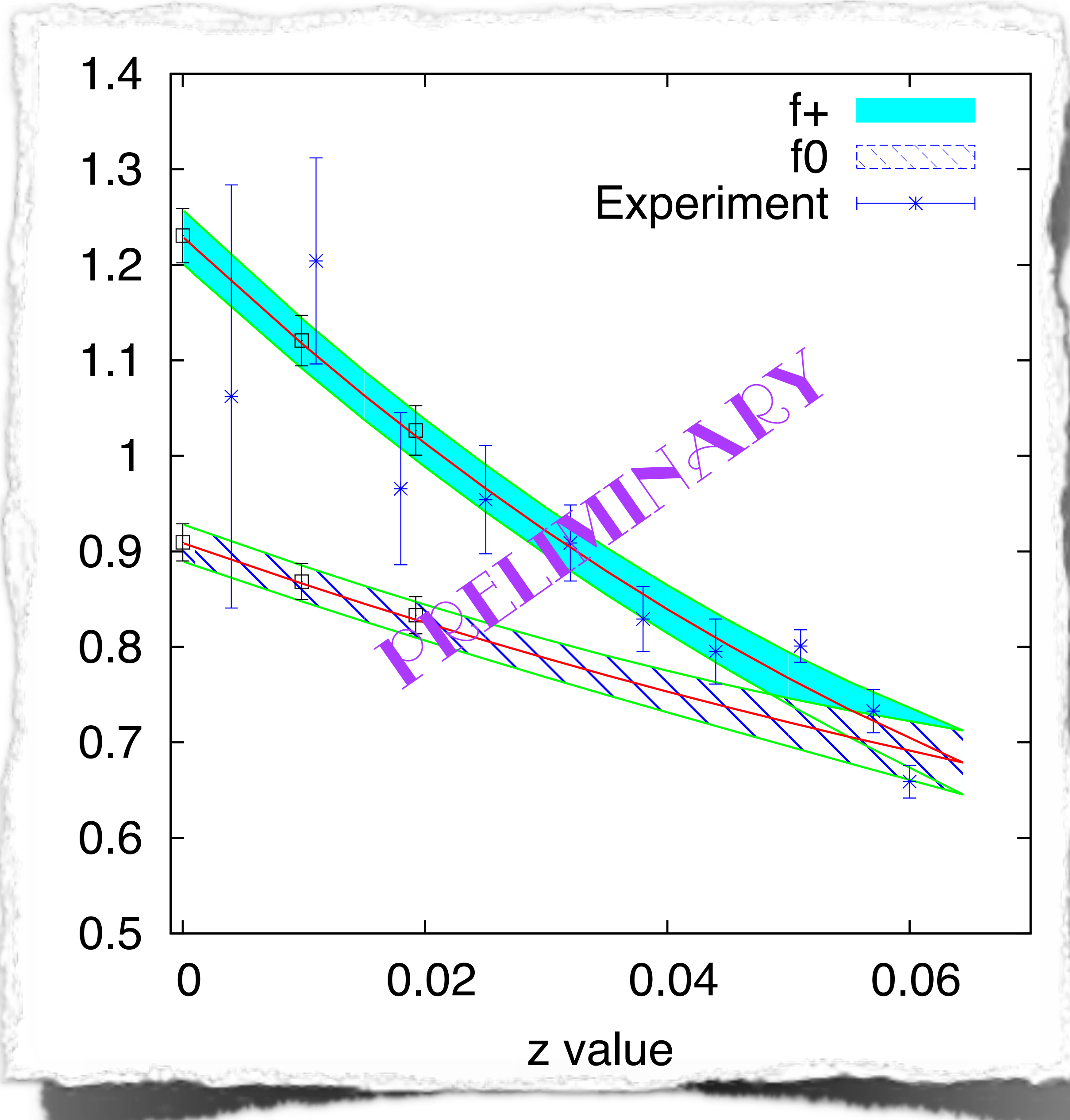
} w ≡ v$_B$ · v$_D$

✦ Comparing theory & experiment at zero recoil (w=1) leads to large experimental errors in $|V_{cb}|$ because decay rate kinematically suppressed

✦ Fermilab/MILC presented **first dynamical result for G(w) over full kinematic range at Lattice 2013**, and analysis is now almost finalized

✦ Following standard method for B→πℓυ, obtain $|V_{cb}|$ from combined fit of lattice & experimental data to z-expansion

✦ **Use of nonzero recoil data reduces (anticipated) error on $|V_{cb}|$ by ~20%**



$B \to D^{(*)}\tau\nu$

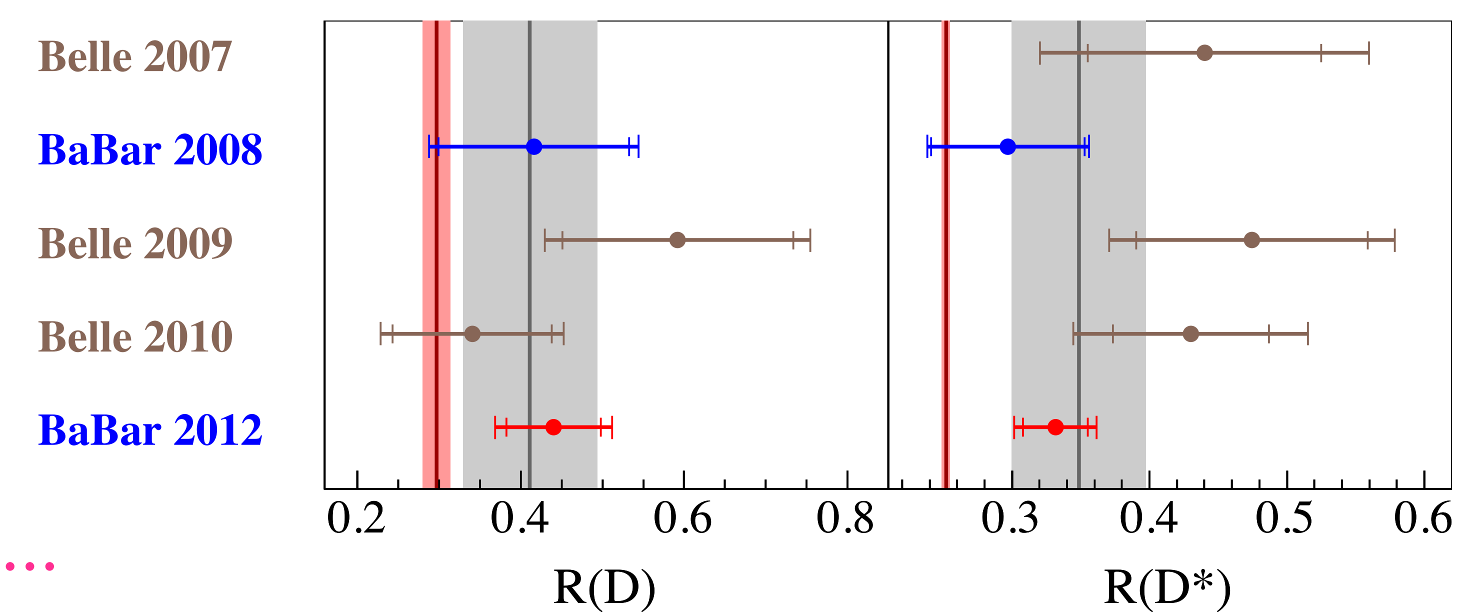
- Many beyond-the-Standard Model theories contain an extended Higgs sector

SM
h^0

- Because Higgs boson(s) couple more strongly to heavy particles, search for charged Higgs in decays involving third generation quarks and leptons, *e.g.* $B_s \to \mu^+\mu^-$, $B \to \tau\nu$, and $B \to D^{(*)}\tau\nu$

- Recently BaBar measured the ratios $R(D) = BR(B \to D\tau\nu)/BR(B \to Dl\nu)$, $R(D^*) = BR(B \to D^*\tau\nu)/BR(B \to D^*l\nu)$ and observed excessed in both channels that **disagree with the Standard-Model by 3.4σ** [PRL 109 (2012) 101802, arXiv:1303.0571]

    - **Standard-Model prediction relies on hadronic form factors** obtained using dispersive bounds, heavy-quark symmetry, and quenched lattice QCD (neglecting u, d, and s quark loops) **with unquantifiable model uncertainties...**



