Some wake field calculations for pill box cavities

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Input values for wakefield calculations

I got the following from Diktys for the VCC channes:

"Pre6D"	1.	2.	3.	4.	""	"Post6D"	1.
"f(MHz)"	325.	325.	650.	650.	""	"f(MHz)"	325.
"L(m)"	0.255	0.25	0.1349	0.1349	""	"L(m)"	0.25
"E (MV/m)"	22.	22.	28.	30.	""	"E(MV/m)"	19.
"Phase(deg.)"	14.	15.	20.	16.	""	"Phase(deg.)"	41.
""	""	""	""	""	""	"Bunch (cm)"	8.31

From HCC

21 bunches   1 bunch each sign

[Start]====325======650====[Recomb]====325======650=====[End 6D] 4.7 e12

eff | 0.84 | 0.84 | 0.8 | 0.84 | 0.84 |

N m   11.8 e12   9.9 e12   8.3 e12   6.6 e12   5.6 e12

Reference: Greg Werner arXiv:0906.1007v1 [physics.acc-ph] 4 Jun 2009.
This is a calculation for bunch velocity v = c.

Analytical equation

$$V_z(s;r_b,r_t,\theta_t) \;=\; 2H(s)\sum_{m,n,p}\frac{2-\delta_{p0}}{1+\delta_{m0}}$$

Off axis beam/test particle

Transit time

$$\times\frac{J_m\left(\frac{j_{m,n}r_b}{R}\right)J_m\left(\frac{j_{m,n}r_t}{R}\right)\cos(m\theta_t)\cdot 2\left[1-(-1)^p\cos(\omega_{mnp}\ell/c)\right]}{\pi\epsilon_0\ell j_{m,n}^2 J_m'(j_{m,n})^2}$$

$$\times\frac{1}{2}\exp\left(-\frac{\sigma^2\omega_{mnp}^2}{2c^2}\right)\mathrm{Re}\left[e^{i\omega_{mnp}s/c}\mathrm{erfc}\left(-\frac{s+i\sigma^2\omega_{mnp}/c}{\sqrt{2}\sigma}\right)\right].$$

