Inclusive Electron Scattering from Nuclei and Scaling

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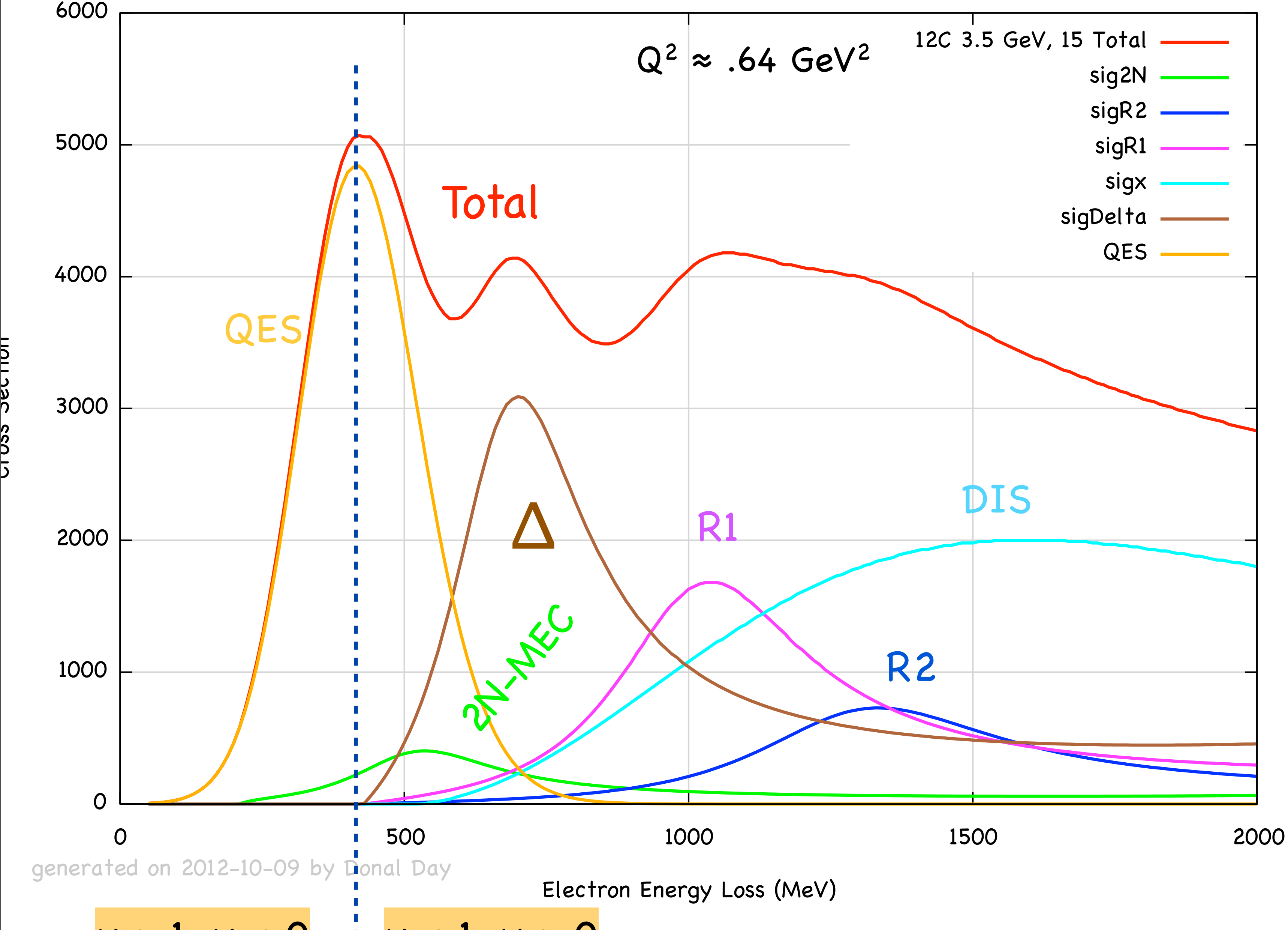
Outline

- Inclusive Electron Scattering from Nuclei
  - General features
- Scaling
  - In QES region in terms of nucleons
  - In DIS region in terms of partons
- 2N Short Range Correlations
  - Connection to the EMC effect
- Finish

General Features of the Inclusive Spectrum, A(e,e′)

Faster

2N-MEC
Δ
QES
R1
R2
DIS

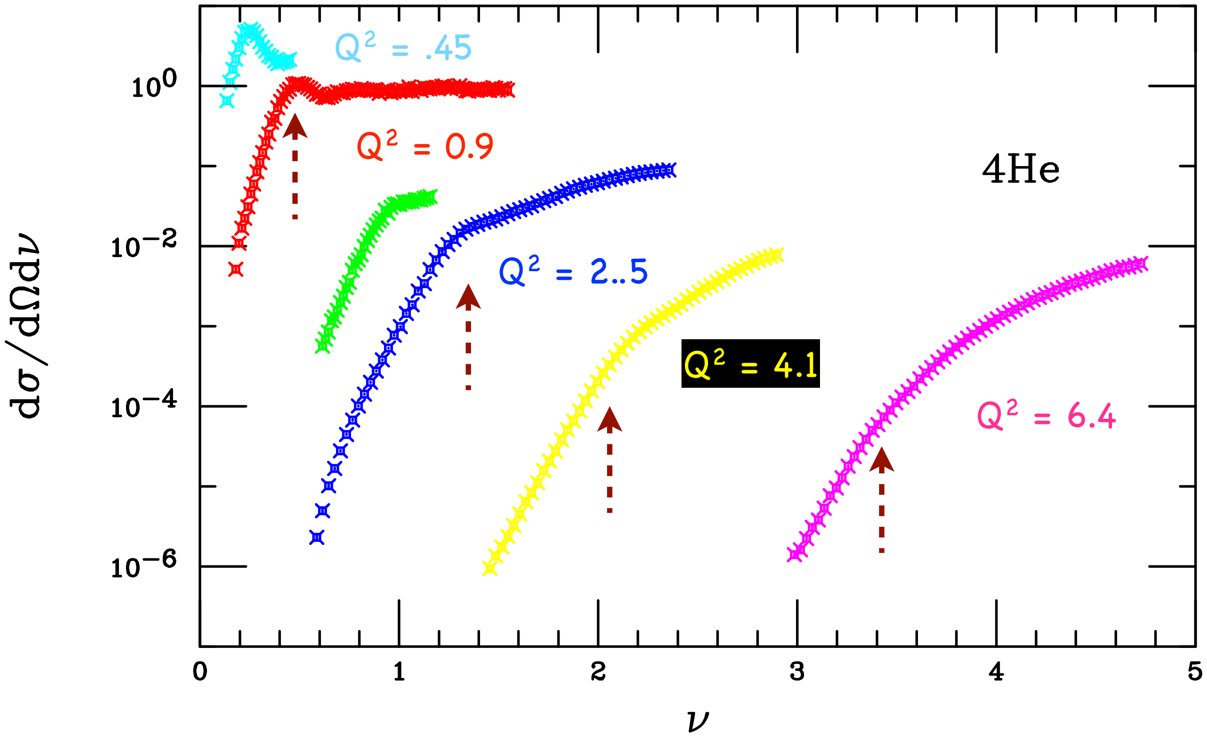


x > 1, y < 0

x < 1, y > 0

qfs.f, Lightbody and O'Connell Computers in Physics May/June 1988

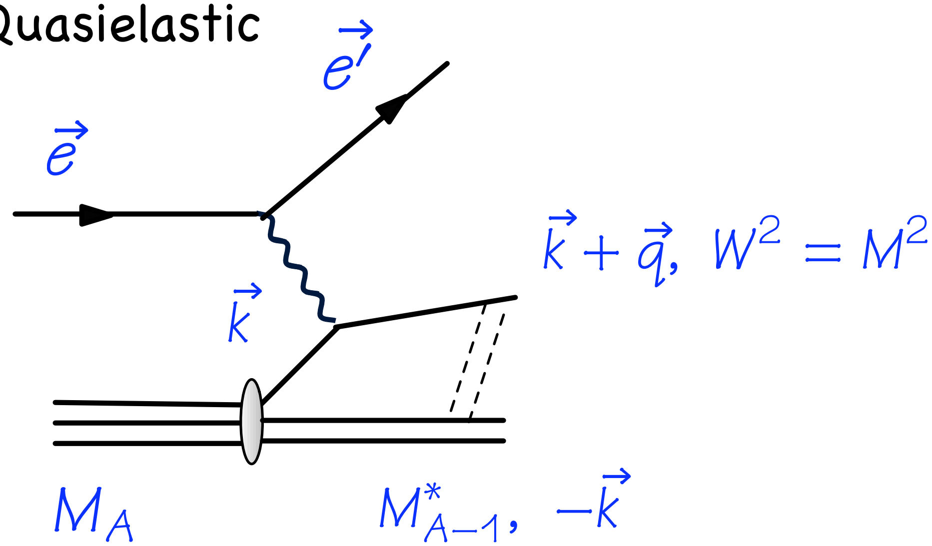
General Features of the Inclusive Spectrum



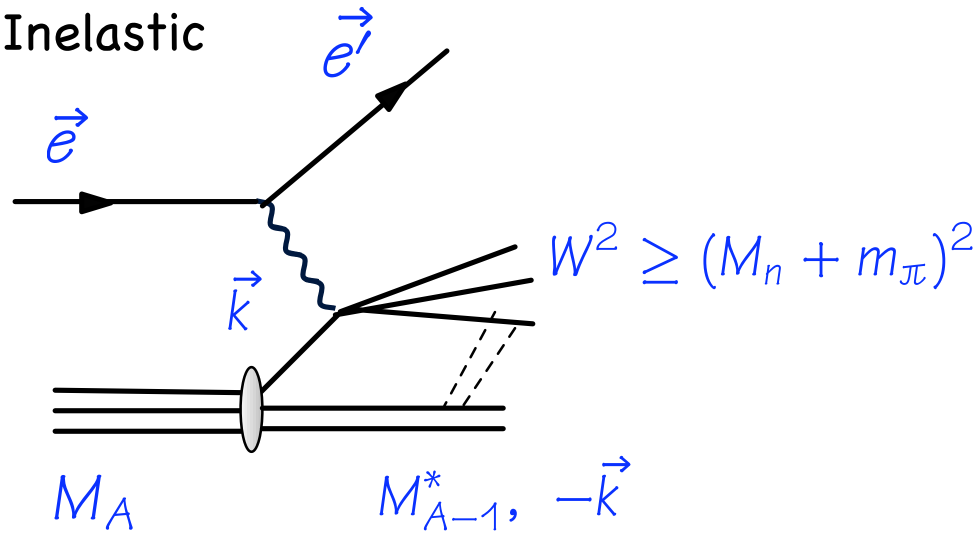
Structures in the spectra diminish faster with Q^2 than in the nucleon because of motion - they give way to smooth and structureless shape

Two dominant processes

Quasielastic



Inelastic



These dominant processes share the same initial state but have very different Q^2 dependencies

The limits on the integrals are determined by the kinematics. Specific (x, Q^2) select specific pieces of the spectral function.

QES in IA $$\frac{d^2\sigma}{d\Omega d\nu} \propto \int d\vec{k} \int dE \sigma_{ei} \underbrace{S_i(k,E)}_{\text{Spectral function}} \delta()$$

$$n(k) = \int dE\, S(k,E)$$

DIS $$\frac{d^2\sigma}{d\Omega d\nu} \propto \int d\vec{k} \int dE\, W_{1,2}^{(p,n)} \underbrace{S_i(k,E)}_{\text{Spectral function}}$$

$\sigma_{ei} \propto$ elastic (form factor)^2

$W_{1,2}$ scale with <u>ln Q^2</u> dependence in DIS region, resonances fall quickly with Q^2

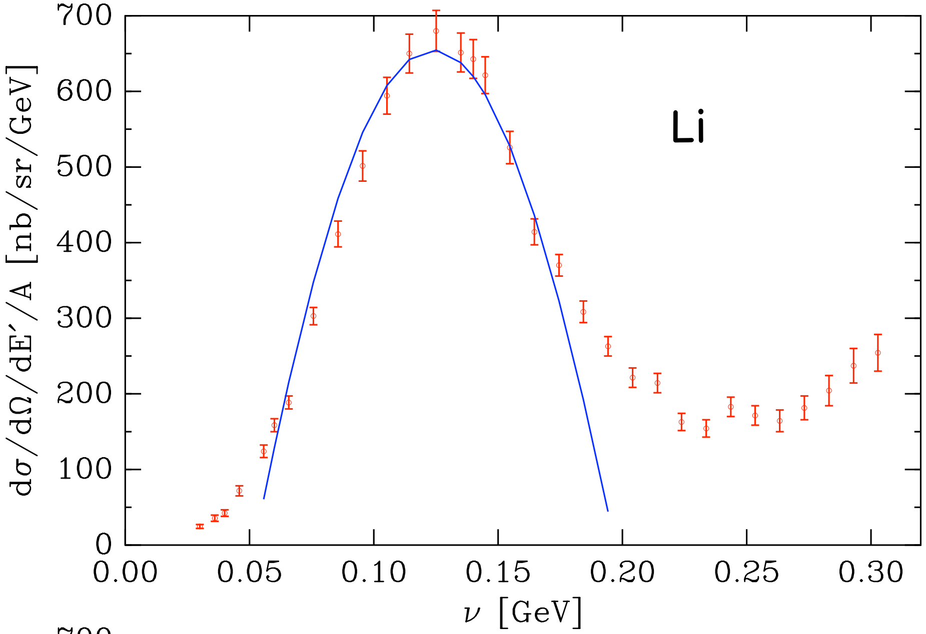
Charge-changing neutrino reaction cross sections for the nucleons in the nucleus for example CCQES

