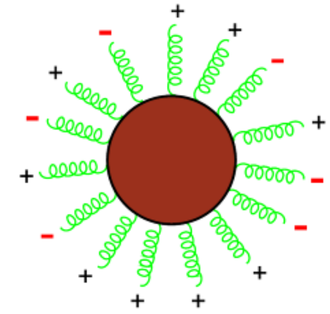

QCD and Monte Carlo
3. Jets, Parton Shower MC

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Fermilab-CERN Hadron Collider Physics Summer School

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JET ALGORITHMS

Cone v Seq, IR Safety, Efficiency, Shapes, Jet Substructure

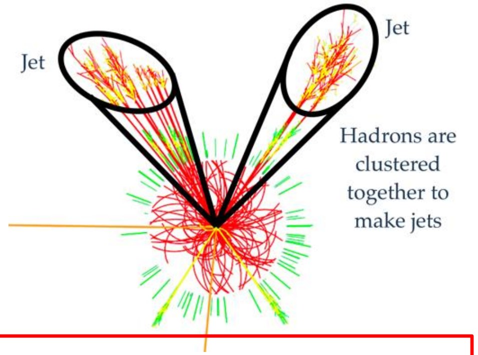
HADRON-COLLIDER EVENTS

Particle multiplicity, Limits of Parton Level, QCD needs Showers

PARTON-SHOWER MONTE CARLOS

Emission, Unitarity, Sudakov Factors, FSR/ISR, Hadronization

Cone vs. Sequential Jet Algorithms at Hadron Colliders



Cone Based Algoritms
(for example JetClu, Midpoint, SISCone, MCFM-Seedless)

1) Cluster particles within a cone of radius *R* in rapidity and azimutal angle space around a given seed *i*

$$\Delta R_{ij}^2 = (y_i - y_j)^2 + (\phi_i - \phi_j)^2 < R^2$$

2) Recombine the momentum of particles contained in given cones
3) Iterate with resulting objects until stable

Overlapping cones would have a prescription for merging them if they share a fraction of energy greater than a parameter *f*

Sequential Algorithms
(anti-kT, kT, Cambridge/Aachen)

1) For each pair particles/Beam define the distances:

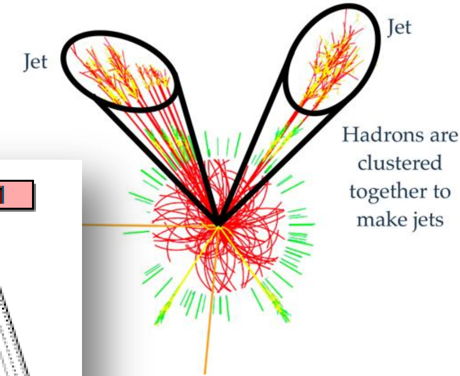
$$d_{ij} = \min(p_{ti}^{2p}, p_{tj}^{2p}) \frac{\Delta R_{ij}^2}{R^2}, \quad d_{iB} = p_{ti}^{2p}$$

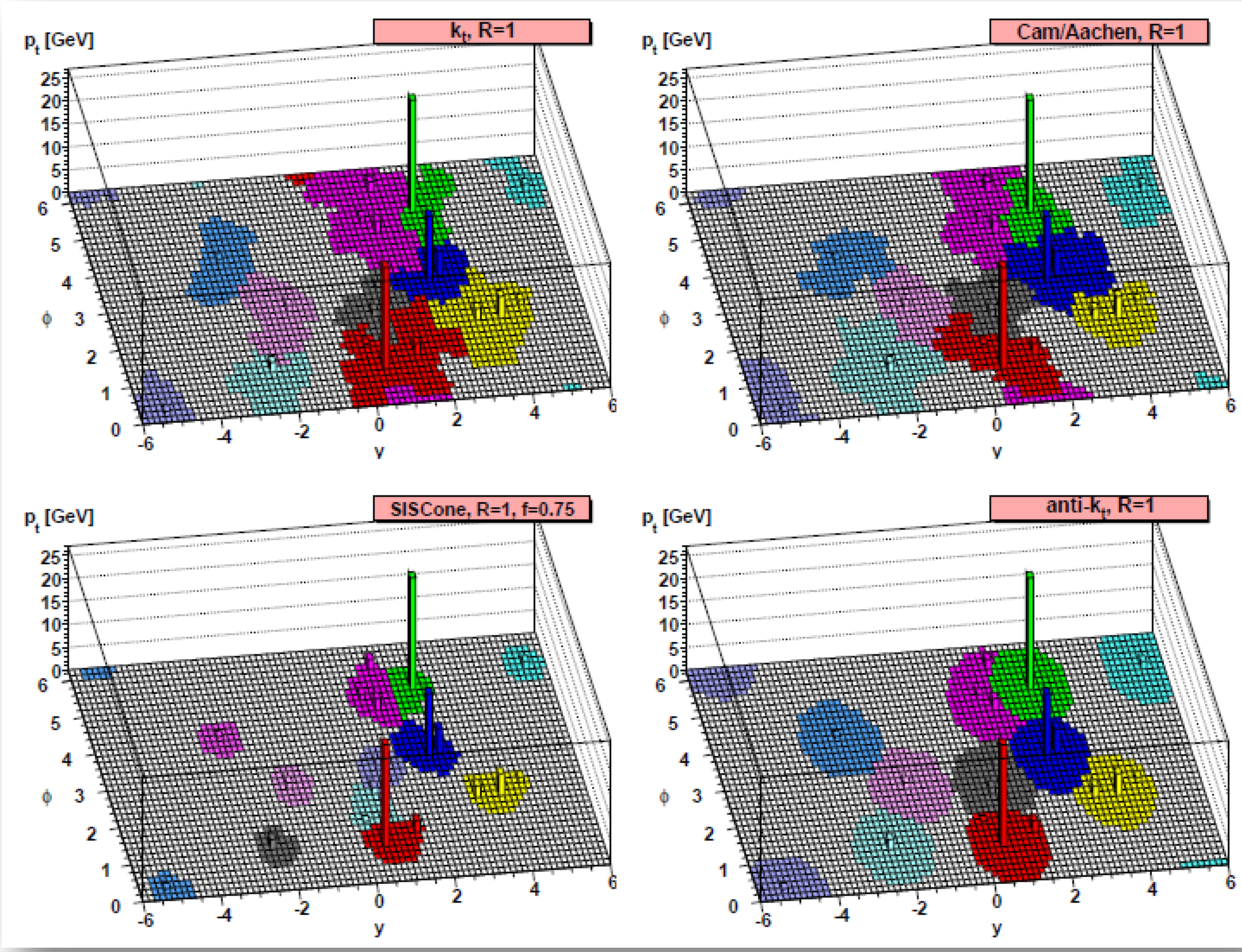
2) Find the minimum distance. If between *i* and *j* combine. If between *i* and *Beam* promote to a jet and remove from list of particles
3) Iterate until no particle left

p=1 for kT, *p=0* for Cambridge/Aachen and *p=-1* for anti-kT

"*Particles*" can be detector cells, tracks, hadrons, partons…

Jet Areas





[Salam arXiv:0906.1833]

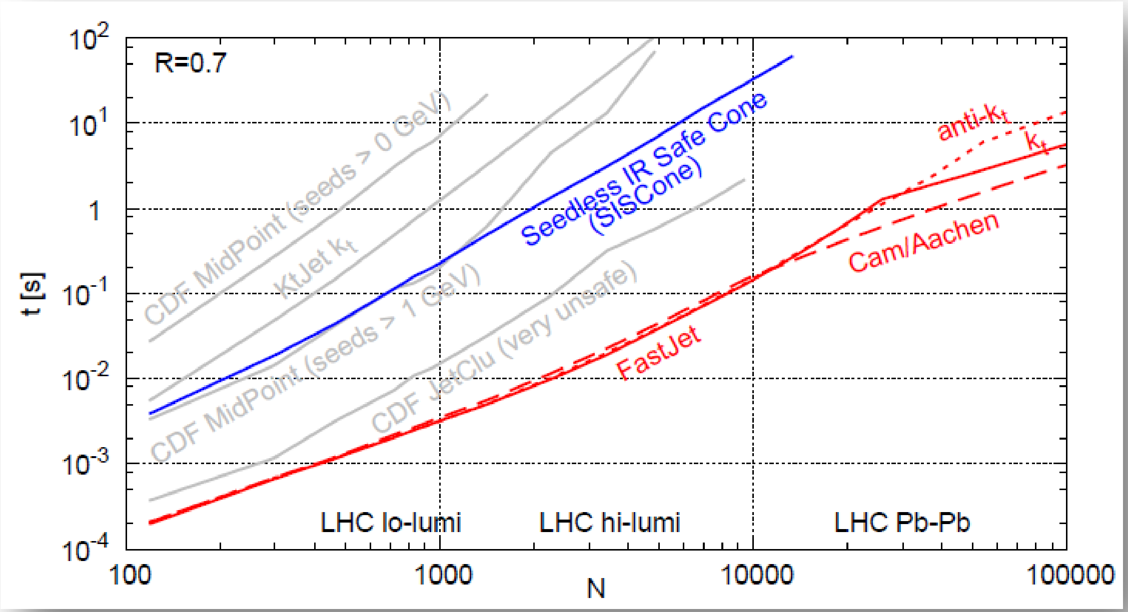
IR-Safe Jet Algorithms (and Fast!)