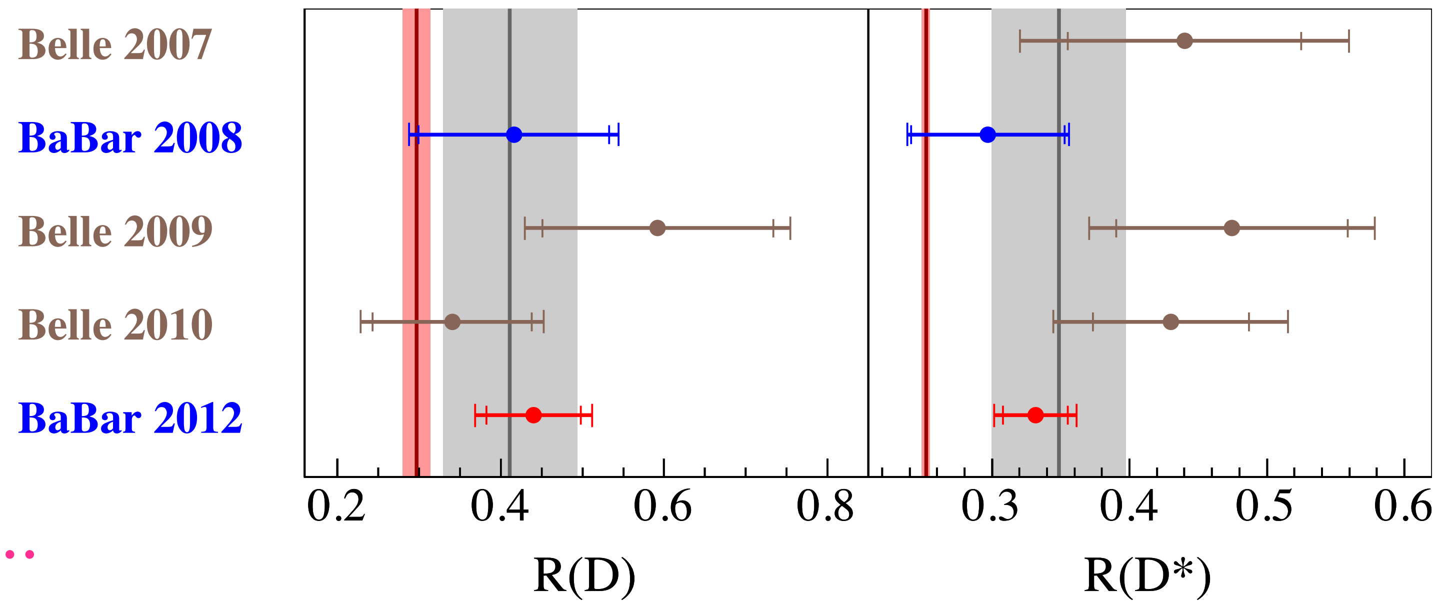
$B \to D^{(*)}\tau\nu$

- Many beyond-the-Standard Model theories contain an extended Higgs sector

- Because Higgs boson(s) couple more strongly to heavy particles, search for charged Higgs in decays involving third generation quarks and leptons, *e.g.* $B_s \to \mu^+\mu^-$, $B \to \tau\nu$, and $B \to D^{(*)}\tau\nu$

- Recently BaBar measured the ratios $R(D) = BR(B \to D\tau\nu)/BR(B \to Dl\nu)$, $R(D^*) = BR(B \to D^*\tau\nu)/BR(B \to D^*l\nu)$ and observed excessed in both channels that **disagree with the Standard-Model by 3.4σ** [PRL 109 (2012) 101802, arXiv:1303.0571]

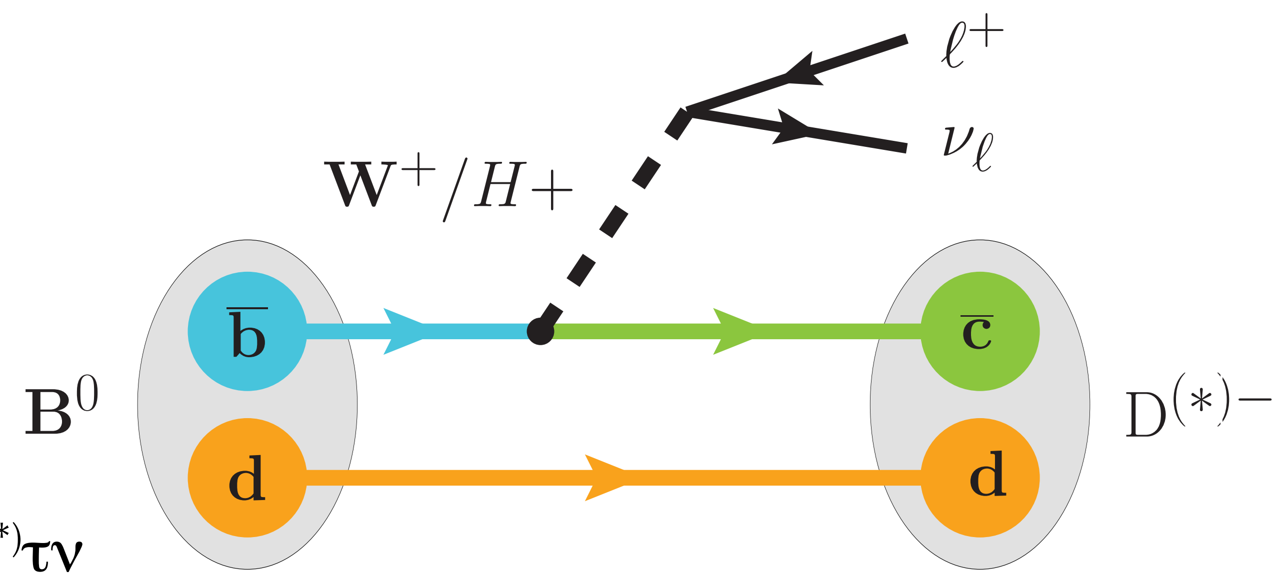
  - **Standard-Model prediction relies on hadronic form factors** obtained using dispersive bounds, heavy-quark symmetry, and quenched lattice QCD (neglecting u, d, and s quark loops) **with unquantifiable model uncertainties...**



$B \to D^{(*)}\tau\nu$

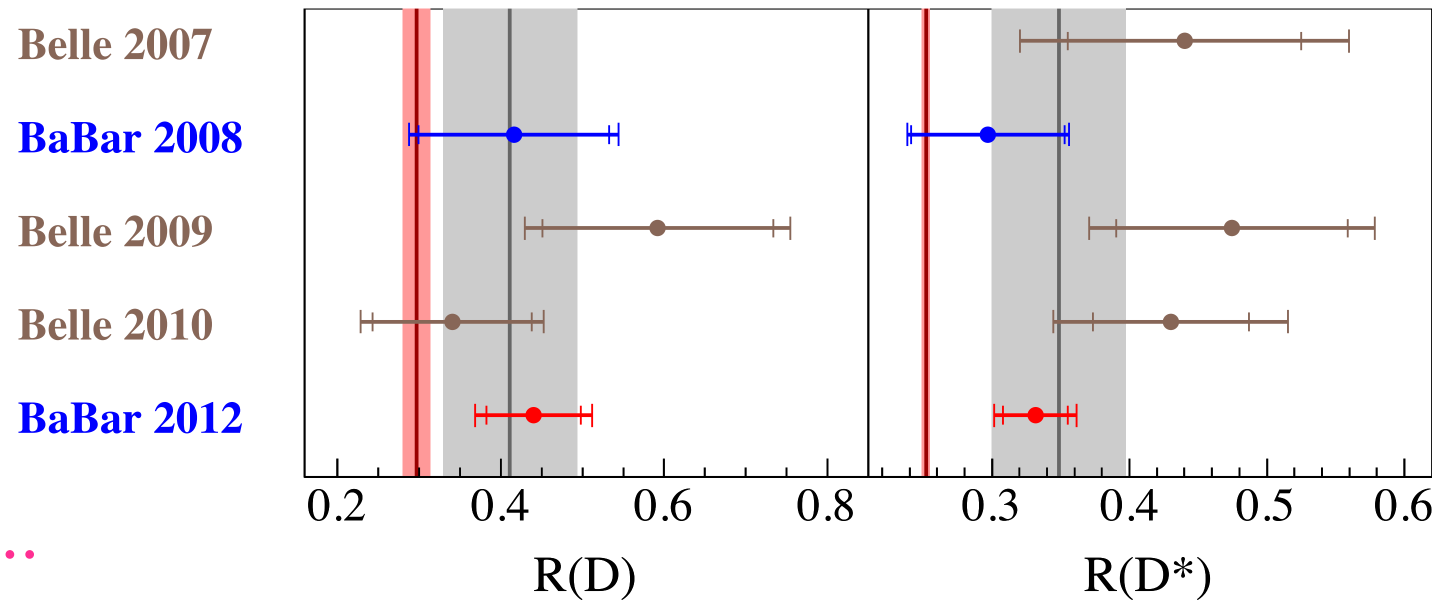
- Many beyond-the-Standard Model theories contain an extended Higgs sector

- Because Higgs boson(s) couple more strongly to heavy particles, search for charged Higgs in decays involving third generation quarks and leptons, *e.g.* $B_s \to \mu^+\mu^-$, $B \to \tau\nu$, and $B \to D^{(*)}\tau\nu$



- Recently BaBar measured the ratios R(D) = BR(B → Dτν)/BR(B → Dlν), R(D*) = BR(B → D*τν)/BR(B → D*lν) and observed excessed in both channels that **disagree with the Standard-Model by 3.4σ** [PRL 109 (2012) 101802, arXiv:1303.0571]