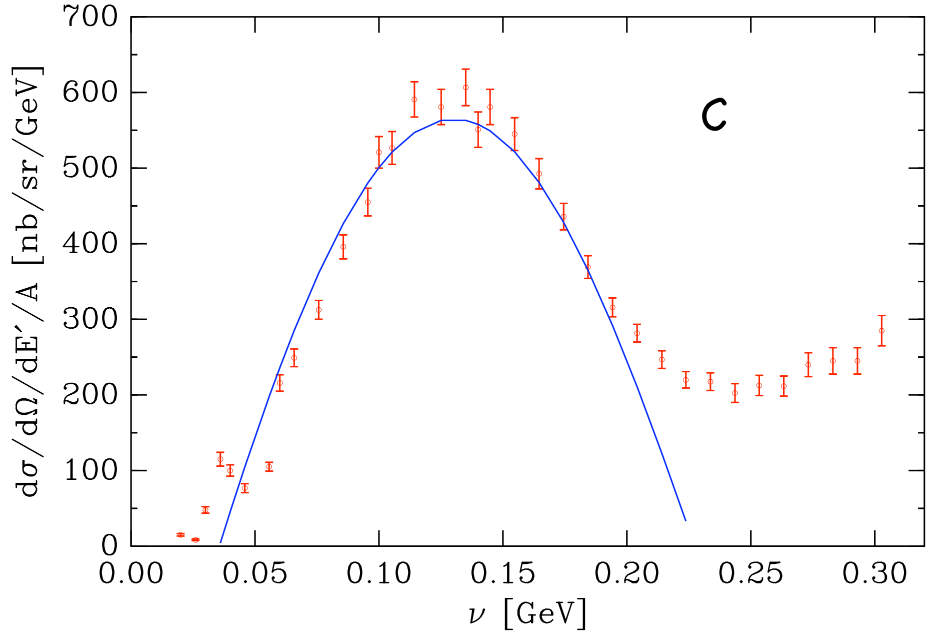
$\sigma_{ei} \longrightarrow \sigma_{\nu i}$ weak charged current interaction with a nucleon

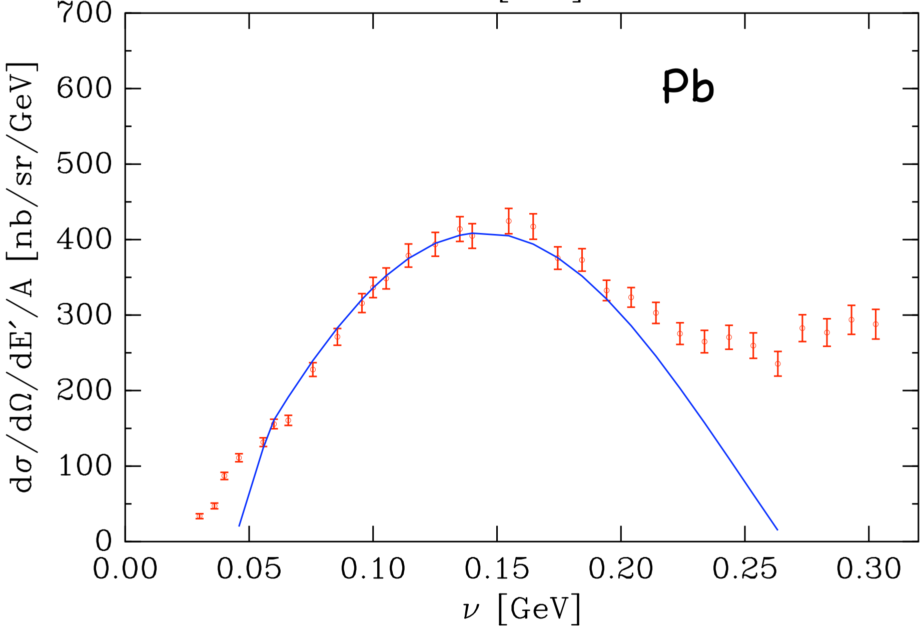
Early 1970's Quasielastic Data

500 MeV, 60 degrees

$\vec{q} \simeq 500 MeV/c$







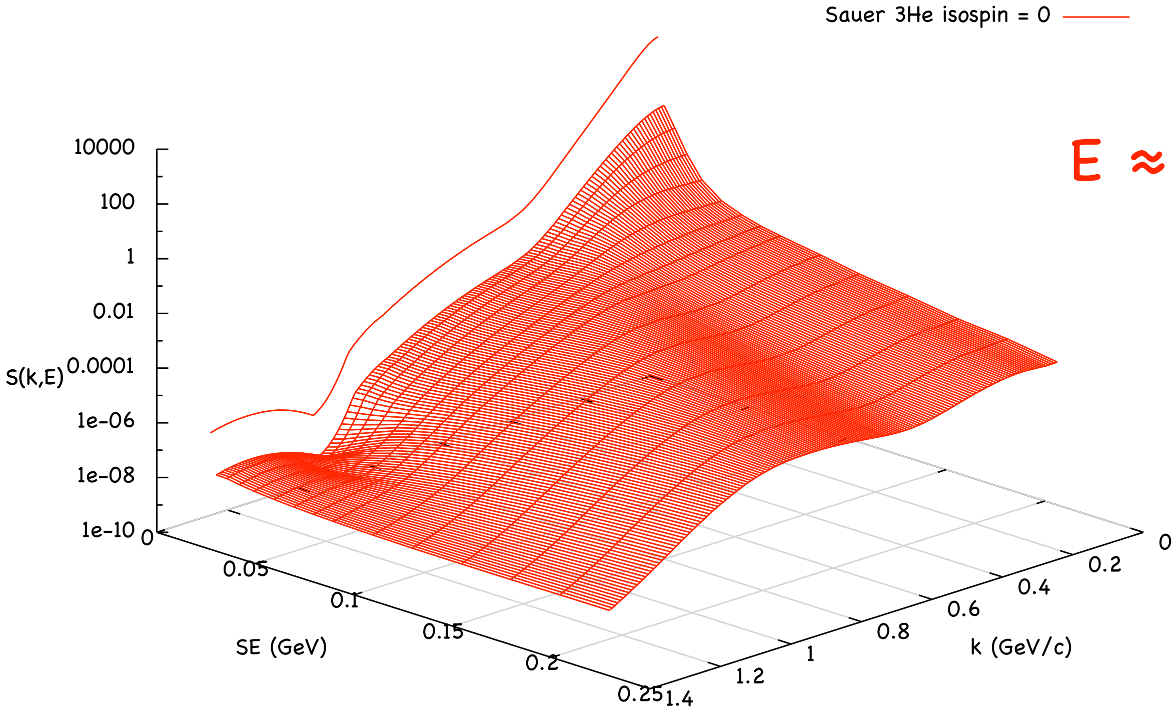
Nucleus	k_F	$\bar{\epsilon}$
^{6}Li	169	17
^{12}C	221	25
^{24}Mg	235	32
^{40}Ca	251	28
^{nat}Ni	260	36
^{89}Y	254	39
^{nat}Sn	260	42
^{181}Ta	265	42
^{208}Pb	265	44

Fermi gas model Moniz, ...

Spectral function S(E, k), not n(k) describes nuclei: probability of finding a nucleon with initial momentum k and energy E in the nucleus

$$n(k) = \int S(E_s, k)\, dE_s$$

S(k,E) for ^{3}He



$E \approx k^2/2M$

S(k,E) for NM

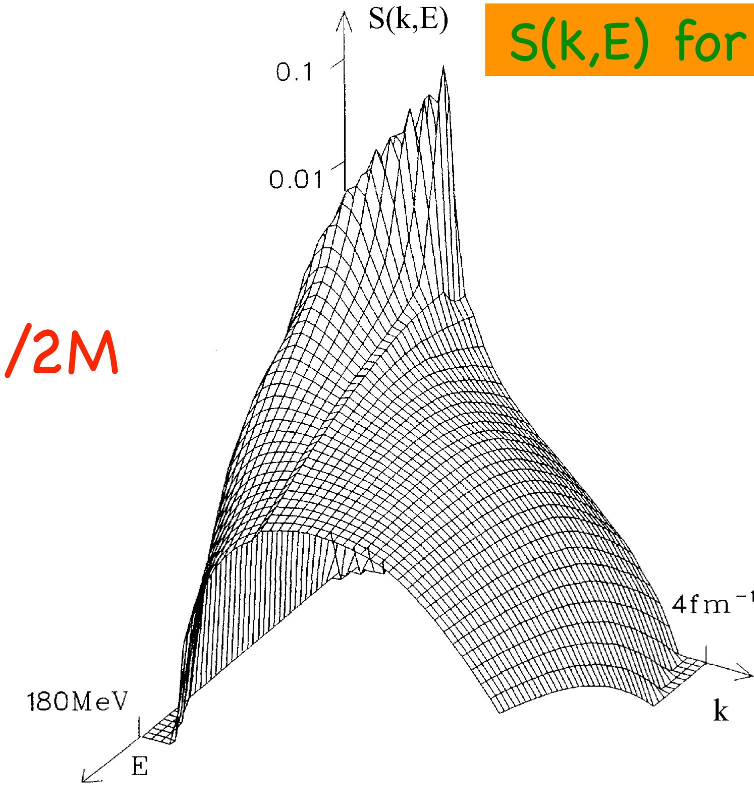


FIG. 6 Nuclear matter spectral function calculated using correlated basis function perturbation theory (Benhar *et al.*, 1989).

For finite nuclei, LDA is used, with experiment

$$S(\vec{k}, E) = S_{MF}(\vec{k}, E) + S_{corr}(\vec{k}, E)$$

$$S_{Corr}(\vec{k}, E) = \int d^3 r \rho_A(\vec{r}) S^{NM}_{corr}(\vec{k}, E; \rho = \rho_A(\vec{r}))$$

$$S_{MF}(\vec{k}, E) = \sum_n Z_n \mid \Phi_n \mid^2 F_n(E - E_n)$$

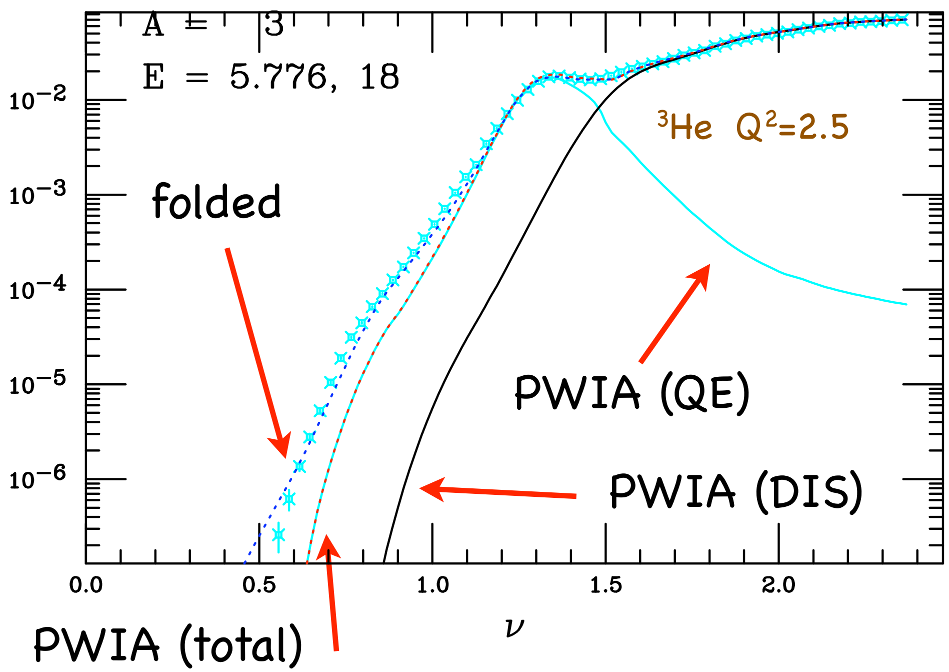
What role FSI?

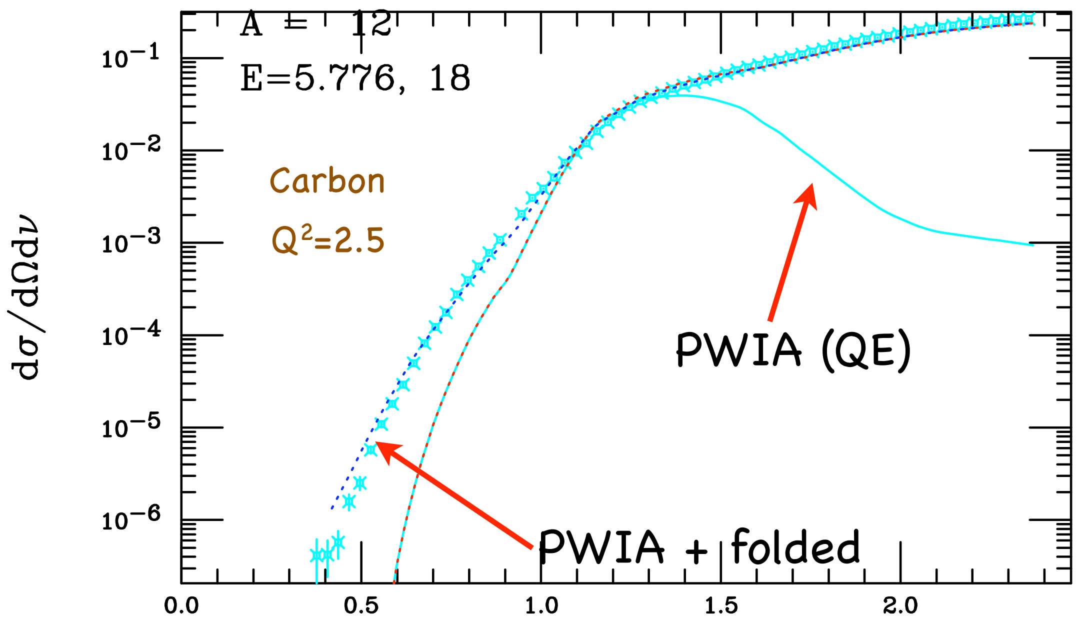
In (e,e′p) flux of outgoing protons strongly suppressed:
20-40% in C, 50-70% in Au

In (e,e′) the failure of IA calculations to explain dσ at small energy loss

Some of this could be resolved by a rearrangement of strength in SE

Old problem: real/complex optical potential.
Real part generates a shift, imaginary part a folding of cs, reduction of qep.
Can FSI ever be neglected? - scaling suggests they can.





O.Benhar, with CGA for FSI

Issues about CGA FSI

- Extreme sensitivity to hole size
- On-shell cross sections: nucleon is off-shell by in E by $\hbar/\Delta t = \hbar$ W: modification of NN interaction
- total cross section?
- Unitarity? Folding function is normalized to one.
- Role of momentum dependent folding function (Petraki et al, PRC 67 014605, 2003) has lead to a quenching of the tails.
  - Comparison to data with this new model for a range of A and Q^2 be very useful

"The discrepancy with the measured cross sections increases as q increases, while the suppression of FSI's due to the momentum dependence of the folding function appears to be larger at lower momentum transfer.

A different mechanism, leading to a quenching of FSI's and exhibiting the opposite momentum-transfer dependence still seems to be needed to reconcile theory and data."

