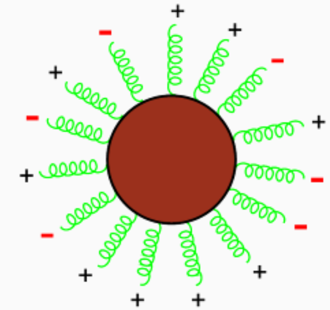

QCD and Monte Carlo
2. Lepton Machines, DIS and PDFs

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HADRONS IN LEPTON COLLIDERS

R ratio, Probabilities, Measurements, NLO, Emerging Jets

DEEP INELASTIC SCATTERING

Structure Funcs, Parton Model, QCD Improved, PDFs, Factorization

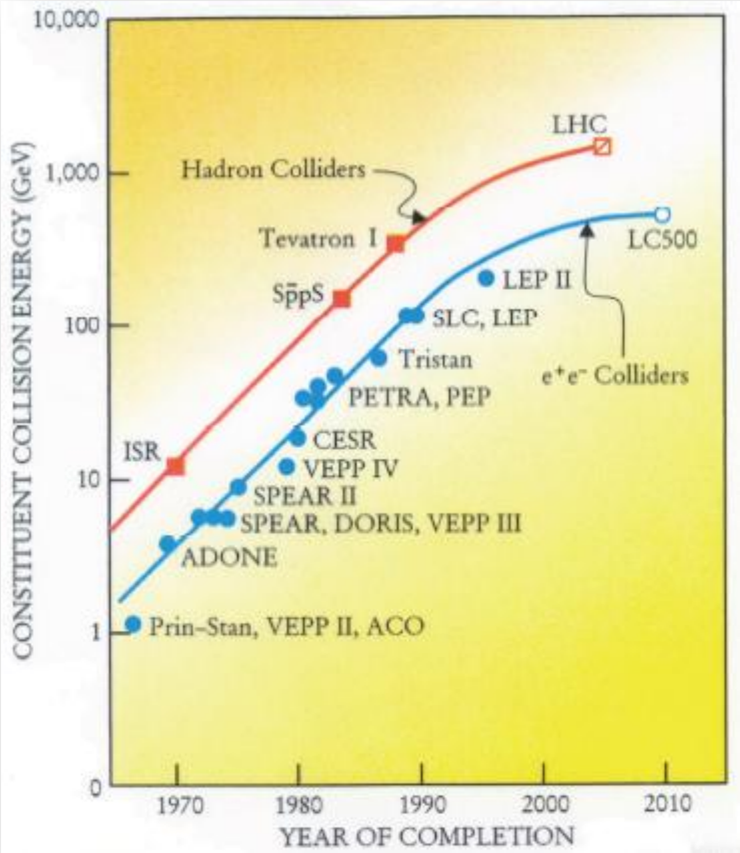
PARTON DISTRIBUTION FUNCTIONS

Global Fits, Q2 vs. x, x Parameterizations, Sets, Uncertainties

Hadron at Lepton Machines

The finding of events with spread of hadrons when reaching energies above the GeV at Lepton-Lepton colliders was a momentous occasion for particle physics

Hadronic Event at TASSO in the PETRA e^+e^- collider



Energy Historical Progress at Colliders

Hadrons at Lepton Machines

We are interested in studying the mechanism for production of these hadrons. Yet (light) hadrons are inherently QCD non-perturbative finite-size objects, how can we compute their rate of production?!

Interestingly, it is found that the production mechanism mimics closely a process like $e^+e^- \rightarrow \mu^+\mu^-$

Consider the following important argument!

- Hadrons are composite particles (quarks and gluons form them)
- When an electron-positron pair annihilates they form a photon of virtuality Q that can fluctuate into a quark-antiquark pair
- Subsequently at energy scales of the order of the QCD scale Λ, hadrons are formed via *hadronization*
- We expect these two processes to occur at time scales of the order $\tau_1 \sim (1/Q) << \tau_2 \sim (1/\Lambda)$, as long as $Q >> \Lambda \sim 400$ MeV

So, when we compute probabilities to produce quarks in e^+e^- annihilation, although the interactions that confine quarks and gluons into hadrons can modify the outgoing state, they occur too late to modify the overall probability!

We can employ perturbation theory to obtain meaningful cross sections!

Producing Colored Partons Perturbatively

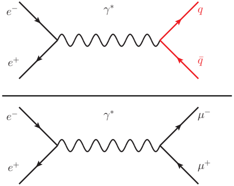
Let's introduce the observable:

$$R = \frac{\sigma(e^+e^- \to \mathrm{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} \tag{1}$$

which easily can be studied experimentally at an e^+e^- collider.
We first will approximate R by:

$$R = \frac{\sigma(e^+e^- \to q\bar{q})}{\sigma(e^+e^- \to \mu^+\mu^-)} \tag{2}$$

We will keep only photon exchange processes, effectively thinking that we do not excite Z exchanges. Pictorially we represent it like:

$R =$ 

$$= \delta_{ij}\delta_{ji}\sum_q Q_q^2 = N\sum_q Q_q^2 \tag{3}$$

At LO in QCD R is a constant which depends on the number of *active* quarks!

Hadrons at e^+e^- Colliders

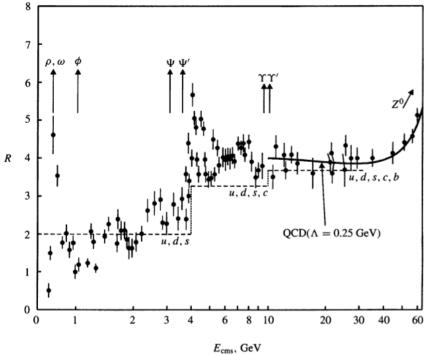
For example, for energies below the *charm* threshold and above the *bottom* threshold we would have:

$$R(\sqrt{s} \lesssim 3GeV) = 3\left(2\left(\frac{1}{3}\right)^2 + \left(\frac{2}{3}\right)^2\right) = 2$$

$$R(\sqrt{s} \gtrsim 10GeV) = 3\left(3\left(\frac{1}{3}\right)^2 + 2\left(\frac{2}{3}\right)^2\right) = \frac{11}{3}$$

Notice

- Qualitative agreement
- Non-perturbative effects
- QCD improvements
- *Z* boson *tail*

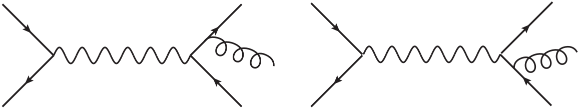


$e^+e^- \to q\bar{q}g$

Let's study the mechanism of production of three partons $q\bar{q}g$.

$\sigma(e^+e^- \to q\bar{q})$ is $\mathcal{O}(\alpha_s^0)$, while $\sigma(e^+e^- \to q\bar{q}g)$ is $\mathcal{O}(\alpha_s^1)$.

$\mathcal{A}_{e^+e^- \to q\bar{q}g} \quad \to$



The color factor of these two diagrams are the same. So when we square the amplitude, we get the color factor:

$$\mathrm{Tr}(t^A t^A) = t^A_{ij} t^A_{ji} = C_F N \qquad (4)$$

For this special case, color factorizes from the full amplitude!