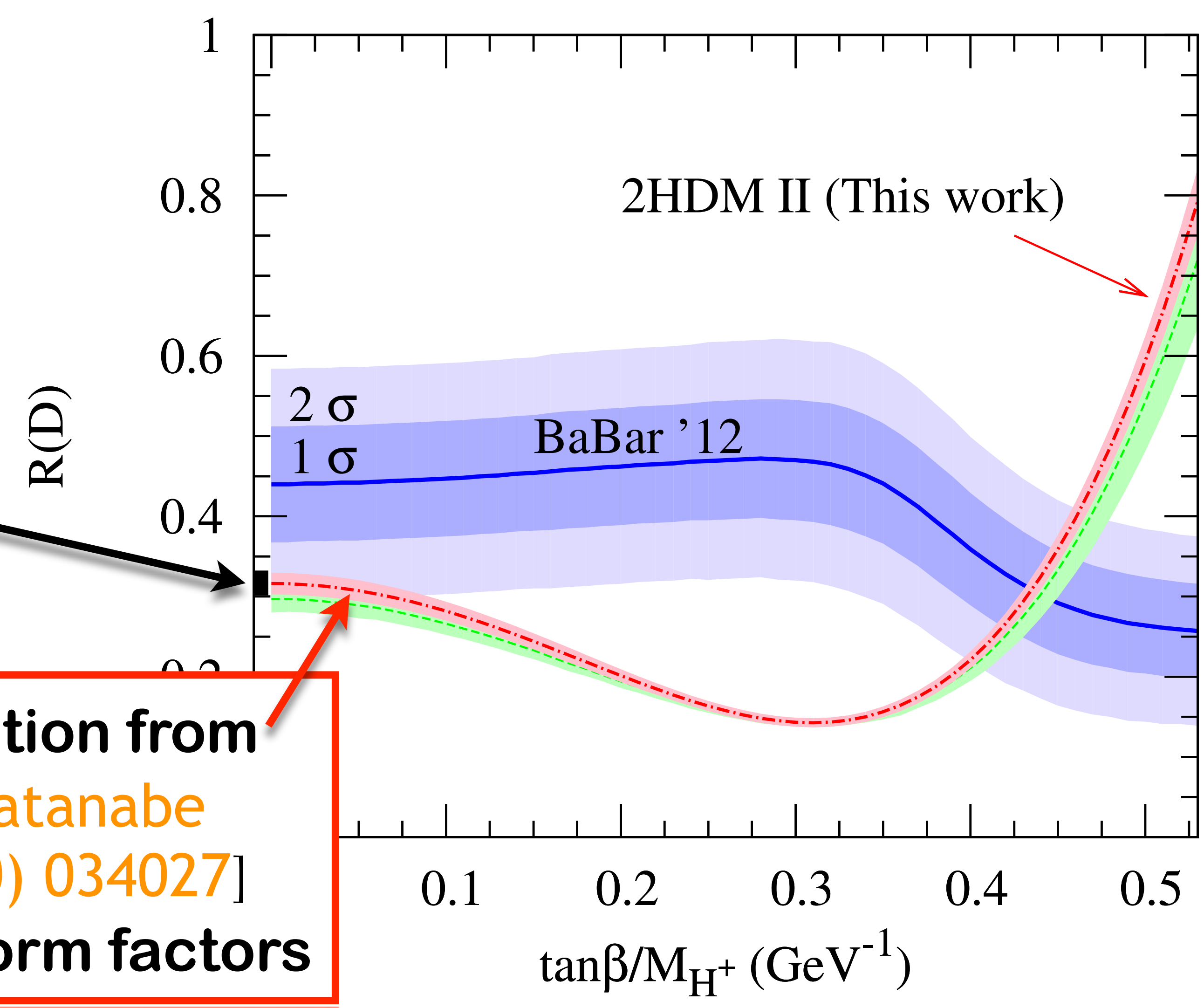
  - Standard-Model prediction relies on hadronic form factors obtained using dispersive bounds, heavy-quark symmetry, and quenched lattice QCD (neglecting u, d, and s quark loops) with unquantifiable model uncertainties...



Lattice-QCD calculation of R(D)

- FNAL/MILC Collaboration [PRD 85 (2012) 114502] responded quickly to the BABAR result and obtained the first Standard-Model calculation of R(D) from *ab initio* lattice-QCD using results for the form factors $f_+(q^2)$ and $f_0(q^2)$ at nonzero recoil from Phys.Rev. D85 (2012) 114502

Standard Model

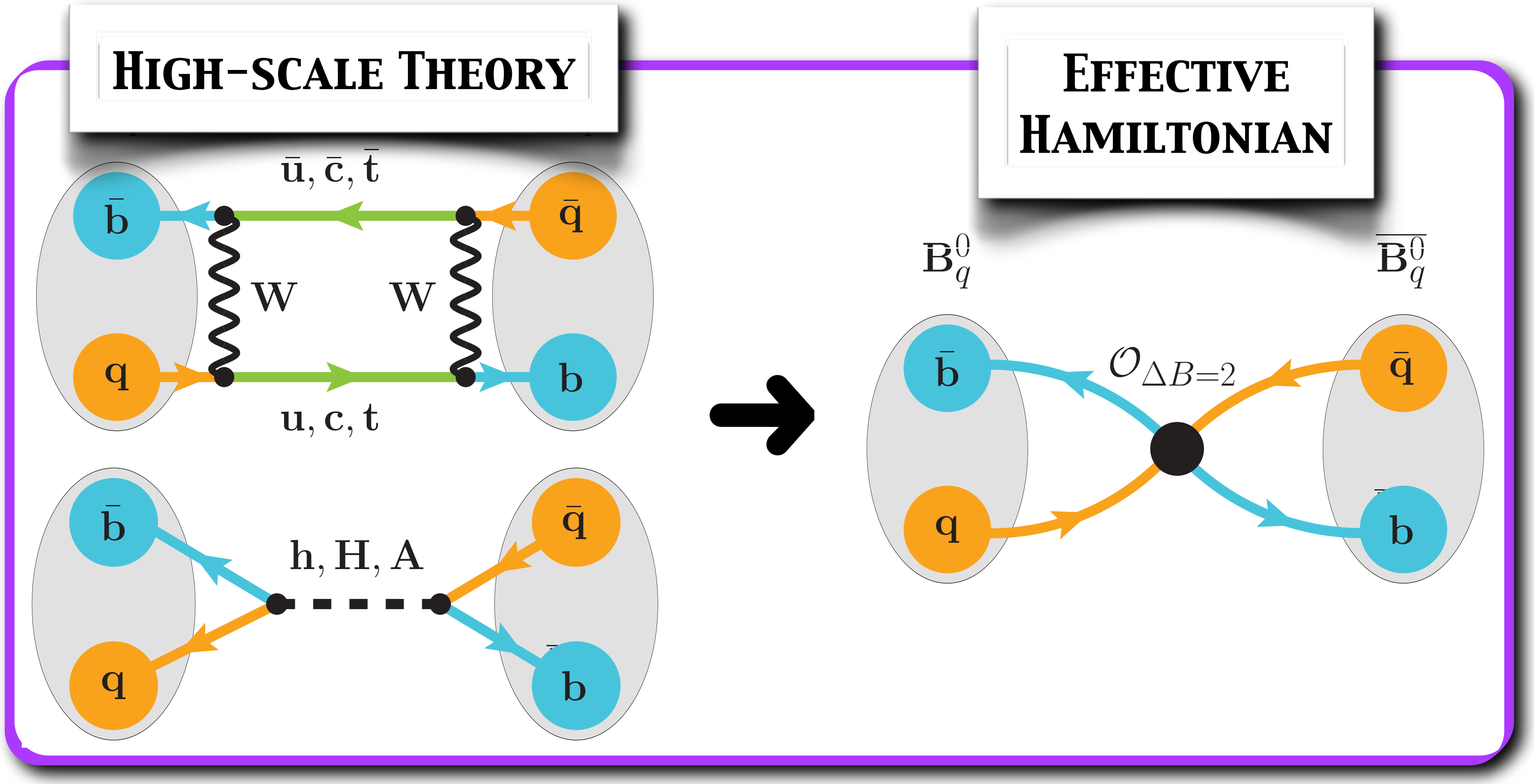


2HDM prediction from Tanaka & Watanabe [PRD 82 (2010) 034027] + FNAL/MILC form factors

- Error smaller than previous estimate, primarily due to reduced uncertainty in scalar form factor $f_0(q^2)$
- R(D) approximately 1σ higher and slightly reduces the tension with experiment
- Lattice-QCD calculation of $R(D^*)$ in progress...

B mixing beyond-the-Standard Model

- QCD contribution to B-mixing parameterized by **matrix elements of ΔB=2 four-fermion effective operators**
- New heavy particles can **change short-distance coefficients** of ΔB=2 operators or **introduce new operator structures**



- **Lattice efforts ongoing to calculate matrix elements for full basis of ΔB=2 operators that appear in beyond-the Standard Model theories** ➜ expect first N_f=2+1 results within a year [Chang *et al.* [FNAL/MILC] PoS LATTICE 2013, 477 (2013); Davies *et al.* [HPQCD] Lattice 2013]
- **Can combine with (model-dependent) perturbative calculation of Wilson coefficients to constrain (or even rule out) extensions of the Standard Model**

Series expansion of B→πℓν form factor

- For most precise $|V_{ub}|$, extend lattice-QCD numerical form-factor data to full kinematic range *(including accurate estimate of uncertainty due to q^2 extrapolation...)*
    - ➡ **Use general properties of analyticity, unitarity, and crossing-symmetry to constrain the shape** [Bourrely *et. al.* (1981); Boyd, Grinstein, & Lebed (1996);...]

- Change variable q^2 to new variable "z" that maps $q^2<(m_B+m_\pi)^2$ (the $B\pi$ production threshold) onto z=[-1,1]
- In terms of z, form factor has simple form:

$$\boxed{f_+(q^2) = \frac{1}{1-q^2/m_{B^*}^2}\sum_{k=0}^{K-1} b_+^{(k)}\left[z^k - (-1)^{k-K}\frac{k}{K}z^K\right]}$$

[Bourrely, Caprini, Lellouch,PRD79 (2009) 013008]

- Bound on coefficients + small range of |z| ➜
  only need few parameters to obtain form factors to high precision

Series expansion of $B \to \pi \ell \nu$ form factor

- For most precise $|V_{ub}|$, extend lattice-QCD numerical form-factor data to full kinematic range *(including accurate estimate of uncertainty due to q^2 extrapolation...)*
  - ➡ **Use general properties of analyticity, unitarity, and crossing-symmetry to constrain the shape** [Bourrely *et. al.* (1981); Boyd, Grinstein, & Lebed (1996);...]