Spectrum analysis gaussian bunch

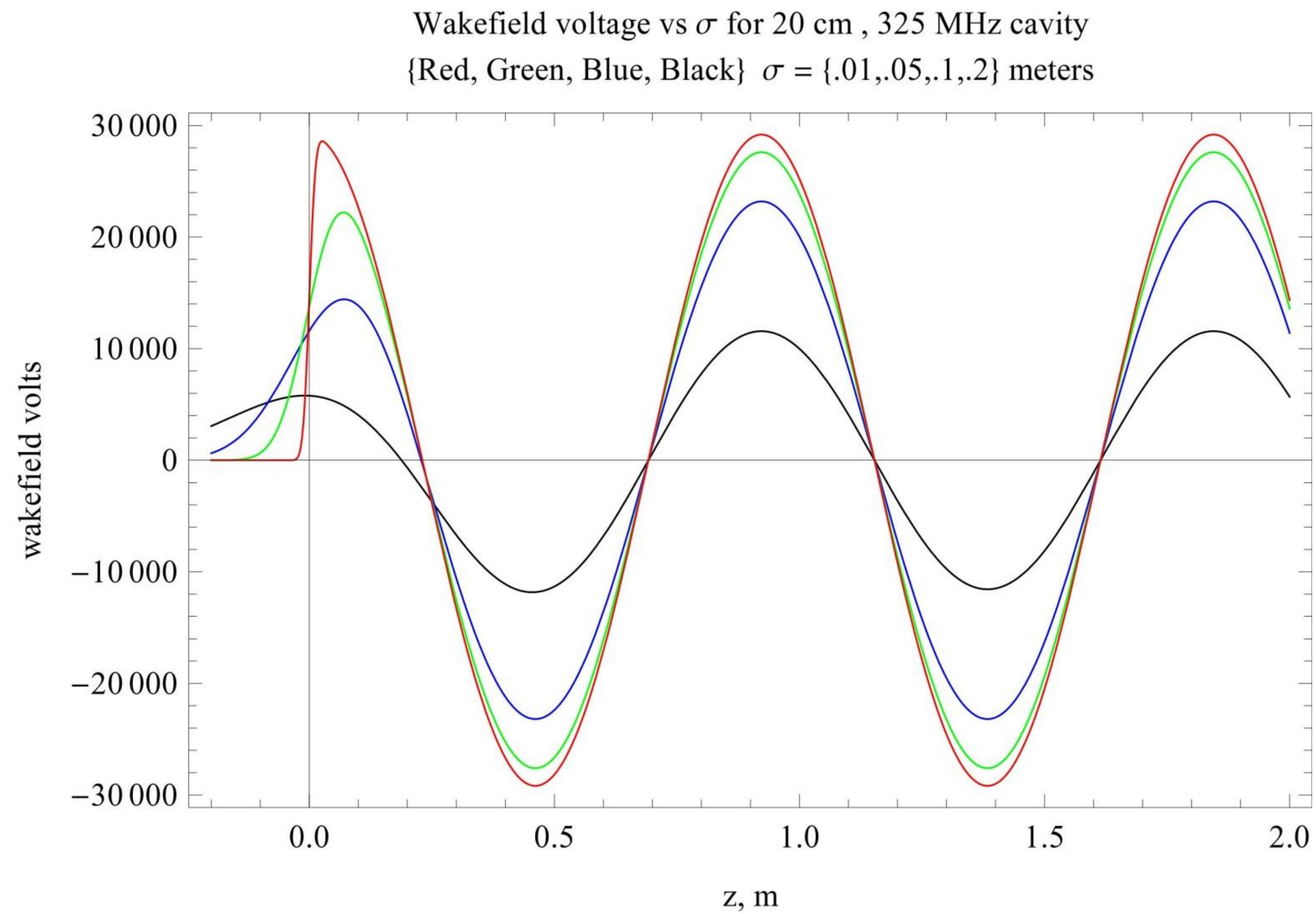
Sine wave

Equals 2 except near z = 0

What the plots will show

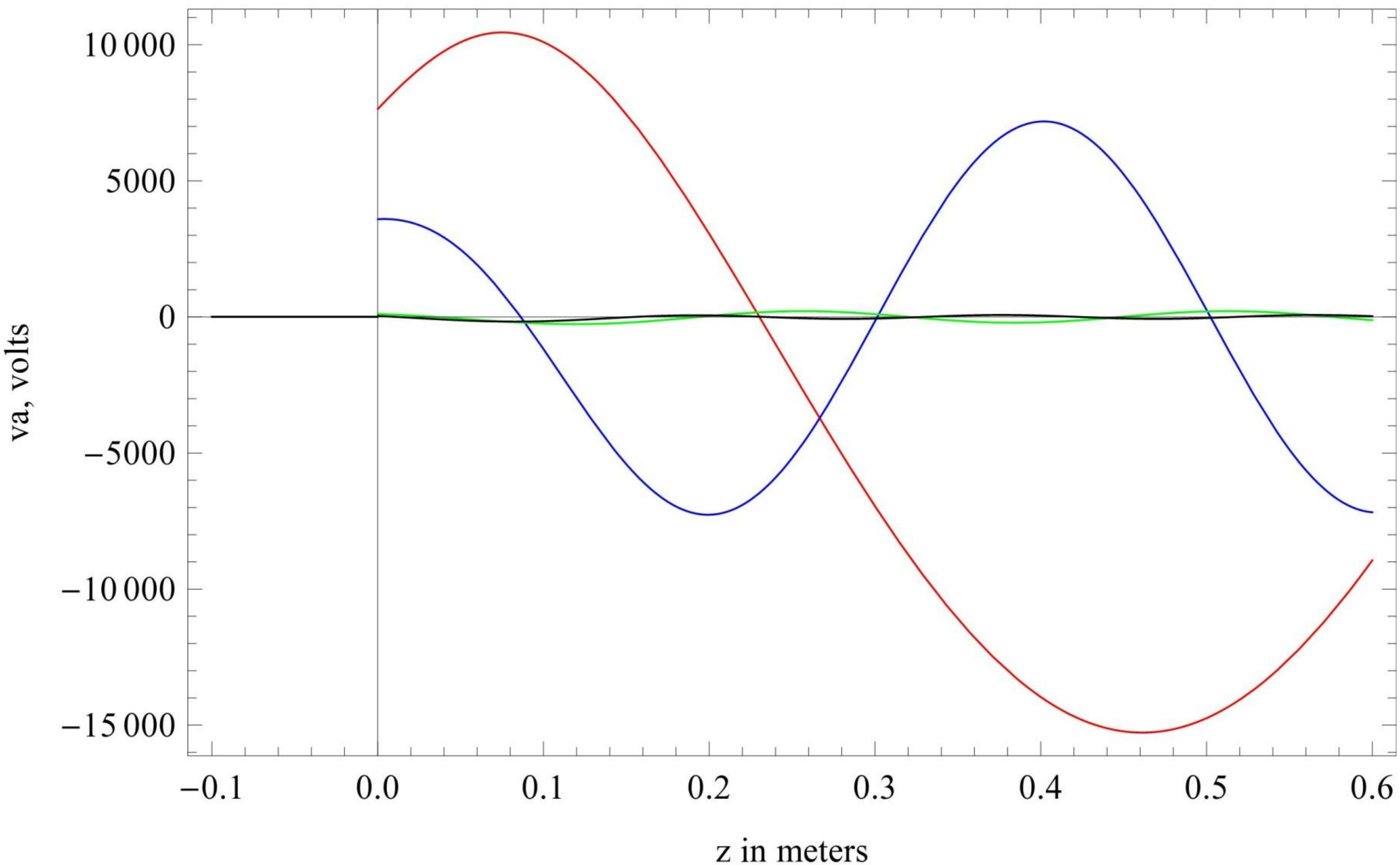
1. We assume a bunch train of 21 bunches going thru a single cavity.
2. Each bunch leaves a residual wake field in the cavity that adds linearly to the cavity drive voltage plus the residual wake fields of all proceeding bunches.
3. The cavity properties are VCC taken from slide #1 as representative examples.
