Neutrino Mass Models

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Theoretical Challenges

(i) Absolute mass scale: Why $m_\nu << m_{u,d,e}$?

- seesaw mechanism: most appealing scenario ⇒ Majorana
  - GUT scale (type-I, II) vs TeV scale (type-III, double seesaw)
- TeV scale new physics (SUSY, extra dimension, U(1)′) ⇒ Dirac or Majorana

(ii) Flavor Structure: Why neutrino mixing large while quark mixing small?

- neutrino anarchy: no parametrically small number — Hall, Murayama, Weiner (2000); de Gouvea, Murayama (2003)
  - near degenerate spectrum, large mixing
  - still alive and kicking — de Gouvea, Murayama (2012)
  - possible heterotic string connection — Buchmüller, Hamaguchi, Lebedev, Ramos-Sánchez, Ratz (2007); Feldstein, Klemm (2012)
- family symmetry: there's a structure, expansion parameter (~~symmetry~~ effect)
  - mixing result from dynamics of underlying symmetry
  - for leptons only (normal or inverted)
  - for quarks and leptons: quark-lepton connection ↔ GUT (normal)
- Alternative?

- In this talk: assume 3 generations, no LSND/MiniBoone/Reactor Anomaly

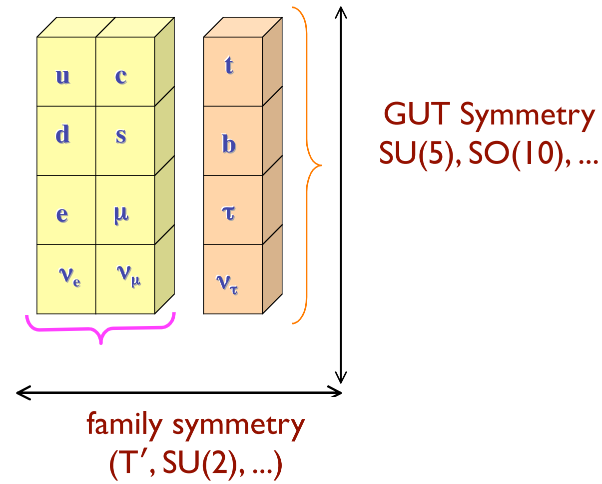
Origin of Mass Hierarchy and Mixing

- In the SM: 22 physical quantities which seem unrelated
- Question arises whether these quantities can be related
- No fundamental reason can be found in the framework of SM
- less ambitious aim ⇒ reduce the # of parameters by imposing symmetries
  - Grand Unified Gauge Symmetry
    - seesaw mechanism naturally implemented
    - GUT relates quarks and leptons: quarks & leptons in same GUT multiplets
      - one set of Yukawa coupling for a given GUT multiplet ⇒ intra-family relations
  - Family Symmetry
    - relate Yukawa couplings of different families
      - inter-family relations ⇒ further reduce the number of parameters

⇒ Experimentally testable correlations among physical observables

Origin of Flavor Mixing and Mass Hierarchy

- Several models have been constructed based on
  - GUT Symmetry [SU(5), SO(10)] ⊕ Family Symmetry G_F
- Family Symmetries G_F based on continuous groups:
  - U(1)
  - SU(2)
  - SU(3)



- Recently, models based on discrete family symmetry groups have been constructed
  - A_4 (tetrahedron)
  - T´ (double tetrahedron)
  - S_3 (equilateral triangle)
  - S_4 (octahedron, cube)
  - A_5 (icosahedron, dodecahedron)
  - Δ_{27}
  - Q_4

Motivation: Tri-bimaximal (TBM) neutrino mixing

Tri-bimaximal Neutrino Mixing

- Latest Global Fit (3σ)

$$\sin^2\theta_{23} = 0.437\ (0.374 - 0.626)$$

Capozzi, Fogli, Lisi, Marrone, Montanino, Palazzo (2014)

$$\sin^2\theta_{12} = 0.308\ (0.259 - 0.359)$$

$$\sin^2\theta_{13} = 0.0234\ (0.0176 - 0.0295)$$

- Tri-bimaximal Mixing Pattern

Harrison, Perkins, Scott (1999)

$$U_{TBM} = \begin{pmatrix} \sqrt{2/3} & \sqrt{1/3} & 0 \\ -\sqrt{1/6} & \sqrt{1/3} & -\sqrt{1/2} \\ -\sqrt{1/6} & \sqrt{1/3} & \sqrt{1/2} \end{pmatrix}$$

$$\sin^2\theta_{\mathrm{atm,\,TBM}} = 1/2 \qquad \sin^2\theta_{\odot,\mathrm{TBM}} = 1/3$$

$$\sin\theta_{13,\mathrm{TBM}} = 0.$$

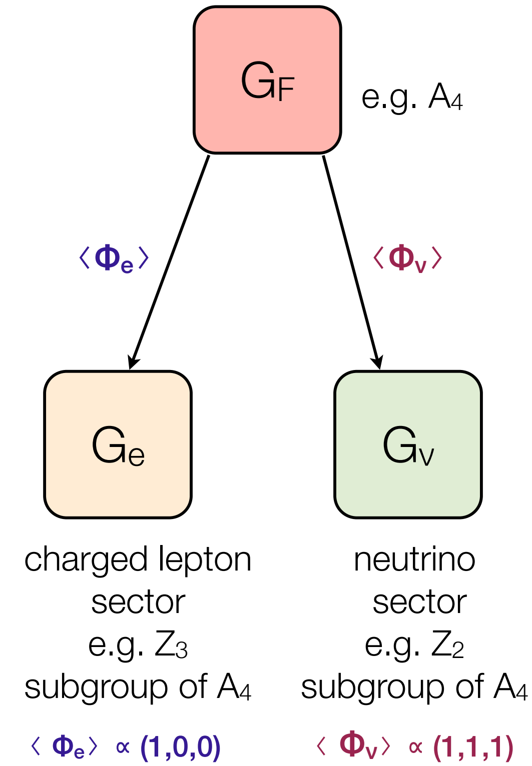
- Leading Order: TBM (from symmetry) + higher order corrections/contributions
- Is TBM a good starting point?