Petraki et al, PRC 67 014605

Scaling

- Scaling refers to the dependence of a cross section (a structure function), in certain kinematic regions, on a single variable.
- At moderate Q^2 inclusive data from nuclei has been well described in terms y-scaling, one that arises from the assumption that the electron scatters from moving, quasi-free nucleons.

Assumptions & Potential Scale Breaking Mechanisms

- No FSI
- No internal excitation of (A-1)
- Full strength of spectral function can be integrated over at finite q
- No inelastic processes
- No medium modifications

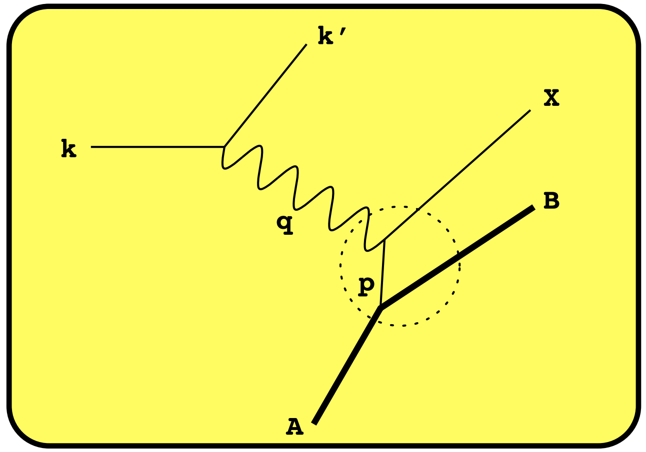
Direct access to the momentum distribution

$$F(y) = \frac{\sigma^{exp}}{(Z \cdot \sigma_{ep} + N \cdot \sigma_{en})} \cdot K \qquad n(k) = -\frac{1}{2\pi y}\frac{dF(y)}{dy}$$

y is the momentum of the struck nucleon parallel to the q-vector: $y \approx -q/2 + m\nu/q$

Derived in straightforward way in the PWIA (next two slides)

y-scaling in PWIA



$$\frac{d^2\sigma}{dE d\Omega_{e'}} = \sum_{i=1}^{A} \int d\vec{k} \int dE_s \, \sigma_{ei} \, S_i(E_s, k)$$

$$\times \, \delta(\omega - E_s + M_A - (M^2 + \vec{k'}^{\,2})^{1/2} - (M_{A-1}^2 + \vec{k}^2)^{1/2}),$$

$$\frac{d^2\sigma}{dE d\Omega_{e'}} = 2\pi \sum_{i=1}^{A} \int_{E_{min}}^{E_{max}} dE_s \int_{k_{min}}^{k_{max}} dk \, k \, \overline{\sigma}_{ei} \, S_i(E_s, k) \; \underbrace{k \left(\left| \frac{\partial \omega}{\partial \cos\theta_{kq}} \right| \right)^{-1}}_{K}$$

$$\sigma_{ei} = f(q, \omega, \vec{k}, E_s) = \overline{\sigma}_{ei}(q, \omega, y, E_{min})$$

$$E_{min} = M_{A-1} + M - M_A, \;\; E_{max} = M_A^* - M_A \qquad K = q/(M^2 + (\vec{k} + \vec{q})^2)^{1/2}$$

$$M_A^* = [(\omega + M_A)^2 - q^2]^{1/2}$$

k_{min} and k_{max} are determined from $\cos\theta = \pm 1$, in energy conserving δ function:

$y = k_{min}$

$$\omega - E_s + M_A = (M^2 + q^2 + k^2 \pm 2kq)^{1/2} + (M_{A-1}^2 + k^2)^{1/2}$$

y-scaling in PWIA

- lower limit becomes $y = y(q,\omega)$
- upper limits grows with q and because momentum distributions are steeply peaked, can be replaced with ∞
- Assume $S(E_s,k)$ is isospin independent and neglect E_s dependence of σ_{ei} and kinematic factor K and pull outside
- At very large q and ω, we can let $E_{max} = \infty$, and integral over E_s can be done

$$n(k) = \int S(E_s, k)\, dE_s$$

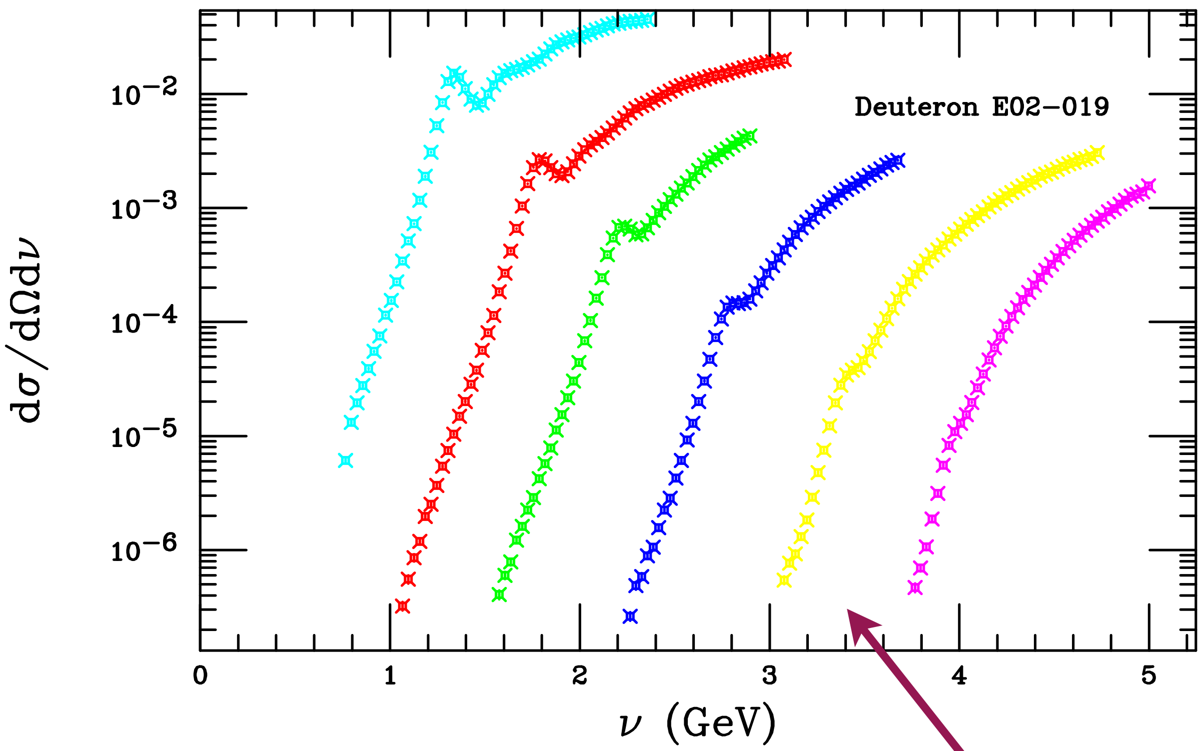
Now we can write

$$\boxed{\frac{d^2\sigma}{dE d\Omega_{e'}} = (Z\overline{\sigma}'_{ep} + N\overline{\sigma}'_{en})K' F(y)}$$

where

$$F(y) = 2\pi \int_{|y|}^{\infty} n(k) k dk$$

Scaling (independent of Q^2) of QES provides direct access to momentum distribution



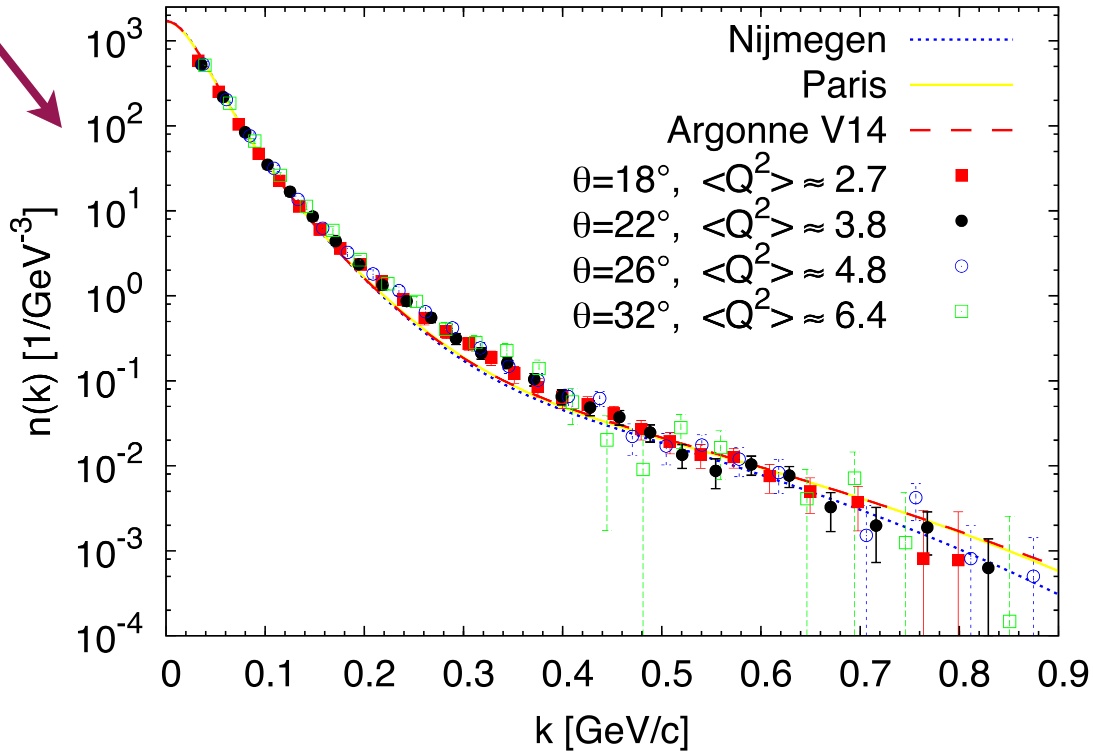
y-scaling - simplest system

No internal excitation of (A-1) system

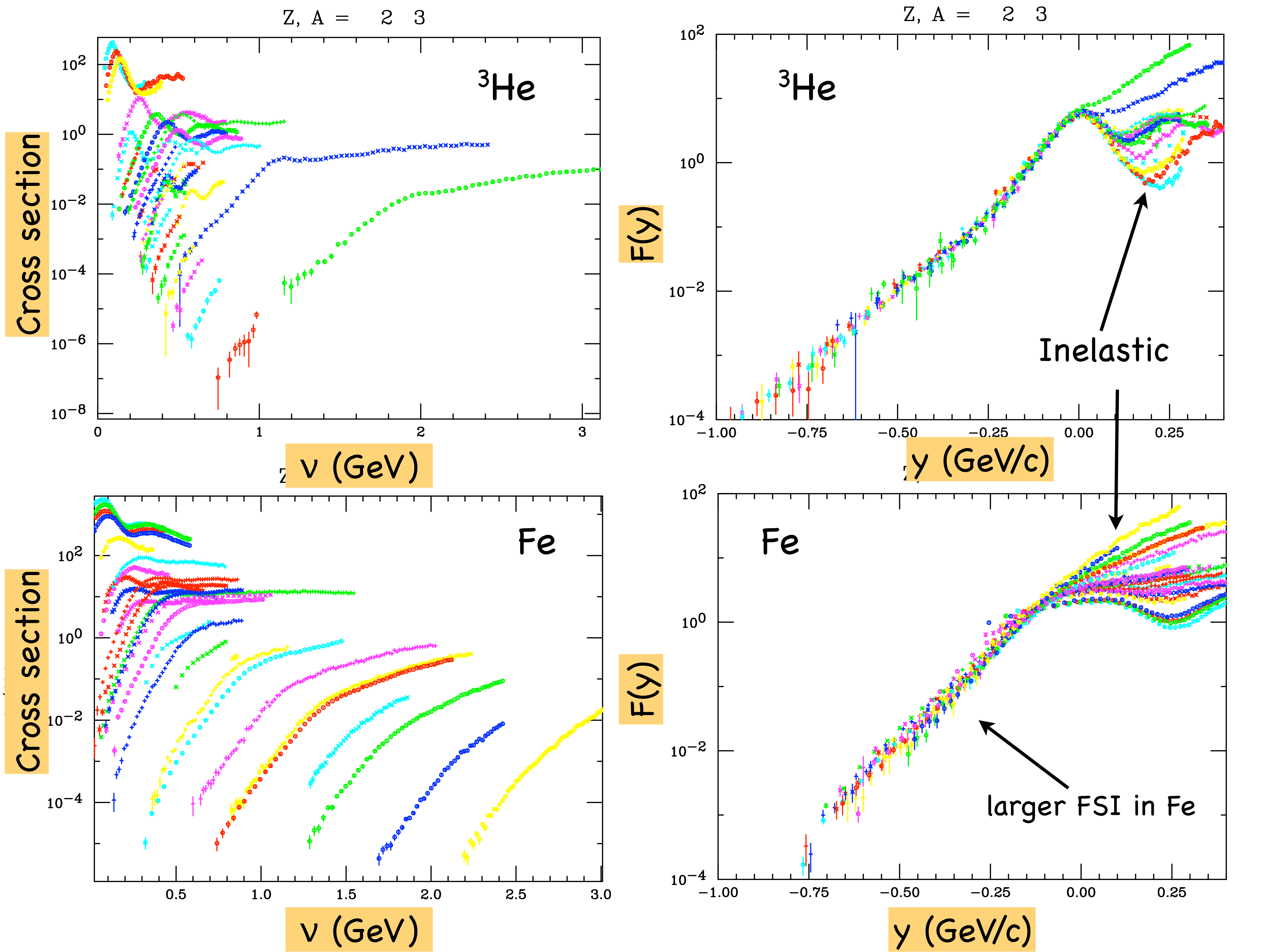
First use: compare to exact calculations to set limits on FSI and extract high momentum (> 500 MeV/c!) piece of gs wave function

$$F(y) = \frac{\sigma^{exp}}{(Z \cdot \sigma_{ep} + N \cdot \sigma_{en})} \cdot K$$