After a little of algebra, we find:

$$\overline{\sum} |\mathcal{A}_{e^+e^- \to q\bar{q}g}|^2 = C_F N \frac{16 e^4 Q_q^2 g_s^2}{s(p_q \cdot p_g)(p_{\bar{q}} \cdot p_g)}$$
$$\times \left((p_q \cdot p_{e^+})^2 + (p_q \cdot p_{e^-})^2 + (p_{\bar{q}} \cdot p_{e^+})^2 + (p_{\bar{q}} \cdot p_{e^-})^2 \right) \qquad (5)$$

Useful Variables for Phase Space

An n particle Lorentz Invariant Phase Space can be written as:

$$d\mathrm{LIPS}_n = \prod_{i=1}^{n} \frac{d^4 p_i}{(2\pi)^4} (2\pi)\delta_+(p_i^2 - m_i^2)(2\pi)^4 \delta^{(4)}(P_{ini} - \sum_j p_j)$$
$$= \prod_{i=1}^{n} \frac{d^3 p_i}{(2\pi)^3} \frac{1}{2E_i} (2\pi)^4 \delta^{(4)}(P_{ini} - \sum_j p_j) \quad (6)$$

Let's explore the behavior of the process $e^+e^- \to q\bar{q}g$ over phase space!

$$d\mathrm{LIPS}_3 = \frac{d^3 p_q}{(2\pi)^3} \frac{1}{2E_q} \frac{d^3 p_{\bar{q}}}{(2\pi)^3} \frac{1}{2E_{\bar{q}}} \frac{d^3 p_g}{(2\pi)^3} \frac{1}{2E_g} (2\pi)^4 \delta^{(4)}(p_{e^+e^-} - p_q - p_{\bar{q}} - p_g)$$

Employing the delta function, we can reduce our integration variables to 5. It is convenient to parametrize them as follows:

$$d\mathrm{LIPS}_3 = \frac{s}{32(2\pi)^5} \underbrace{d\alpha\, d\cos\beta\, d\gamma}_{\text{Euler angles}} \underbrace{dx_1 dx_2}_{x_{1,2(,3)} = 2E_{q,\bar{q}(,g)}/\sqrt{s}} \quad (7)$$

$e^+e^- \to q\bar{q}g$ Phase Space Integration

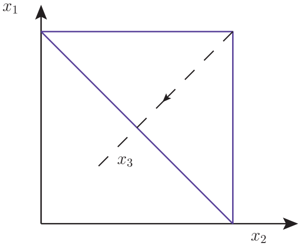
Euler angles can be integrated out, and we find:

$$\sigma_{q\bar{q}g} = \sigma_{q\bar{q}} C_F \frac{\alpha_s}{2\pi} \int dx_1 dx_2 \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)} \qquad (8)$$

where from the kinematical relations $E_q + E_{\bar{q}} + E_g = \sqrt{s}$, $0 \leq 2p_q \cdot p_{\bar{q}} = s - 2p_g \cdot (p_q + p_{\bar{q}}) = s - 2E_k\sqrt{s} = s(1 - x_3)$, we can extract the integration boundaries:

$$0 \leq x_1, x_2 \leq 1 \;, \qquad x_1 + x_2 \geq 1 \qquad (9)$$

Notice $x_1 + x_2 + x_3 = 2$. In Eq. (8) the two dimensional integral $dx_1 dx_2$ cover the blue triangle:



The denominators $(1 - x_1)(1 - x_2)$ lead to a divergent integral!

Seems familiar??

$e^+e^- \to q\bar{q}g$ Over Phase Space

Our familiar *collinear* (and *soft*) regions!

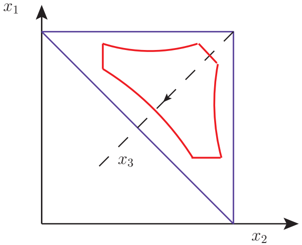
$$1-x_1 = \frac{1}{\sqrt{s}} x_2 E_g (1-\cos\theta_{2g})), \quad 1-x_2 = \frac{1}{\sqrt{s}} x_1 E_g (1-\cos\theta_{1g}) \quad (10)$$

But what happens is that when any of the separation angles (or E_g) become small, we would expect the event to be characterized as a two-*object* events.

In 1977 Sterman and Weinberg proposed (the birth of the jet algorithms!): *if you can draw two cones of half angle δ such that all but a fraction ϵ of the total energy is contained in them, classify the configuration as a two-jet-like event*

Breaking PS

- If outside red area, two-jet-like
- Inside red area, *hard* three-jet event
- See more later on modern jet algorithms



$e^+e^- \to q\bar{q}g$ as a Three-Jet Process

When restricting our attention to events with three jets, we can easily compute

$$\sigma_{q\bar{q}g} = \sigma_{q\bar{q}} C_F \frac{\alpha_s}{2\pi} \int_{\mathrm{A_{3j}}} dx_1 dx_2 \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)} \tag{11}$$

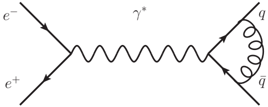
Where $\mathrm{A_{3j}}$ is the three jet area in $x_1 x_2$, which have trimmed all divergent collinear regions.

This would be our (finite!) parton level $\mathcal{O}(\alpha_s)$ LO prediction to the production of three jets at an e^+e^- collider!

$e^+e^- \to q\bar{q}g$ (*real*) contributions to the Two Jet Process

Basically $e^+e^- \to q\bar{q}g$ can be seen as an $\mathcal{O}(\alpha_s)$ correction to the two-jet process $e^+e^- \to q\bar{q}$. A real-emission correction.

But at this order, virtual one-loop diagrams also contribute:



$+ \quad \cdots$

One-loop diagrams are also divergent!

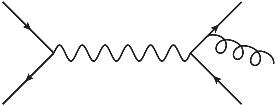
The Kinoshita-Lee-Nauenberg (KLN) Theorem

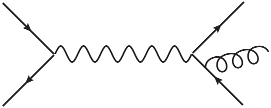
IR divergences in physical observables, at each order in perturbation theory, cancel when summing over (all) degenerate states

Now we dimensionally regulate the $d\mathrm{LIPS}_3$ of the real contributions, either including the $\mathrm{A}_{3\mathrm{j}}$ hard three-jet region (*inclusive*) or not (*exclusive*), and compare to the corresponding virtual contribution!