- Change variable q^2 to new variable "z" that maps $q^2<(m_B+m_\pi)^2$ (the $B\pi$ production threshold) onto z=[-1,1]

- In terms of z, form factor has simple form:

Accounts for subthreshold B* pole

$$f_+(q^2) = \frac{1}{1 - q^2/m^2_{B^*}} \sum_{k=0}^{K-1} b_+^{(k)} \left[z^k - (-1)^{k-K} \frac{k}{K} z^K \right]$$

[Bourrely, Caprini, Lellouch, PRD79 (2009) 013008]

- Bound on coefficients + small range of |z| ➜ **only need few parameters to obtain form factors to high precision**

Series expansion of $B\to\pi\ell\nu$ form factor

- For most precise $|V_{ub}|$, extend lattice-QCD numerical form-factor data to full kinematic range *(including accurate estimate of uncertainty due to q^2 extrapolation...)*

  ➡ **Use general properties of analyticity, unitarity, and crossing-symmetry to constrain the shape** [Bourrely *et. al.* (1981); Boyd, Grinstein, & Lebed (1996);...]

- Change variable q^2 to new variable "z" that maps $q^2<(m_B+m_\pi)^2$ (the $B\pi$ production threshold) onto z=[-1,1]

- In terms of z, form factor has simple form:

Accounts for subthreshold B* pole

Unitarity bounds sizes of coefficients $\sum_{j,k=0}^{K} B_{jk}(t_0) b_i^{(j)}(t_0) b_i^{(k)}(t_0) \lesssim 1$

$$f_+(q^2) = \frac{1}{1-q^2/m_{B^*}^2} \sum_{k=0}^{K-1} b_+^{(k)} \left[z^k - (-1)^{k-K} \frac{k}{K} z^K \right]$$

[Bourrely, Caprini, Lellouch, PRD79 (2009) 013008]

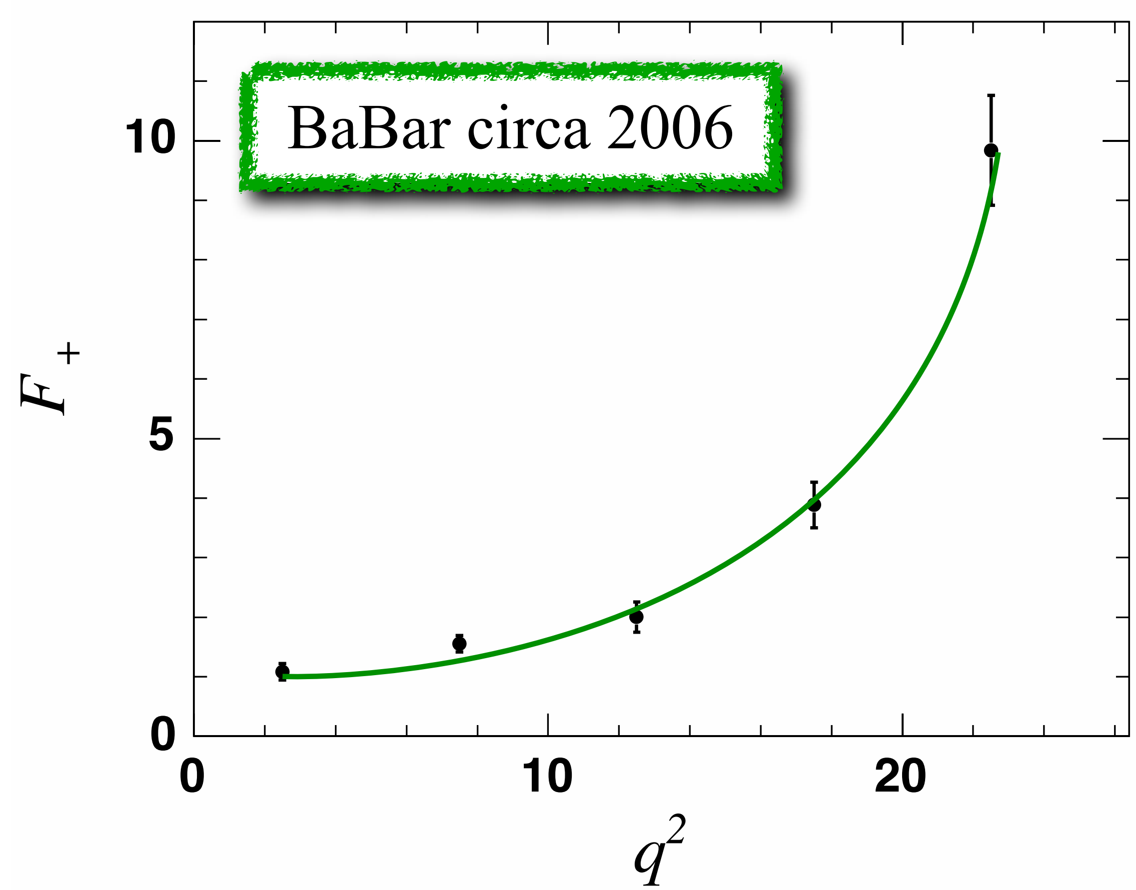
- Bound on coefficients + small range of |z| ➜

  only need few parameters to obtain form factors to high precision

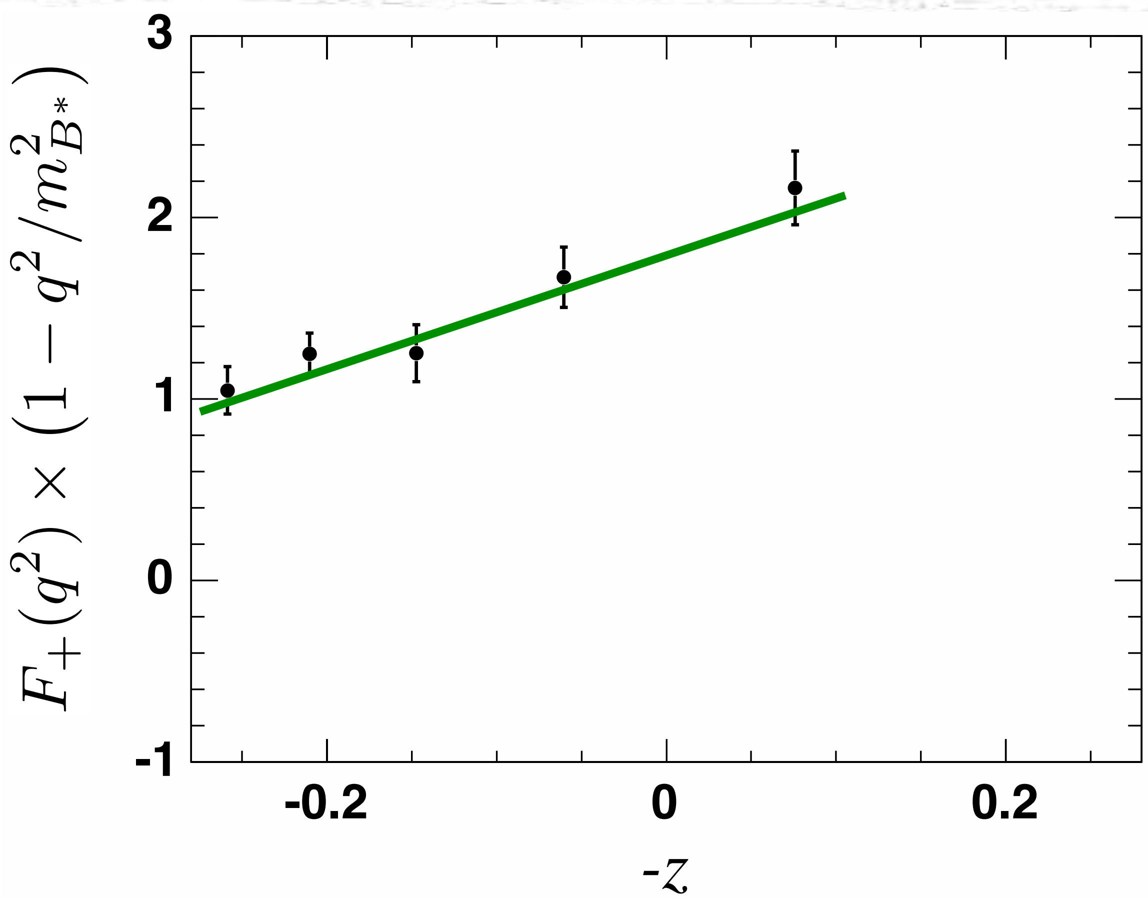
After removing pole and plotting vs. z...

- Curvature in data due primarily to B^* pole
- **Experimental data completely described by a normalization and slope**, and constrains the size of possible curvature

[*from R. Hill*]