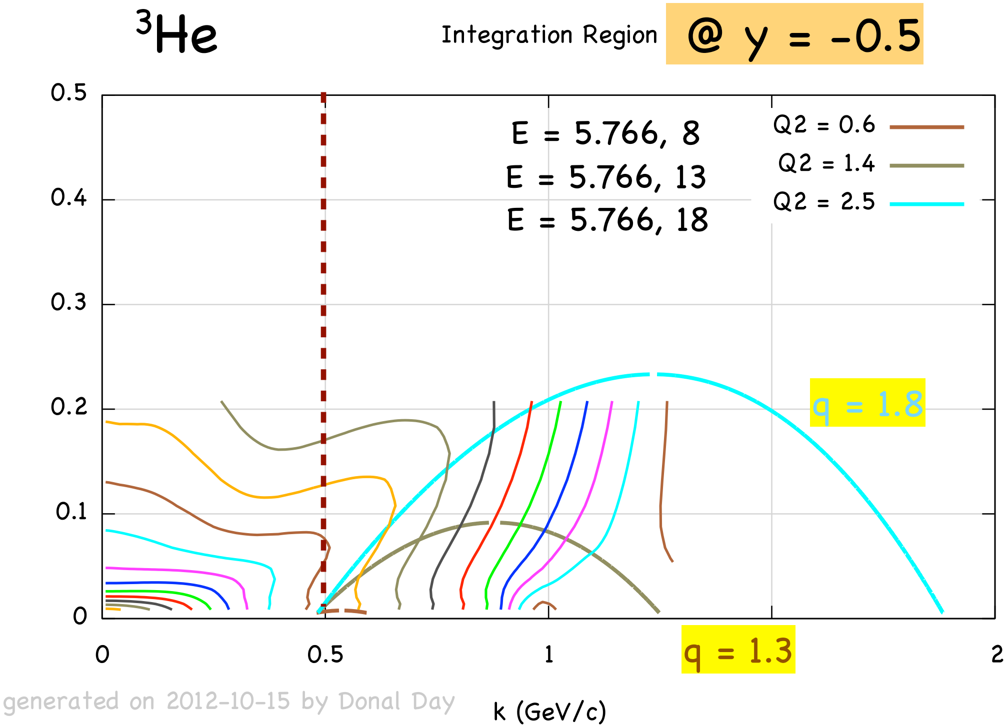
$$n(k) = -\frac{1}{2\pi y}\frac{dF(y)}{dy}$$

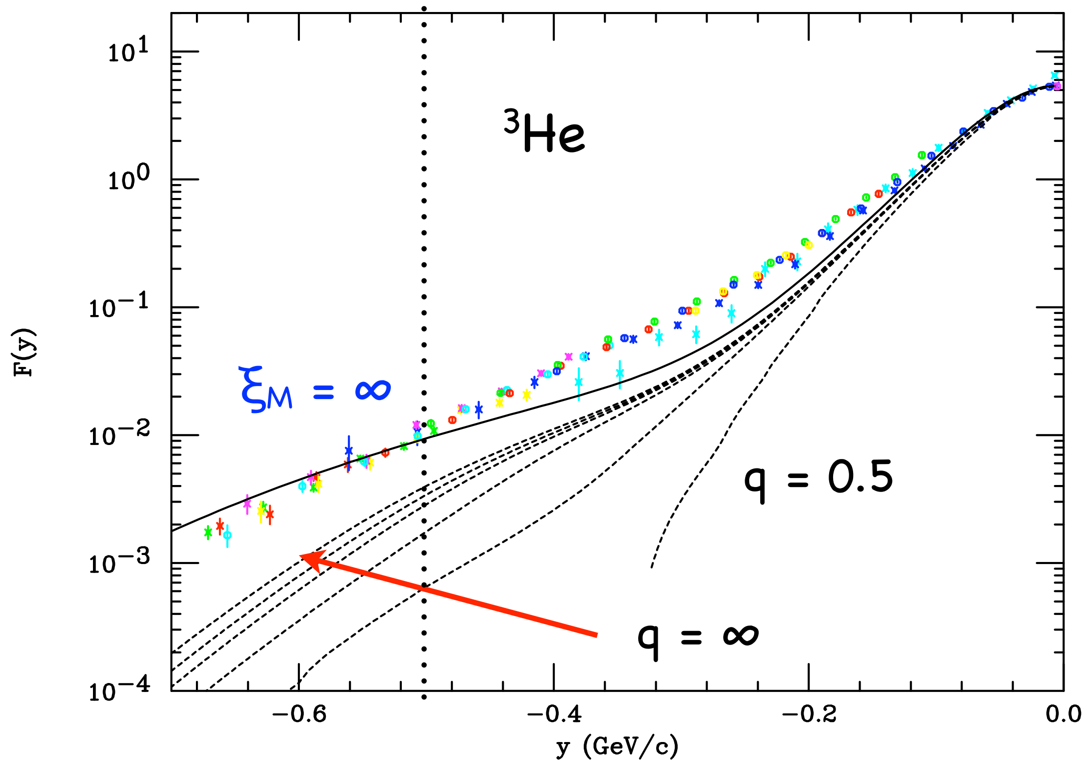


Second use: turn around: predict σ at any E, θ and ν, using parametrization of F(y); convert to n(k) and smear inelastic ==> convolution model.


Z, A = 2 3
^{3}He
Cross section
ν (GeV)
Z, A = 2 3
^{3}He
F(y)
y (GeV/c)
Inelastic
Fe
Cross section
ν (GeV)
Fe
F(y)
y (GeV/c)
larger FSI in Fe

Spectral function integration regions grows with q





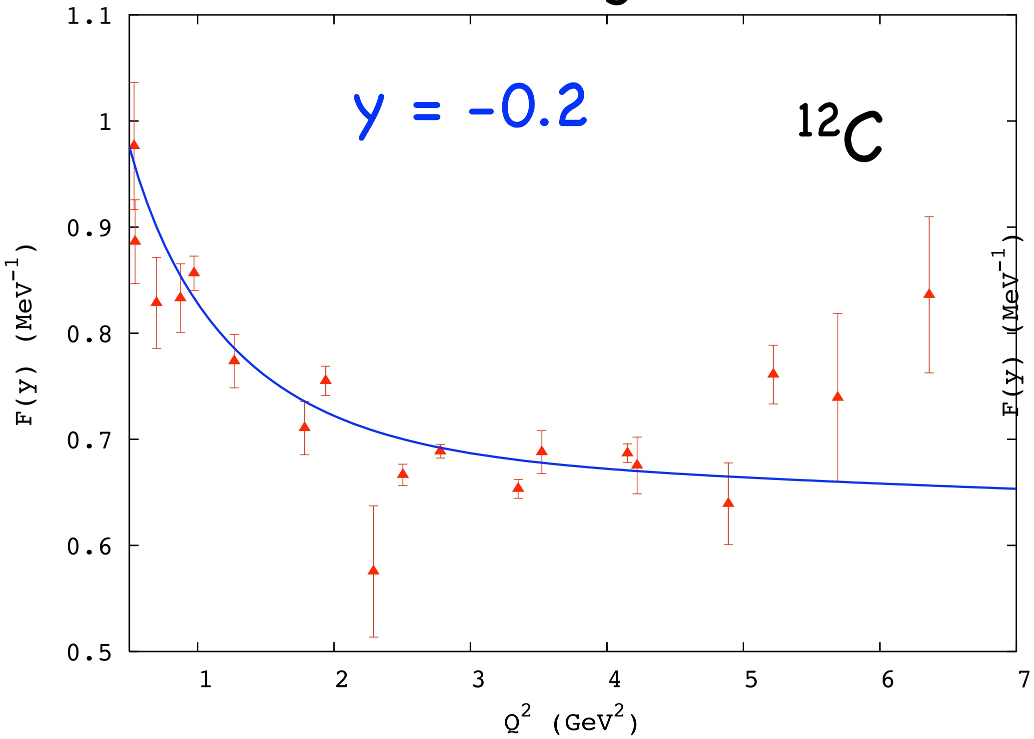
As q increases, more and more of the spectral function S(k,E) is integrated, convergence from below.

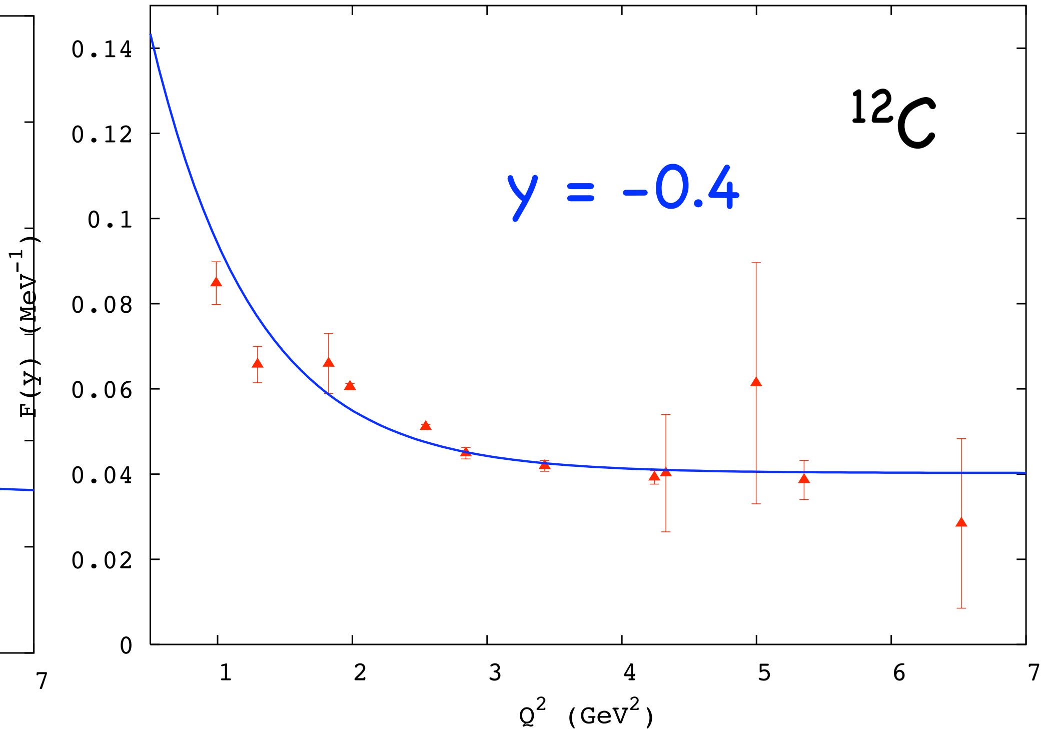
Is the energy distribution as calculated (scaling occurs at much lower q)? Do other processes play a role? FSI or/and DIS - what role

Nonetheless, inclusive data in the quasi-elastic region display scaling - Q^2 independence: - scaling of the 1st kind. Can be used to accurately estimate cross sections. A independence and Q^2 independence: superscaling

Convergence of F(y,q)

Convergence from above, not below suggests that FSI, known to contribute, die out with increasing momentum transfers





Questions:

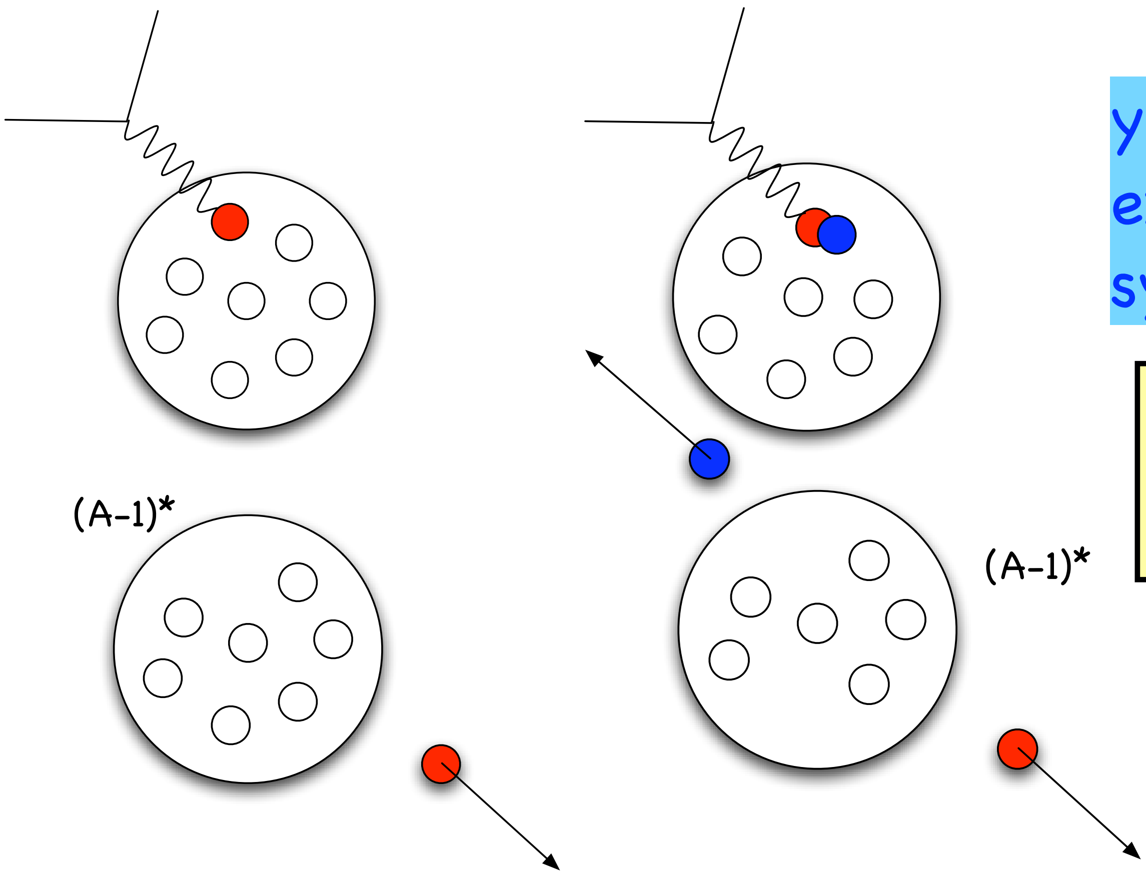
- How to account for the fact the binding (the distribution of strength in S(k,E)) in a y-scaling analysis
- Account for the change in the energy balance when scattering from a nucleon in a SRC

y-scaling indicates very high-momenta: model incomplete - strength is spread out in E

$$\frac{d^2\sigma}{d\Omega d\nu} \propto \int d\vec{p} \int dE \sigma_{ei} \underbrace{S_i(p,E)}_{\text{Spectral function}} \delta()$$

Single nucleon knock-out, $E \neq E_{min}$, A-1 system excited

$$\nu + M_A = \sqrt{M^2 + (p+q)^2} + M_{A-1} + \frac{p^2}{2M} + \underbrace{b_A - c_A \mid p \mid}_{\text{CM motion}} - < E_{gr} >$$