In the past, performance of implementations of IR safe jet algorithms, made them impractical at hadron colliders: for example with the “standard” N^3 scaling of the kt algorithm or the naive 2^N of seedless cone algorithms

Settled recently:

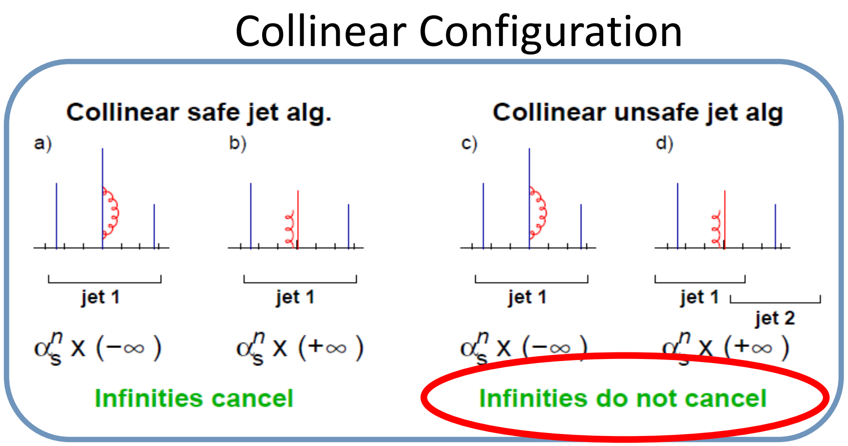
- Sequential recombination algorithms as kt / Cambrige-Aachen / anti-kt have been implemented with *N ln (N)* scaling [Cacciari, Salam hep-ph/0512210]
- A seedless infrared-safe cone algorithm, SISCone, has appeared with *N^2 ln (N)* scaling [Salam, Soyez arXiv:0704.0292]

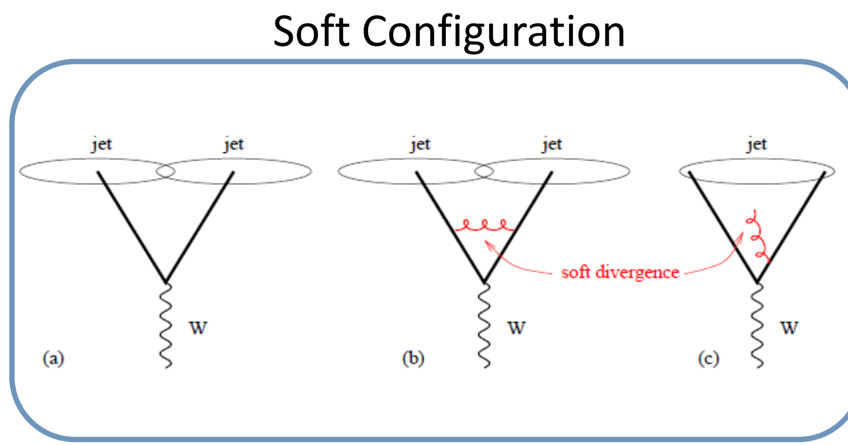
Available within FatJet http://fastjet.fr



[Salam arXiv:0906.1833]

The need for IR safety





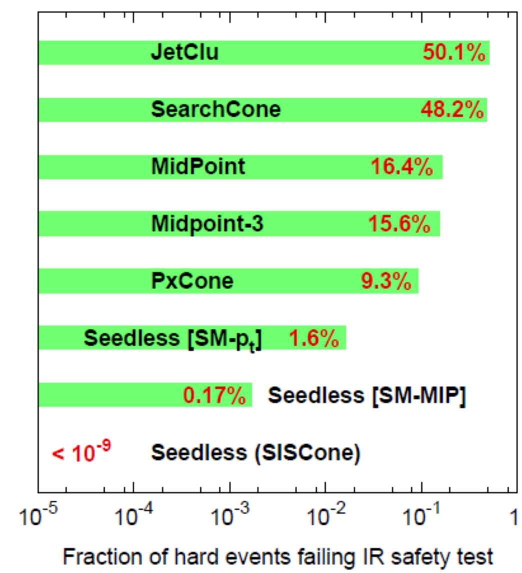
→ IRC unsafety makes data / pertubative calculation comparison hard (if at all meaningful)

→ Indeed, quantum corrections become useless for large enough multiplicity!

[Salam, Soyez arXiv:0704.0292]

Observable	1st miss cones at	Last meaningful order
Inclusive jet cross section	NNLO	NLO
$W/Z/H + 1$ jet cross section	NNLO	NLO
3 jet cross section	NLO	LO
$W/Z/H + 2$ jet cross section	NLO	LO
jet masses in 3 jets, $W/Z/H + 2$ jets	LO	none

Testing IR safety of some commonly used cone algorithms



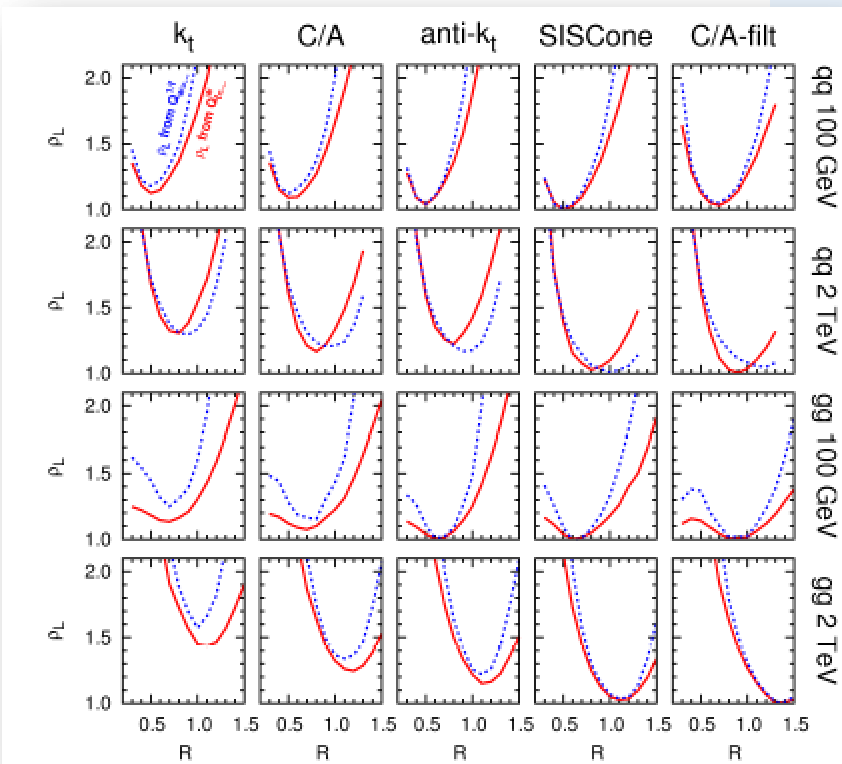
Both ATLAS and CMS already include IR safe algorithms in their standard software!

Towards "jetography"

[Salam arXiv:0906.1833]

Many ideas like *variable-R algorithms* [Krohn, Thaler, Wang arXiv:0903.0392], *filtering* [Butterworth, Davison, Rubin, Salam arXiv:0802.2470], *pruning* [Ellis, Vermilion, Walsh arXiv:0903.5081], among others, and the availability of *many practical IR safe jet algorithms*, have opened the possibility of **optimizing jet definitions for a given physical study**

See also for example: [Buge, Heinrich, Klein, Rabbertz; Cacciari, Rojo, Salam, Soyez arXiv:0803.0678], [Olness, Soper arXiv:0907.5052]



Different jet algorithms perform differently:

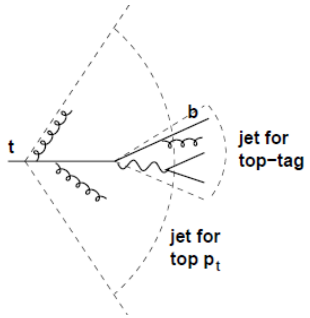
- Too small R → hadronization effects
- Too large R → Underlying Event and Pile-Up

[Cacciari, Rojo, Salam, Soyez arXiv:0810.1304]
See also: [Rojo arXiv:0910.1449]

Jet Substructure

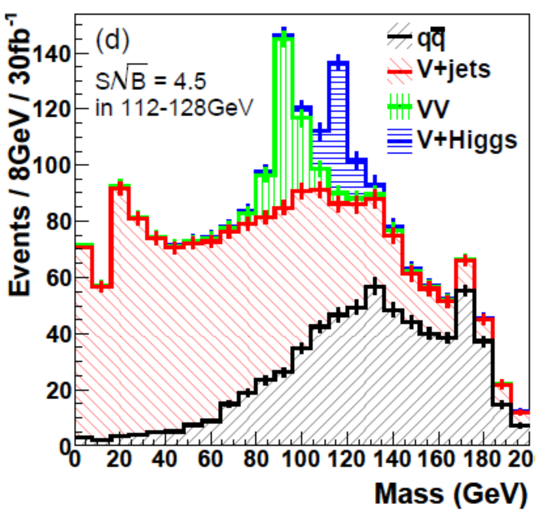
Given the large amount of energy accessible at the LHC, it'll be common to find highly boosted heavy objects (eg. top or a Higgs) whose decaying products will appear in a single jet!