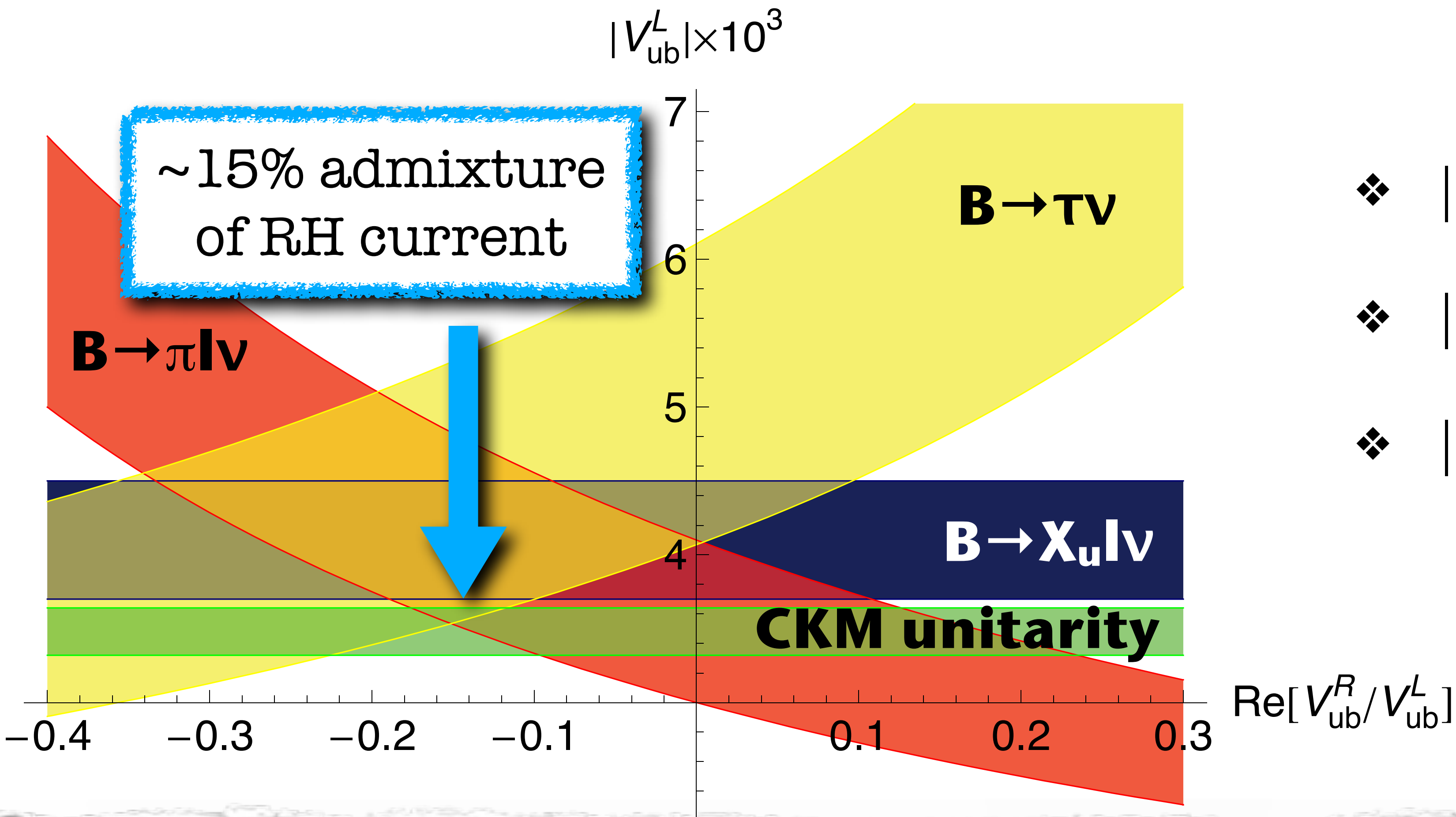
Striking curvature in $B\to\pi l\nu$ form factor data versus q^2



No visible curvature after remapping

Right-handed currents?

- Elegant solution provided by introducing right-handed weak current with coupling V_{ub}^R [Crivellin, Phys.Rev. D81 (2010) 031301], which enters as:



- $|V_{ub}^L+V_{ub}^R|^2$ in $B\to\pi l\nu$
- $|V_{ub}^L-V_{ub}^R|^2$ in $B\to\tau\nu$,
- $|V_{ub}^L|^2+|V_{ub}^R|^2$ in $B\to X_u l\nu$

- In practice, however, cannot realize this scenario in simplest left-right asymmetric models while satisfying other flavor constraints, especially from ϵ_K [Blanke, Buras, Gemmler, Heidsieck,JHEP 1203 (2012) 024]

a_μ^{HVP} from $e^+e^- \to$ hadrons

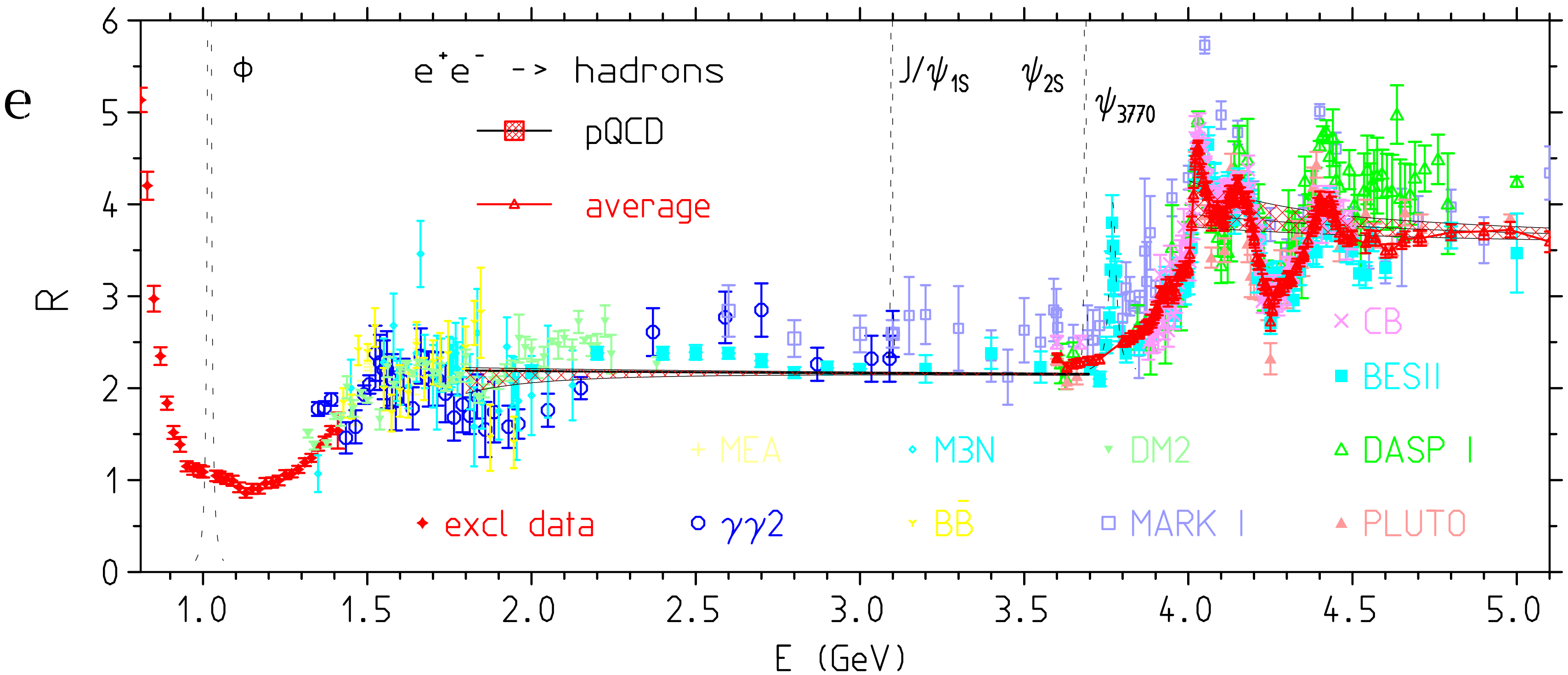
- Standard-Model value for a_μ^{HVP} obtained from experimental measurement of $\sigma_{total}(e^+e^- \to \text{hadrons})$ via optical theorem:

$$a_\mu^{\mathrm{HVP}} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \int_{m_{\pi^0}^2}^{\infty} \mathrm{d}s \frac{R(s)K(s)}{s^2} \qquad R \equiv \frac{\sigma_{\mathrm{total}}(e^+e^- \to \mathrm{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

  - (Away from quark thresholds, use four-loop pQCD)

- Includes >20 multi-particle channels with up to six final-state hadrons

- Multi-hadron channels represent a small absolute contribution to a_μ^{HVP}, but contribute a significant fraction of the total uncertainty



[Jegerlehner and Nyffeler, Phys.Rept. 477 (2009) 1-110]