4. The bunch intensities are taken from slide #1 but using HCC values.   Again, representative example. They are scaled up from MCAC values.
5. The wake fields are calculated for V=C.

Example wake fields for 325 MHz cavity at channel start



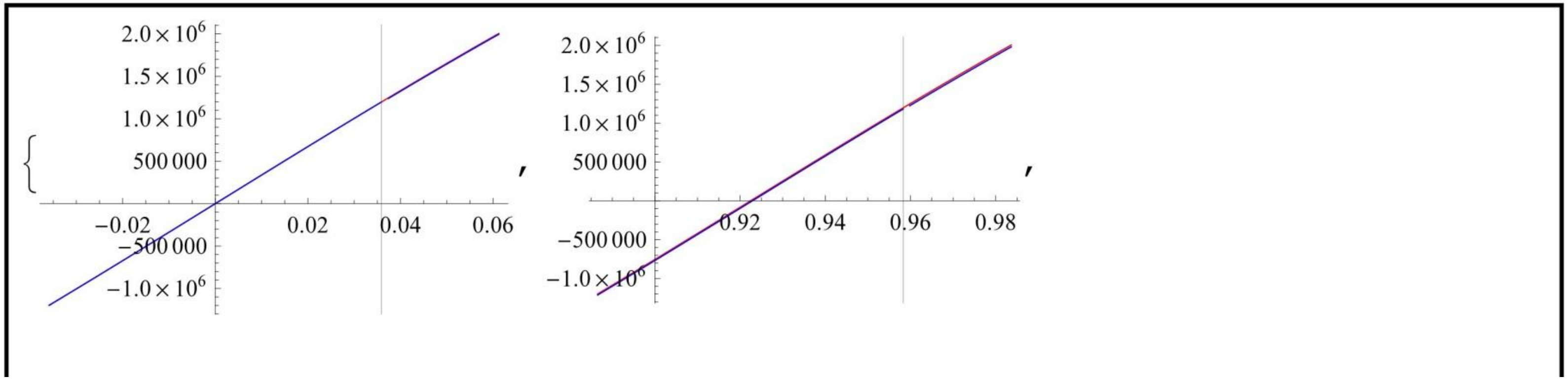
Single Bunch Wake Field for Various bunch lengths, n=1, p=0, 10e12 muons

Plot of the wakefields of the first 3 radial modes,p=0, n =1,2,3 (Red,Blue, Green)
11.8/21 10^{12} particles, lc = .225 m