Then the need to ***look inside jets!***



See for example: [Butterworth, Cox, Forshaw hep-ph/0702150], [Ellis, Vermilion, Walsh arXiv:0903.5081] , [Almeida, Lee, Perez, Sterman, Sung, Virzi arXiv:0807.0234], [Plehn, Salam, Spannowsky arXiv:0910.5472]

An example: **Two-pronged decays**, LHC $Z/W+H(\rightarrow b\bar{b})$



With a highly boosted Higgs there is the possibility of measuring this combination of production and decay mode

[Butterworth, Davison, Rubin, Salam arXiv:0802.2470]

JET ALGORITHMS

Cone v Seq, IR Safety, Efficiency, Shapes, Jet Substructure

HADRON-COLLIDER EVENTS

Particle multiplicity, Limits of Parton Level, QCD needs Showers

PARTON-SHOWER MONTE CARLOS

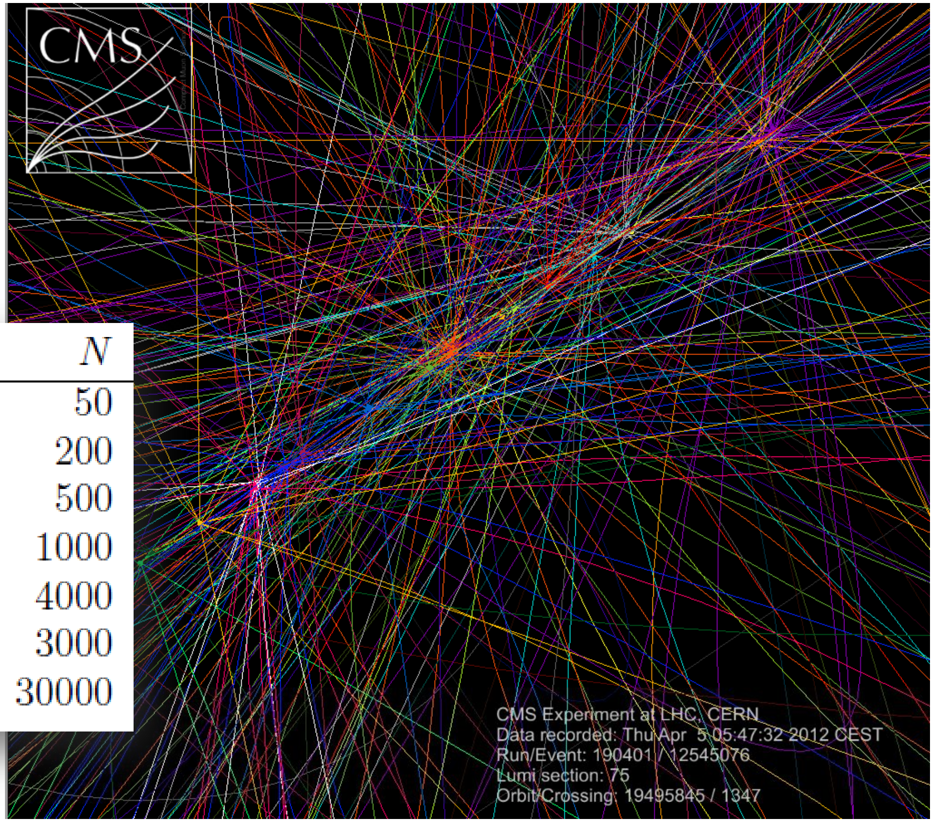
Emission, Unitarity, Sudakov Factors, FSR/ISR, Hadronization

Particle Multiplicity

Some typical event multiplicities at colliders:

Type of event	N
e^+e^- event on the Z peak	50
Tevatron ($\sqrt{s} = 1.96$ TeV) dijet event	200
LHC ($\sqrt{s} = 14$ TeV) dijet event	500
LHC low-luminosity event (5 pileup collisions)	1000
LHC high-luminosity event (20 pileup collisions)	4000
RHIC Au Au event ($\sqrt{s} = 200$ GeV/nucleon)	3000
LHC Pb Pb event ($\sqrt{s} = 5.5$ TeV/nucleon)	30000

[Salam arXiv:0906.1833]



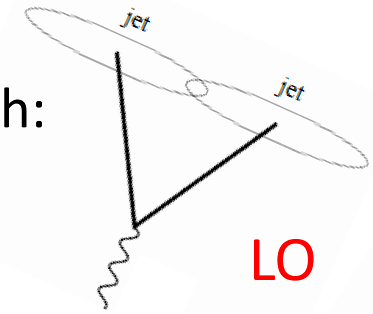
Compare with state-of-the art Fixed Order perturbative calculations:

- QCD LO number of particles in final state ≤ 9
- QCD NLO number of particles in final state ≤ 6
- QCD NNLO number of particles in final state ≤ 2
- QCD N^{3}LO number of particles in final state ≤ 1

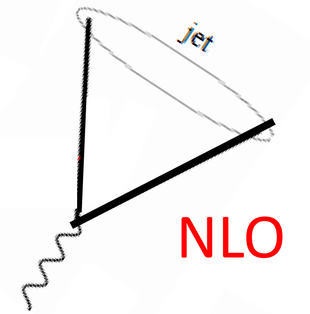
Clearly we need an alternative to Fixed Order to simulate *realistic* hadron collider events!

Limited Jet Structure at Fixed Order

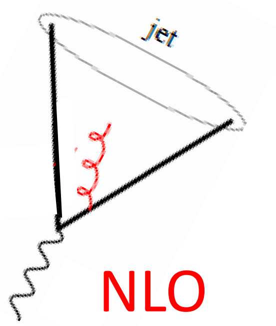
- At QCD LO producing *m* jets we find Events with:
  - *m* jets with 1 parton per jet (1 p/j)



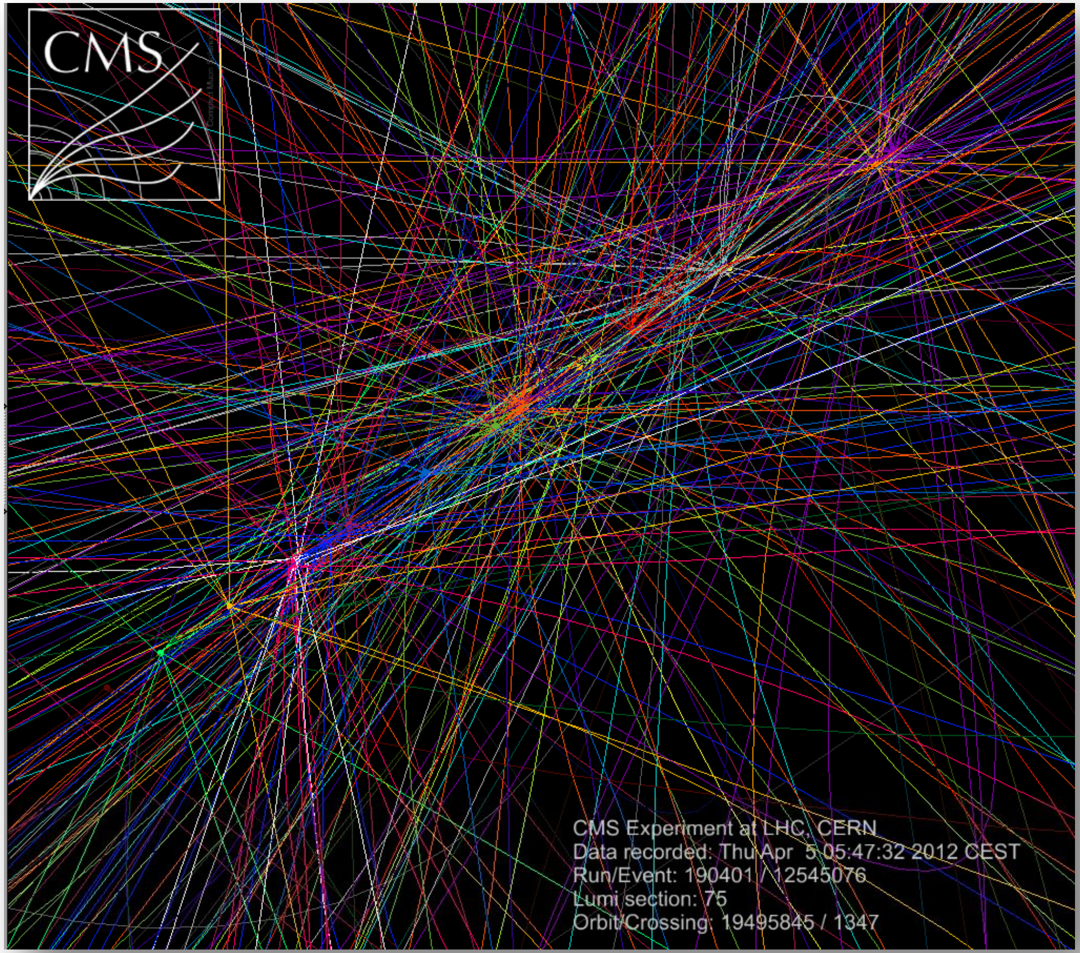
- At QCD NLO producing *m* jets we find Events with:
  - *m* jets with 1 jet with 2 p/j and *m-1* jets with 1 p/j
  - *m+1* jets with 1 p/j



- At QCD NNLO producing *m* jets we find Events with:
  - *m* jets with 1 jet with 3 p/j and *m-1* jets with 1 p/j
  - *m+1* jets with 2 jets with 2 p/j and *m-1* jets with 1 p/j
  - *m+2* jets with with 1 p/j