$\mathcal{O}(\alpha_s)$ QCD Corrections to R

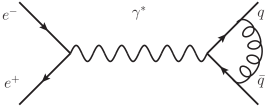
Real contributions:





$$R_{\text{real}} = R_{LO} \times \frac{\alpha_s}{2\pi} C_F \left(\frac{s}{4\pi\mu^2}\right)^{-\epsilon} \left\{ \frac{2}{\epsilon^2} + \frac{3}{\epsilon} + \frac{19}{2} + \mathcal{O}(\epsilon) \right\}$$

Virtual (renormalized) contributions:



$+ \quad \cdots$

$$R_{\text{virt}} = R_{LO} \times \frac{\alpha_s}{2\pi} C_F \left(\frac{s}{4\pi\mu^2}\right)^{-\epsilon} \left\{ -\frac{2}{\epsilon^2} - \frac{3}{\epsilon} - 8 + \mathcal{O}(\epsilon) \right\}$$

From which we obtain the finite NLO QCD correction to R:

$$R_{NLO} = R_{LO} \times \left\{ 1 + \frac{3}{2} C_F \frac{\alpha_s(\mu)}{2\pi} \right\} \qquad (12)$$

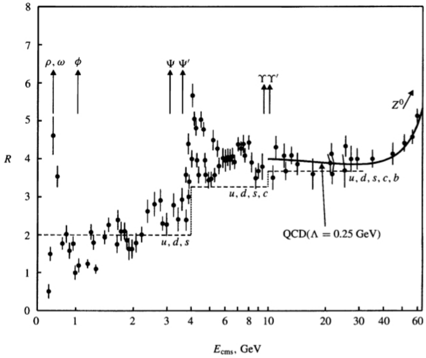
R at NLO QCD

$$R_{NLO} = R_{LO} \times \left\{ 1 + \frac{3}{2} C_F \frac{\alpha_s(\mu)}{2\pi} \right\} \tag{13}$$

QCD increases slightly the value of R. Above the *bottom* threshold this is approximately a factor of 1.11.

Notice

- Better quantitative agreement
- QCD enhances the cross section
- Still non-perturbative effects relevant

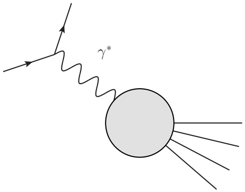


Brief Recount

- Whether regions of phase space of the process $e^+e^- \to q\bar{q}g$ contribute to two-jet or three-jet-like events has to be decided on the basis of a jet definition
- Modern jet algorithms will be reviewed later
- IR safety of definitions (to assure cancellation of divergences) is fundamental
- In general, one *defines* first the process of interest and this settles subprocess contributions at all orders
- Collinear and Soft divergences will appear in real contributions and will cancel with corresponding virtual contributions

What if we want to describe Hadron Production?

Directly from our QCD Lagrangian we would have a hard time. But nevertheless, we can indeed parameterize the interaction of the *blob* with the photon current!



Contract the lepton current with a Lorentz tensor (vector) L^μ to write the amplitude:

$$\mathcal{A}_{e^+e^-\to\text{hadrons}} = e\overline{u}(p_{e^-})\gamma_\mu \overline{v}(p_{e^+})\frac{-g^{\mu\nu}}{q^2}L_\nu(q;p_1,\cdots,p_n) \quad (14)$$

$$\Downarrow$$

$$\begin{aligned}\sigma_{e^+e^-\to\text{hadrons}} &\propto \mathrm{Tr}\left(\not{p}_{e^+}\gamma^\mu \not{p}_{e^-}\gamma^\nu\right) \qquad (15)\\ &\times \sum \int d\mathrm{LIPS}_n L_\mu(q;p_1,\cdots,p_n)L^*_\nu(q;p_1,\cdots,p_n)\\ &\equiv \mathrm{Tr}\left(\not{p}_{e^+}\gamma^\mu \not{p}_{e^-}\gamma^\nu\right) H_{\mu\nu}(q)\end{aligned}$$

Constrain the form of the $H_{\mu\nu}(q)$ Tensor

Based on Lorentz invariance we write:

$$H_{\mu\nu}(q) = A(q^2)g_{\mu\nu} + B(q^2)q_\mu q_\nu \tag{16}$$

Which by Ward identity (gauge invariance) we can simplify:

$$0 = q^\mu H_{\mu\nu}(q) = A(q^2)q_\nu + q^2 B(q^2)q_\nu \;\Rightarrow H_{\mu\nu} = A(q^2)\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) \tag{17}$$

The scalar function $A(q^2)$ ends up parameterizing the hadronic current. It can be interpreted as a function of the structure of the final hadronic state.

Although inherently non-perturbative, we could learn about $A(q^2)$ from experiments!

This reasoning is helpful to study hadronic τ decays, and also it is our bridge to study processes with initial-state hadrons!

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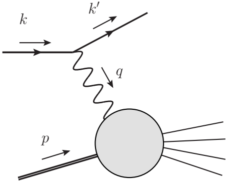
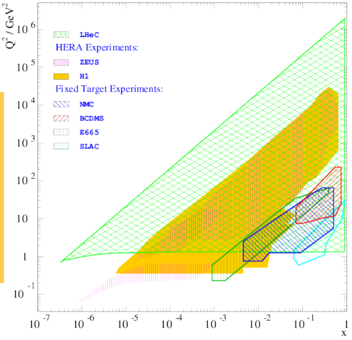
Standard Deep-Inelastic-Scattering (DIS) Variables



- $Q^2 = -q^2 > 0$
- $\nu = p \cdot q$
- $x = \frac{Q^2}{2p \cdot q}$
- $y = \frac{q \cdot p}{k \cdot p}$



Notice the kinematical constraints:

$$Q^2 < s \ , \qquad x > \frac{Q^2}{s} \tag{18}$$

We will be assuming that the incoming proton is massless (high-energy limit).

The *Bjorken limit* is defined by $Q^2, \nu \to \infty$ with x fixed

DIS Cross Section

Following our discussion for hadronic production at e^+e^- colliders, we parameterize the DIS cross section as:

$$d\sigma = \frac{\pi Q^2}{2s^2x^2} dQ^2 dx \sum_Y \int d\mathrm{LIPS}_Y \frac{1}{4} \sum |\mathcal{A}|^2 \qquad (19)$$

Y representing the hadronic final states. Even more:

$$L^{\mu\nu} H_{\mu\nu} = \frac{e^4}{Q^4} \mathrm{Tr}\left(\not{k}' \gamma^\mu \not{k} \gamma^\nu\right) H_{\mu\nu} = \sum_Y \int d\mathrm{LIPS}_Y \frac{1}{4} \sum |\mathcal{A}|^2$$

And based on Lorentz invariance and gauge symmetry we constrain $H_{\mu\nu}$ as follows:

$$H_{\mu\nu} = -g_{\mu\nu} h_1 + \frac{p_\mu p_\nu}{Q^2} h_2 + \frac{q_\mu q_\nu}{Q^2} h_3 + \frac{p_\mu q_\nu + p_\nu q_\mu}{Q^2} h_4 \qquad (20)$$

But after contracting we need only two functions

$$L^{\mu\nu} H_{\mu\nu} = 4k \cdot k' H_1 + 4 \frac{(p \cdot k)(p \cdot k')}{Q^2} H_2$$

DIS Structure Functions

Define $H1 = 4\pi F_1$ and $H_2 = 8\pi x F_2$ and then we find:

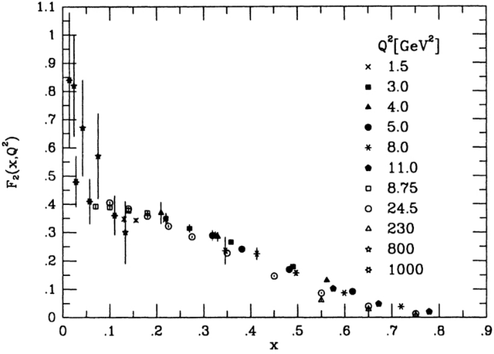
$$\frac{d^2\sigma}{dxdQ^2} = \frac{4\pi\alpha}{Q^4}\left[y^2 F_1(x,Q^2) + \frac{(1-y)}{x}F_2(x,Q^2)\right] \tag{21}$$

The functions $F_1(x,Q^2)$ and $F_2(x,Q^2)$ are known as *structure functions*: They parameterize the structure of the incoming proton

The structure functions are observed to obey an approximate *scaling law* in the Bjorken limit: $F_i(x,Q^2) \rightarrow F_i(x)$

Point-like constituents!