SU(5) Compatibility ⇒ T′ Family Symmetry

M.-C.C, K.T. Mahanthappa (2007, 2009)

- Double Tetrahedral Group T´: double covering of A4
- Symmetries ⇒ 10 parameters in Yukawa sector ⇒ 22 physical observables
- neutrino mixing angles from group theory (CG coefficients)
- TBM: misalignment of symmetry breaking patterns
  - neutrino sector: T′ → G_{TST2} ,
  - charged lepton sector: T′ → G_T
- GUT symmetry ⇒ contributions to mixing parameters from charged lepton sector

  ⇒ deviation from TBM related to Cabibbo angle θ_c



$$\theta_{13} \simeq \theta_c / 3\sqrt{2}$$ ← CG's of SU(5) & T´

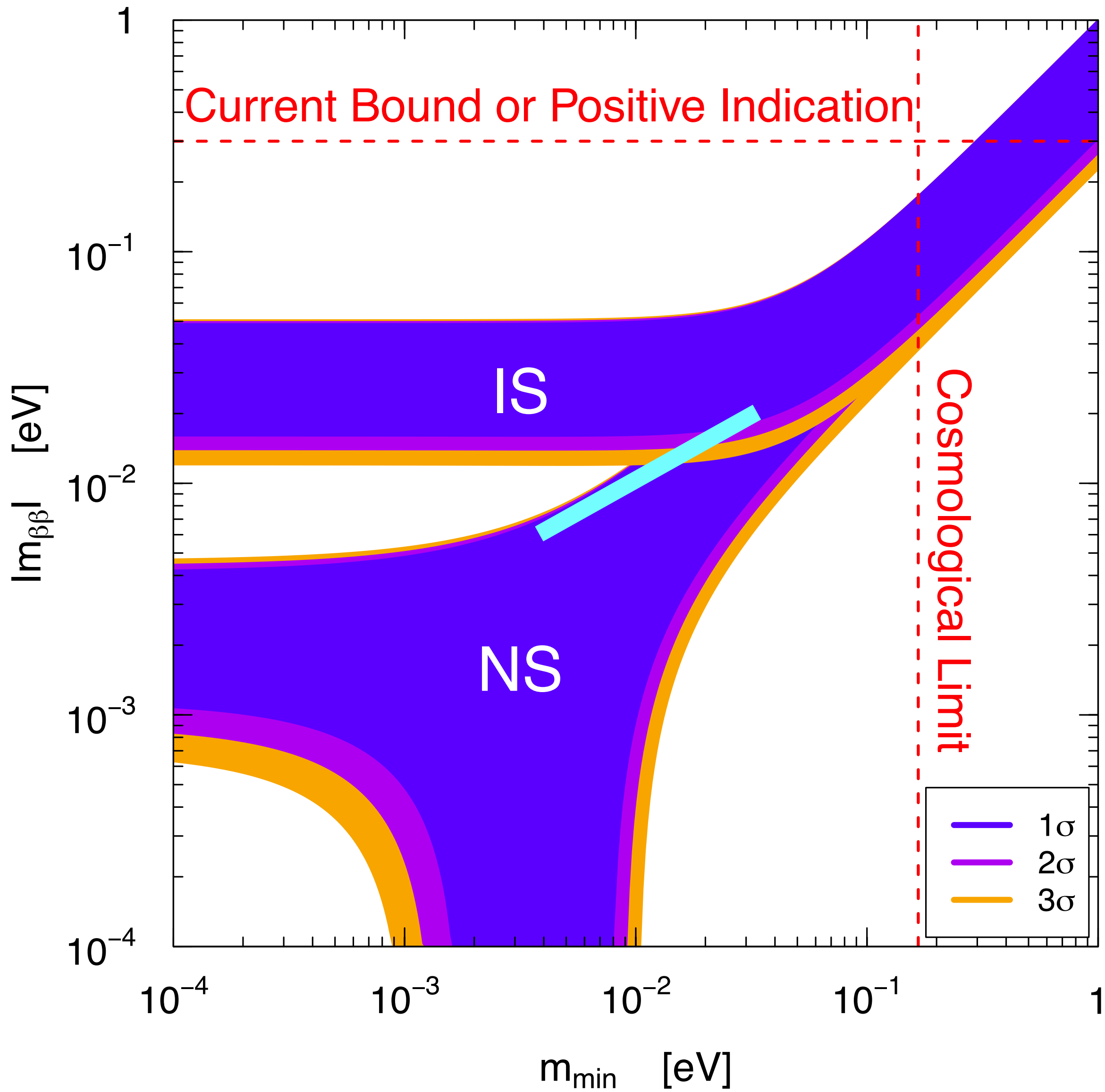
$$\delta \simeq 227^o$$

$$\tan^2\theta_\odot \simeq \tan^2\theta_{\odot,TBM} + \frac{1}{2}\theta_c \cos\delta$$

- large θ_{13} possible with one additional singlet flavon

  M.-C. C., J. Huang, K.T. Mahanthappa, A. Wijiangco (2013)

Neutrinoless Double Beta Decay



our model prediction

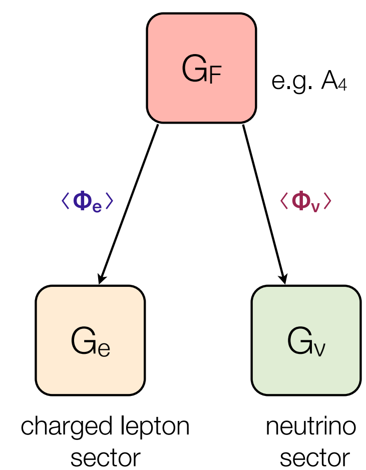
sum rule among masses
⇒ small predicted region

[Plot taken from C. Giunti, LIONeutrino2012]