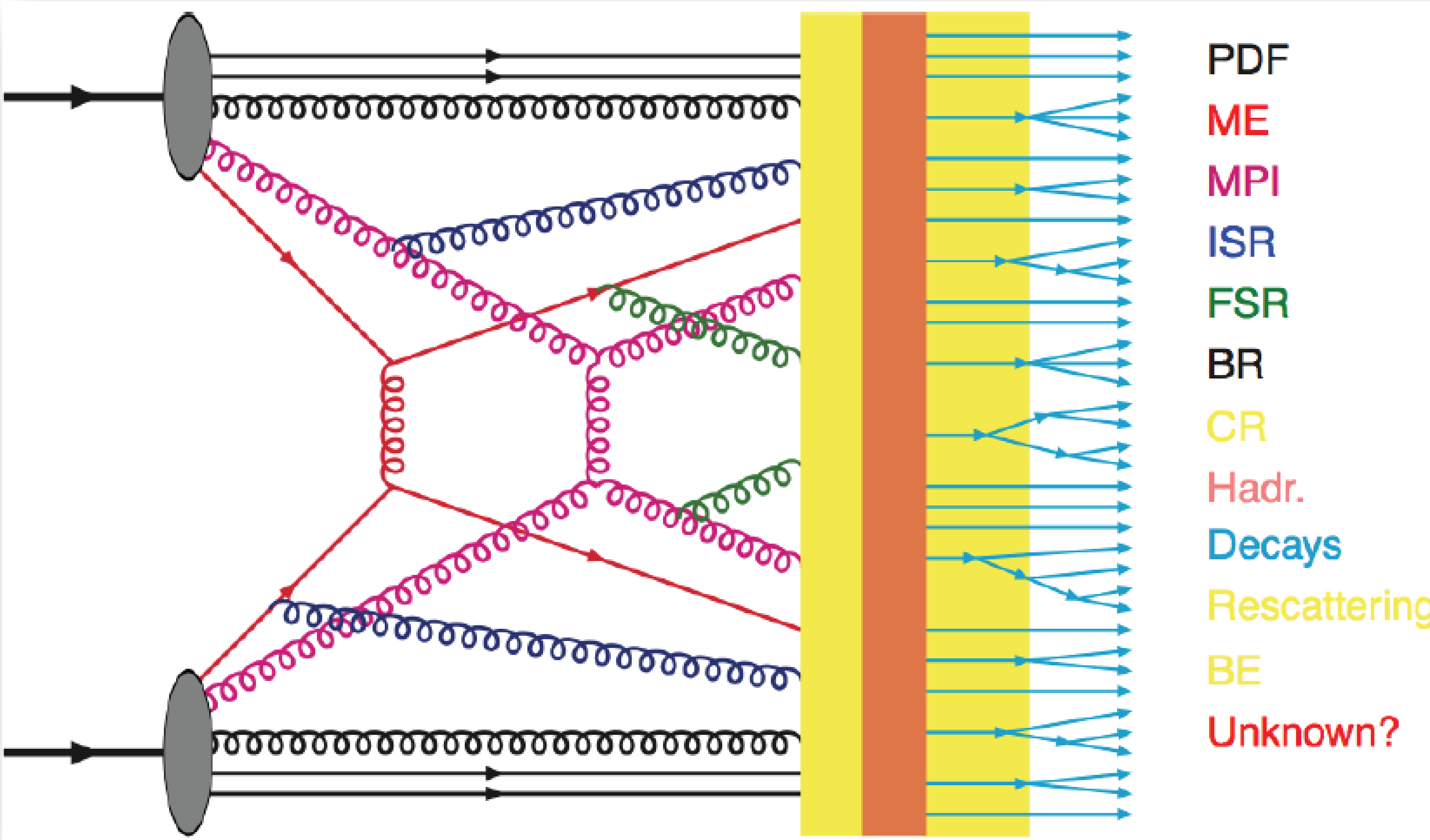


Go Beyond Fix order



Although fixed-order results describe well (very) inclusive observable at high energy, we need an alternative approach that would allow to simulate events similar to the ones at hadron colliders!

QCD Needs Parton-Shower Monte Carlos



Taken from Torbjörn Sjöstrand

JET ALGORITHMS

Cone v Seq, IR Safety, Efficiency, Shapes, Jet Substructure

HADRON-COLLIDER EVENTS

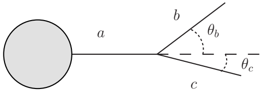
Particle multiplicity, Limits of Parton Level, QCD needs Showers

PARTON-SHOWER MONTE CARLOS

Emission, Unitarity, Sudakov Factors, FSR/ISR, Hadronization

Final-State Parton Branching

Consider the final state branching of a colored parton a into two other partons b and c.



Let's assume that

$$p_b^2, p_c^2 \ll p_a^2 \equiv t > 0 \tag{1}$$

So then this is a *timelike branching*. The opening angle we will write as $\theta = \theta_b + \theta_a$ and the energy fractions:

$$z = \frac{E_b}{E_a} = 1 - \frac{E_c}{E_a} \tag{2}$$

In the small angle limit we find:

$$t = 2E_bE_c(1 - \cos\theta) \approx z(1-z)E_a^2\theta^2 \tag{3}$$

We want to make use of the *splitting functions* to simulate emissions from a hard parton

Polarization Correlations

We found out that Altarelli-Parisi splitting functions that govern unpolarized emissions, for example:

$$\tilde{P}_{qq}(z) = C_F \left(\frac{1+z^2}{1-z} \right)$$

But you might remember that these were summed over polarizations. Defining as ϕ the polarization angle of a with respect to the plane bc we would replace $\tilde{P}_{qq}$ by:

$$C_F F_{qq} = C_F \frac{1+z^2}{1-z} + \frac{2z}{1-z} \cos 2\phi \tag{4}$$

And similarly we can do with other splittings.

The idea would be to find out the evolution of splittings in fraction of energy x and t and then generate the splitting momenta including this angular correlation!

Branched Phase Space

If we want to describe the PS after the splitting, we basically need to think in the connection:

$$d\mathrm{LIPS}_n = \cdots \frac{d^3 p_a}{(2\pi)^3} \frac{1}{2E_a}$$

$$\updownarrow$$

$$d\mathrm{LIPS}_{n+1} = \cdots \frac{d^3 p_b}{(2\pi)^3} \frac{1}{2E_b} \frac{d^3 p_c}{(2\pi)^3} \frac{1}{2E_c} \tag{5}$$

Which you can get in the small angle limit with a change of variables:

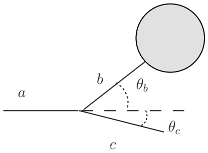
$$d\mathrm{LIPS}_{n+1} = d\mathrm{LIPS}_n \frac{1}{4(2\pi)^3} dt dz d\phi \tag{6}$$

And we gather this info at the level of differential cross sections with:

$$d\sigma_{n+1} = d\sigma_n \frac{dt}{t} dz \frac{d\phi}{2\pi} \frac{\alpha_s}{2\pi} \underbrace{C}_{\text{color factor}} F_{ba} \tag{7}$$

Initial-State Splitting

Initial-state branching of a into two other partons b and c.



Here we have:

$$|p_a^2|, p_c^2 \ll |p_b^2| \equiv t \tag{8}$$

And we have $z = E_b/E_a = 1 - E_c/E_a$, and for small angles:

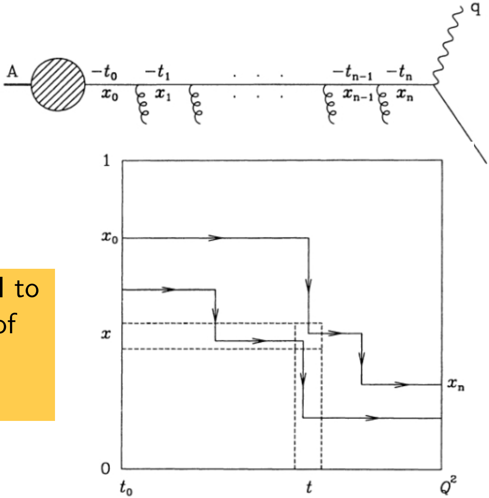
$$t = E_a E_c \theta_c^2 \tag{9}$$

The relation between differential cross sections results into:

$$d\mathrm{LIPS}_{n+1} = d\mathrm{LIPS}_n \frac{1}{4(2\pi)^3} dt \frac{dz}{z} d\phi \tag{10}$$

Evolution Equations

For simplicity, consider the case of only one type of branching like in the process:



We will find very useful to think of the evolution of these splittings in the (t, x) plane

Let's take the regulated splitting function (to avoid soft complications) as the probability densities for a parton to branch:

$$d\mathcal{P}_{a\to bc} = \frac{\alpha_s}{2\pi}\frac{dt}{t}P_{ba}(z)dz \tag{11}$$

Branching probability and unitarity

Using unitarity we can write the relation:

$$\mathcal{P}(\text{no emission}) = 1 - \mathcal{P}(\text{emission}) \tag{12}$$

$\mathcal{P}(\text{no emission})$ has the nice feature of compositeness. Say $T_i = \frac{i}{n} T$ with $0 \leq i \leq n$:

$$\begin{aligned}
\mathcal{P}_{\text{no}}(0 \leq t < T) &= \lim_{i \to \infty} \prod_{i=0}^{n-1} \mathcal{P}_{\text{no}}(T_i \leq t < T_{i+1}) \\
&= \lim_{i \to \infty} \prod_{i=0}^{n-1} (1 - \mathcal{P}_{\text{em}}(T_i \leq t < T_{i+1})) \\
&= \exp\left(- \lim_{n \to \infty} \sum_{i=0}^{n-1} \mathcal{P}_{\text{em}}(T_i \leq t < T_{i+1}) \right) \\
&= \exp\left(- \int_0^T \frac{d\mathcal{P}_{\text{em}}(t)}{dt} dt \right)
\end{aligned} \tag{13}$$

The Sudakov Form Factor

We write the probability of a parton not to branch between the scale t_0 and t according to:

$$\Delta(t) = \exp\left[-\int_{t_0}^{t} \frac{dt'}{t'} \int dz \frac{\alpha_s}{2\pi} P_{ba}(z)\right] \tag{14}$$

$\Delta(t)$ is the so called Sudakov form factor, which will guide the evolution of our parton shower

Notice that we have to regularize the soft divergence in order for $\Delta(t)$ to be well defined. We can at this stage just remove the divergence at $z = 1$ by integrating between $z = 0$ and $z = 1 - \epsilon$, with a cutoff that defines *resolvable* radiation.