Otherwise a characteristic scale $1/Q_0$ should be evident in the F_i's



A simple model for Structure Functions

$$\frac{d^2\sigma}{dxdQ^2} = \frac{4\pi\alpha}{Q^4}\left[(1-(1-y))^2 F_1 + \frac{(1-y)}{x}(F_2 - 2xF_1)\right] \quad (22)$$

Suppose now that the DIS process proceeds through an interaction with a constituent with momentum $p_q = \xi p$. A simple calculation returns:

$$\frac{d\hat{\sigma}(e^-(k)q(p_q) \to e^-(k')q(p'_q))}{dQ^2} = \frac{2\pi\alpha^2 Q_q^2}{Q^4}(1+(1-y)^2) \quad (23)$$

And imposing the mass-shell condition on the outgoing q:

$$p_q'^2 = q^2 + 2p_q \cdot q = -2p \cdot q(x-\xi) = 0 \quad (24)$$

allows to write the distribution:

$$\frac{d\hat{\sigma}}{dxdQ^2} = \frac{4\pi\alpha^2}{Q^4}(1+(1-y)^2)\frac{1}{2}Q_q^2\delta(x-\xi) \quad (25)$$

$$\boxed{\hat{F}_2 = xQ_q^2\delta(x-\xi) = 2x\hat{F}_1}$$

The Naive Parton Model

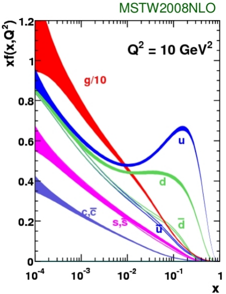
Of course, data shows that F_2 is not a delta function, so the *quark* constituents should carry a range of momenta

The Parton Model of Hadrons

- Introduce a function $q(\xi)$, such that $q(\xi)d\xi$ represents the probability that a quark q carries a momentum fraction of the hadron between ξ and $\xi + d\xi$
- Assume that the virtual photon scatters incoherently off the quark constituents

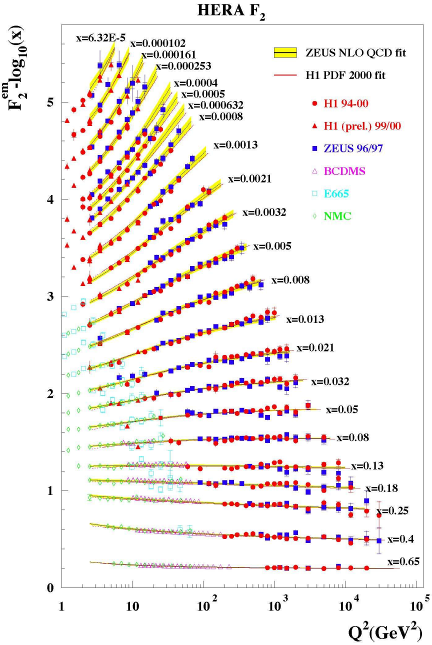
- $F_2(x) = \sum_{q,\bar{q}} \int d\xi q(\xi) x Q_q^2 \delta(x - \xi)$
- $F_2^{ep} = x(\frac{4}{9}(u+\bar{u}+c+\bar{c})+\frac{1}{9}(d+\bar{d}+s+\bar{s}))$
- $F_2^{\nu p} = 2x(d + s + \bar{u} + \bar{c})$
- $\cdots$ (more processes)
- $2xF_1 = F_2$

$\Rightarrow$ *invert*



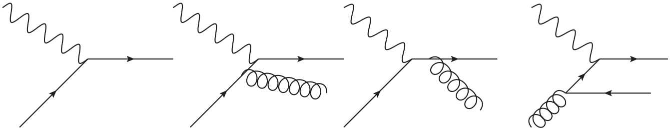
Breaking of Bjorken Scaling

- Careful study of structure functions show explicit dependence on Q^2
- For x below ~ 0.01, F_2 shows a positive derivative with respect to Q^2
- For x above ~ 0.3, F_2 shows a negative derivative with respect to Q^2
- Bjorken scaling breaking can't be explained within the naive parton model



QCD Improved Parton Model

In the context of QFT a more formal description of DIS gives the basis for the so called QCD Improved Standard Model



Quantum QCD corrections are key when studying virtual photon interactions with hadron constituents

These diagrams bring about divergences, which are reabsorbed at a scale μ_f, the *factorization scale*, into *bare* distributions for the quarks and the gluon. It is found:

$$F_2(x, Q^2) = x \sum_{q,\bar{q}} Q_q^2 \int_x^1 \frac{d\xi}{\xi} f_q(\xi, \mu_f^2) \times \left[\delta\left(1 - \frac{x}{\xi}\right) + \frac{\alpha_s}{2\pi} \underbrace{P\left(\frac{x}{\xi}\right)}_{\text{Splitting}} \log \frac{Q^2}{\mu_f^2} + \underbrace{\cdots}_{\mathcal{O}(\alpha_s^2)} \right] \tag{26}$$

Parton Distribution Functions

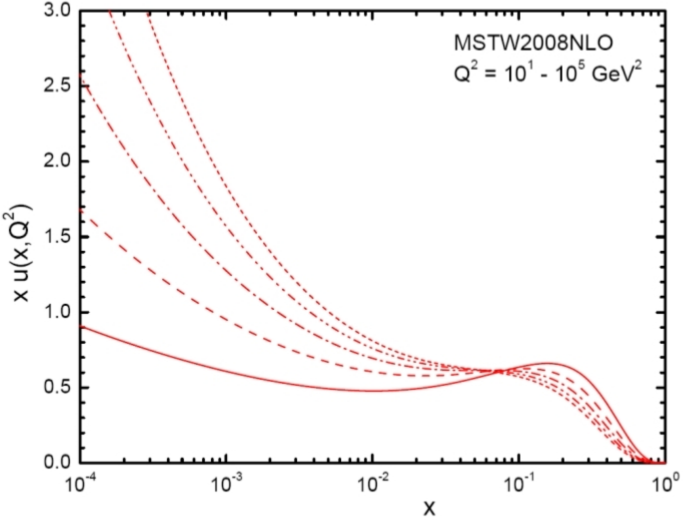
The functions $f_i(\xi, \mu_f^2)$ are defined as the *parton distribution functions* (not *density*!).