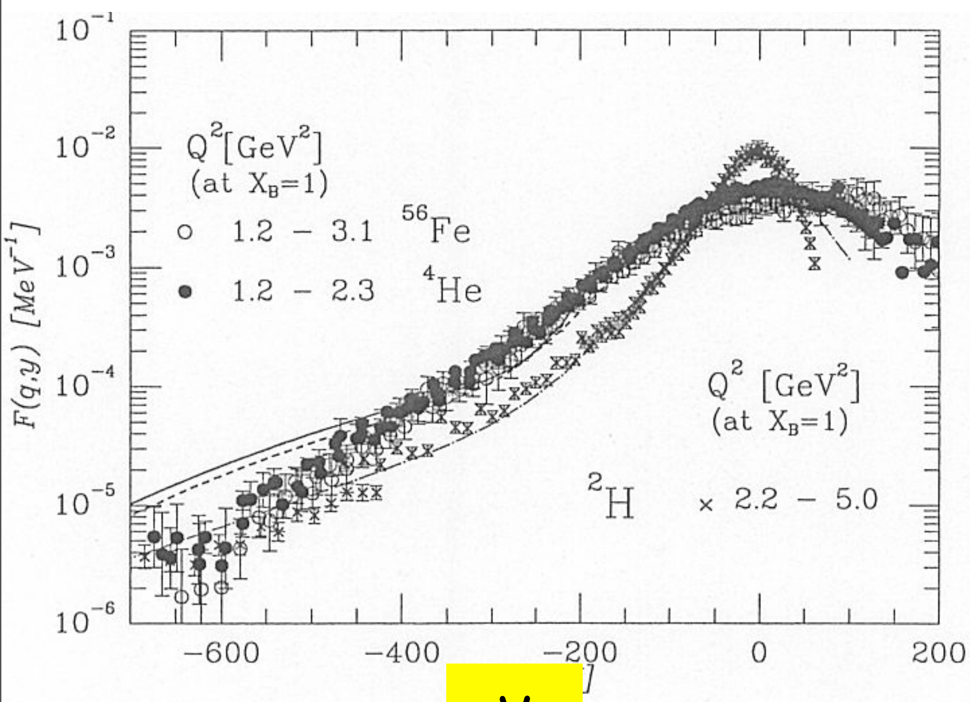


y_{cw} : Like y but accounting for excitation energy of residual system

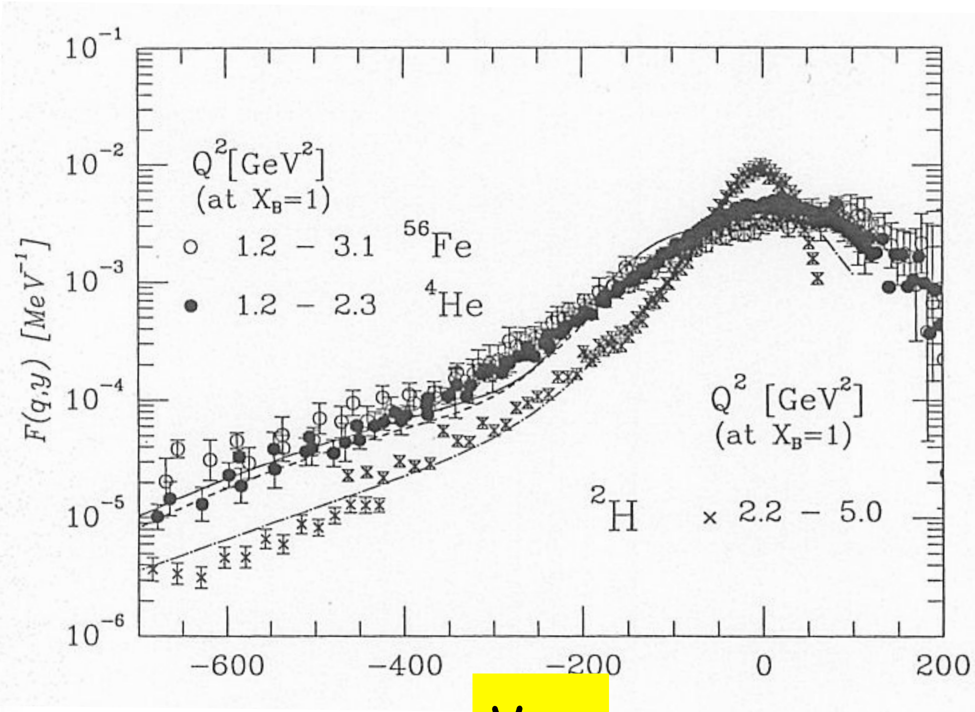
$$\boxed{F(y_{cw}) = \frac{\sigma^{exp}}{(Z\tilde{\sigma}_p + N\tilde{\sigma}_n)} \cdot K}$$

$$F(y_{cw}) = 2\pi \int_{|y_{cw}|}^{\infty} p dp n(p)$$

Faralli, Ciofi degli Atti & West, Trieste 1999



y



y_{CW}

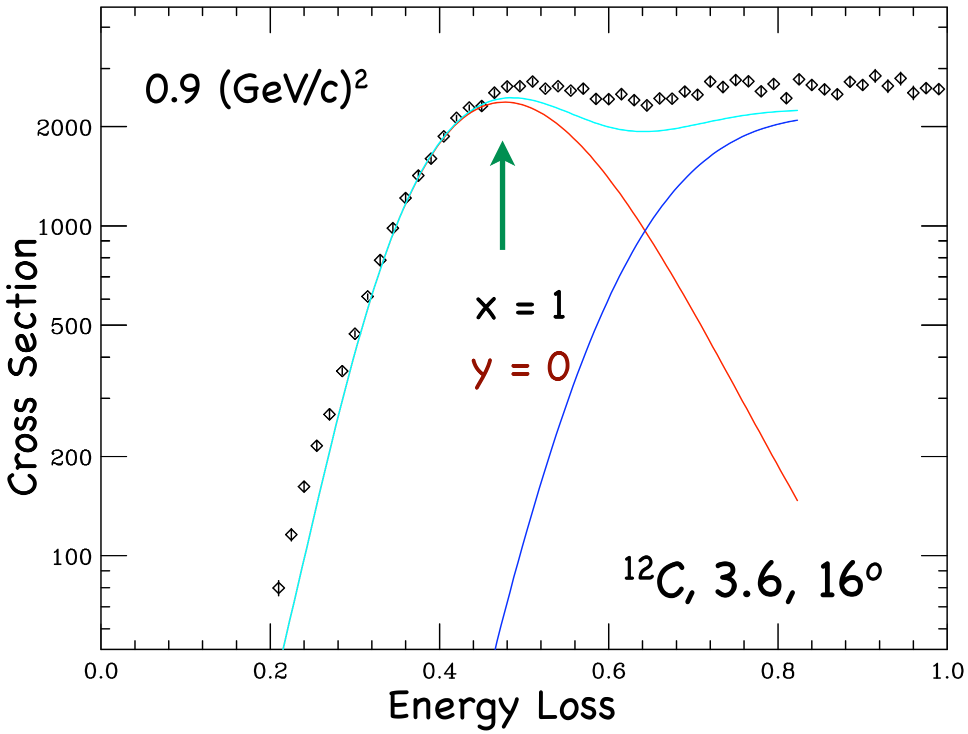
Farali, Ciofi degli Atti & West, Trieste 1999

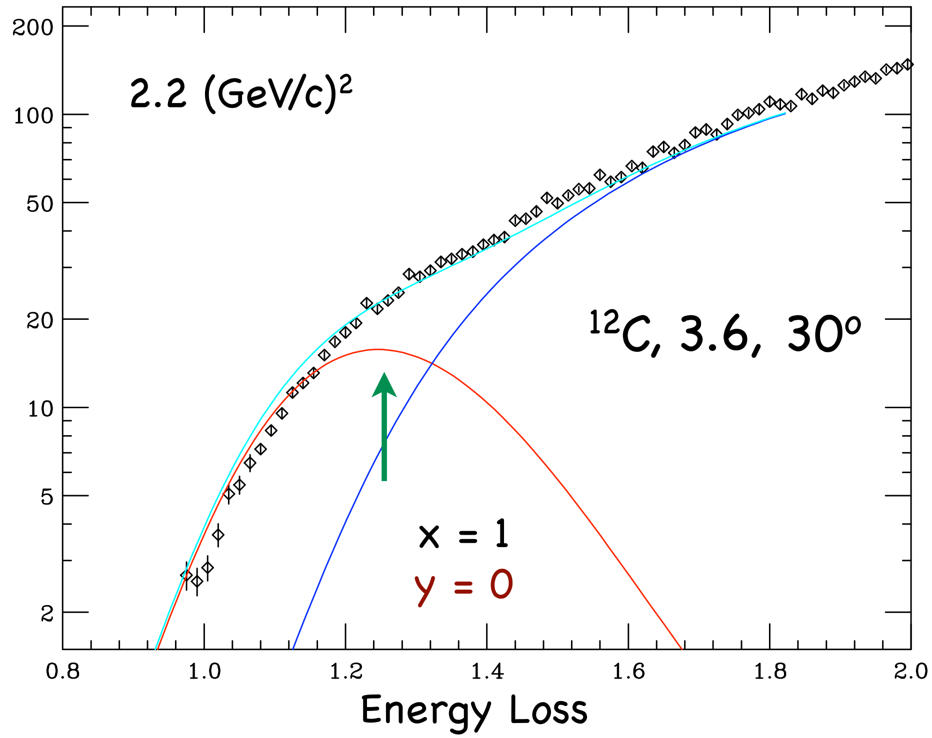
Many body calculations at high momenta indicate that nuclear momentum distributions are rescaled versions of the deuteron

$$n_A(p) \approx C_A n_D(p)$$

$$F_A(q, y_{CW}) \approx C_A F_D(q, y_{CW})$$

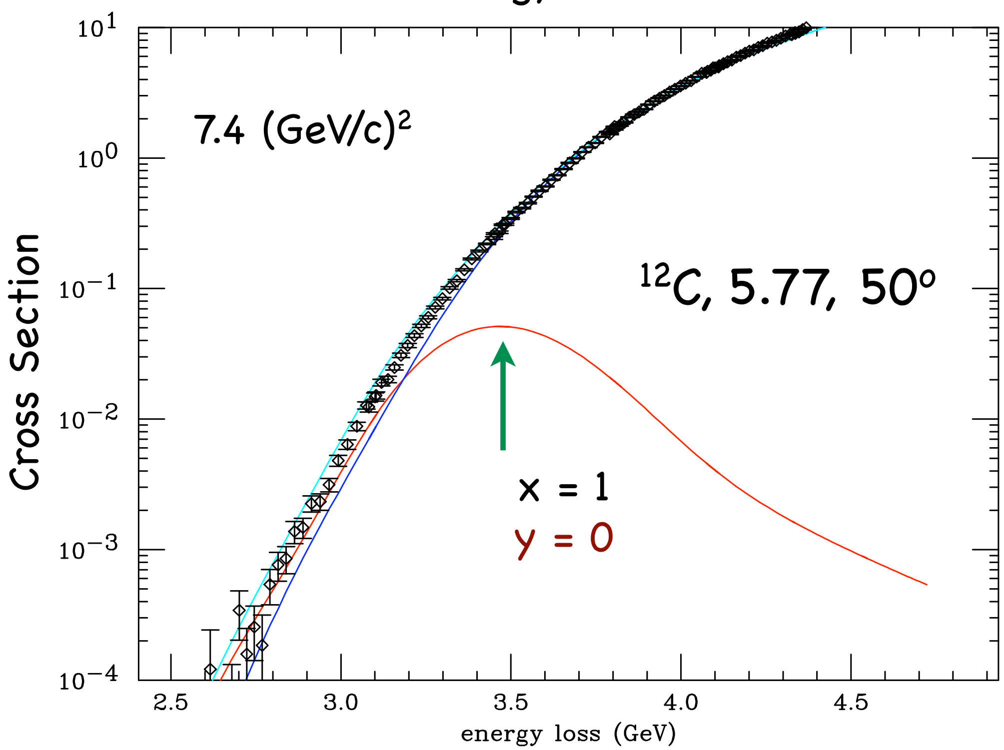
Inelastic contribution increases with Q^2





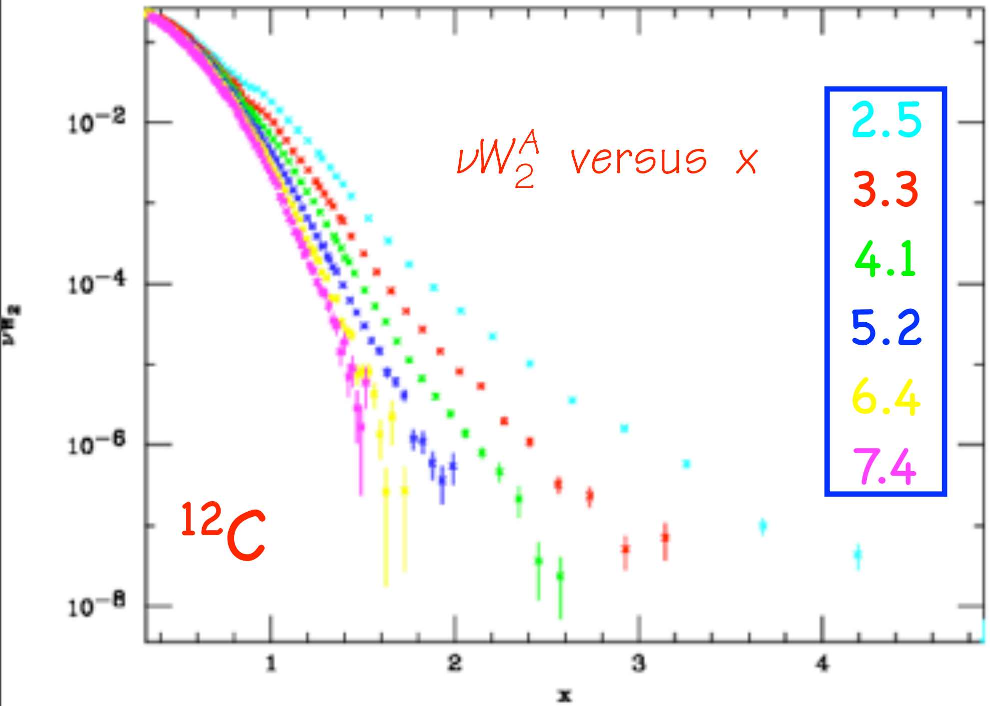
DIS begins to contribute at $x > 1$

Convolution model



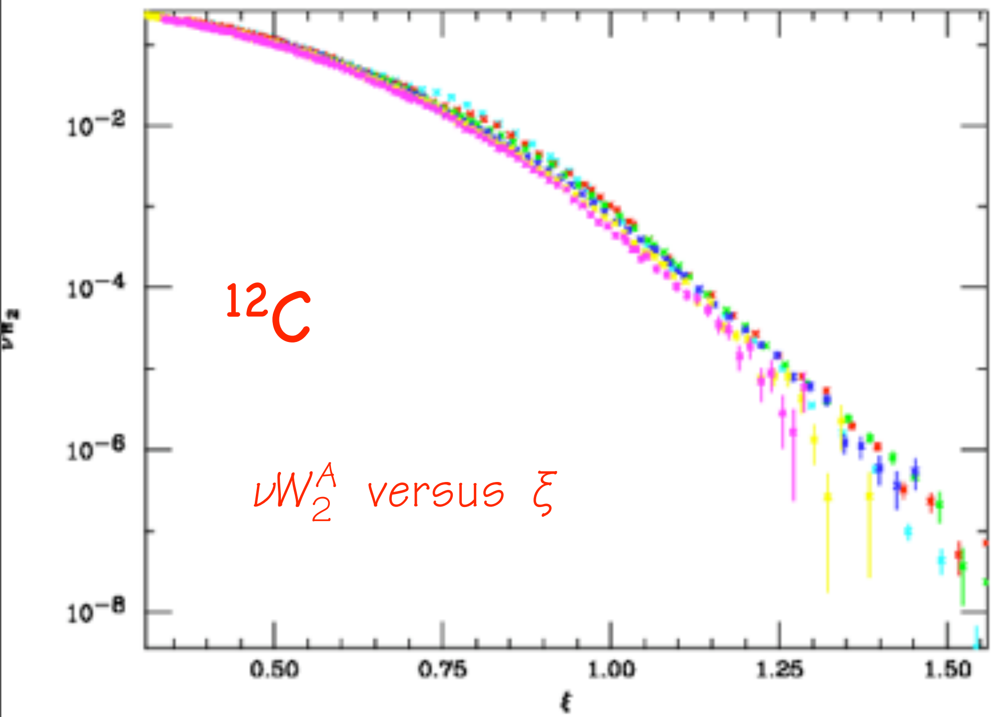
We expect that as Q^2 increases to see evidence for x-scaling – and Q^2 independence.