Actually a detailed combined analysis of soft and collinear radiation would return a similar picture with a modified Sudakov form factor!

Monte-Carlo Method

Suppose you start your shower from a configuration in (t_1, x_1). Define a scale Q^2 at which your spacelike shower will stop. Follow the steps:

1. Find the value of t_2, next scale for branching, by solving the equation:
$$\frac{\Delta(t_2)}{\Delta(t_1)} = \mathcal{R} \tag{15}$$
in which $\mathcal{R}$ is a random number distributed in $[0, 1]$
2. If $t_2 > Q^2$ stop the shower
3. Else solve for the x_2 fraction according to the *resolvable* probability distribution $(\alpha_s/2\pi)P(z)$, for example by solving:
$$\int_{\epsilon}^{x_2/x_1} dz \frac{\alpha_s}{2\pi} P(z) = \mathcal{R}' \int_{\epsilon}^{1-\epsilon} dz \frac{\alpha_s}{2\pi} P(z) \tag{16}$$
4. Generate momenta according to t_2, x_2 and ϕ (weighted by F_{ba})
5. Repeat until the shower stops

Comments

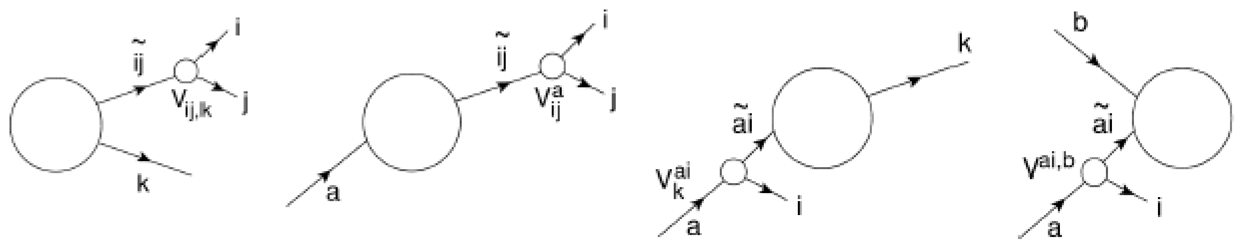
We have presented a very simplified picture of the algorithms implemented in modern Monte-Carlo Programs. Notice that:

- We have considered a shower with only a type of branching. In QCD you need to add all possible branchings
- It is customary to make *backward evolution* of initial state showers, this to avoid high inefficiency due to the unknown nature of the final-parton energy fraction in the forward evolution
- Timelike parton branching has analogous evolution
- We have presented a chain of branches in a given initial parton. In realistic simulations you start with a number of initial- and final-state colored partons, which can all produce their associated showers
- Daughter partons can start secondary branchings, and so on
- FSR showers are usually evolved until $Q^2 \sim 1\ \mathrm{GeV}^2$

What about kinematics??

Dipole showers

- In parton showers, the collinear/soft limit is never reached
  But who absorbs recoil when a splitting parton goes off mass-shell?
- No answer in DGLAP evolution equations $\leftrightarrow$ collinear limit
  Ambiguity introduces large uncertainties, especially at large t
- Natural solution provided by $2 \to 3$ splittings
  Spectator kinematics enters splitting probability
- Basic concept of dipole showers



final-final | final-initial | initial-final | initial-initial

Taken from Stefan Höche

Multi Purpose Monte Carlo Programs

HERWIG, PYTHIA and SHERPA offer convenient frameworks for LHC physics studies, but with slightly different emphasis:

PYTHIA (successor to JETSET, begun in 1978):

- originated in hadronization studies: the Lund string
- leading in development of MPI for MB/UE
- pragmatic attitude to showers & matching

HERWIG (successor to EARWIG, begun in 1984):

- originated in coherent-shower studies (angular ordering)
- cluster hadronization & underlying event pragmatic add-on
- large process library with spin correlations in decays

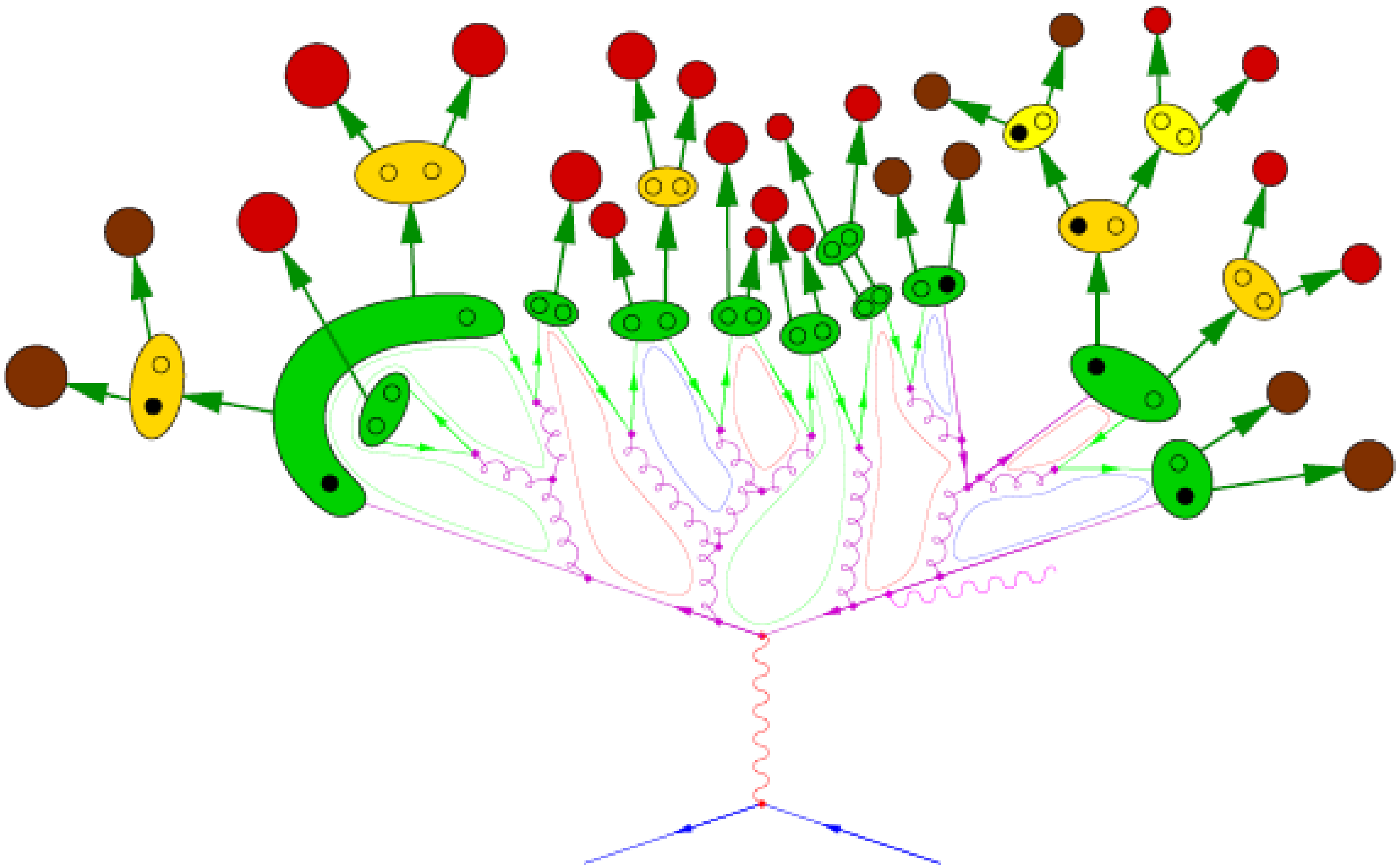
SHERPA (APACIC++/AMEGIC++, begun in 2000):

- own matrix-element calculator/generator
- extensive machinery for CKKW ME/PS matching
- hadronization & min-bias physics under development

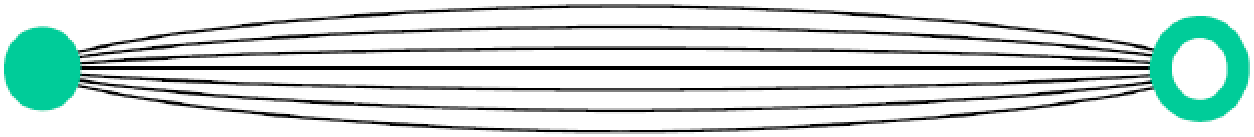
PYTHIA and HERWIG originally in Fortran, but now all in C++.