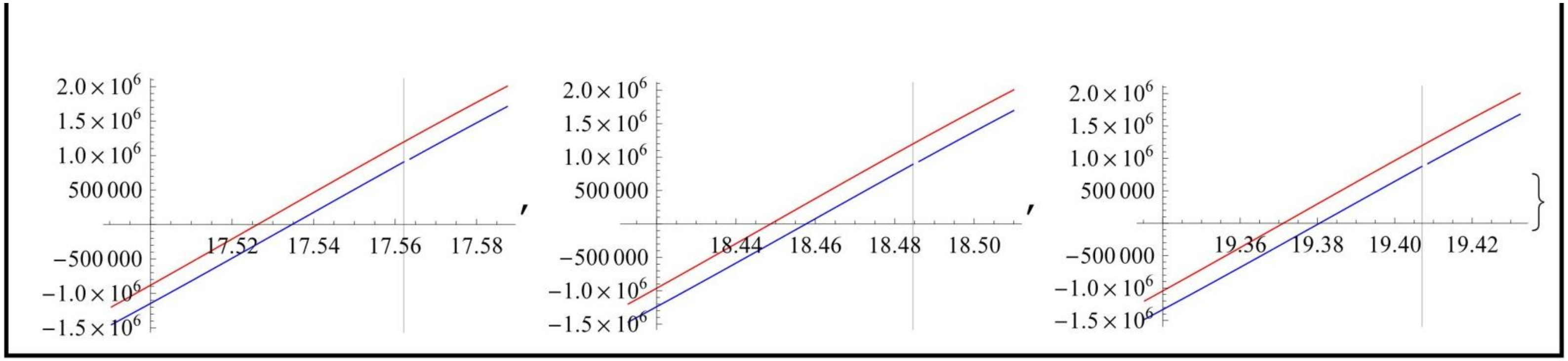


Plot of first 3 radial modes form single bunch of 5.6 10e11

Blue line: 325 MHz wakefield after (1,21} previous bunches
n=1, p = 0, nb=21, 5.6 10^{11} ppb, lc = 22.5 cm , σ = 8 cm
The red curve is equal to the RF drive,
the verticle line is the bunch crossing time



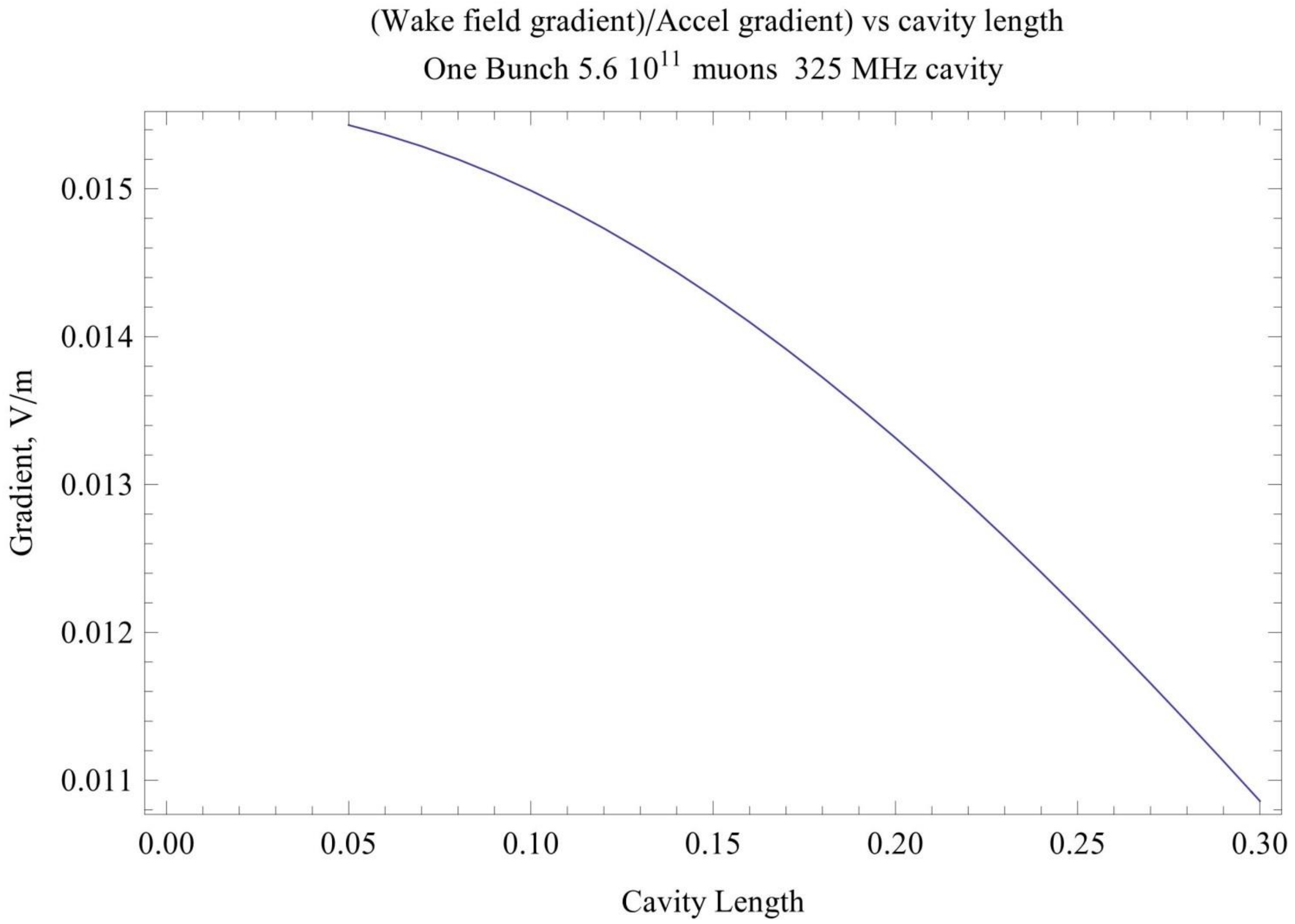
Above: first two bunches at channel beginning
Below: Last three bunches in 21 bunch train
Red: Drive field, Blue: drive field + wake field.