“Large” Deviations from TBM in A_4

- Generically: corrections on the order of $(\theta_c)^2$
  - from charged lepton sector:
    - through GUT relations
  - from neutrino sector:
    - higher order contributions in superpotential

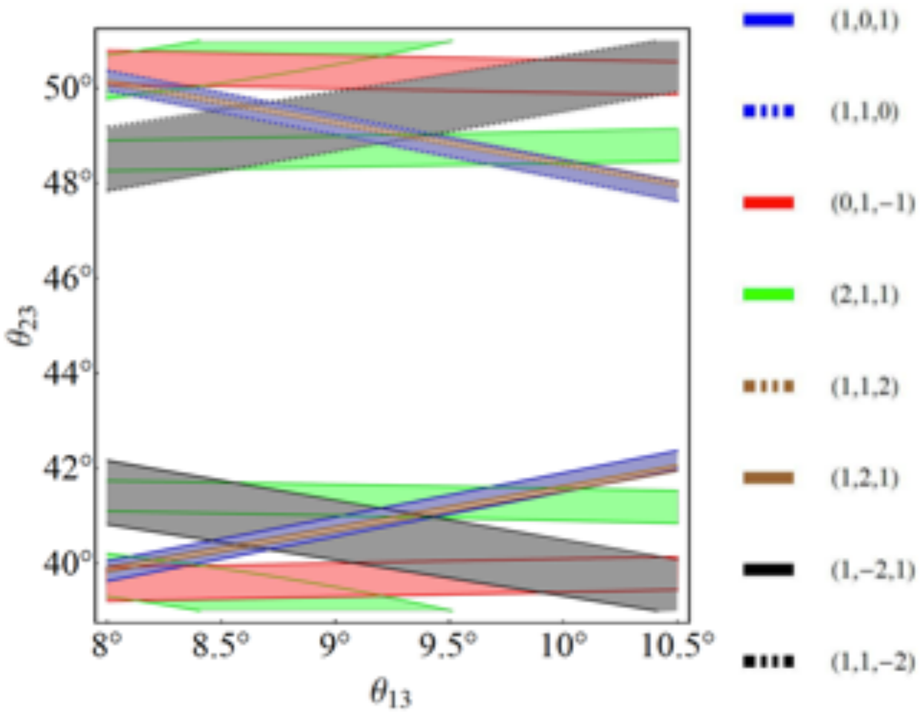
- Modifying the Neutrino sector: Different symmetry breaking patterns
  - TBM: misalignment of M.-C.C, J. Huang, J. O’Bryan, A. Wijangco, F. Yu, (2012)
    - $A4 \to G_{TST2}$ and $A4 \to G_T$
  - A4: group of order 12 ⇒ many subgroups
  - systematic study of breaking into other A4 subgroups



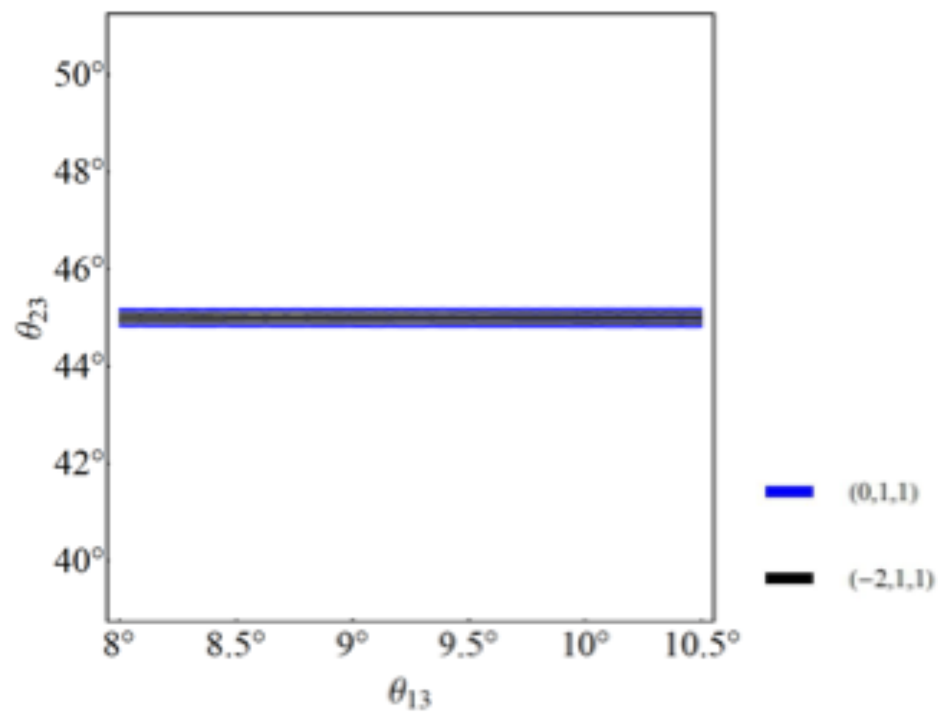
“Large” Deviations from TBM in A_4

M.-C.C, J. Huang, J. O’Bryan, A. Wijangco, F. Yu, (2012)

- Different A4 breaking patterns:



normal



inverted

deviations correlated

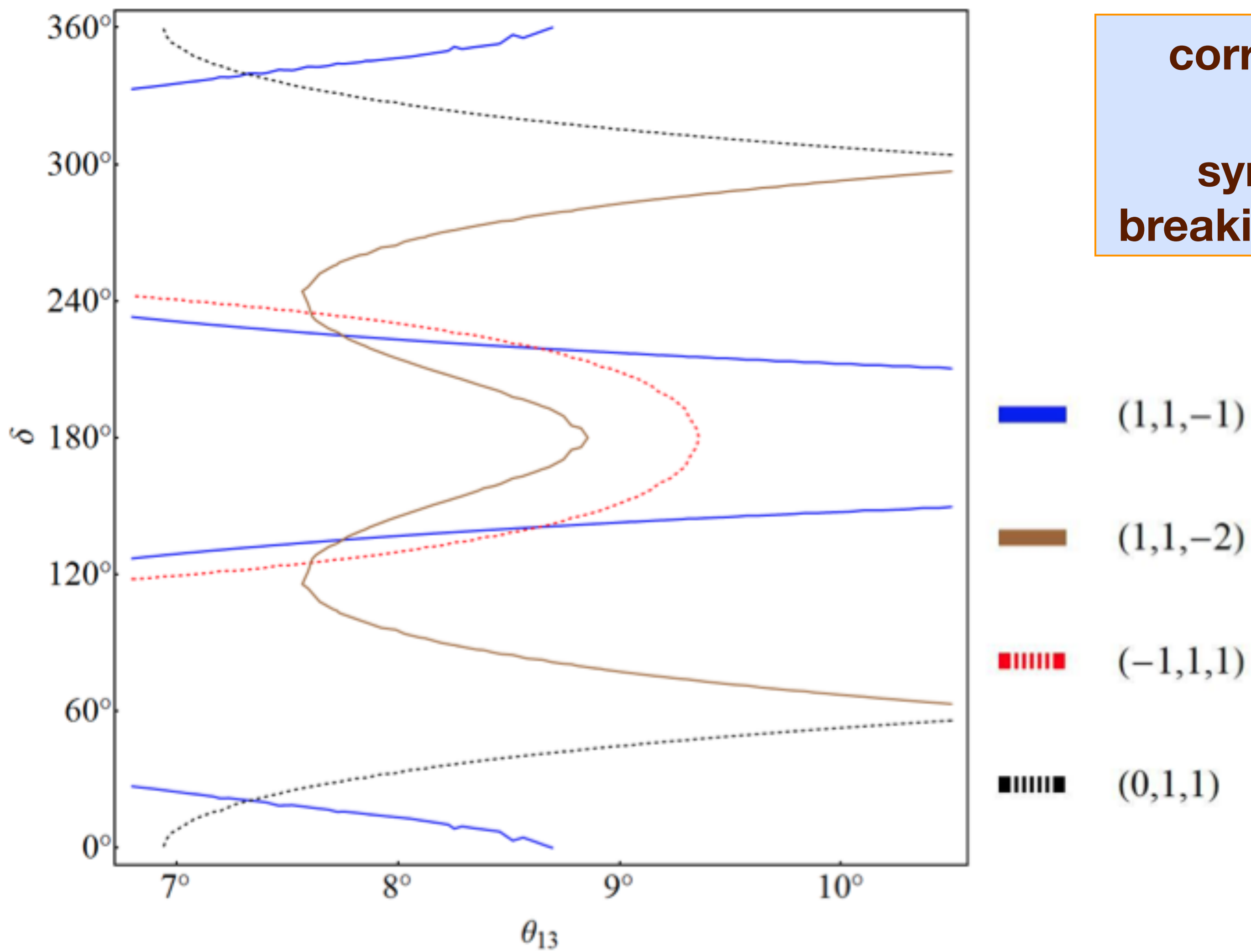
non-maximal θ_{23} ⇨ normal hierarchy

mass ordering ⇨ symmetry breaking patterns

"Large" Deviations from TBM in A_4

M.-C.C, J. Huang, J. O'Bryan, A. Wijangco, F. Yu, (2012)

- Correlation between Dirac CP phase and θ_{13}:



correlations

⇳

symmetry breaking pattern

Another Example: A_5

P. Ballett, S. Pascoli, J. Turner (2015)

- Correlations among different mixing parameters

<table>
<tr><th>G_e</th><th>θ_{12}</th><th>θ_{23}</th><th>$\sin\alpha_{ji}$</th><th>δ</th></tr>
<tr><td rowspan="2">$\mathbb{Z}_3$</td><td rowspan="2">$35.27^\circ + 10.13^\circ\, r^2$</td><td rowspan="2">$45^\circ$</td><td rowspan="2">0</td><td>90°</td></tr>
<tr><td>270°</td></tr>
<tr><td rowspan="4">$\mathbb{Z}_5$</td><td rowspan="4">$31.72^\circ + 8.85^\circ\, r^2$</td><td rowspan="2">$45^\circ \pm 25.04^\circ\, r$</td><td rowspan="2">0</td><td>0°</td></tr>
<tr><td>180°</td></tr>
<tr><td rowspan="2">45°</td><td rowspan="2">0</td><td>90°</td></tr>
<tr><td>270°</td></tr>
<tr><td rowspan="4">$\mathbb{Z}_2 \times \mathbb{Z}_2$</td><td rowspan="4">$36.00^\circ - 34.78^\circ\, r^2$</td><td rowspan="2">$31.72^\circ + 55.76^\circ\, r$</td><td rowspan="2">0</td><td>0°</td></tr>
<tr><td>180°</td></tr>
<tr><td rowspan="2">$58.28^\circ - 55.76^\circ\, r$</td><td rowspan="2">0</td><td>0°</td></tr>
<tr><td>180°</td></tr>
</table>

TABLE I. Numerical predictions for the correlations found in this paper. The dimensionless parameter $r \equiv \sqrt{2}\sin\theta_{13}$ is constrained by global data to lie in the interval $0.19 \lesssim r \lesssim 0.22$ at 3σ. The predictions for θ_{12} and θ_{23} shown here ne-