Taken from Torbjörn Sjöstrand

Modeling Hadron Production



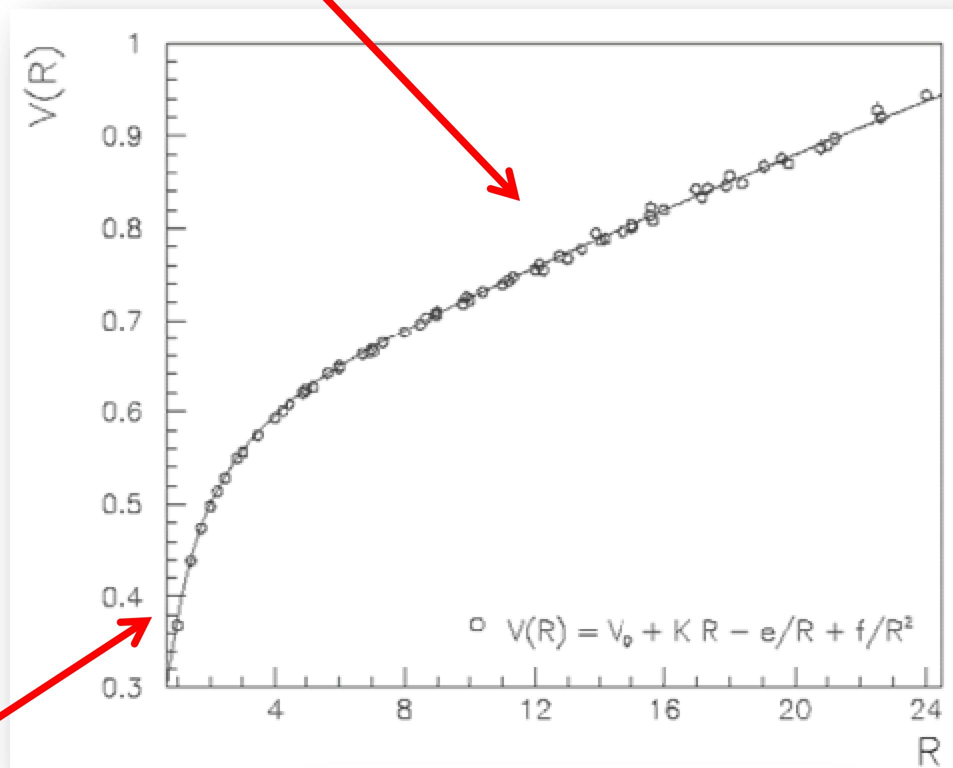
QCD Confinement



Chromomagnetic field lines are tight due to gluon self interactions

Linear

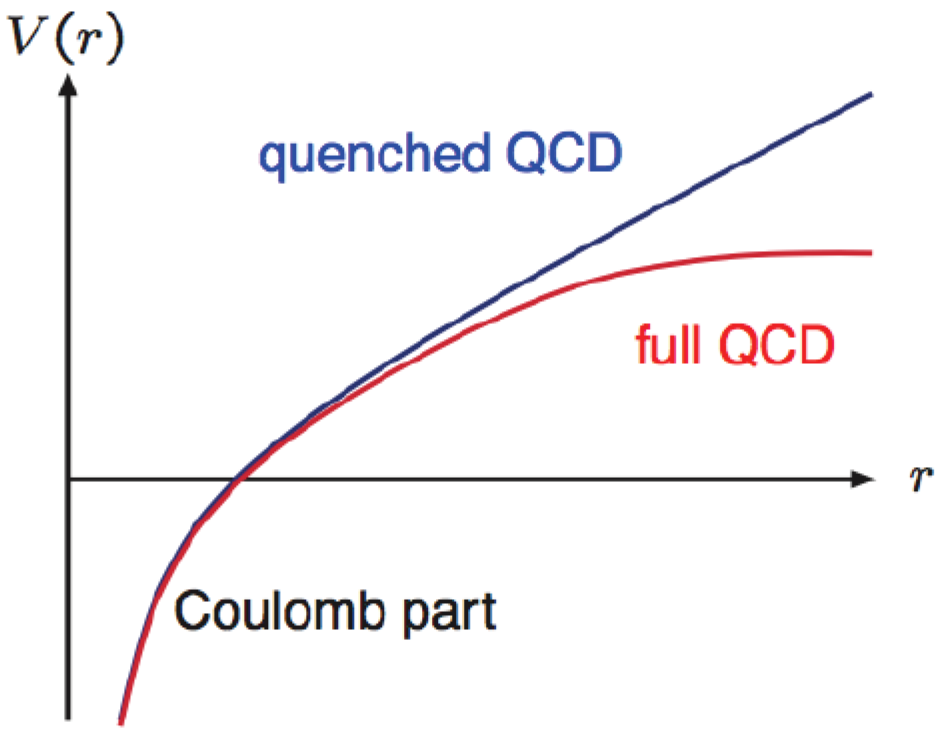
Lattice QCD in fact shows us that between a static color-anticolor pair, at large distances a *confining* linear potential emerges!

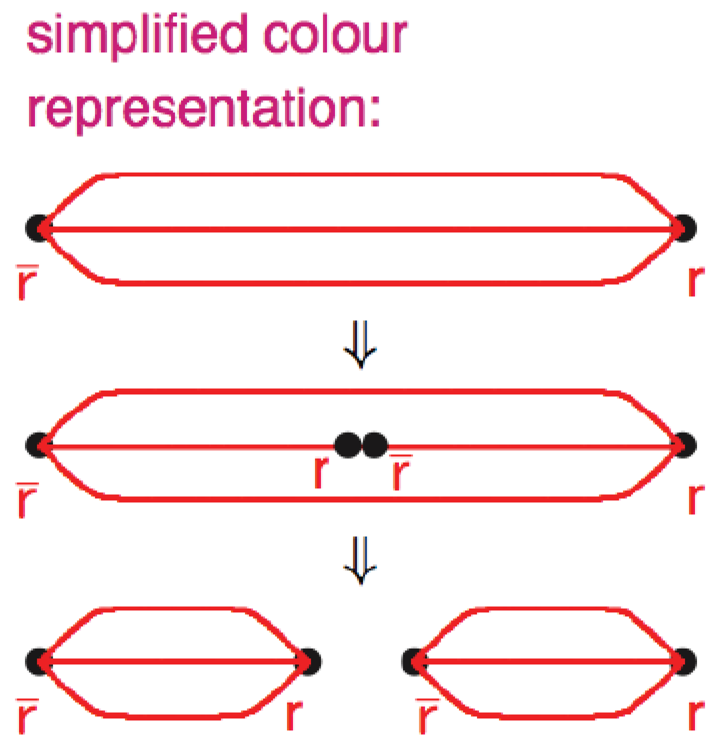


Coulomb

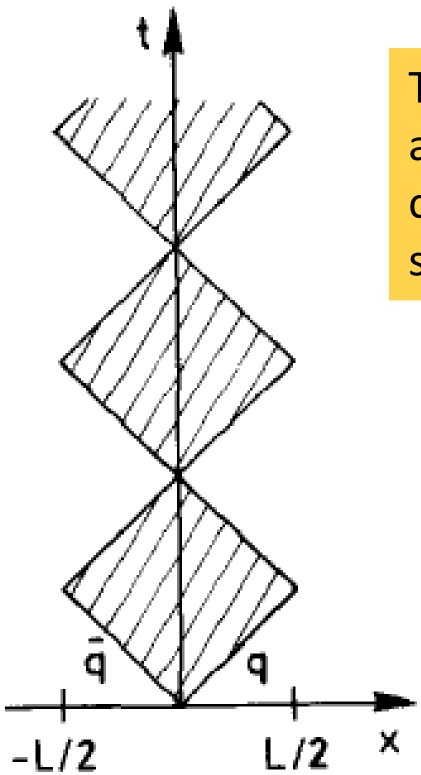
$$F(r) \approx \text{const} = \kappa \approx 1\,\text{Gev/fm}$$