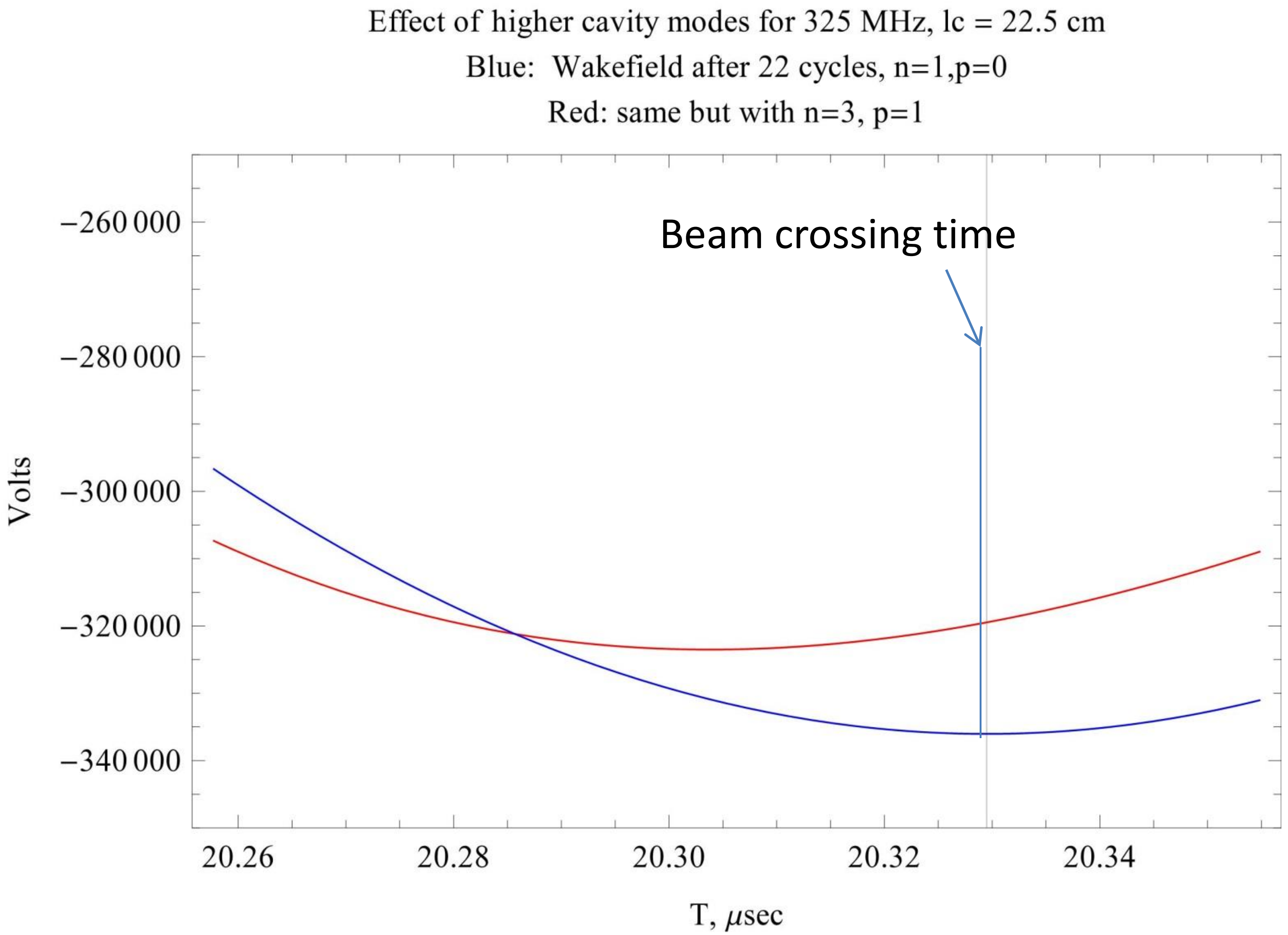
z Cycle of bunch crossing

Two effects from wake field:

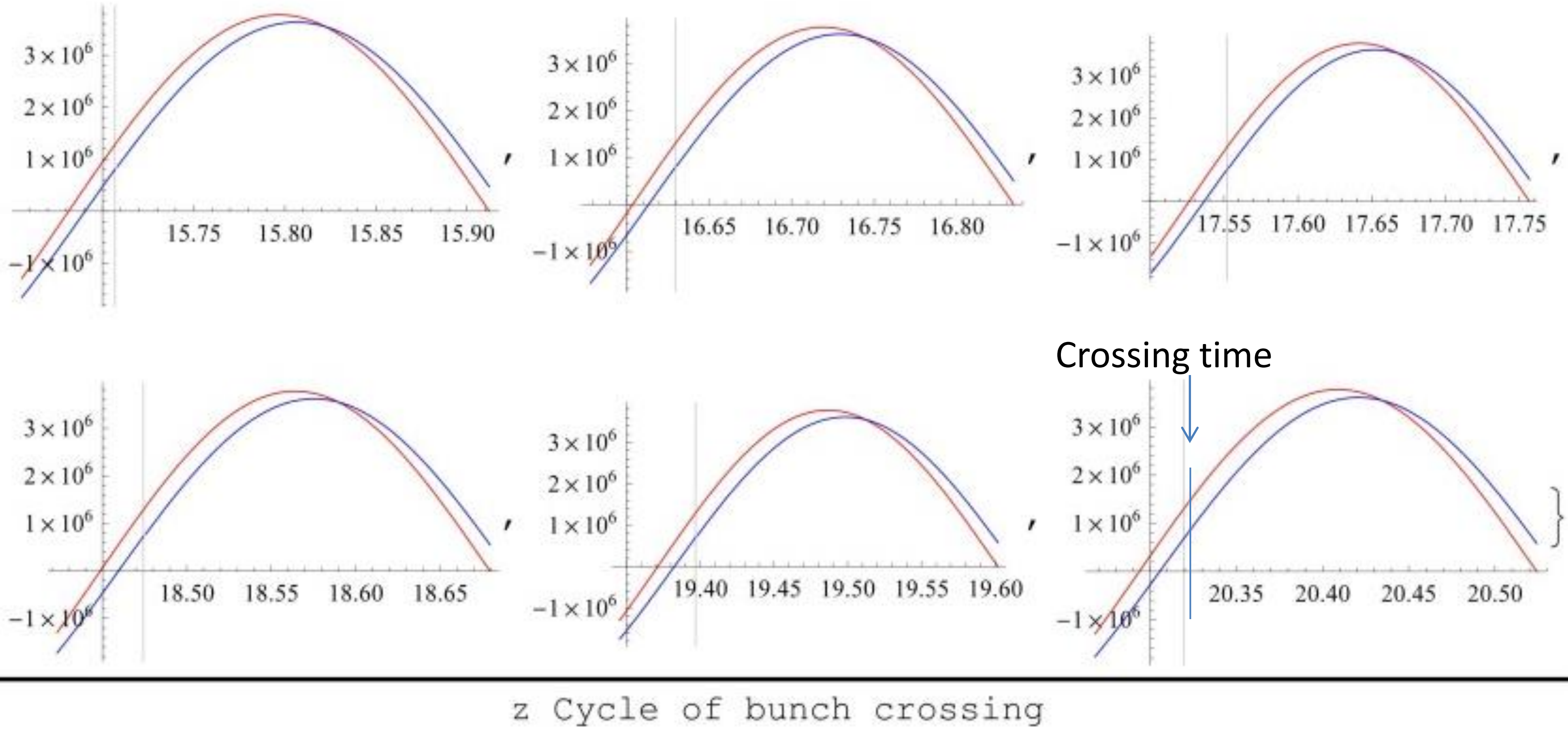
1. Energy gain at central crossing time is decreased
2. Slope of cavity voltage is changed which changes synchrotron frequency for particles in the bunch.