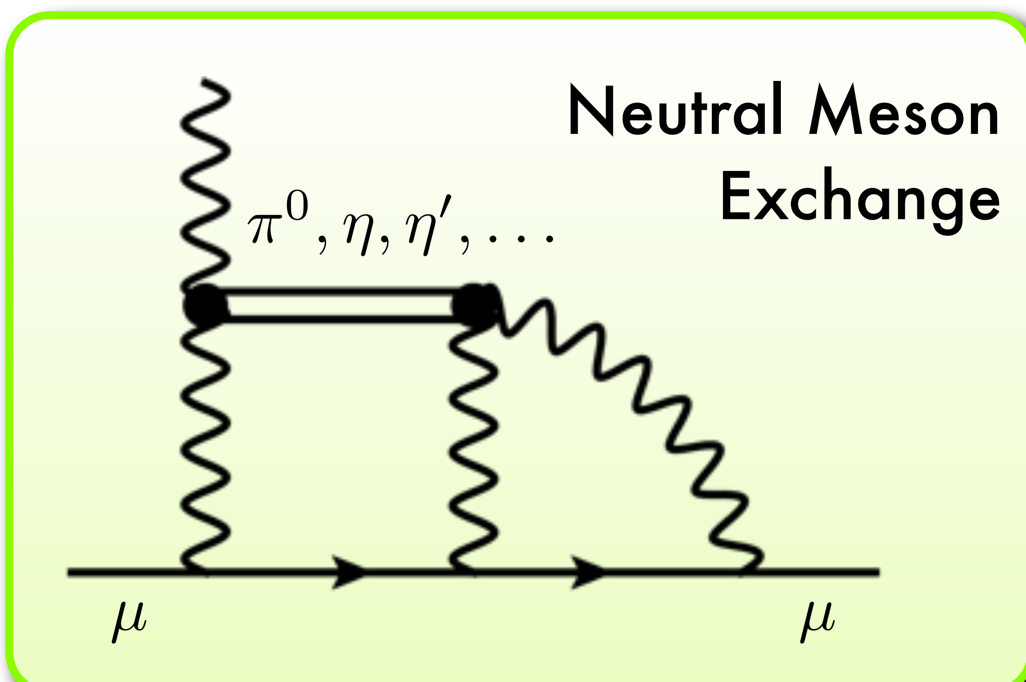
a_μ^{HLbL} from QCD models

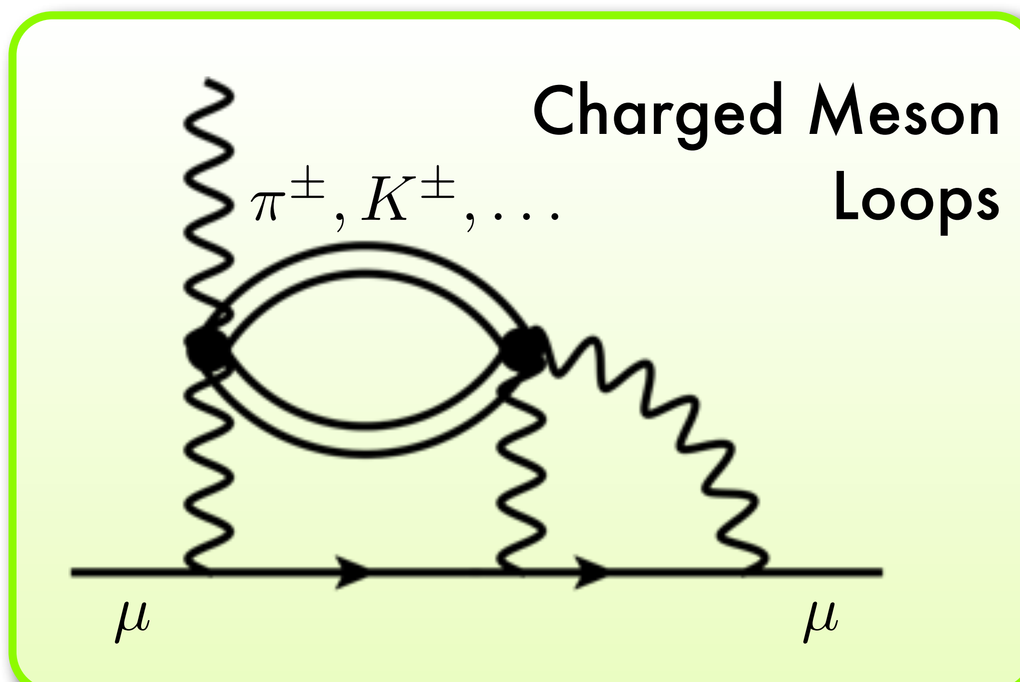
[Jegerlehner & Nyffeler, Phys.Rept. 477 (2009) 1-110; Prades, de Rafael, Vainshtein, 0901.0306]

- Hadronic light-by-light contribution cannot be expressed in terms of experimental quantities and must be obtained from theory: present model estimates report errors in the 25-40% range

- All recent calculations compatible with constraints from large-N_c and chiral limits and normalize dominant π^0-exchange contribution to measured $\pi^0 \to \gamma\gamma$ decay width

  - Differ for form factor shape due to different QCD-model assumptions such as vector-meson dominance, chiral perturbation theory, and the large N_c limit





- Error estimate more subjective than for HVP and somewhat controversial

The Glasgow consensus for a_μ^{HLbL}

[Prades, de Rafael, Vainshtein, 0901.0306]

- Quoted error for a_μ^{HLbL} is based on model estimates, but does not cover spread of values
  - π^0-exchange contribution estimated to be ~10 times larger than others
  - Largest contribution to uncertainty ($\pm 1.9\times 10^{-10}$) attributed to charged pion and kaon loop contributions

Table 1: *Contribution to a^{HLbL} from π^0, η and η' exchanges*

Result	Reference
$(8.5 \pm 1.3) \times 10^{-10}$	[7, 8]
$(8.3 \pm 0.6) \times 10^{-10}$	[4, 5, 6]
$(8.3 \pm 1.2) \times 10^{-10}$	[1]
$(11.4 \pm 1.0) \times 10^{-10}$	[9]

$\Rightarrow$ $a^{\mathrm{HLbL}}(\pi, \eta, \eta') = (11.4 \pm 1.3) \times 10^{-10}$

- **Error estimate more subjective than for HVP and somewhat controversial**

New lattice method for a_μ^{HVP}

[Chakraborty et al. (HPQCD), 1403.1778]

- Sidestep $q^2 \to 0$ extrapolation by expressing a_μ^{HVP} in terms of derivatives of vacuum polarization function $\Pi(q^2)$ at $q^2=0$

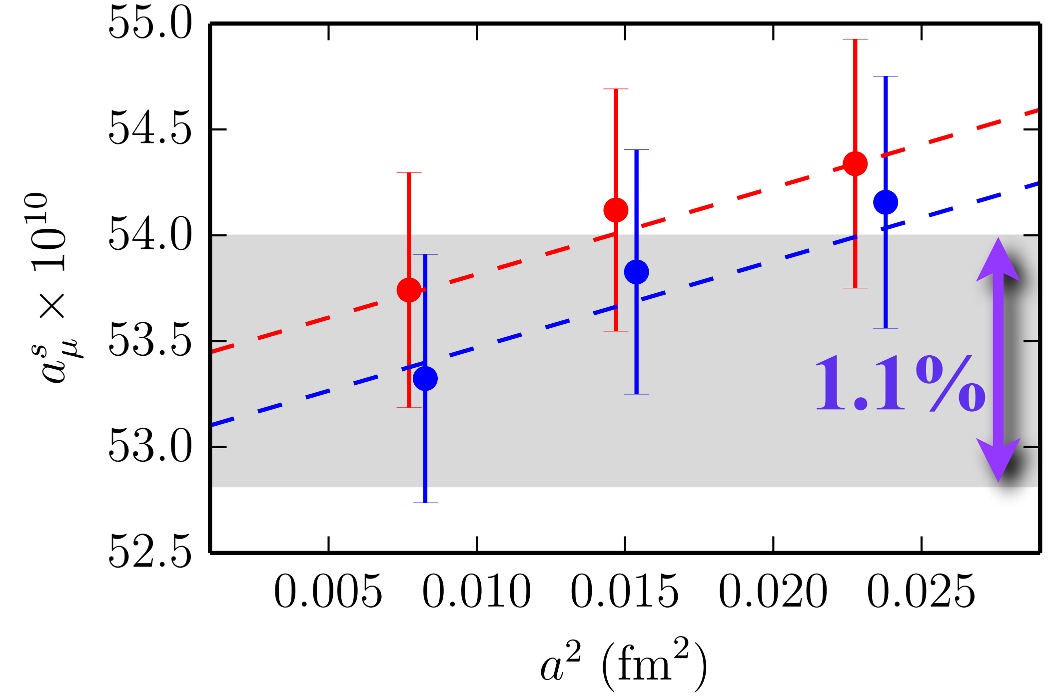
- Derivatives easily computed on lattice to high statistical precision from time-moments of the electromagnetic current-current correlator at $q^2 = 0$

$$
\begin{aligned}
G_{2n} &\equiv a^4 \sum_t \sum_{\mathbf{x}} t^{2n} Z_V^2 \langle j^i(\mathbf{x},t) j^i(\mathbf{0},0)\rangle \\
&= (-1)^n \frac{\partial^{2n}}{\partial q^{2n}} q^2 [\Pi(q^2) - \Pi(0)]\Big|_{q^2=0}
\end{aligned}
$$

- Illustrate method with strange and charm-quark contributions and obtain a_μ^s to ~1%

- Correlator noisier for light quarks, but **estimate that similar precision can be obtained for $a_\mu^{u,d}$ with 10× larger gauge-field ensembles**

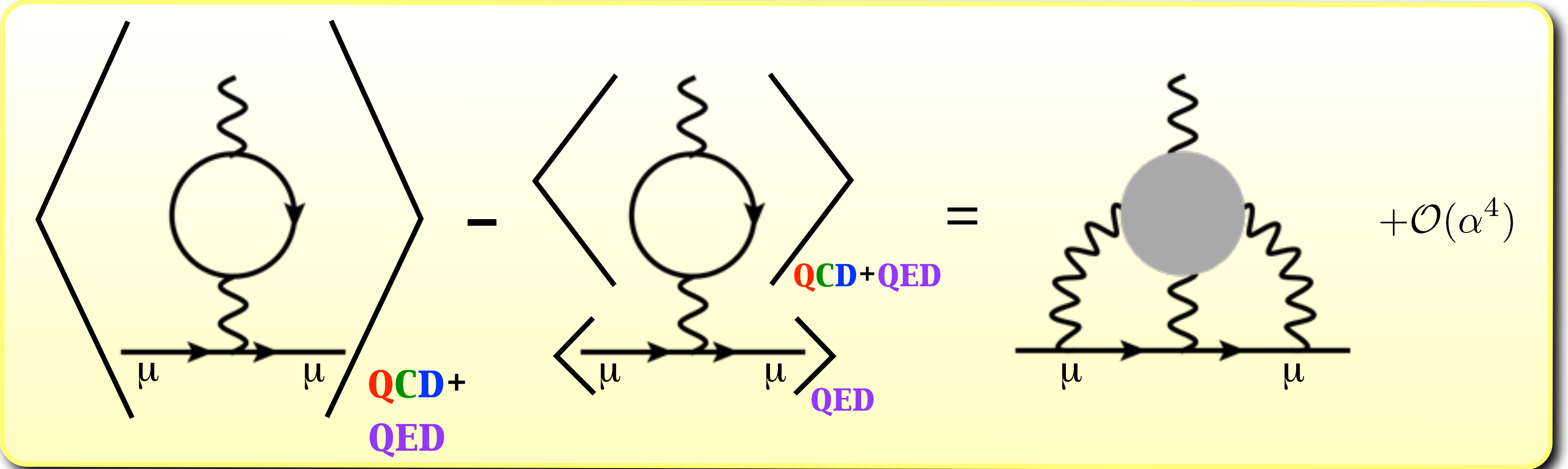
- Beyond ~1%, likely need to directly include EM and isospin-breaking in simulations



	a_μ^s	a_μ^c
lattice spacing (w_0, r_1):	1.0%	0.6%
Uncertainty in Z_V:	0.4%	2.5%
Monte Carlo statistics:	0.1%	0.1%
$a^2 \to 0$ extrapolation:	0.1%	0.4%
QED corrections:	0.1%	0.3%
Quark mass tuning:	0.0%	0.4%
Finite lattice volume:	$< 0.1\%$	0.0%
Padé approximants:	$< 0.1\%$	0.0%
Total:	1.1%	2.7%