The scale dependence of the $f_i(\xi, \mu_f^2)$ can be derived from perturbative QCD, in a similar way to the running of $\alpha_s(\mu)$.

$$\mu^2 \frac{\partial}{\partial \mu^2} f_i(x, \mu^2) = \frac{\alpha_s(\mu^2)}{2\pi} \sum_j \int_x^1 \frac{dy}{y} f_j(y, \mu^2) P_{ij}(x/y, \alpha_s(\mu^2)) \tag{27}$$

This is the so called *DGLAP equation*. The $f_i(x, \mu_0^2)$ can be determined from fits to data (and in principle from lattice QCD calculations also)

u Quark PDF Evolution in Q^2



Partonic Cross Section in Perturbation Theory

$$\hat{\sigma}(\alpha_s, \mu_F, \mu_R) \;=\; [\alpha_s(\mu_R)]^{n_\alpha}\left[\hat{\sigma}^{(0)} + \frac{\alpha_s}{2\pi}\hat{\sigma}^{(1)}(\mu_F, \mu_R) + \left(\frac{\alpha_s}{2\pi}\right)^2 \hat{\sigma}^{(2)}(\mu_F, \mu_R) + \cdots\right]$$

LO NLO NNLO

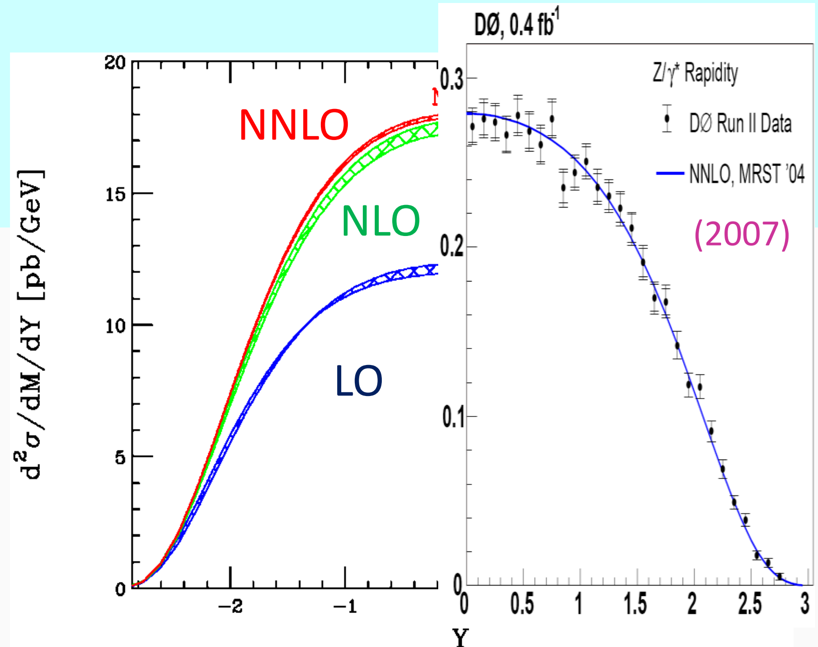
We can make a perturbative calculation of the hard interaction based on the parton constituents of the initial-state hadrons

Example: Z production at the Tevatron
Distribution in rapidity Y

$$Y = \frac{1}{2}\ln\left(\frac{E+p_z}{E-p_z}\right)$$

$\frac{d\sigma}{dY}$ has $n_\alpha = 0$

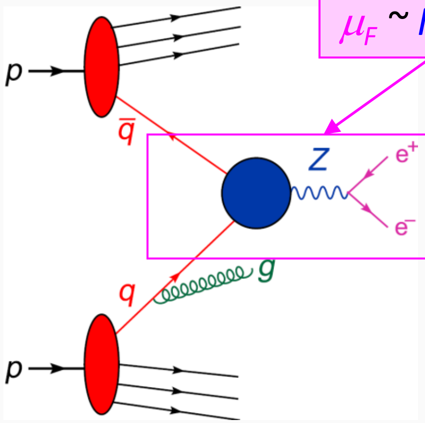
still ~50% corrections, LO → NLO



[Anastasiou, Dixon, Melnikov, Petriello hep-ph/0312266]

QCD Factorization of hadron-hadron cross sections

- PDFs are universal functions: extracted from DIS and used for hadron-hadron machines
- Asymptotic freedom guarantees that at short distances (large transverse momenta), partons in the proton are almost free.
- Sampled "one at a time" in hard collisions.
  → QCD-improved parton model
  → Proven to all orders for Drell-Yan Processes (Collins, Soper, Sterman)



infrared safe final state

Parton distribution functions – known to ~ 4% for x ~ 0.01-0.1

factorization scale

$$\sigma^{pp\to X}(s;\alpha_s,\mu_R,\mu_F) = \sum_{a,b}\int_0^1 dx_1 \int_0^1 dx_2\, f_a(x_1,\alpha_s,\mu_F)\, f_b(x_2,\alpha_s,\mu_F) \times \hat{\sigma}^{ab\to X}(sx_1x_2;\alpha_s,\mu_R,\mu_F)$$

Partonic cross section, computable in perturbative QCD

partonic CM energy²

renormalization scale

HADRONS IN LEPTON COLLIDERS

R ratio, Probabilities, Measurements, NLO, Emerging Jets

DEEP INELASTIC SCATTERING

Structure Funcs, Parton Model, QCD Improved, PDFs, Factorization

PARTON DISTRIBUTION FUNCTIONS

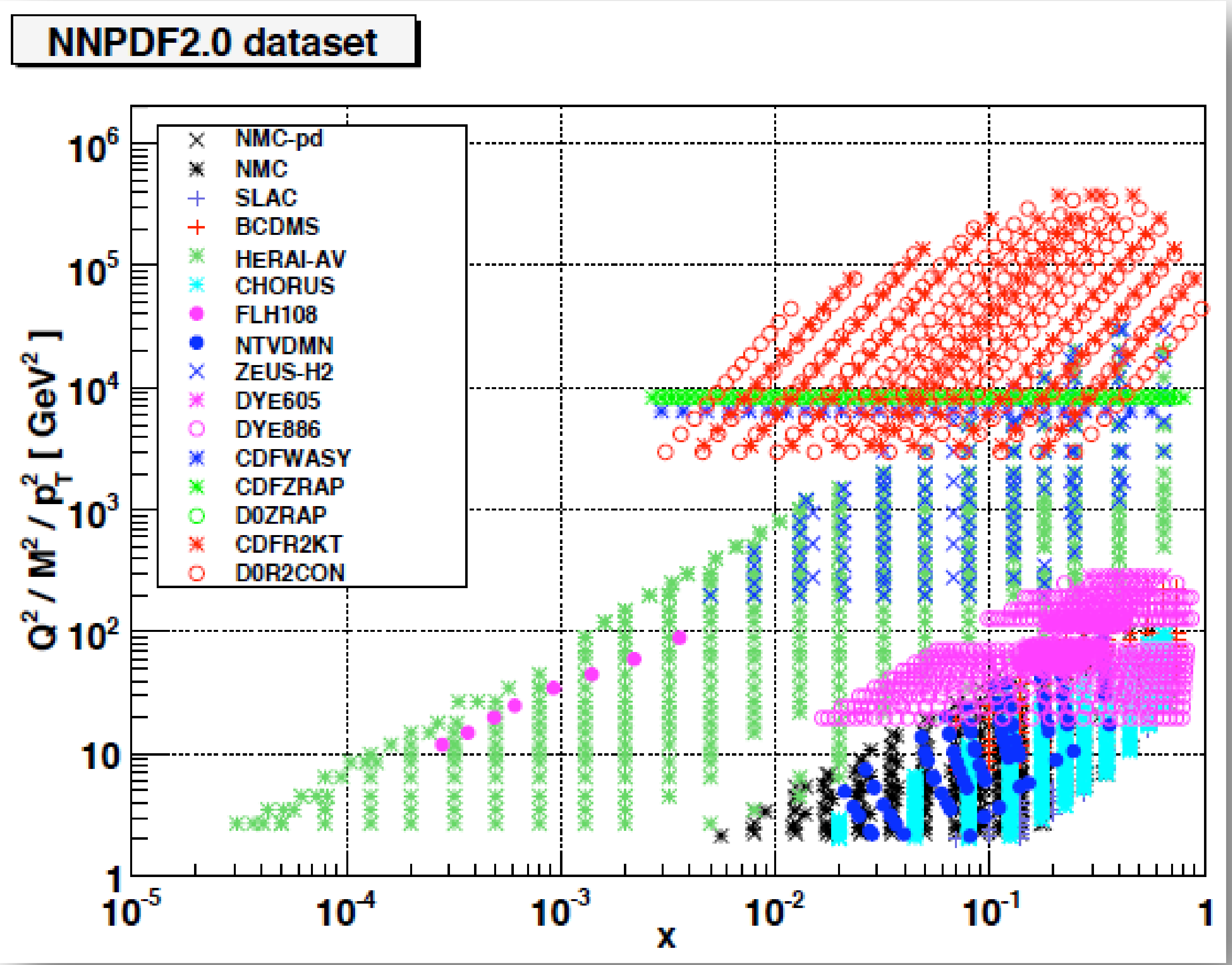
Global Fits, Q2 vs. x, x Parameterizations, Sets, Uncertainties