Corrections to Kinetic Terms

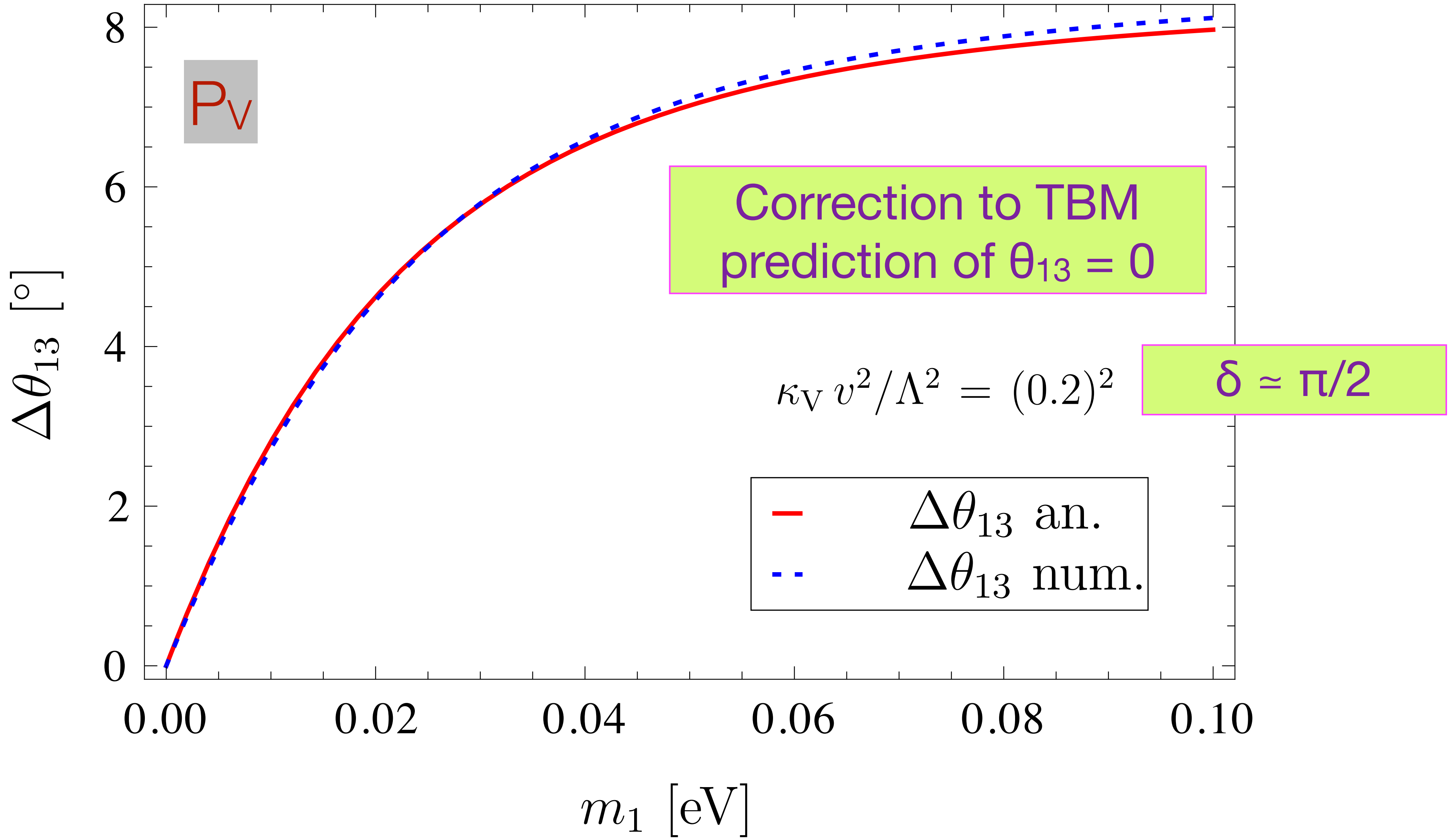
- Corrections to the kinetic terms induced by family symmetry breaking generically are present, should be properly included Leurer, Nir, Seiberg (1993); Dudas, Pokorski, Savoy (1995); Dreiner, Thomeier (2003)
  - can be along different directions than RG corrections
  - dominate over RG corrections (no loop suppression, copious heavy states)
  - only subdominant for quark flavor models
  - sizable for neutrino mass models based on discrete family symmetries, e.g. A_4
    - Contributions from Flavon VEVs (1,0,0) and (1,1,1) M.-C.C, M. Fallbacher, M. Ratz, C. Staudt (2012)
    - five independent "basis" matrices

$$P_{\mathrm{I}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad P_{\mathrm{II}} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad P_{\mathrm{III}} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad P_{\mathrm{IV}} = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}, \quad P_{\mathrm{V}} = \begin{pmatrix} 0 & \mathrm{i} & -\mathrm{i} \\ -\mathrm{i} & 0 & \mathrm{i} \\ \mathrm{i} & -\mathrm{i} & 0 \end{pmatrix}$$

- RG correction: essentially along P_{III} = diag(0,0,1) direction due to y_τ dominance
- kinetic term corrections can be along different directions than RG: P_I - P_V
  - nontrivial flavor structure can be induced
  - non-zero CP phase can be induced