Note: From n=1 mode there is only amplitude change as the wakefield from n=-1 is always at 90 degrees. However for all other modes there is no phase coherence, so they can change the slope.

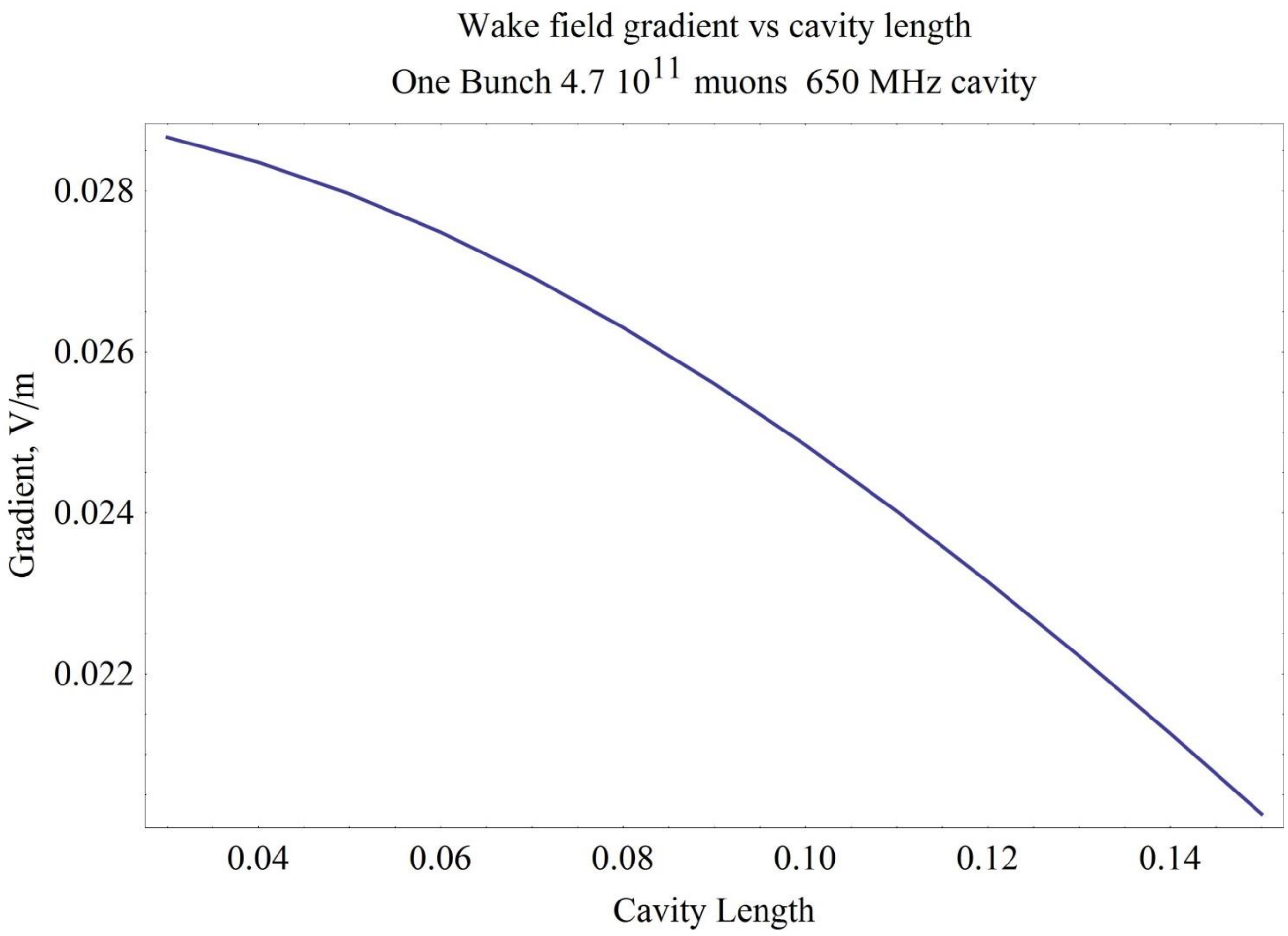


Some results for 650 MHz cavity before recombination

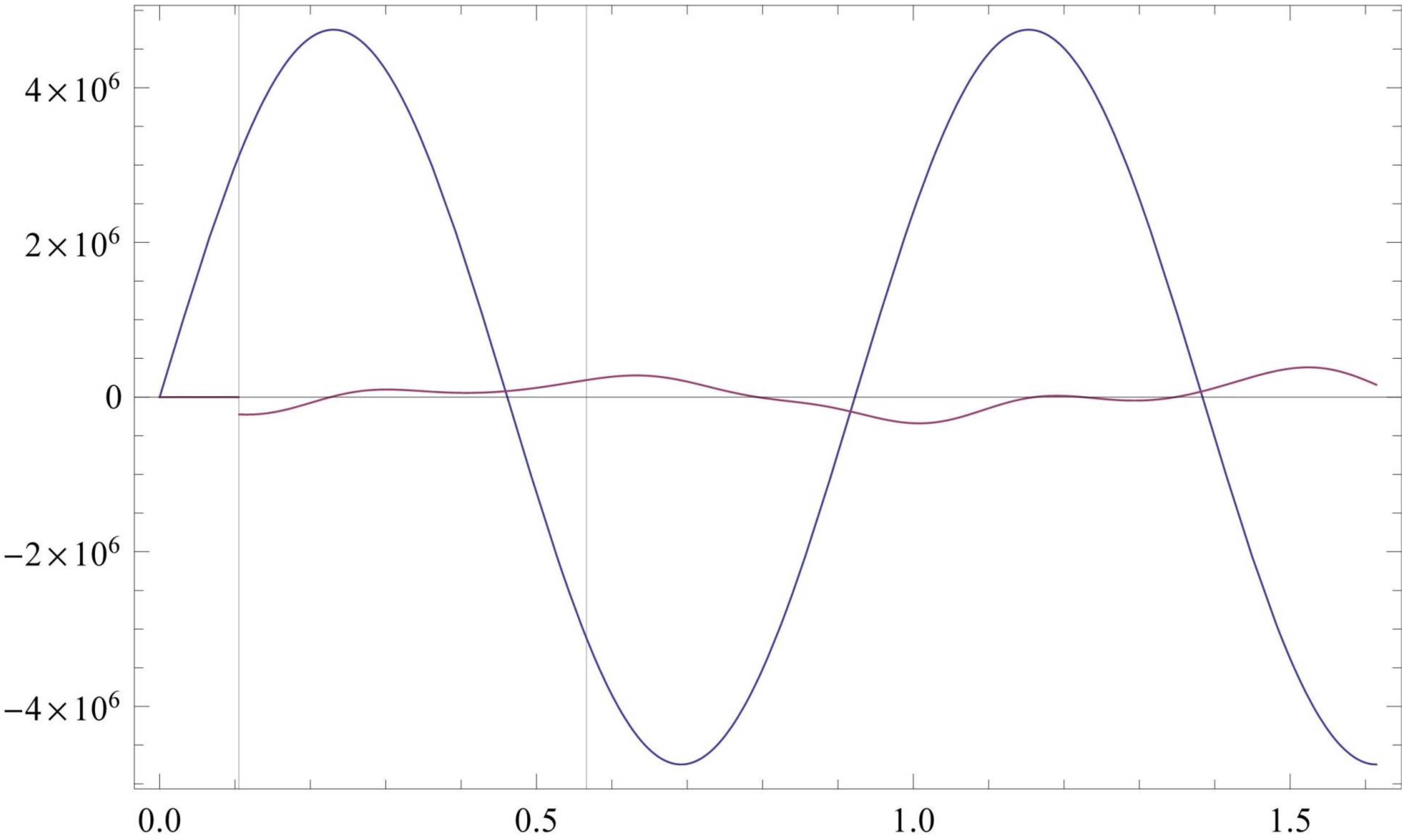


Almost 50% of the voltage is lost

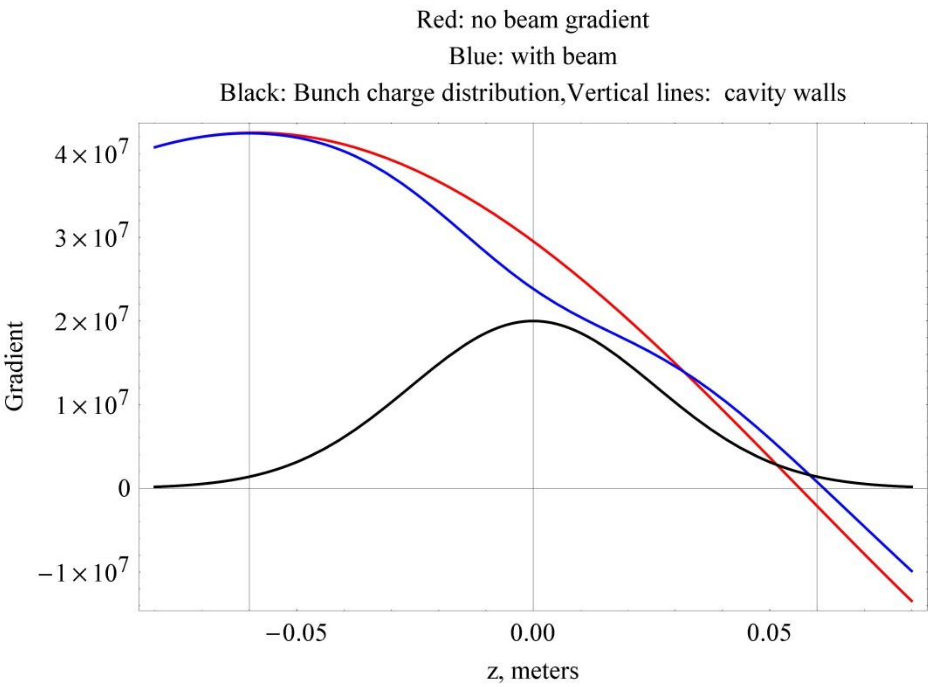
650 MHz, VCC. Change in gradient vs cavity length