Global QCD Analysis

Process	Subprocess	Partons	x range
$\ell^\pm \{p,n\} \to \ell^\pm X$	$\gamma^* q \to q$	$q, \bar{q}, g$	$x \gtrsim 0.01$
$\ell^\pm n/p \to \ell^\pm X$	$\gamma^* d/u \to d/u$	d/u	$x \gtrsim 0.01$
$pp \to \mu^+\mu^- X$	$u\bar{u}, d\bar{d} \to \gamma^*$	$\bar{q}$	$0.015 \lesssim x \lesssim 0.35$
$pn/pp \to \mu^+\mu^- X$	$(u\bar{d})/(u\bar{u}) \to \gamma^*$	$\bar{d}/\bar{u}$	$0.015 \lesssim x \lesssim 0.35$
$\nu(\bar{\nu}) N \to \mu^-(\mu^+) X$	$W^* q \to q'$	$q, \bar{q}$	$0.01 \lesssim x \lesssim 0.5$
$\nu N \to \mu^-\mu^+ X$	$W^* s \to c$	s	$0.01 \lesssim x \lesssim 0.2$
$\bar{\nu} N \to \mu^+\mu^- X$	$W^* \bar{s} \to \bar{c}$	$\bar{s}$	$0.01 \lesssim x \lesssim 0.2$
$e^\pm p \to e^\pm X$	$\gamma^* q \to q$	$g, q, \bar{q}$	$0.0001 \lesssim x \lesssim 0.1$
$e^+ p \to \bar{\nu} X$	$W^+ \{d, s\} \to \{u, c\}$	d, s	$x \gtrsim 0.01$
$e^\pm p \to e^\pm c\bar{c} X$	$\gamma^* c \to c$, $\gamma^* g \to c\bar{c}$	c, g	$0.0001 \lesssim x \lesssim 0.01$
$e^\pm p \to$ jet$+X$	$\gamma^* g \to q\bar{q}$	g	$0.01 \lesssim x \lesssim 0.1$
$p\bar{p} \to$ jet$+X$	$gg, qg, qq \to 2j$	g, q	$0.01 \lesssim x \lesssim 0.5$
$p\bar{p} \to (W^\pm \to \ell^\pm \nu) X$	$ud \to W, \bar{u}\bar{d} \to W$	$u, d, \bar{u}, \bar{d}$	$x \gtrsim 0.05$
$p\bar{p} \to (Z \to \ell^+\ell^-) X$	$uu, dd \to Z$	d	$x \gtrsim 0.05$

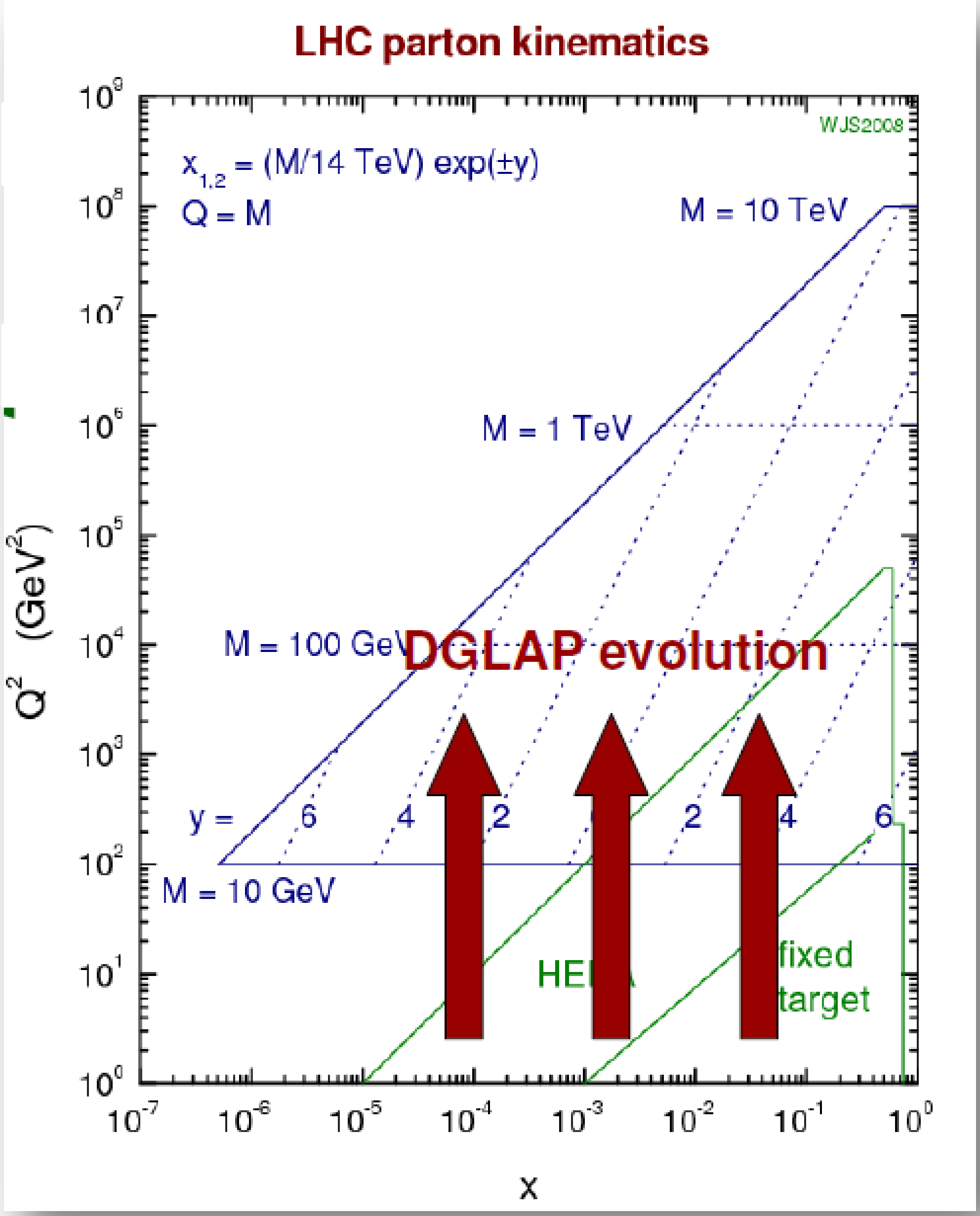
- To extract reliable Parton Distribution Functions use large amount of experimental data
- Pay attention to Q^2 and x ranges
- Use evolution kernel to relate different scales
- Need for theory predictions for consistent fit (LO, NLO, NNLO)

Q^2 vs. x



As an example, the NNPDF collaboration uses Fixed target DIS, HERA NC and CC cross-sections, Fixed-target Drell-Yan production and collider weak boson and jet production to constrain all PDF flavors in the relevant kinematical regime

And run to get predictions!