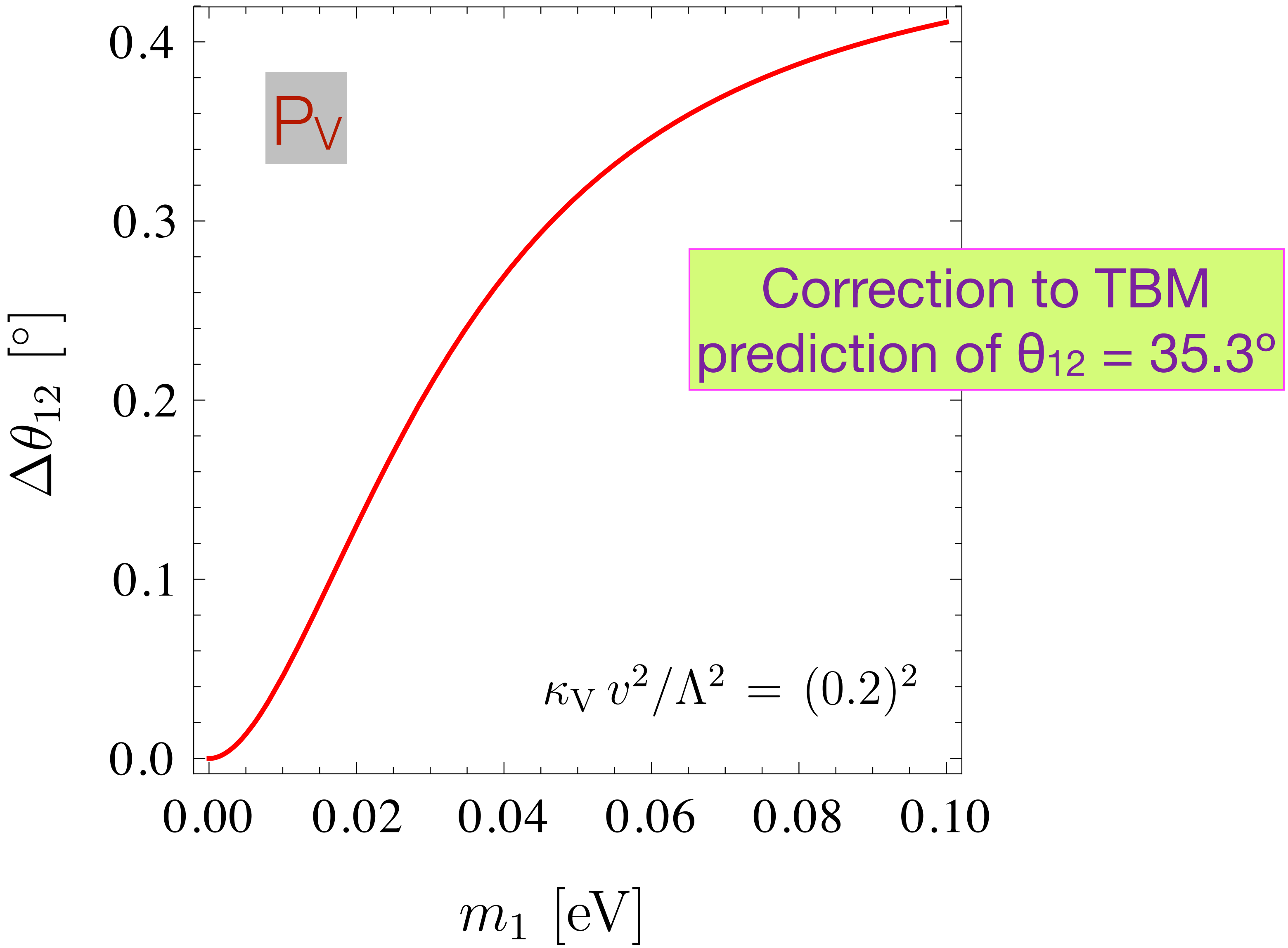
An Example: Enhanced θ_{13} in A_4

M.-C.C., M. Fallbacher, M. Ratz, C. Staudt (2012)



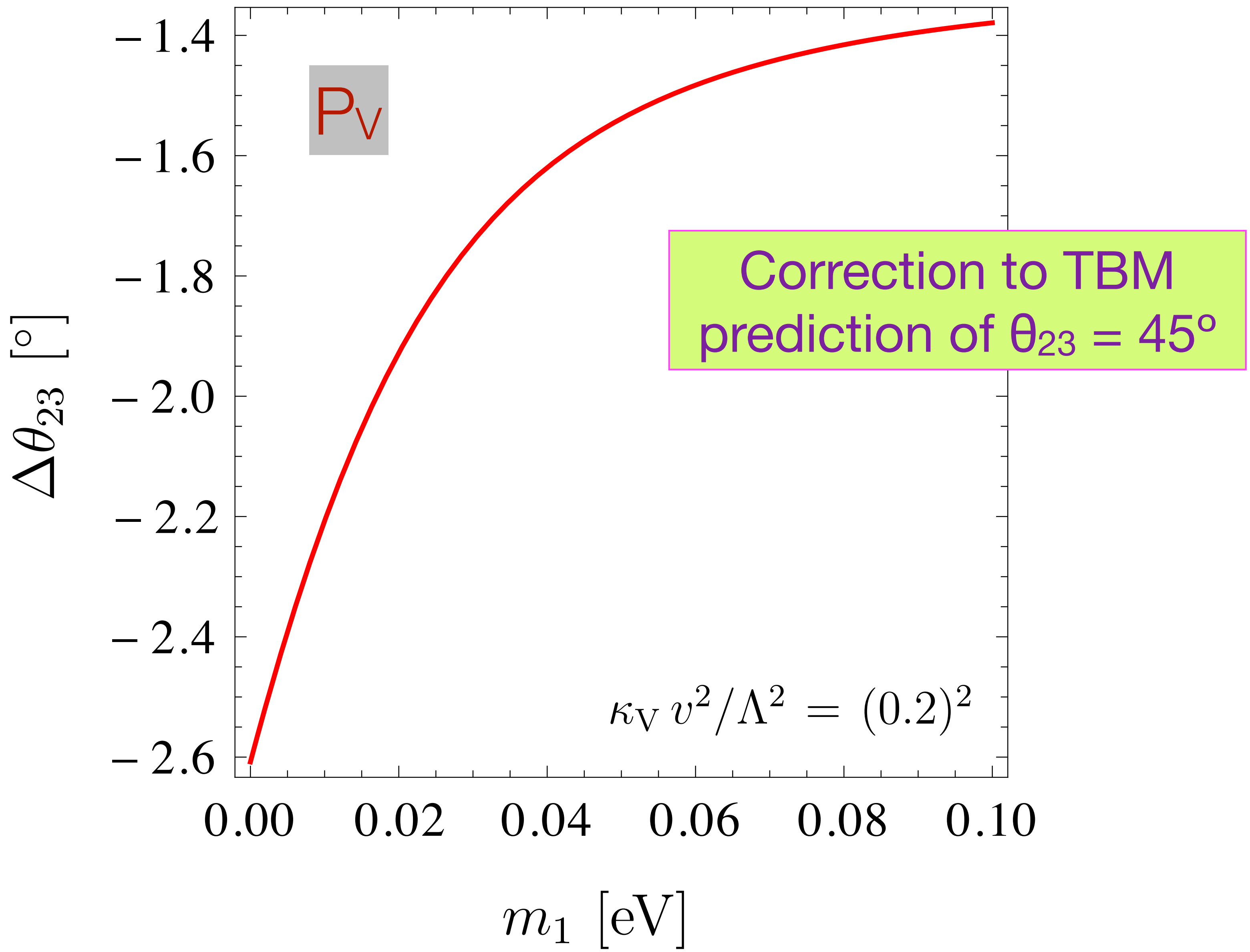
Corresponding Change in θ_{12}

M.-C.C., M. Fallbacher, M. Ratz, C. Staudt (2012)