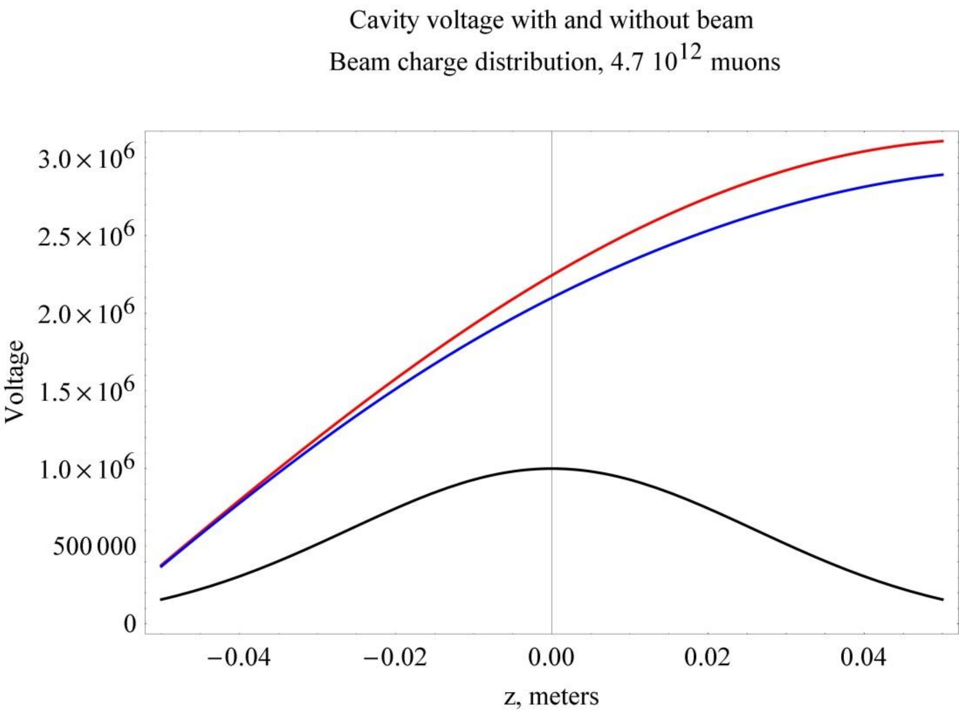


325 MHz after recombination 8.3 e 12 muons/bunch
N=1,2,3 and p=1
Blue: Drive voltage, Red: wake field

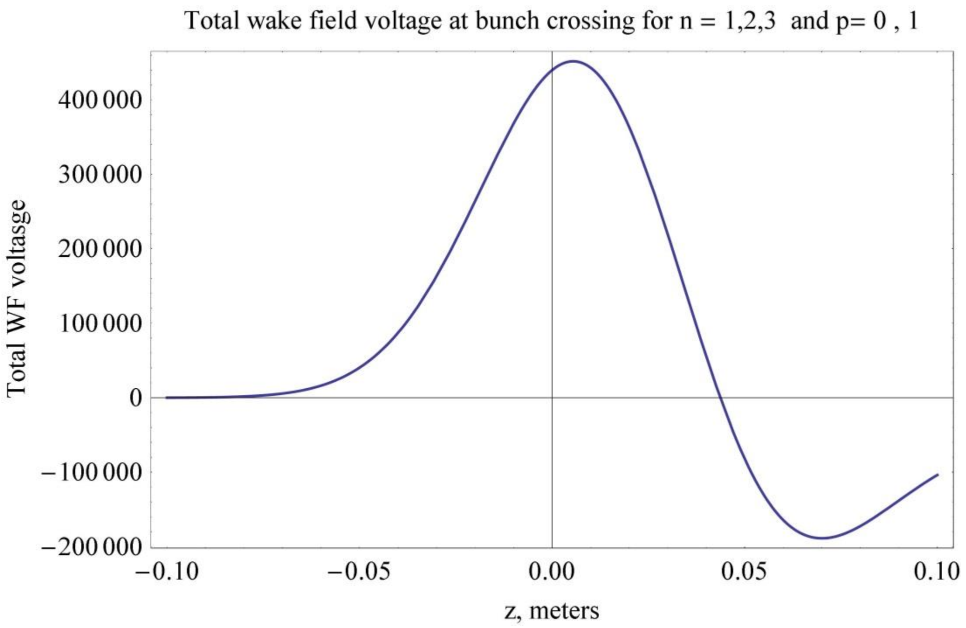


650 MHz after recombination 4.7 e 12 muons/bunch
N=1 and p=0
Red: Drive voltage, Blue: Drive + wake field





Gradient **Voltage**

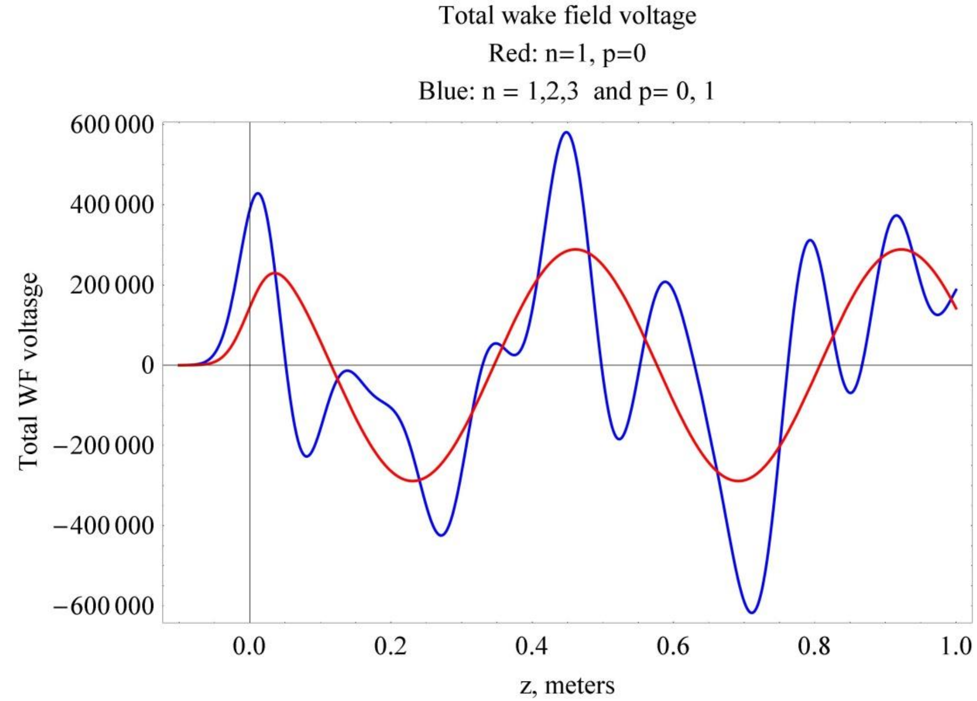


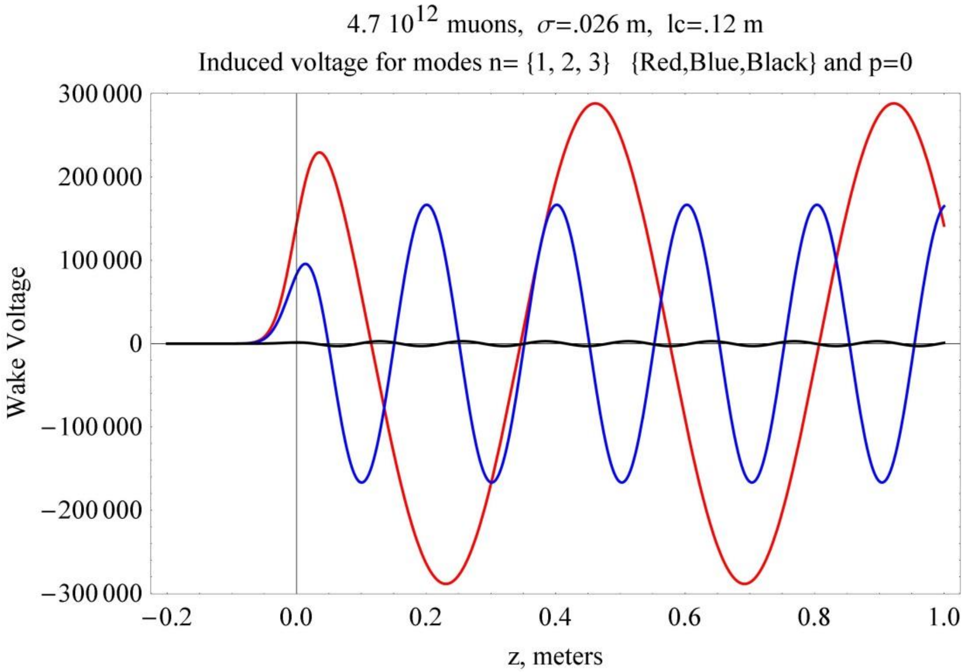
Total Wake field voltage n = 1,2,3 p=0,1