x and ξ scaling



$$\nu W_2^A = \nu \cdot \frac{\sigma^{exp}}{\sigma_M}\left[1 + 2\tan^2(\theta/2)\cdot\left(\frac{1+\nu^2/Q^2}{1+R}\right)\right]^{-1}$$

Evidently the inelastic and quasielastic contributions cooperate to produce ξ scaling. Is this duality?



$$F_2^A(\xi) = \underbrace{\int_\xi^A dz F(z) F_2^n(\xi/z)}_{\text{averaging}}$$

Super-fast quarks

Structure functions: only divide through by σ_{Mott}, not σ_{en}

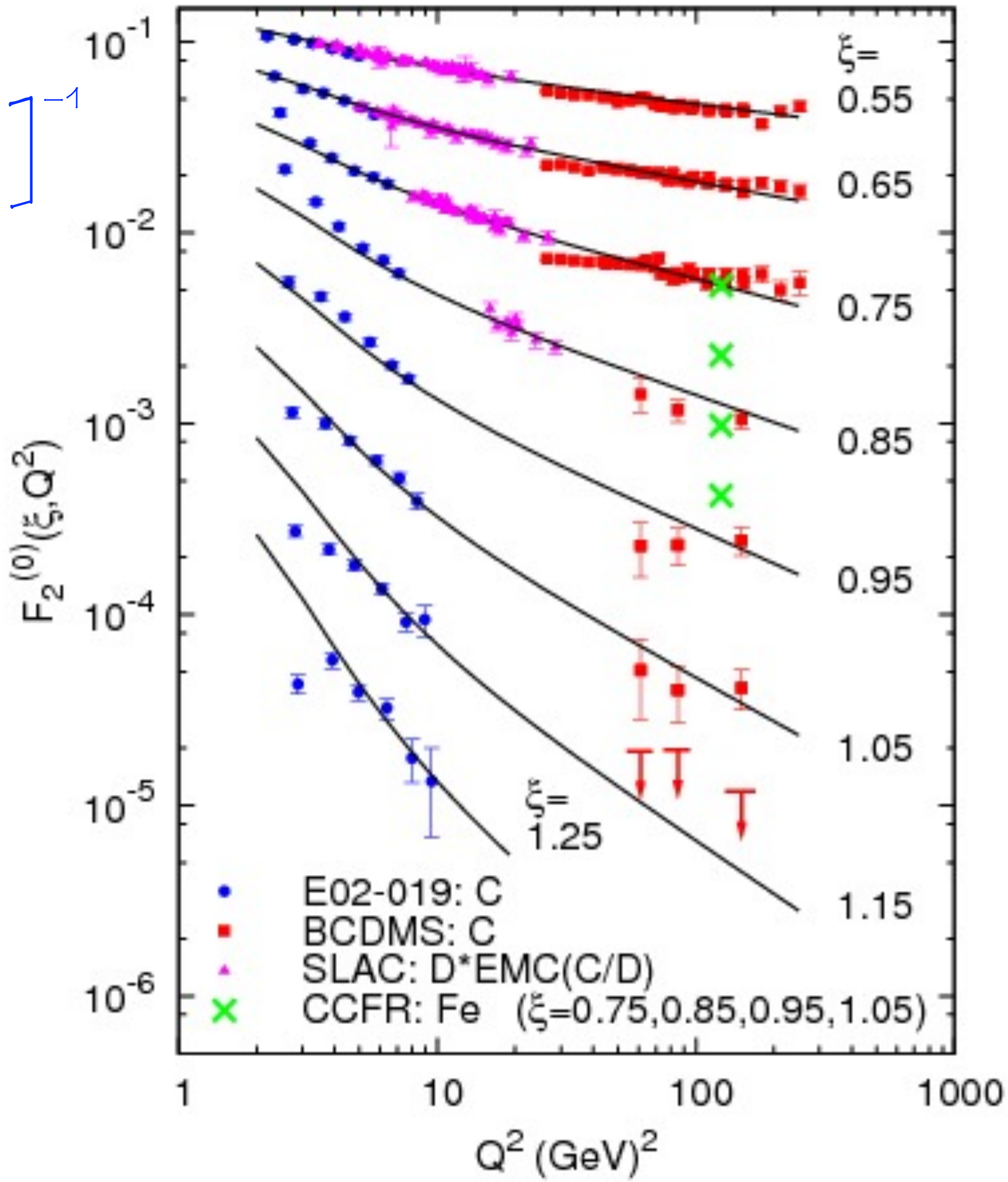
$$\nu W_2^A = \nu \cdot \frac{\sigma^{exp}}{\sigma_M}\left[1 + 2\tan^2(\theta/2)\cdot\left(\frac{1+\nu^2/Q^2}{1+R}\right)\right]^{-1}$$

$$\xi = 2x/\left(1+\sqrt{1+\frac{4m^2x^2}{Q^2}}\right)$$

The Nachtmann variable has been shown to be the variable in which logarithmic violations of scaling in DIS should be studied. Takes care of $1/Q^2$ corrections

Current data at highest Q^2 (JLab E02-019) already show partonic-like scaling behavior at x>1

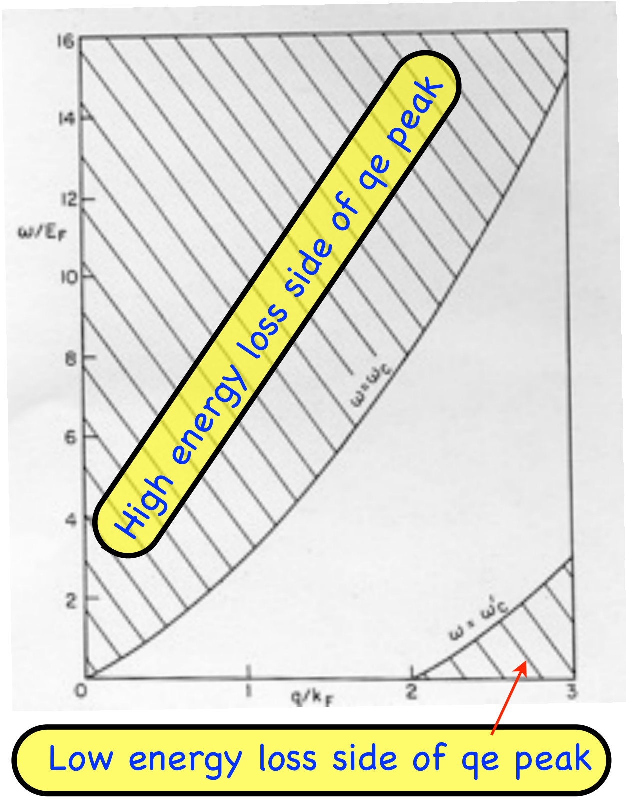
N. Fomin et al, PRL 105, 212502 (2010)



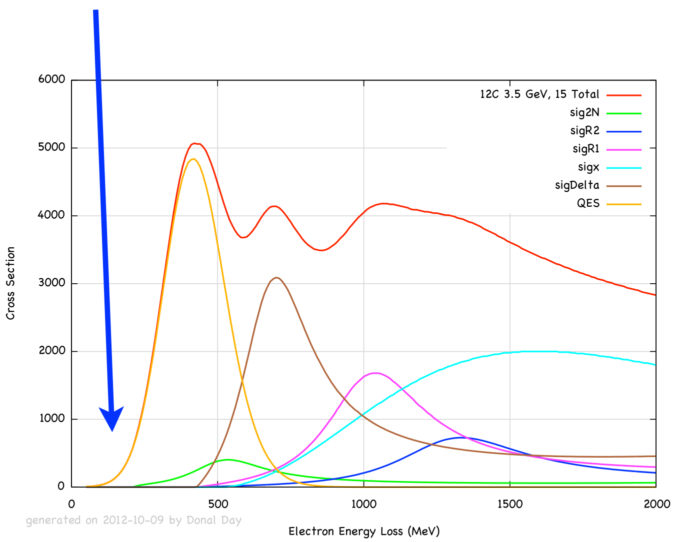
Correlations and Inclusive Electron Scattering

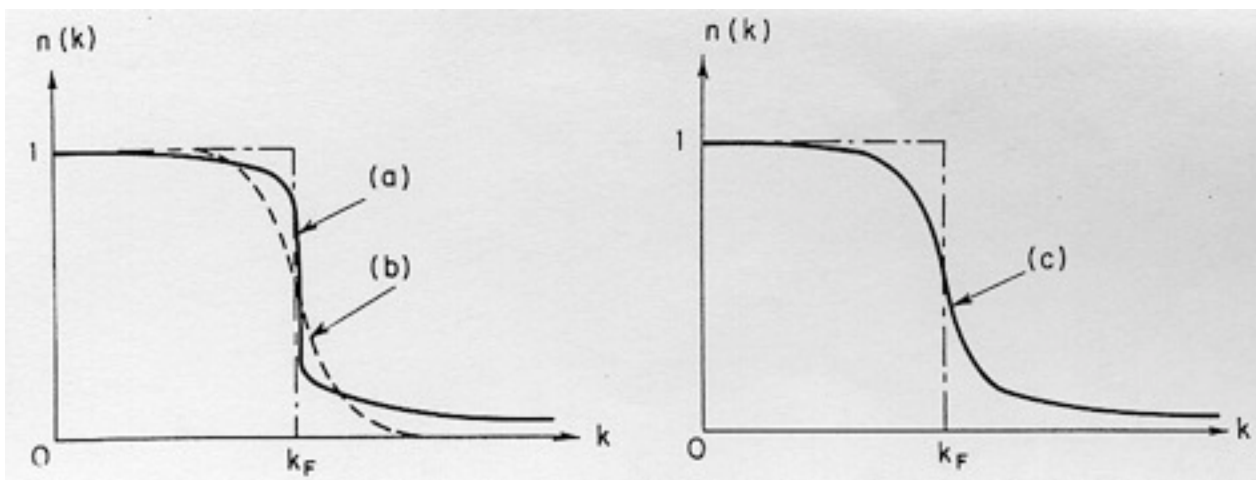
Czyz and Gottfried (1963) suggest electron scattering might reveal presence of correlations between nucleons

$$\omega_c = \frac{(k+q)^2}{2m} + \frac{q^2}{2m} \qquad \omega_c' = \frac{q^2}{2m} - \frac{qk_f}{2m}$$



Czyz and Gottfried proposed to replace the Fermi n(k) with that of an actual nucleus. (a) hard core gas; (b) finite system of noninteracting fermions; (c) actual large nucleus.