Corresponding Change in θ_{23}

M.-C.C., M. Fallbacher, M. Ratz, C. Staudt (2012)

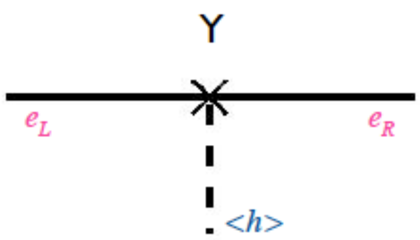


Origin of CP Violation

- CP violation ⇔ complex mass matrices

$$\overline{U}_{R,i}(M_u)_{ij}Q_{L,j} + \overline{Q}_{L,j}(M_u^\dagger)_{ji}U_{R,i} \xrightarrow{\mathcal{CP}} \overline{Q}_{L,j}(M_u)_{ij}U_{R,i} + \overline{U}_{R,i}(M_u)^*_{ij}Q_{L,j}$$

- Conventionally, CPV arises in two ways:
  - Explicit CP violation: complex Yukawa coupling constants Y
  - Spontaneous CP violation: complex scalar VEVs <h>



- **Complex CG coefficients in certain discrete groups ⇒ explicit CP violation**
  - CPV in quark and lepton sectors purely from complex CG coefficients

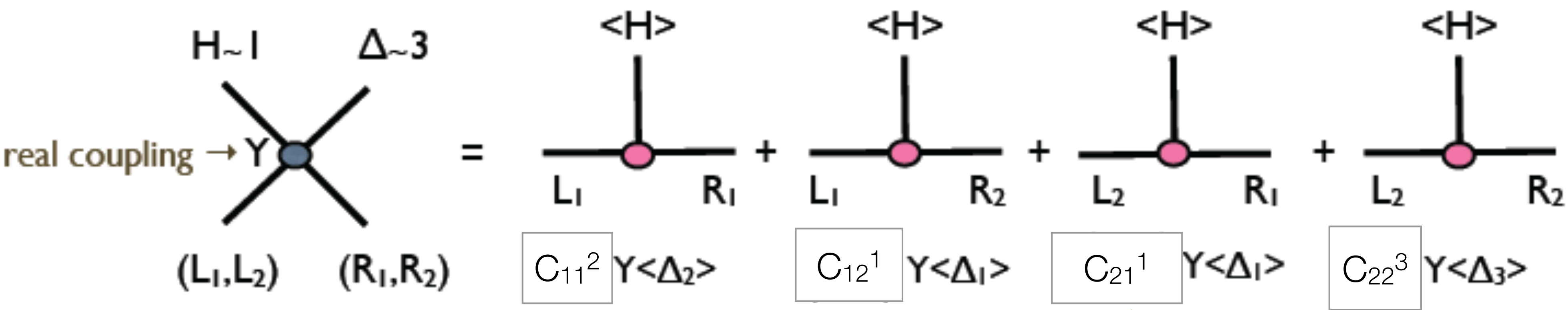
CG coefficients in non-Abelian discrete symmetries
➪ relative strengths and phases in entries of Yukawa matrices
➪ mixing angles and phases (and mass hierarchy)

Group Theoretical Origin of CP Violation

M.-C.C., K.T. Mahanthappa
Phys. Lett. B681, 444 (2009)

Basic idea

Discrete symmetry ***G***



$C_{ij}{}^{k}$: complex CG coefficients of ***G***

- Scalar potential: if Z_3 symmetric ⇒ $\langle\Delta_1\rangle = \langle\Delta_2\rangle = \langle\Delta_3\rangle \equiv \langle\Delta\rangle$ real
- Complex effective mass matrix: **phases determined by group theory**

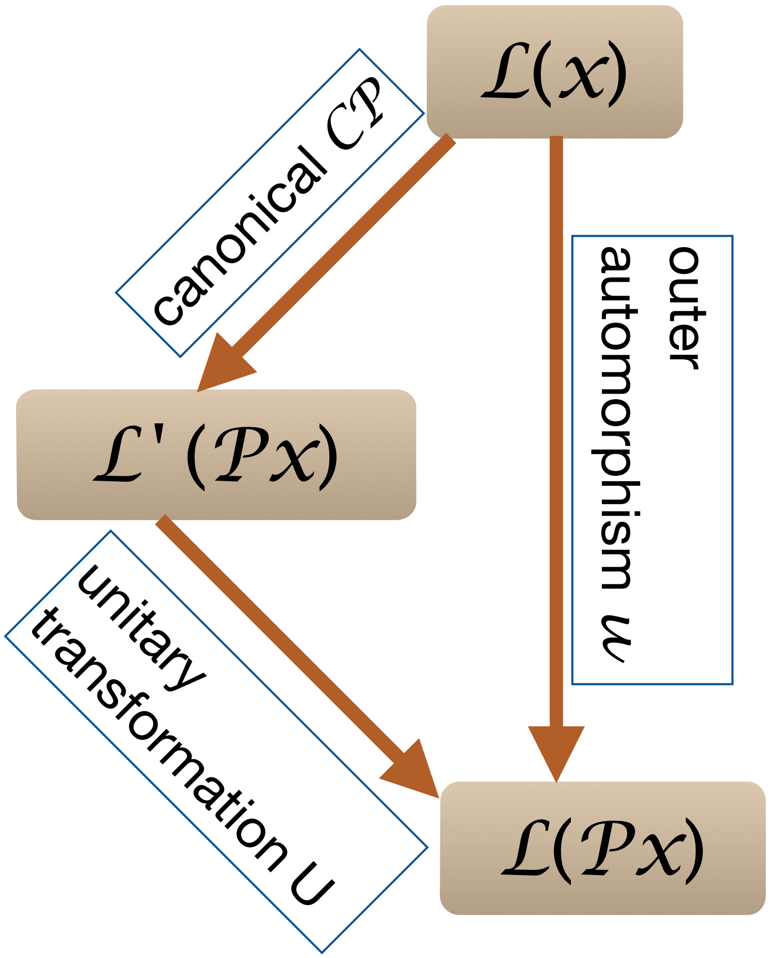
$$(L_1 \quad L_2)\; M = \begin{pmatrix} C_{11}{}^{2} & C_{21}{}^{1} \\ C_{12}{}^{1} & C_{22}{}^{3} \end{pmatrix} Y \langle\Delta\rangle \begin{pmatrix} R_1 \\ R_2 \end{pmatrix}$$