An example for applying the DGLAP equations for the running of the PDFs, Eq. (27)

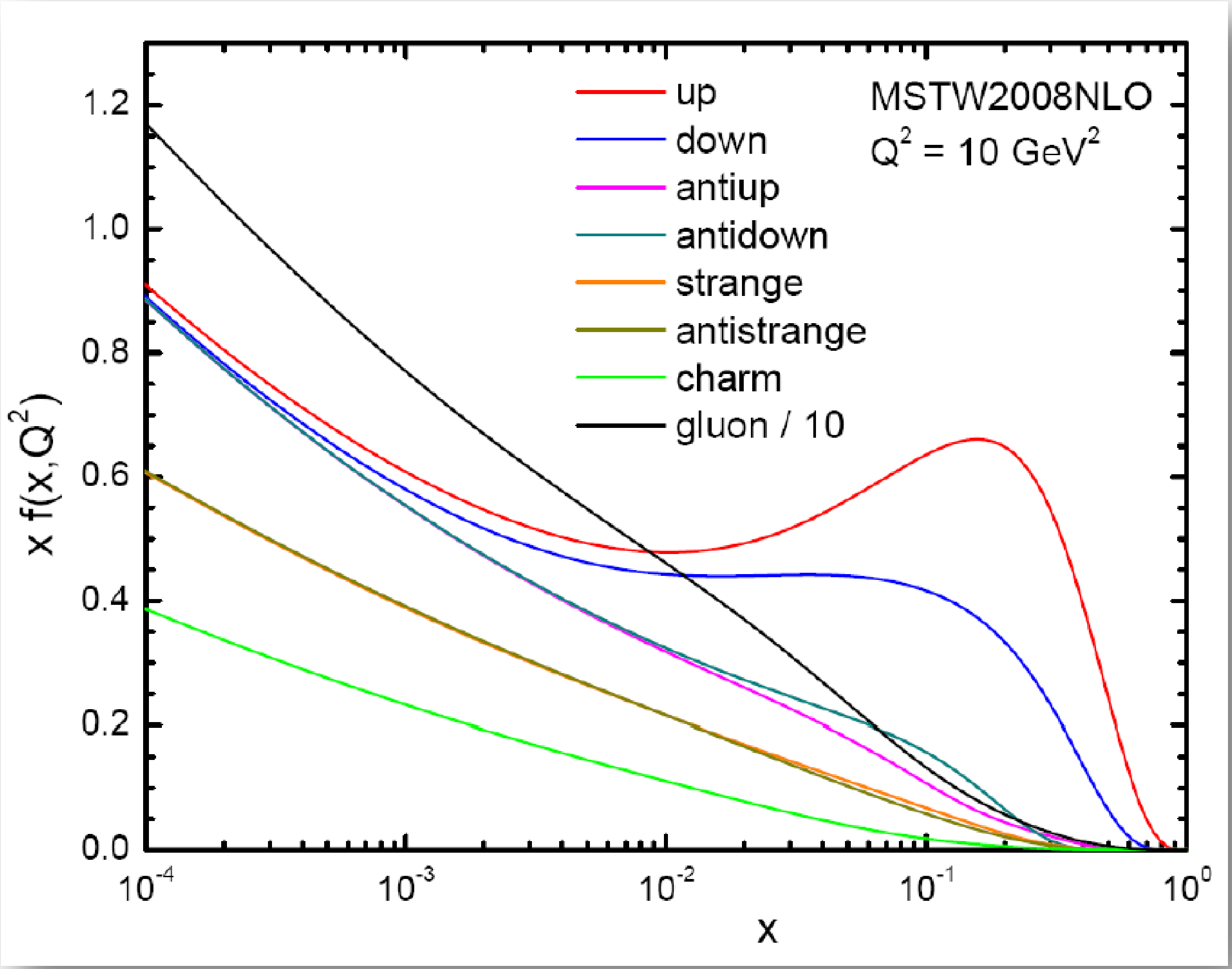
x Parameterizations

$$
\begin{aligned}
u_v(x), d_v(x), g(x) &= A_0\, x^{A_1-1}\,(1-x)^{A_2}\, e^{-A_3(1-x)^2 + A_4 x^2} \\
\bar{u}(x) + \bar{d}(x) &= \tfrac{1}{2}\, A_0\, x^{A_1-1}\,(1-x)^{A_2}\, e^{A_3 x}\,(1 + x e^{A_4})^{A_5} \\
\bar{d}(x)/\bar{u}(x) &= e^{A_1}\, x^{A_2-1}\,(1-x)^{A_3} \;+\; (1 + A_4 x)\,(1-x)^{A_5} \\
s(x) = \bar{s}(x) &= \tfrac{1}{2}\,\kappa\,(\bar{u}(x) + \bar{d}(x))
\end{aligned}
$$

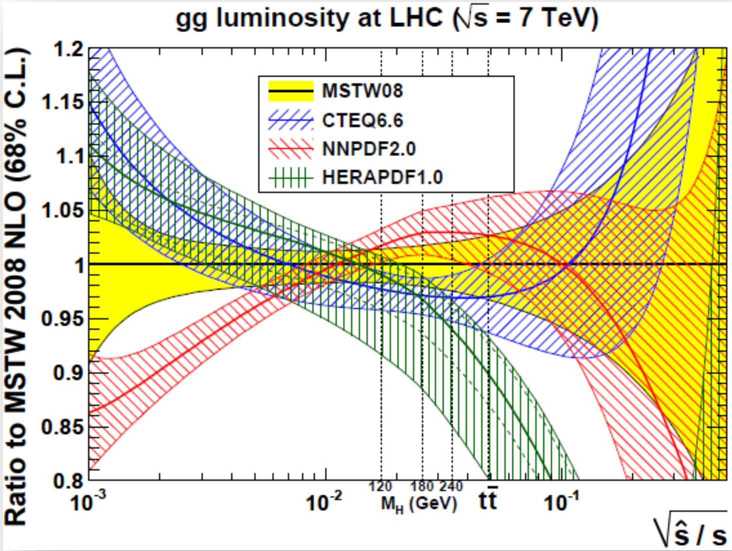
Example of a low-scale parameterization employed by the CTEQ collaboration