Above at beam crossing

Right: Longer time scale. Red is for mode n=1 , p = 0

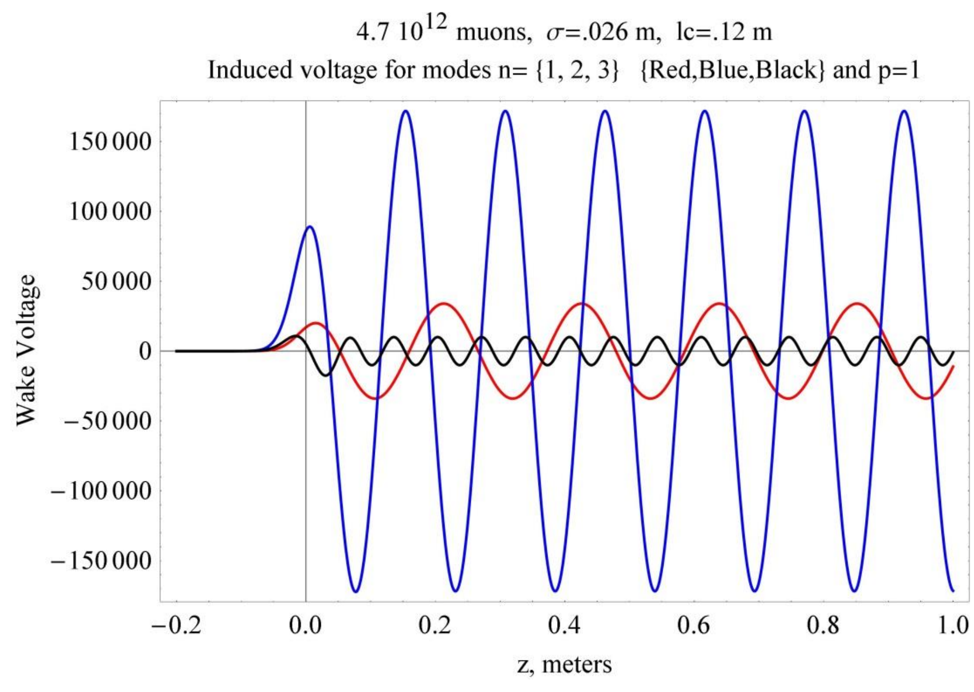
Blue has sume of modes n = 1, 2, 3 and p = 0,1

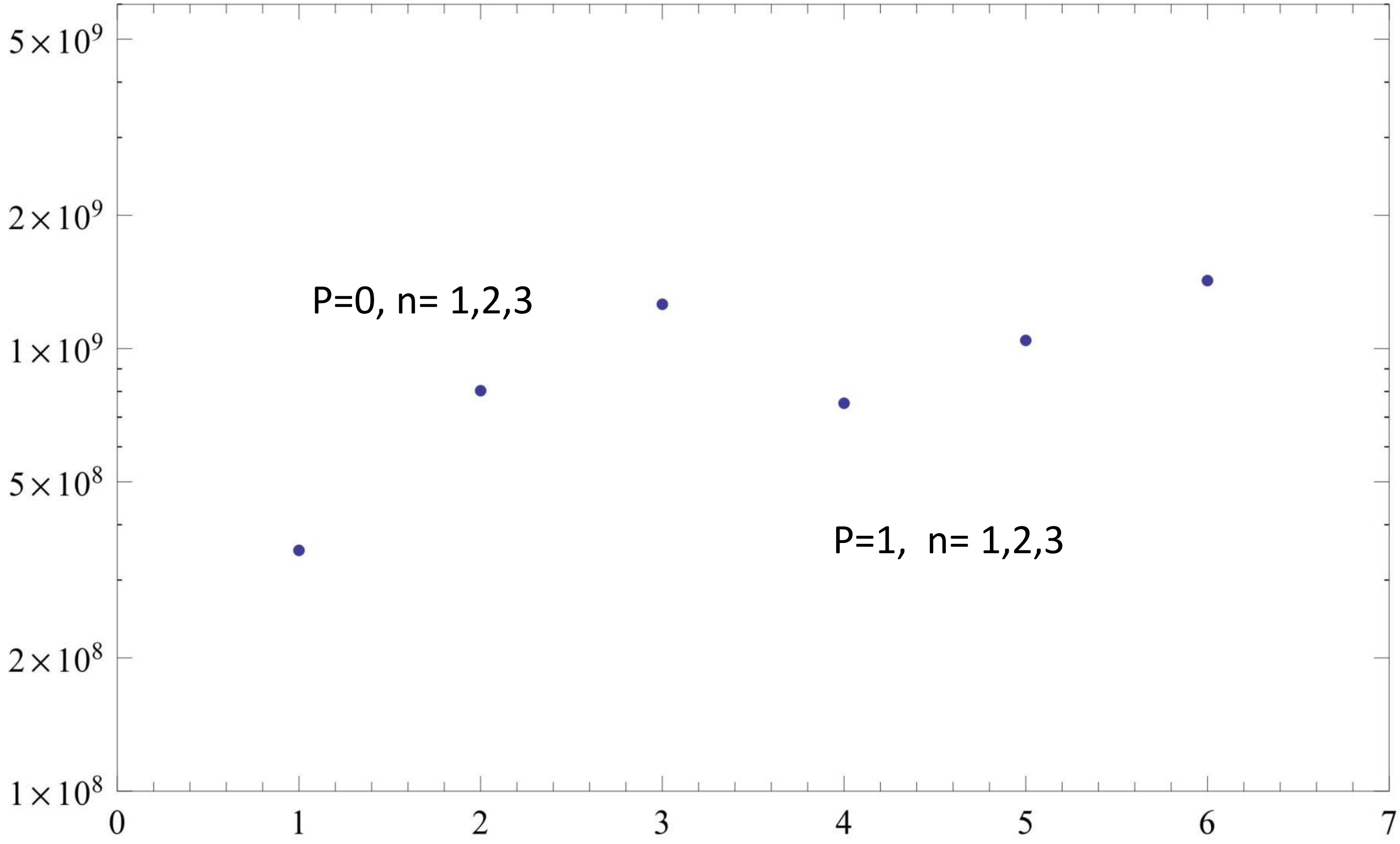




Below:
Modes n = 1,2,3 with p=1

Above:
Modes n = 1,2,3 with p=0





The plot above is for a 325 NHz cavity with lc = 22.5 cm

Conclusions

Need correct simulation with correct velocity in all cases. The down stream bunches always see a lower voltage which will cause synchrotron oscillations which n turn will affect the down stream wake field. It couples the bunches so the picture here is far from complete.

1. **The effect for 650 is larger than for the 325 MHz cavities.**
2. **The higher modes, both radial n modes with p=0 and p=1 seem to be important to include**
3. **Small transit angles produce a phase shift in the cavity voltage, and transit angles near 90 degrees produce an RF voltage reduction**
4. **After recombination the effects are large and we are in a new regime. The single bunch changes the cavity voltage as it crosses the gap. This will require a serious effort to include it in the cooling simulation.**
5. **Accelerating both + and – charges in the same channel looks like a challenge.**
6. **Cavity length is an important parameter mechanically, mode excitation, and power wise and needs study.**