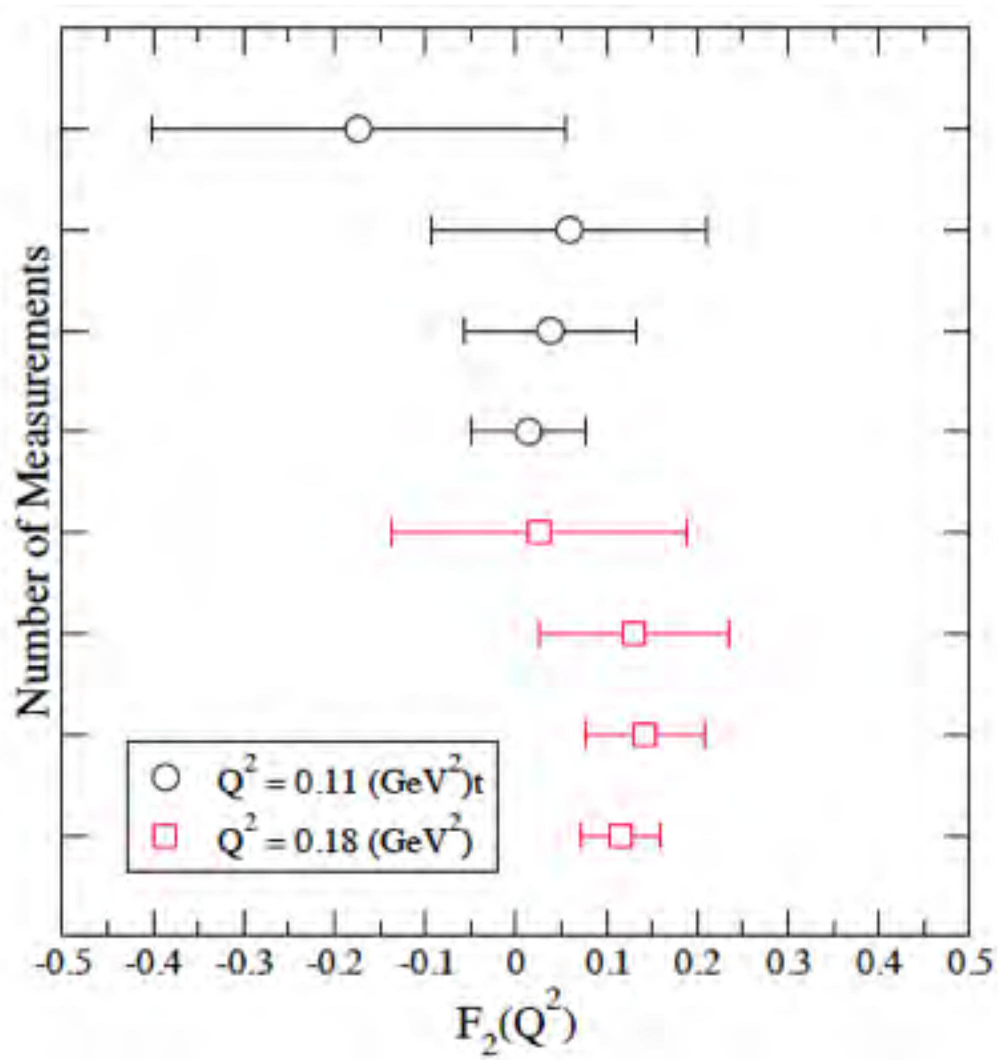
QCD + QED simulations

- Most promising method introduced by Blum and collaborators in which one computes the full hadronic amplitude, including the muon and photons, nonperturbatively [Hayakawa *et al.*, PoS LAT2005 (2006) 353]
- Treat photon field in parallel with gluon field and include in gauge link, so the simulation and analysis follows a conventional lattice-QCD calculation
- In practice, must insert a single valence photon connecting the muon line to the quark loop "by hand" into the correlation function, then perform correlated nonperturbative subtraction to remove the dominant O(α^2) contamination



Preliminary tests

- **Early results appear promising** [Blum *et al.*, PoS LATTICE2012 (2012) 022]
- Stable, statistically-significant signal emerging in the ballpark of model estimates



- $a = 0.114$ fm; $V=(24 \times a)^3$
- $Q^2 = 0.11$ and 0.18 GeV^2
- $m_\pi = 329$ MeV
- $m_\mu = 190$ MeV
- $\alpha=1/4\pi$ to enhance signal

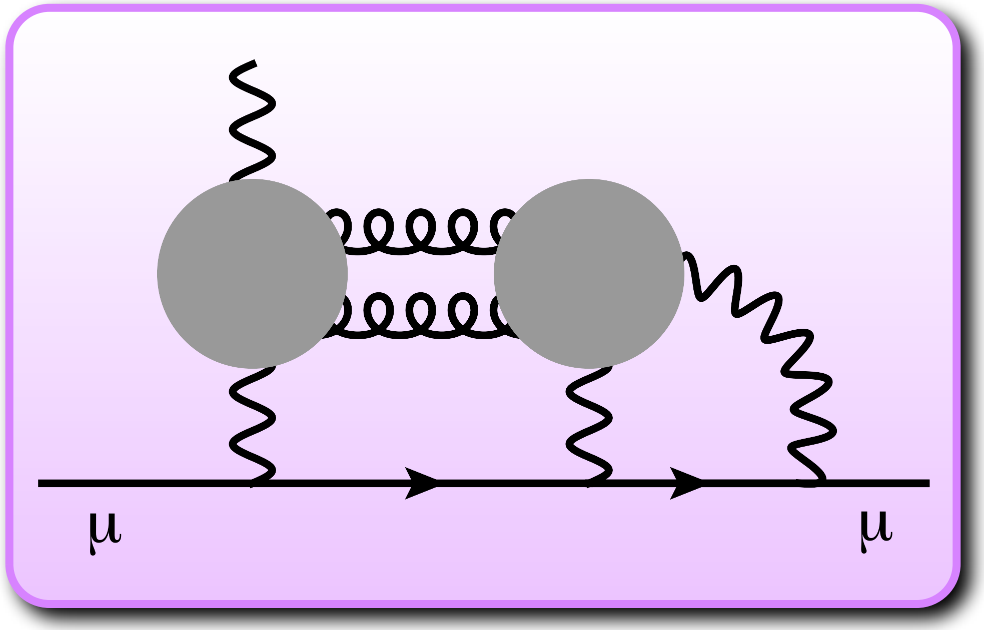
$$a_\mu^{HLbL} = F_2(Q^2 \to 0) \times (\alpha/\pi)^3$$

Other outstanding issues

(1) Finite-volume effects

- QED-only calculations suggest that errors due to the finite lattice size may be significant, but increased computing power is allowing the generation of larger lattices

(2) Quark-disconnected contributions



- Preliminary calculations work in the quenched approximation of QED, so contributions from diagrams with two quark loops only connected by a pair of gluons are not included

- Studying various approaches to include these such as directly simulating dynamical photons

(3) Chiral ($m_q \rightarrow m_q^{phys}$) and continuum ($a \rightarrow 0$) extrapolations

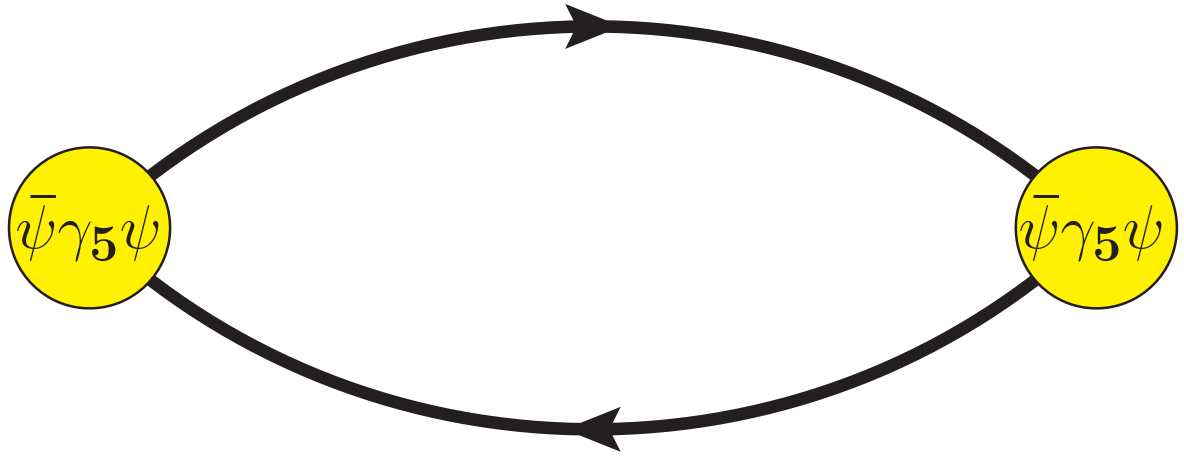
- New large-volume lattices being generated have close-to-physical pion masses

(4) Momentum extrapolation ($Q^2 \rightarrow 0$)

Still quite a bit of work to do...