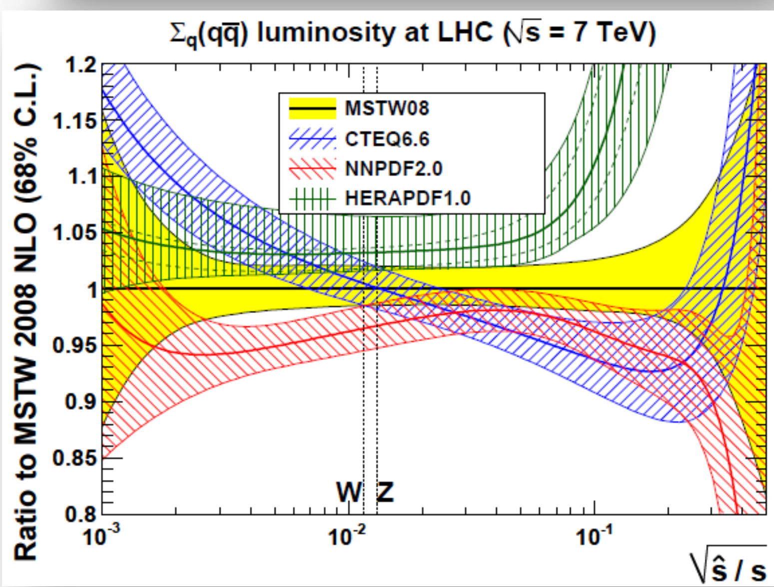
- Although perturbative calculations give us their Q^2 running, their *x* dependence is of non-perturbative nature
- Propose functions for fits: sufficient parameters to give enough freedom, but without loosing predictive power
- Employ known behavior for *x* near 0 and 1
- *Sum rules* constrain the fitting procedure

Example from the MSTW collaboration



PDF4LHC Recommendations for PDF Uncertainty





The CTEQ collaboration provides 44 PDFs set to compute uncertainties for observables. Similarly, the MSTW provide 40 set while the NNPDF 100

Compute PDF envelopes with (*i=CTEQ,MSTW,NNPDF*):

$$
\begin{aligned}
U &= \max_i\{\mathcal{O}_0^i + \sigma^{(i)}(\alpha_s + PDF, +)\} \\
L &= \min_i\{\mathcal{O}_0^i - \sigma^{(i)}(\alpha_s + PDF, -)\} \\
M &= \frac{U+L}{2}
\end{aligned}
$$

Where *U* and *L* are the upper and lower limits of the envelope and *M* the mid point

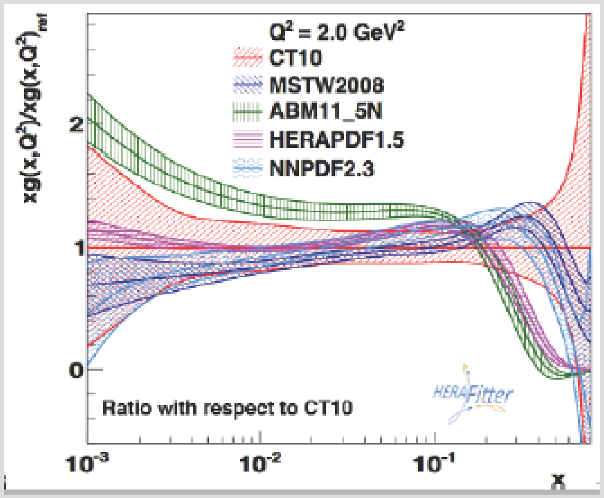
LHC Data Impact on Global Fits

CMS-PAS-SMP-12-028

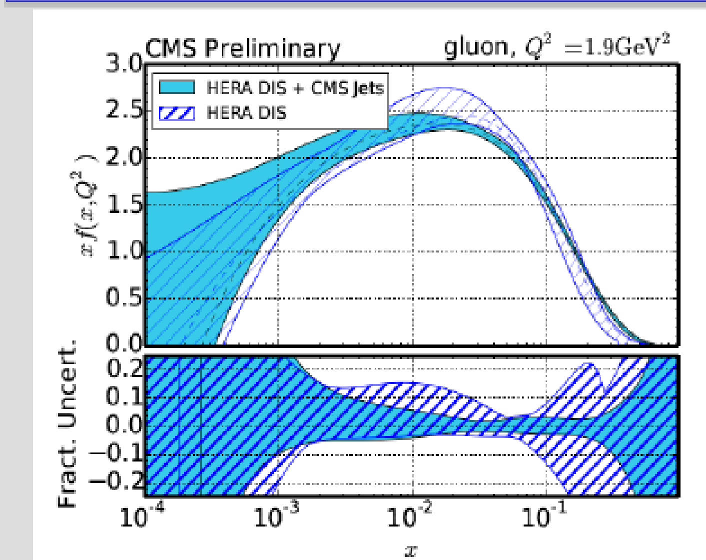
The gluon PDF… and input from LHC

PDF fits using only data from experiments with lower momentum transfer than available at the LHC have large uncertainties for the LHC kinematic region.

Gluon PDF constraint from CMS with 7 TeV inclusive jet cross-section



Data from LHC start to be precise enough to be used for PDF constraint



- CMS fit a harder gluon, with respect to the fit using HERA alone (same results from ATLAS on lower stat. sample)
- Fractional uncertainty is smaller at high *x, partly due to* increased prediction
- Measurements are systematic limited at lower jet p_T, dominated by the JES uncertainty

Slide taken from Chiara Roda, *Probing QCD & Hadron Physics*, ICHEP 2014

LHAPDF: PDF's in one place

https://lhapdf.hepforge.org/

LHAPDF ID	Set name	Number of set members
10800	CT10	53
10860	CT10as	11
10900	CT10w	53
10960	CT10was	11
10980	CT10f3	1
10981	CT10f4	1
21000	MSTW2008lo68cl	41
21050	MSTW2008lo90cl	41
21100	MSTW2008nlo68cl	41
21150	MSTW2008nlo90cl	41
21200	MSTW2008nnlo68cl	41
21250	MSTW2008nnlo90cl	41
22000	MSTW2008nlo_asmzrange	22
22100	MSTW2008nlo68cl_asmz+68cl	41
22150	MSTW2008nlo68cl_asmz-68cl	41
22200	MSTW2008nlo68cl_asmz+68clhalf	41
42330	abm12lhc_4_nnlo	29
42360	abm12lhc_5_nnlo	29
60600	HERAPDF15NNLO_EIG	29
60630	HERAPDF15NNLO_VAR	11
247200	NNPDF23_nlo_as_0119_qed_mc	101
247400	NNPDF23_nnlo_as_0119_qed_mc	101
260000	NNPDF30_nlo_as_0118	101
260200	NNPDF30_nlo_as_0118_nf_3	101

- The LHAPDF compiles into a single library thousands of publically available sets of PDF's
- Current version 6.1.3 is written in C++, with routines to link to Fortran codes
- Simple way to get access to all PDF set, particularly for computing PDF errors

Summary

- Perturbative QCD is successful in describing main features of hadron production at lepton-lepton colliders
- The definition of jets appears fundamental to compute meaningful observables in perturbation theory
- Approximate Bjorken scaling gives rise to the Parton Model. QCD improvements allow a proper definition of Universal Parton Distribution Functions
- PDFs sets are obtained from Global Fits and through perturbative running allow predictions at untested scales