QCD vs. Quenched QCD



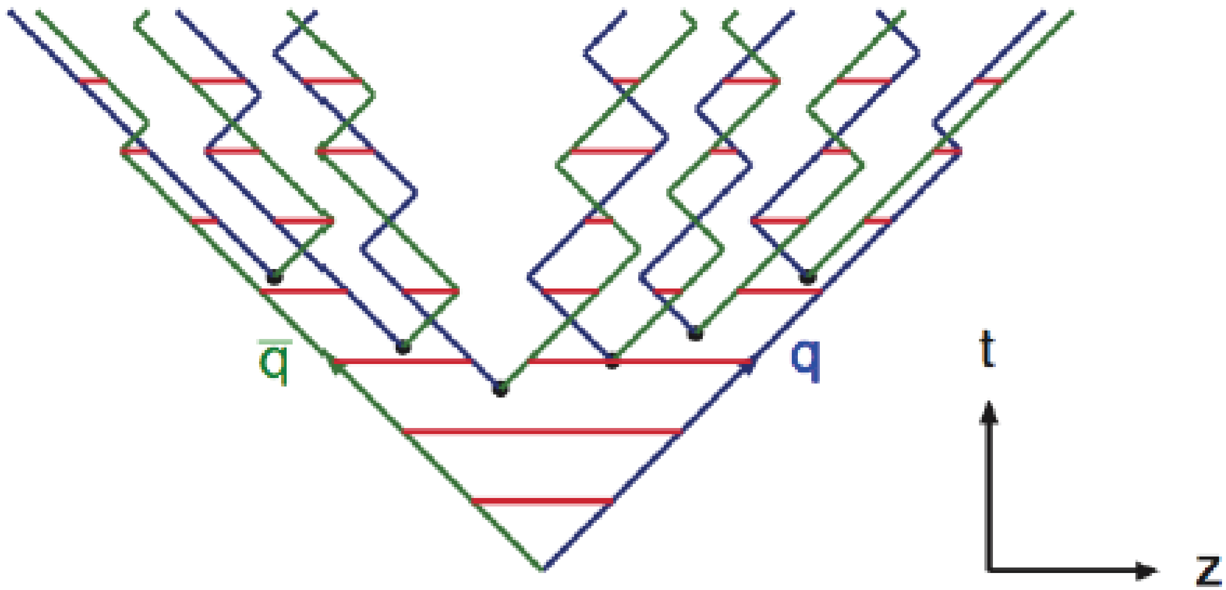


Lund String Model for Hadronization

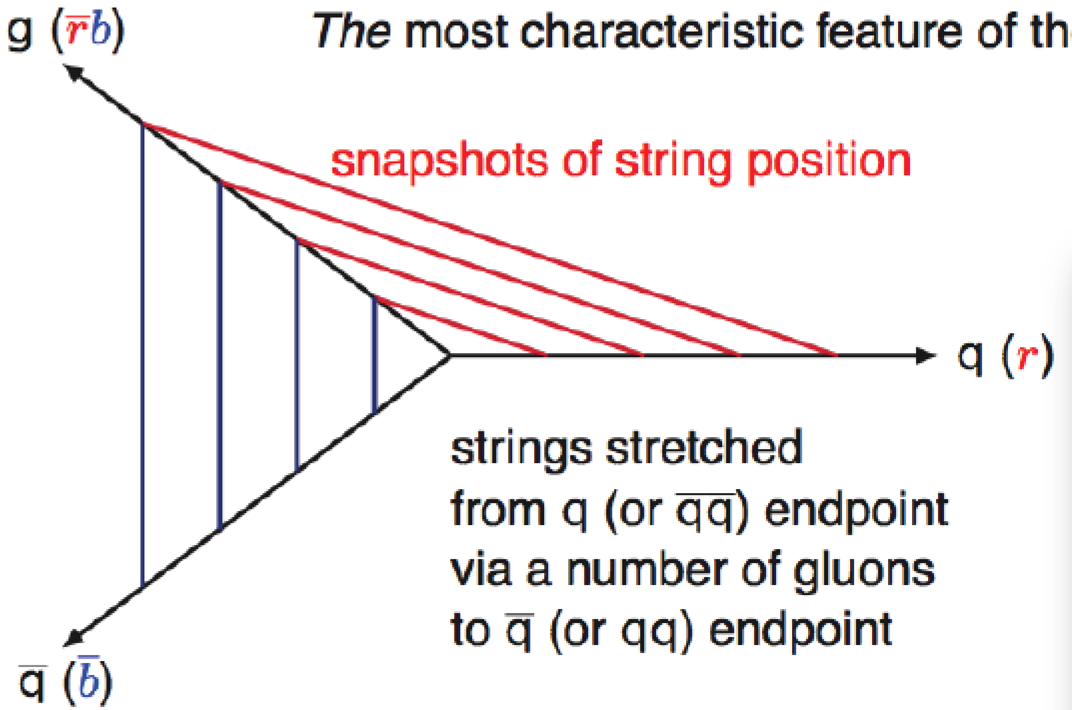


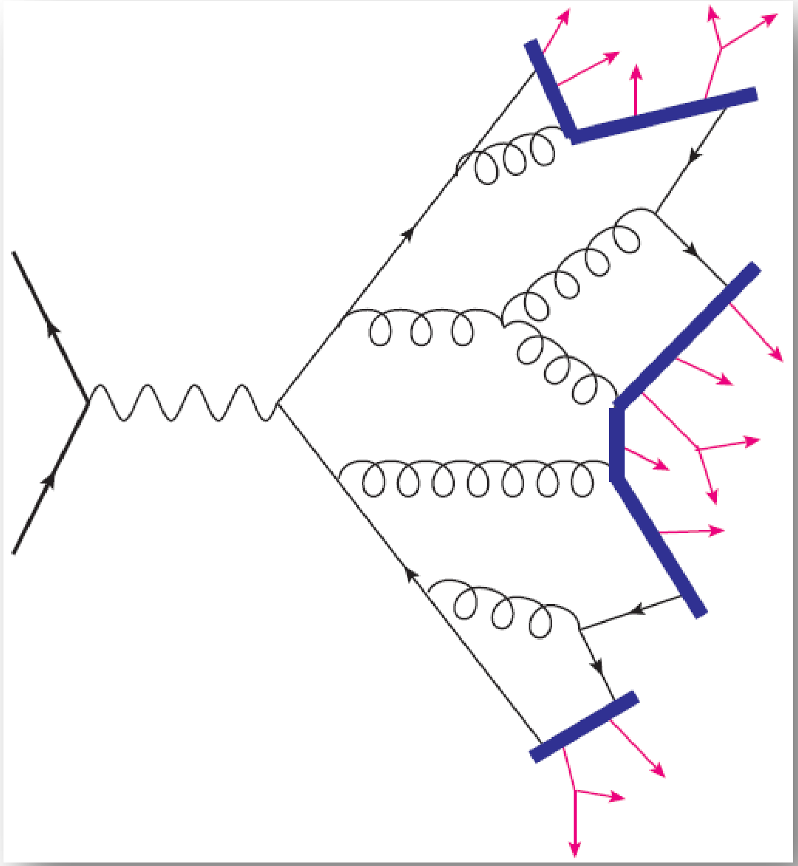
Think of a quark-antiquark pair as connected by a string

When the pair has a large relative transverse momenta, by tunneling the string might break (pair production)