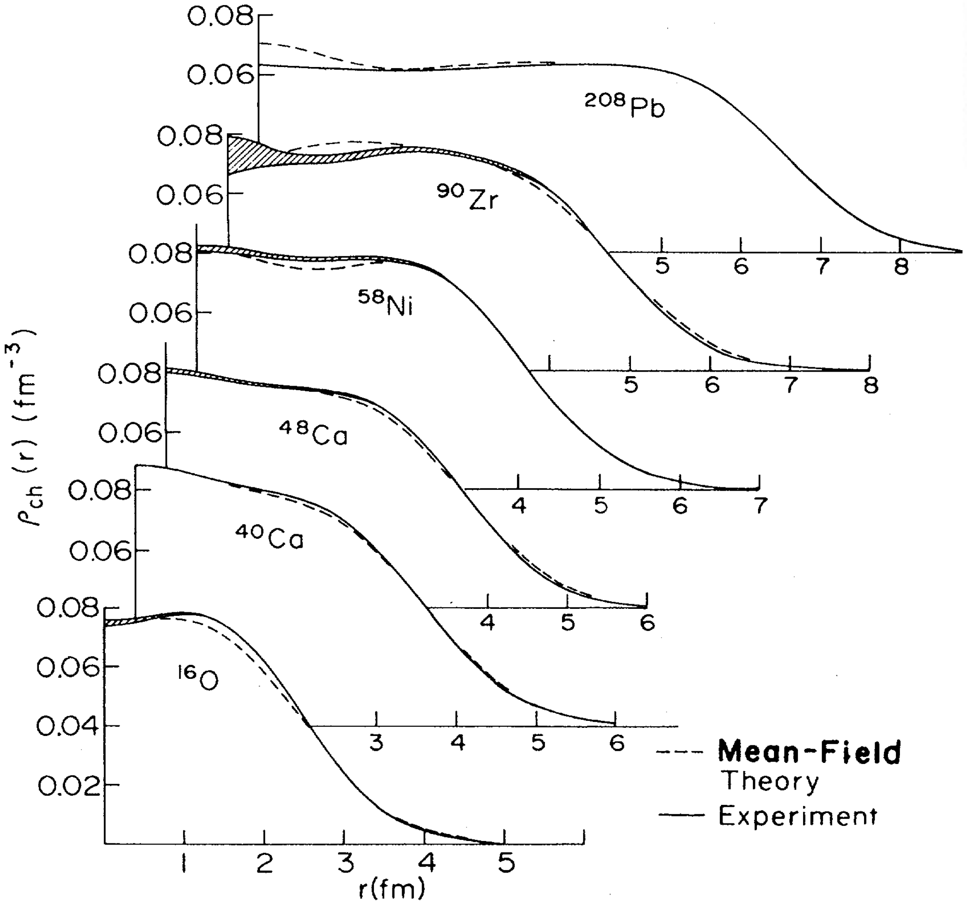


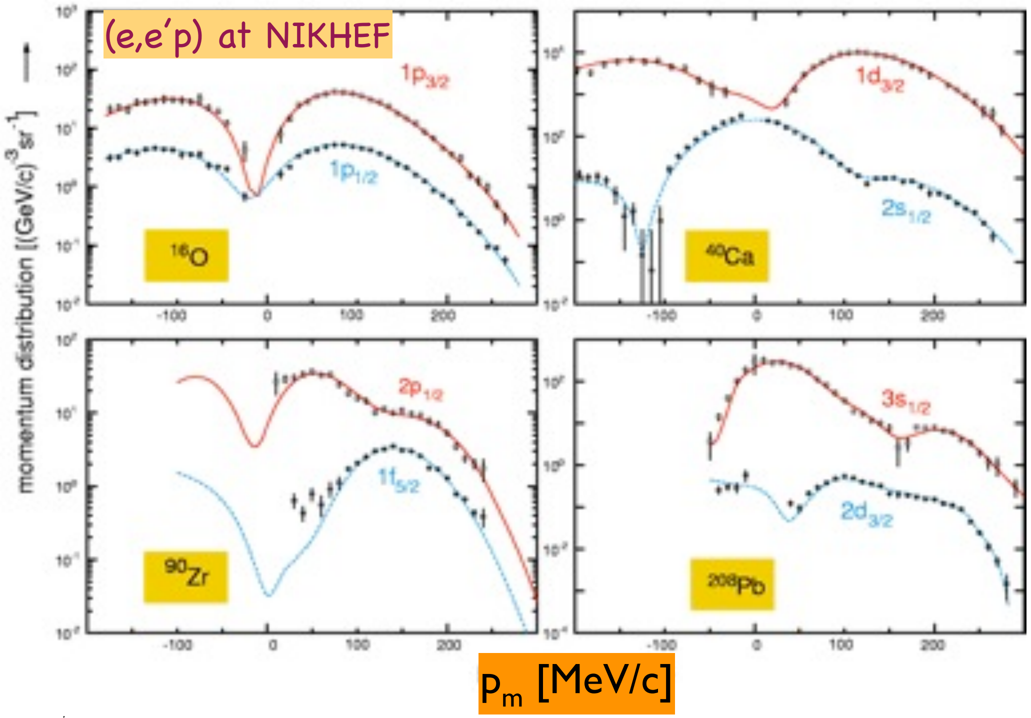


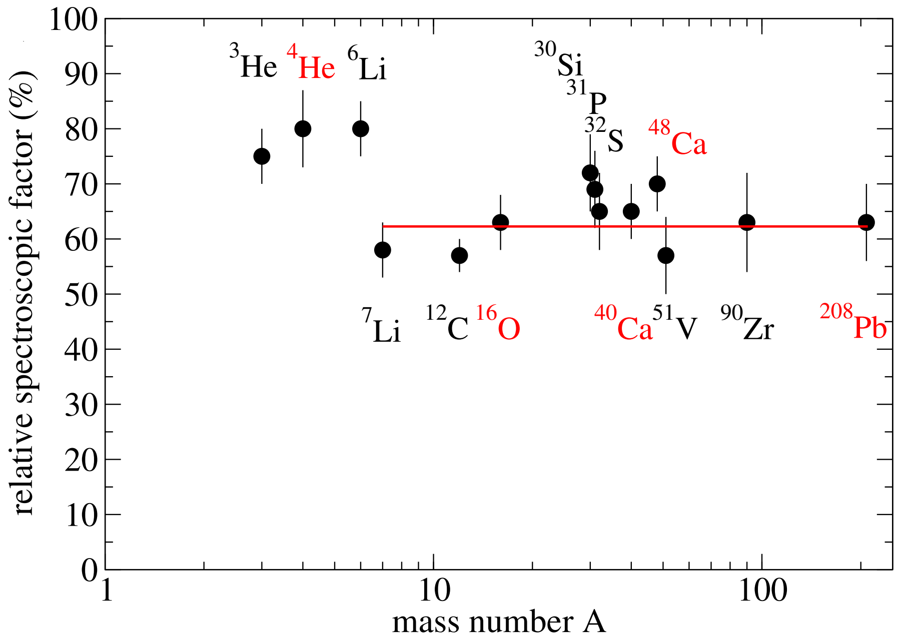
Short range correlations do exist!

Occupation numbers scaled down by a factor ~0.65.

Central density is saturated – nucleons can be packed only so close together: $\rho_{ch} * (A/Z) = \text{constant}$

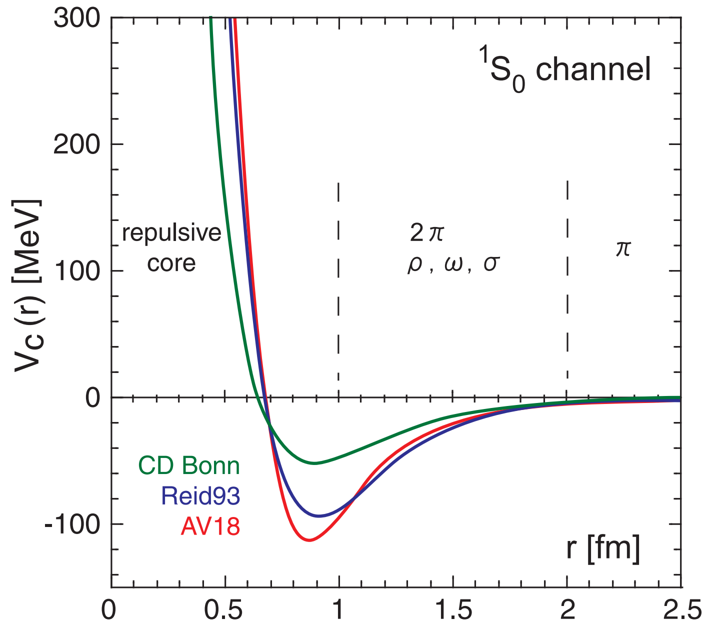


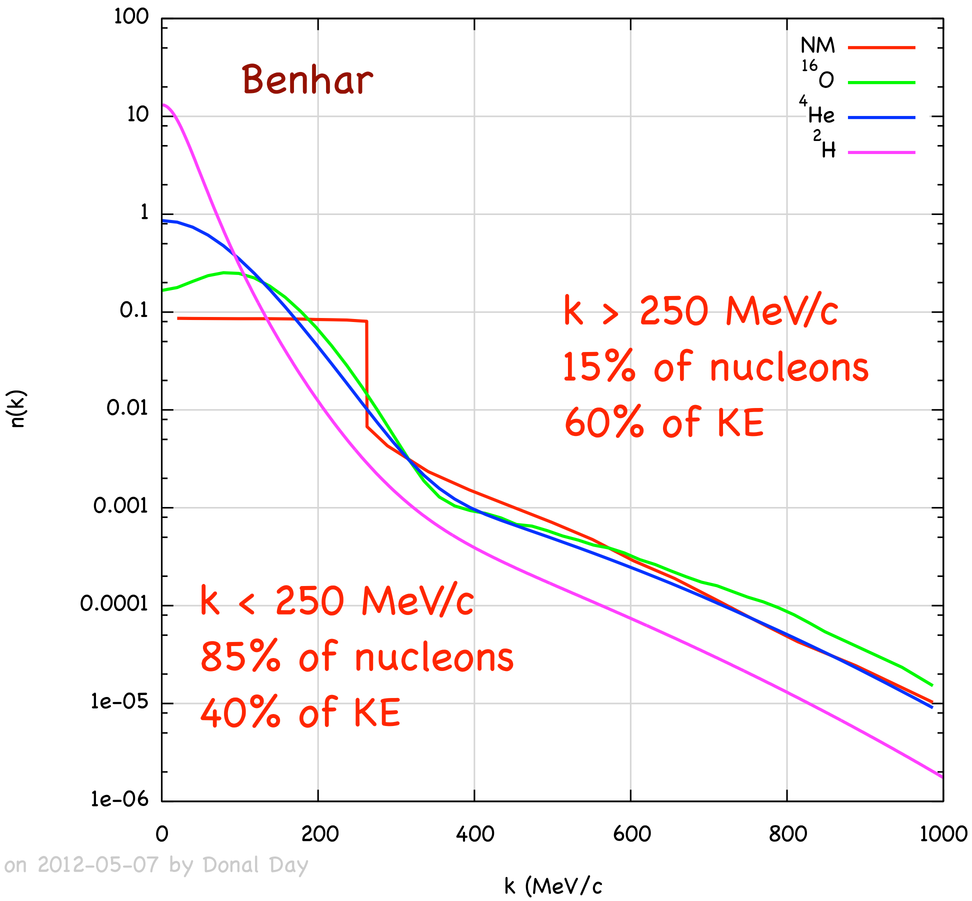


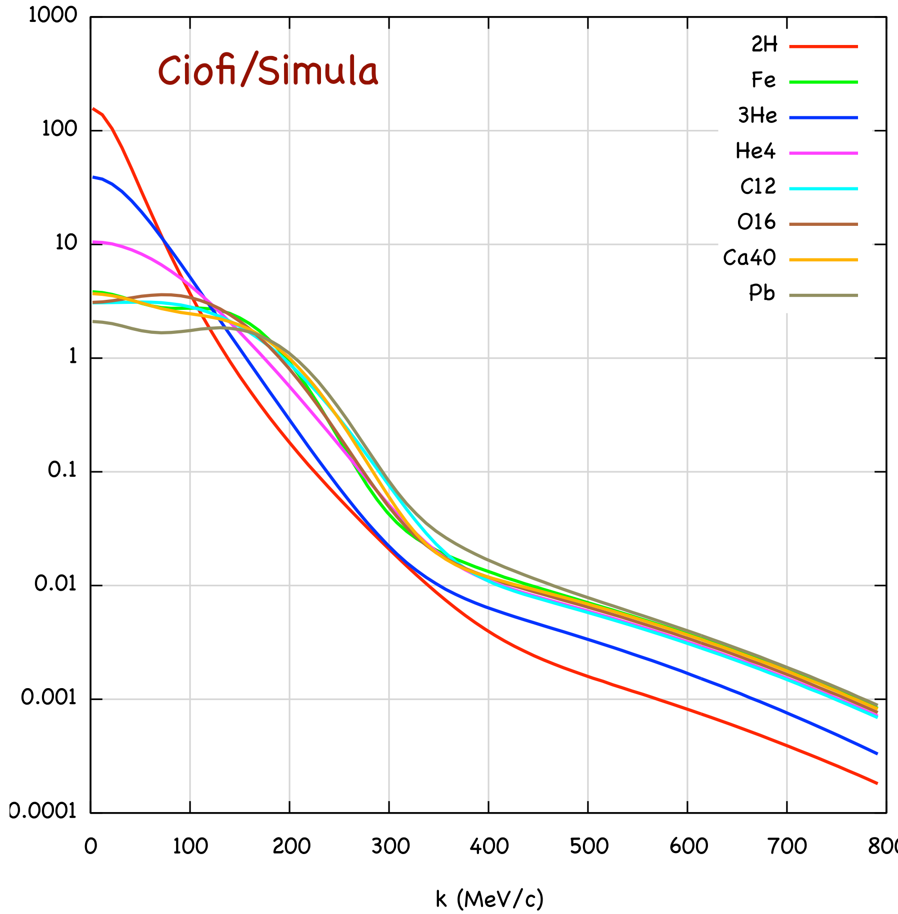


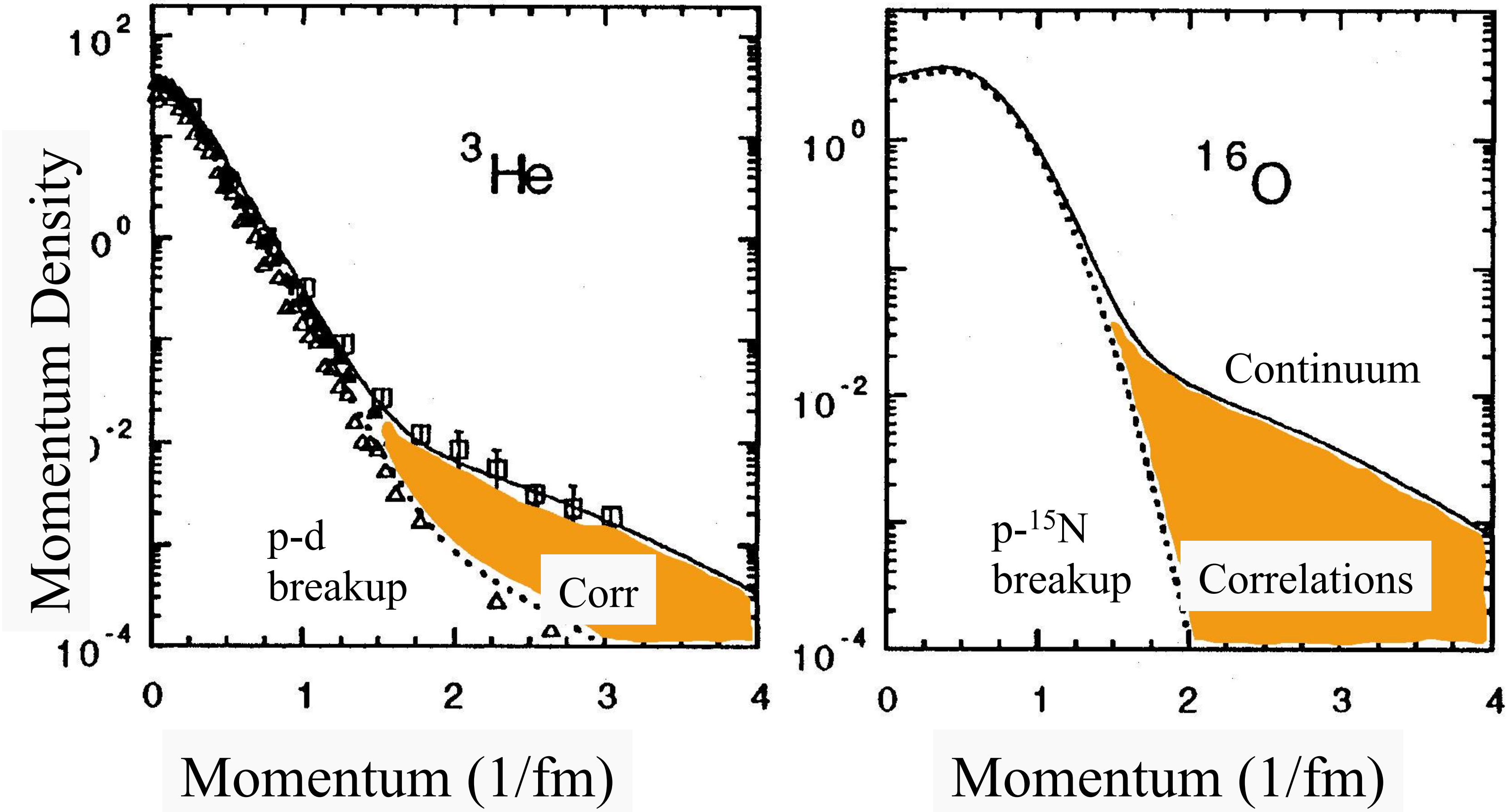
Theory suggests a common feature for all nuclei

What many calculations indicate is that the tail of n(k) for different nuclei has a similar shape - reflecting that the NN interaction, common to all nuclei, is the source of these dynamical correlations. The must be accounted for.