For further insights, see M. Fallbacher, A. Trautner, NPB (2015)

Group Theoretical Origin of CP Violation

M.-C.C, M. Fallbacher, K.T. Mahanthappa, M. Ratz, A. Trautner, NPB (2014)

complex CGs ➪ G and physical CP transformations do not commute



$$\Phi(x) \xmapsto{\widetilde{\mathcal{CP}}} U_{\mathrm{CP}}\,\Phi^*(\mathcal{P}\,x)$$

$$\rho_{\boldsymbol{r}_i}(u(g)) = U_{\boldsymbol{r}_i}\,\rho_{\boldsymbol{r}_i}(g)^*\,U_{\boldsymbol{r}_i}^\dagger \quad \forall\, g \in G \text{ and } \forall\, i$$

u has to be a **class-inverting**, **involuntory** automorphism of G

➪ non-existence of such automorphism in certain groups

➪ explicit physical CP violation in generic setting

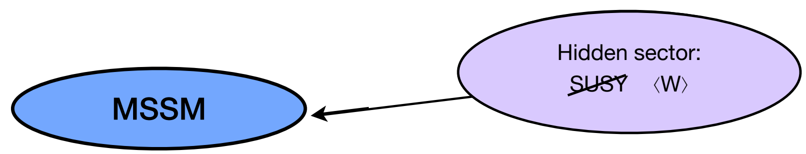
examples: T_7, $\Delta(27)$, …..

Dirac Neutrinos and SUSY Breaking

- naturally small Dirac neutrino masses can arise
  - Randall-Sundrum model: wave function overlap
  - Supersymmetry breaking
- before SUSY breaking: absence of Dirac neutrino masses (as well as Weinberg operator)
- after SUSY breaking: realistic effective Dirac neutrino masses generated

Arkani-Hamed, Hall, Murayama, Tucker-Smith, Weiner (2001)

$$Y_\nu \sim \frac{m_{3/2}}{M_{\mathrm{P}}} \sim \frac{\mu}{M_{\mathrm{P}}}$$



- similar to the Giudice-Masiero Mechanism for the mu problem

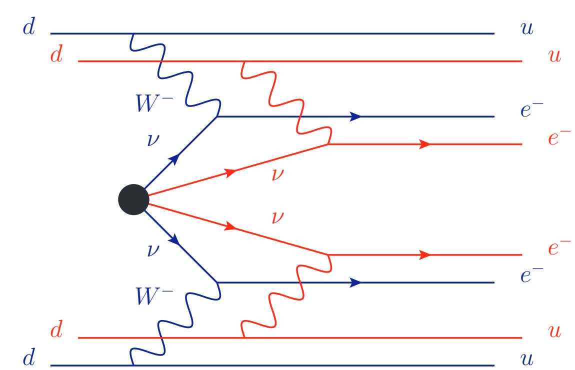
$$\mu \sim \langle \mathscr{W} \rangle / M_{\mathrm{P}}^2 \sim m_{3/2}$$

Giudice, Masiero (1988)

- Need a symmetry reason for the absence of these operators before SUSY breaking

Dirac Neutrinos and SUSY Breaking

- Simultaneous realization of these two scenarios can arise in MSSM with discrete R symmetries, $\mathbb{Z}_M^R$ M.-C. C., M. Ratz, C. Staudt, P. Vaudrevange (2012)
  - **neutrinos are of the Dirac type**, with naturally small masses
  - Δ L = 2 operators forbidden to all orders ⇒ no neutrinoless double beta decay
  - **New signature: lepton number violation ΔL = 4 operators, $(\nu_R)^4$, allowed ⇒ new LNV processes, e.g.** M.-C. C., M. Ratz, C. Staudt, P. Vaudrevange (2012)
    - neutrinoless quadruple beta decay Heeck, Rodejohann (2013)



- mu term is naturally small, simultaneously
- dangerous proton decay operators forbidden/suppressed
- may simultaneously explain the flavor structure with discrete generation dependent R symmetries (even with non-Abelian!) M.-C.C., M. Ratz, A. Trautner (2013)
- Dynamical generation of RPV operators with size predicted, different processes correlated

M.-C.C., M. Ratz, V. Takhistov (2014)

Summary

- Fundamental origin of fermion mass hierarchy and flavor mixing still not known
- Neutrino masses: evidence of physics beyond the SM
- Symmetries: can provide an understanding of the pattern of fermion masses and mixing
  - Grand unified symmetry + discrete family symmetry ⇒ predictive power
  - Symmetries lead to testable predictions:
    - interesting leading order sum rules between quark & lepton mixing parameters
    - lepton flavor violating charged lepton decays
    - proton (nucleon) decay, neutron-antineutron oscillation
  - corrections to kinetic terms need to be properly included
- **Discrete Groups (of Type I) affords a Novel origin of CP violation:**
  - **Complex CGs ⇒ Group Theoretical Origin of CP Violation**
  - **as a R-symmetry: Dirac neutrino + solving problems in MSSM**