Heavy-quark masses from lattice QCD

- Most precise determinations of m_c and m_b are obtained by fitting moments of correlation functions of the quarks' electromagnetic current to $O(\alpha_S^3)$ perturbative expressions



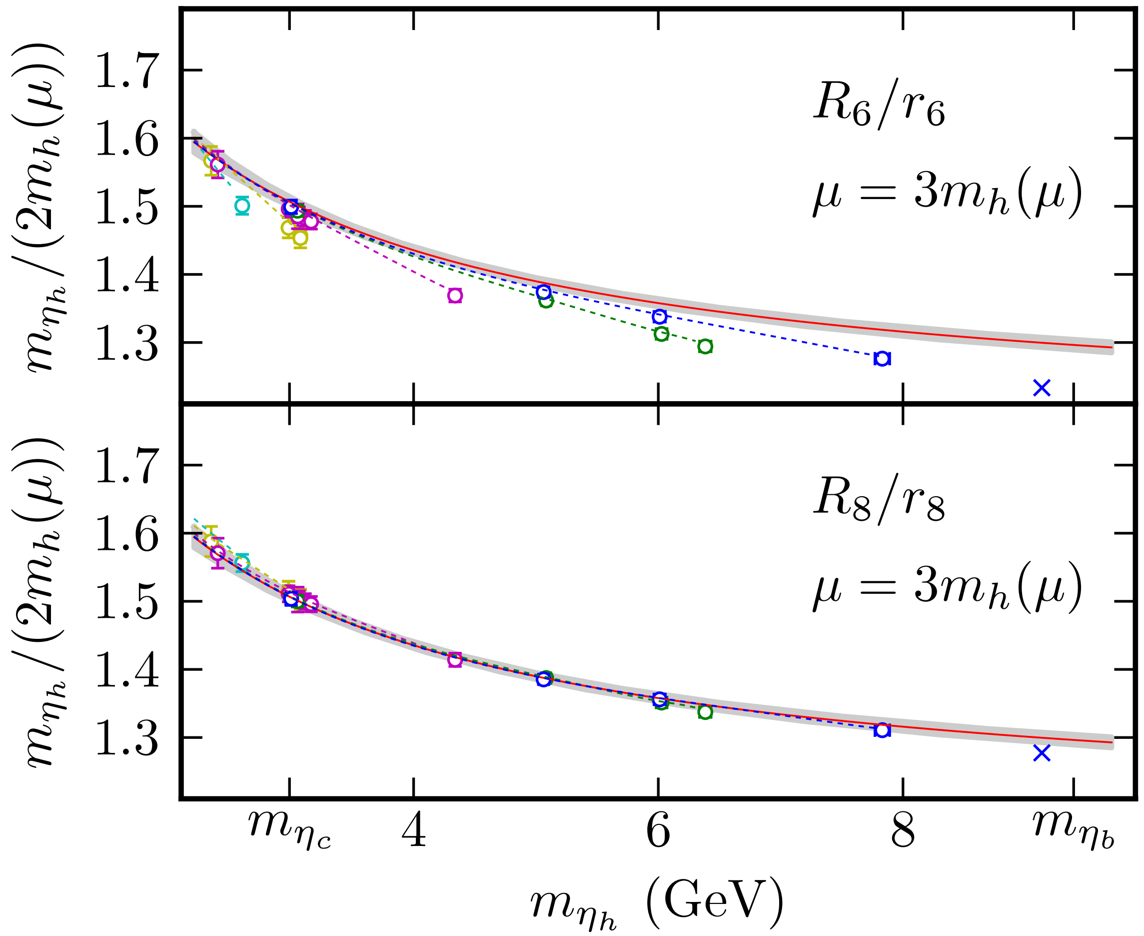
- Such moments can be obtained from experimental e+e- annihilation data [Chetyrkin *et al.* PRD80 (2009) 074010], and also computed numerically with lattice-QCD simulations via

$$G(t) = a^6 \sum_{\mathbf{x}} (am_{0h})^2 \langle 0 | j_5(\mathbf{x}, t) j_5(\mathbf{0}, 0) | 0 \rangle$$

$$G_n \equiv \sum_t (t/a)^n G(t)$$

- Lattice moments have negligible statistical uncertainties, so cleaner than e+e- data

- Lattice simulations can vary quark-mass values between m_c and m_b to control and estimate systematic errors

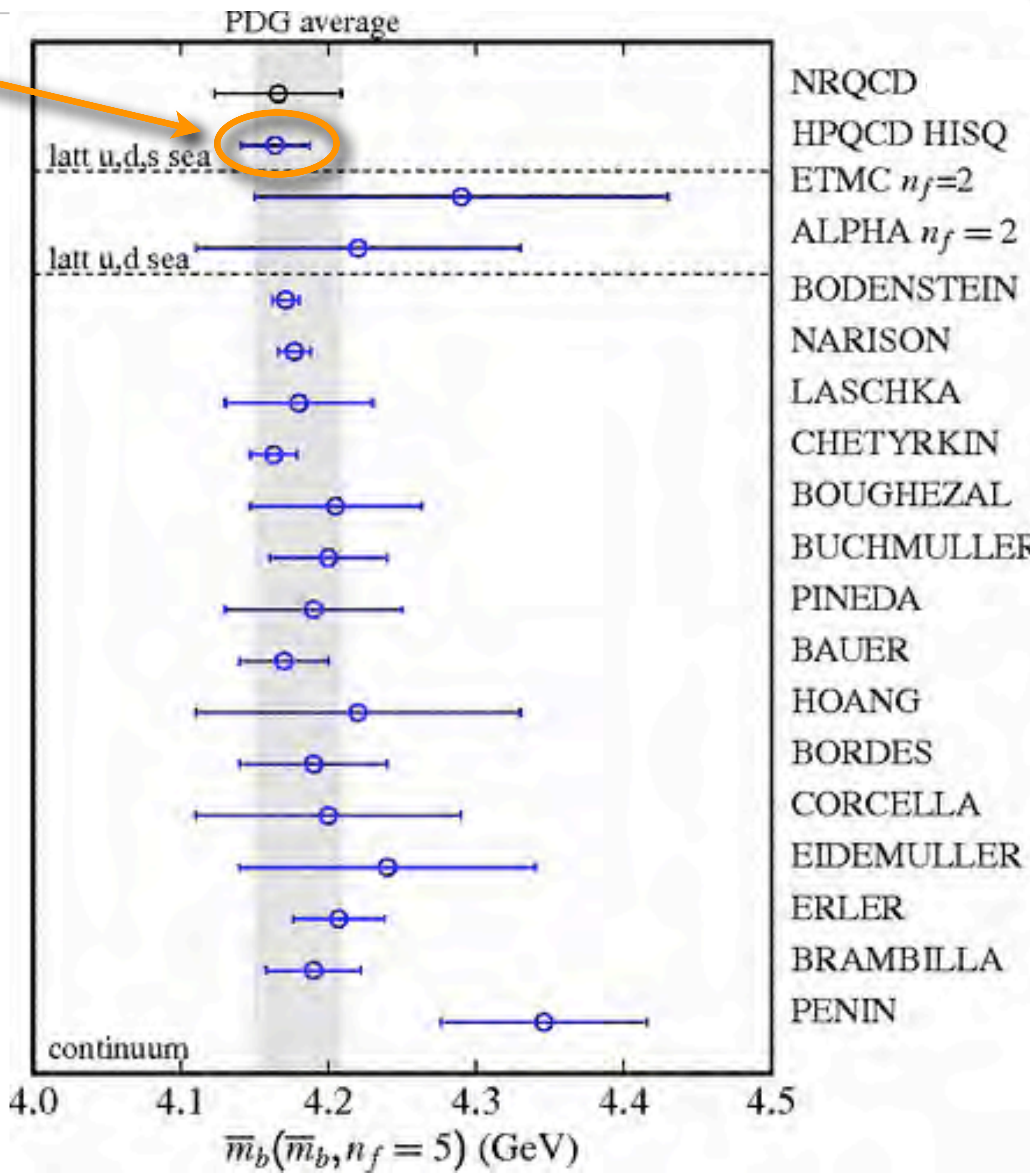


Results for m_b & m_c

[McNeile et al., PRD82 (2010) 034512]

- HPQCD Collaboration obtains sub-percent precision with this method, and **finds good agreement with non-lattice determinations**
- m_c will only improve modestly without higher-order PT calculation
- **m_b will improve significantly with simulations using finer lattice spacings**

	$m_b(10)$	$m_c(3)$
a^2 extrapolation	0.6%	0.2%
perturbation theory	0.1	0.4
statistical errors	0.3	0.2
m_h extrapolation	0.1	0.0
errors in r_1	0.1	0.1
errors in r_1/a	0.3	0.1
errors in m_{η_c},m_{η_b}	0.1	0.0
α_0 prior	0.1	0.1
gluon condensate	0.0	0.2
Total	0.7%	0.6%



[HPQCD, PRD87 (2013) 7, 074018]

Probable U.S. HEP program

(Also ν-less double β-decay & dark-matter detection experiments)

Low Energy ν's (Booster)

μBooNE

MicroBooNE
ν X-sections,
MiniBooNE anomaly

SBNE (LAr on BNB)
sterile neutrinos

ICARUS

Neutrinos
(only Fermilab expts. shown)

High Energy ν's (NuMI/LBNE)

NOvA θ_{13}, mass hierarchy, neutrino mixing

MINERvA ν X-sections

MINOS+ θ_{23}, Δm^2_{32}, sterile νs

LBNE

LBNF
CPV, mass hierarchy,
neutrino mixing,
proton decay,
supernova ν's

Now **Soon** **2020+**

Charged leptons

Muon g-2
(2016)

Mu2e
(2019)