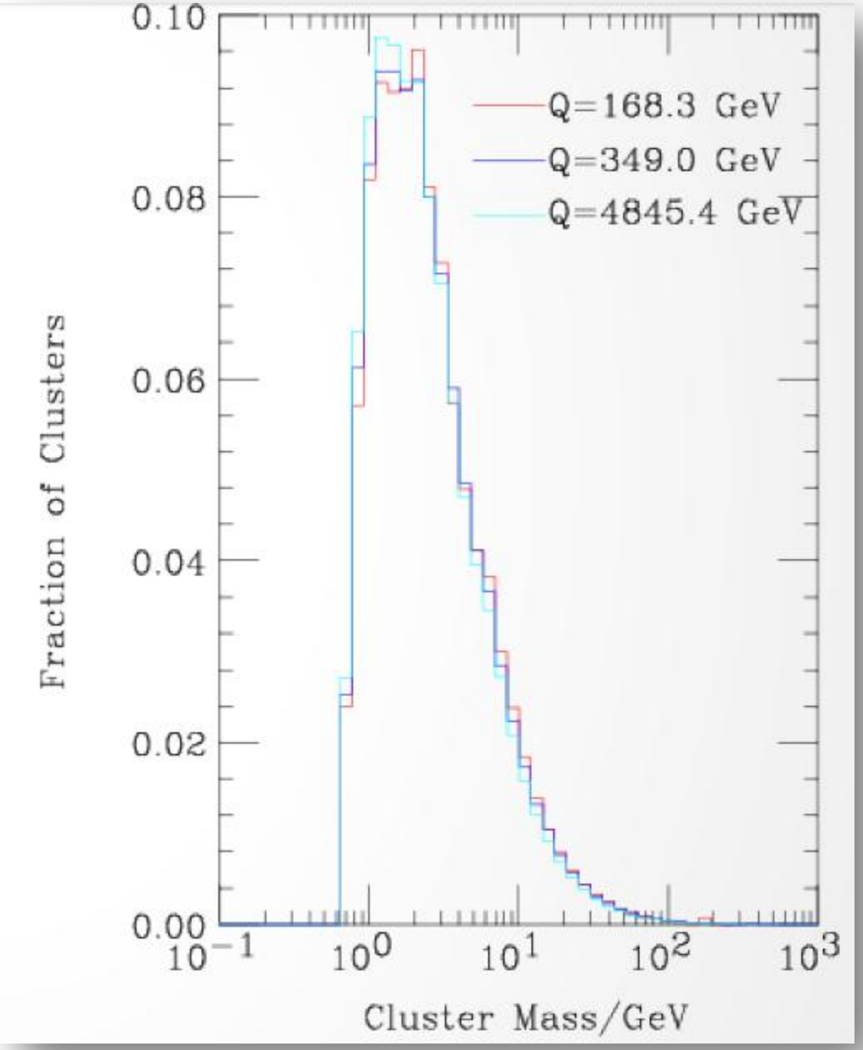
Gluons in the String Model





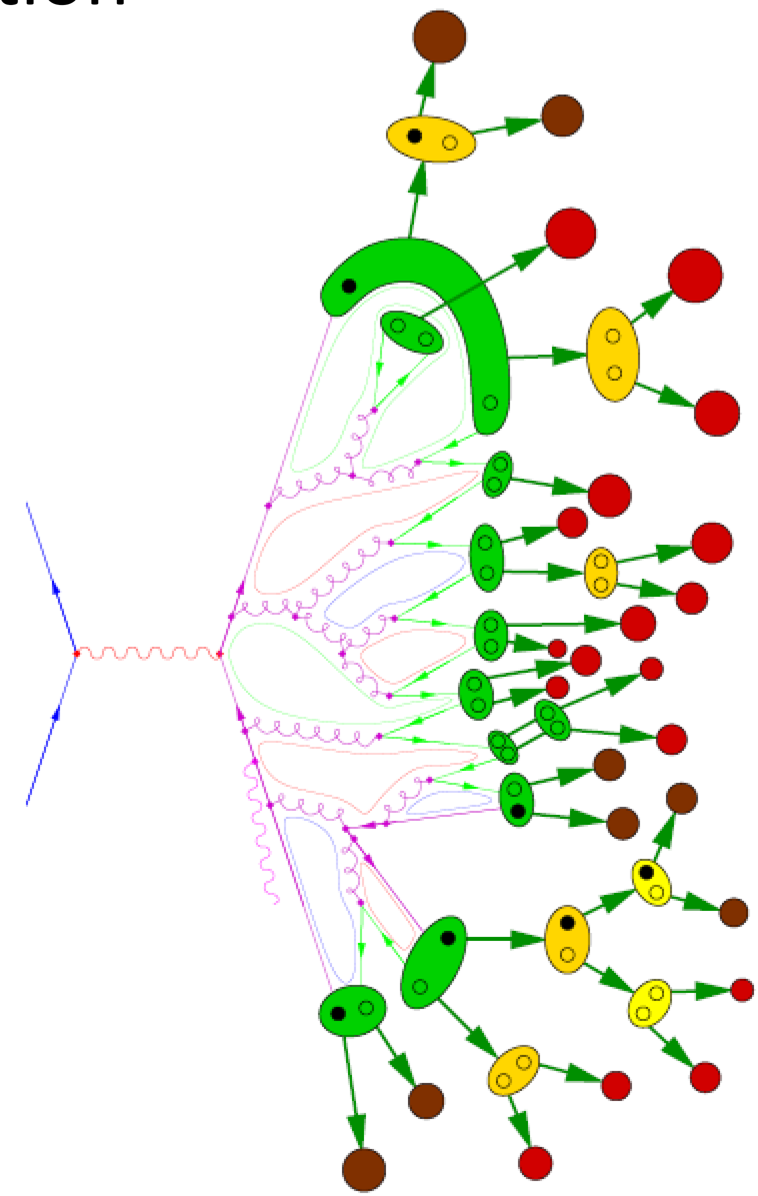
Preconfinement

- Color singlet quark-antiquark pairs after parton shower are found to end up close in phase space
- The mass spectrum of the color singlets is asymptotically independent of the production mechanism
- It peaks at low mass of the order of the PS cutoff



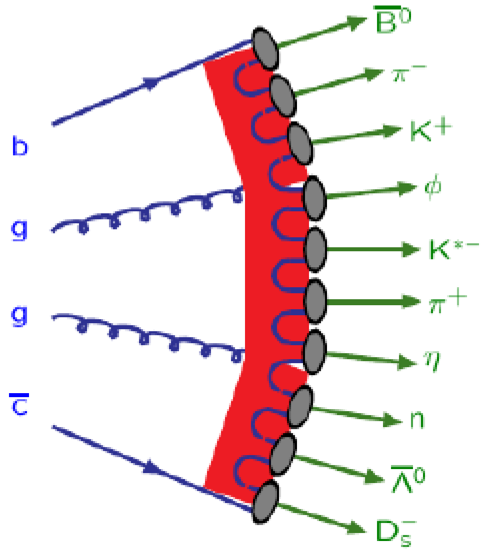
Cluster Model for Hadronization

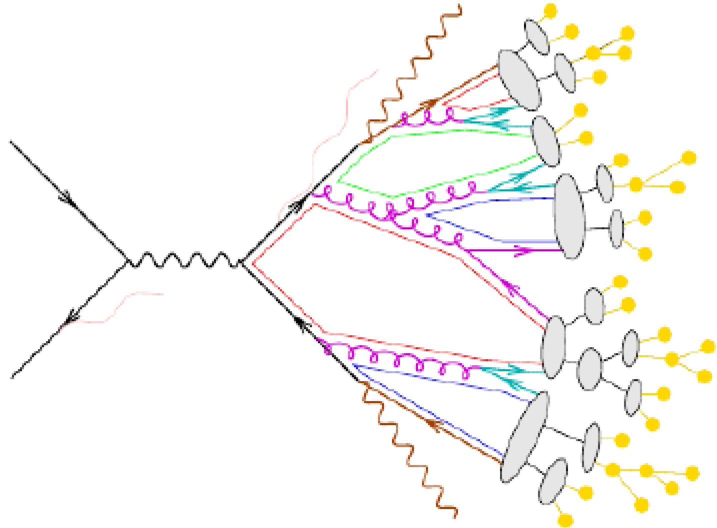
- Split gluons into *qqbar* pairs
- Color adjacent pairs form primordial clusters
- Clusters decay into hadrons according to phase space
- Heavy clusters can decay into lighter ones (C→CC, C→CH, C→HH)



String vs Cluster

[T.Sjöstrand, Durham'09]

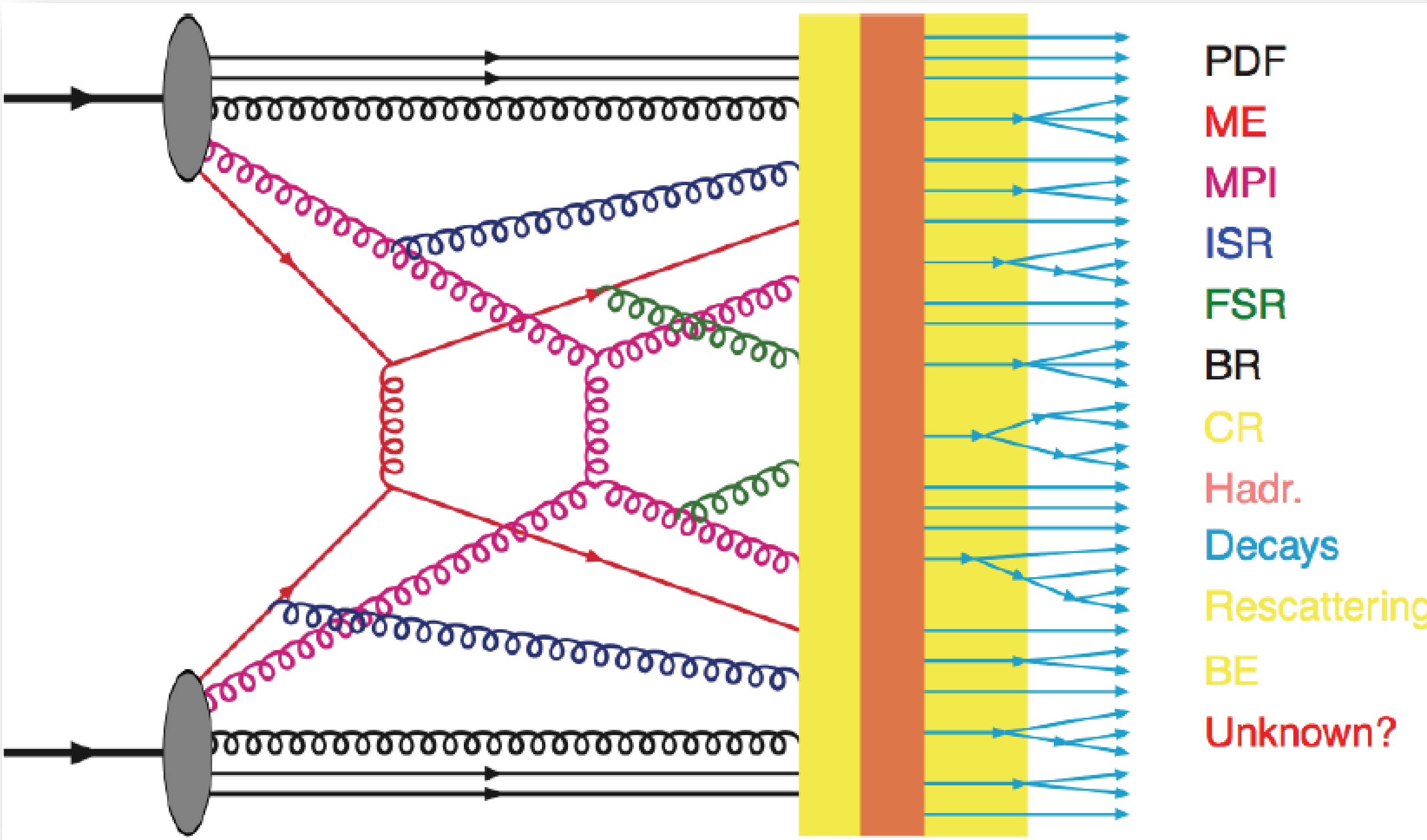




program	PYTHIA	HERWIG
model	string	cluster
energy–momentum picture	powerful	simple
	predictive	unpredictive
parameters	few	many
flavour composition	messy	simple
	unpredictive	in-between
parameters	many	few

"There ain't no such thing as a parameter-free *good* description"

QCD Needs Parton-Shower Monte Carlos



Taken from Torbjörn Sjöstrand

Summary

- Jet algorithms form essential part of the phenomenology at hadron colliders
- Modern jet algorithms are both IR safe and very efficient
- Fixed-order calculations are very reliable, but they produce an oversimplified picture of hadron collider events
- Hadron colliders need Monte-Carlo generators
- Event generation is a multilayered problem, and many aspects of it, although under relative control, would benefit from new (first principles) ideas