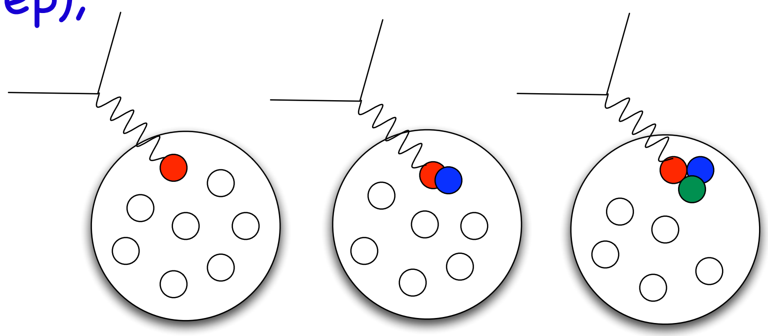


Ciofi degli Atti, PRC 53 (1996) 1689

This strength must be accounted for when trying to predict the cross sections

Access to SRC via CS Ratios

In the region where correlations should dominate, large x (at low energy loss side of qep),



$$\sigma(x,Q^2) = \sum_{j=2}^{A} A\frac{1}{j}a_j(A)\sigma_j(x,Q^2)$$
$$= \frac{A}{2}a_2(A)\sigma_2(x,Q^2) + \frac{A}{3}a_3(A)\sigma_3(x,Q^2) + \vdots$$

$a_j(A)$ are proportional to finding a nucleon in a j-nucleon correlation. It should fall rapidly with j as nuclei are dilute.

$$\sigma_2(x,Q^2) = \sigma_{eD}(x,Q^2) \text{ and } \sigma_j(x,Q^2) = 0 \text{ for } x > j.$$

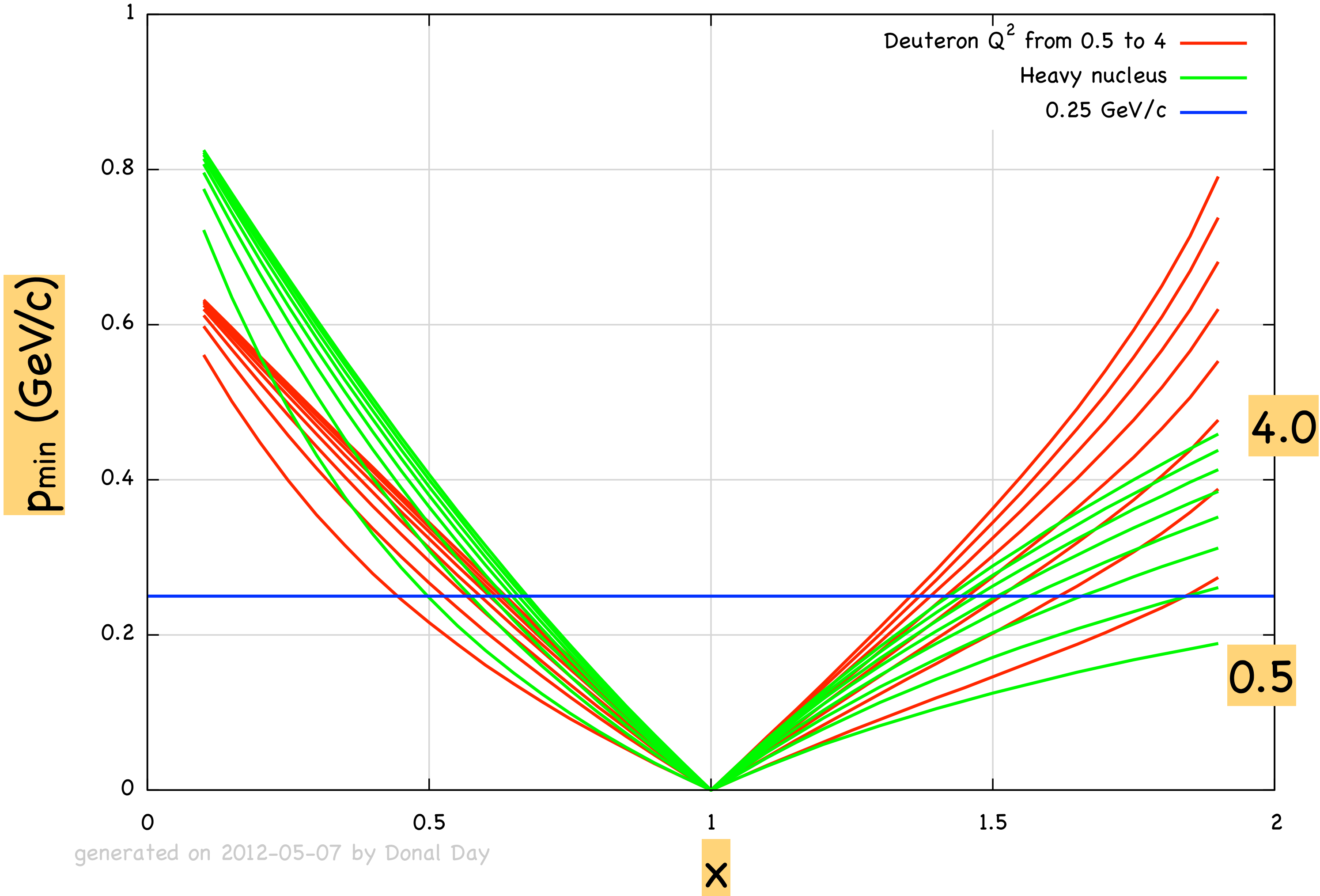
$$\Rightarrow \quad \frac{2}{A}\frac{\sigma_A(x,Q^2)}{\sigma_D(x,Q^2)} = a_2(A)\Big|_{1<x\leq 2}$$

$$\frac{3}{A}\frac{\sigma_A(x,Q^2)}{\sigma_{A=3}(x,Q^2)} = a_3(A)\Big|_{2<x\leq 3}$$

Assumption is that in the ratios, off-shell effects and FSI largely cancel.

"Evidence for Short Range Correlations from high Q2 (e,e') reactions", L. Frankfurt, M. Strikman, D.B. Day, and M. Sargsian, Phys. Rev. C48 2451 (1993)

Selection by kinematics



Appearance of plateaus is A dependent.

Kinematics: heavier recoil systems do not require as much energy to balance momentum of struck nucleon - hence p_{min} for a given x and Q^2 is smaller.

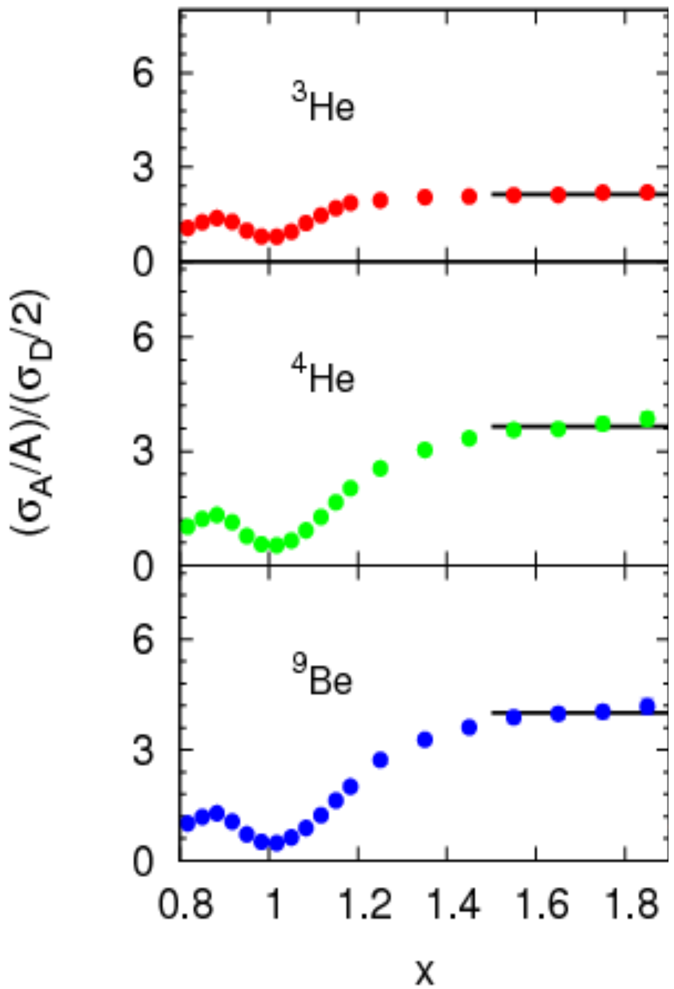
Dynamics: mean field part in heavy nuclei persist to larger values in x

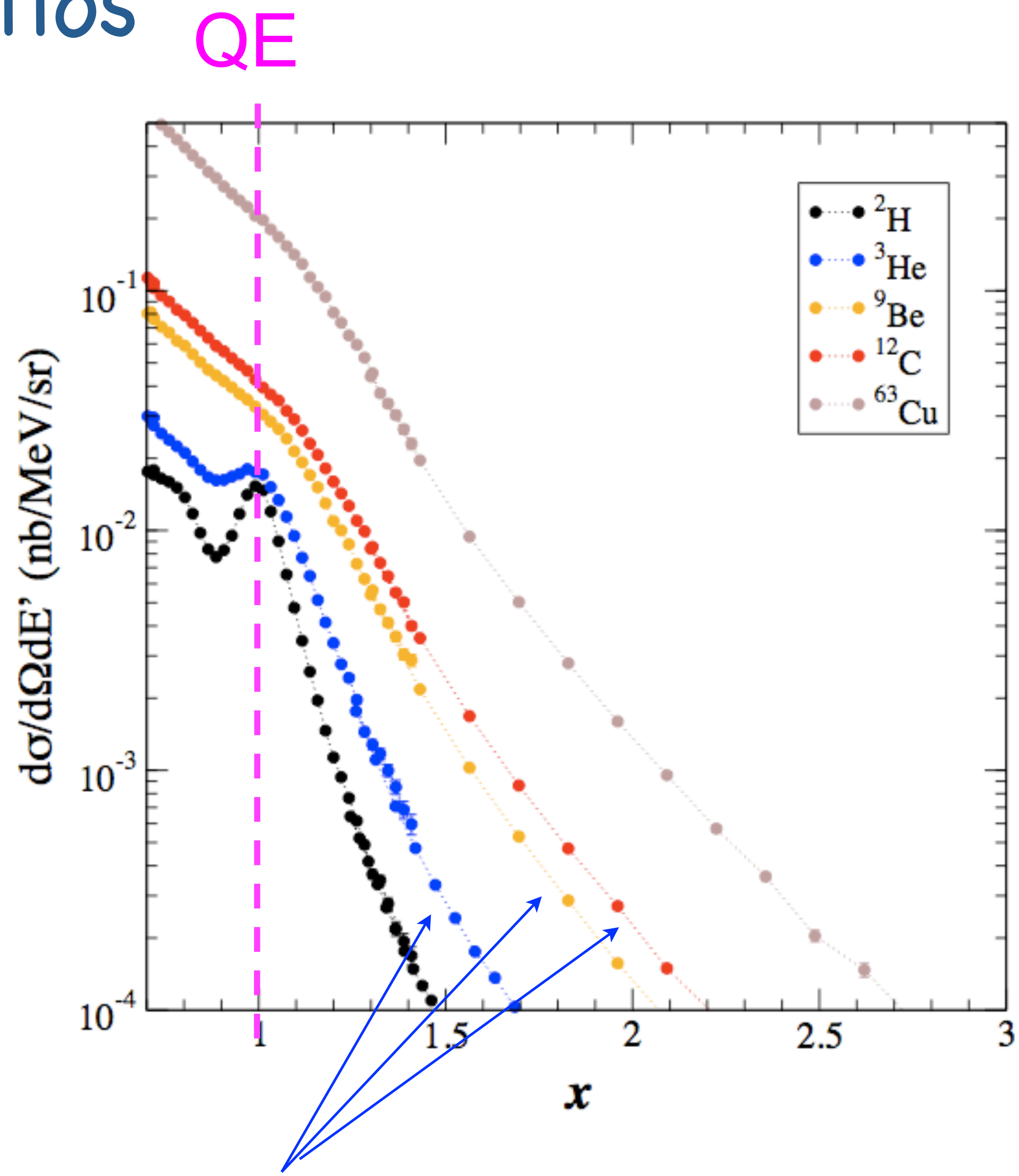
Have to go to higher x or Q^2 to insure scattering is not from mean-field nucleon

Should be similar for ν QES

SRC evidence: A/D ratios

N. Fomin et al., PRL 108, 092052 (2012): E02-019





Ratio of cross section (per nucleon) shows plateau above x ≈ 1.4, as expected if high-momentum tails dominated by 2N-SRCs

High momentum tails should yield constant ratio if seeing SRC (identical 2-body physics)

EMC Effect

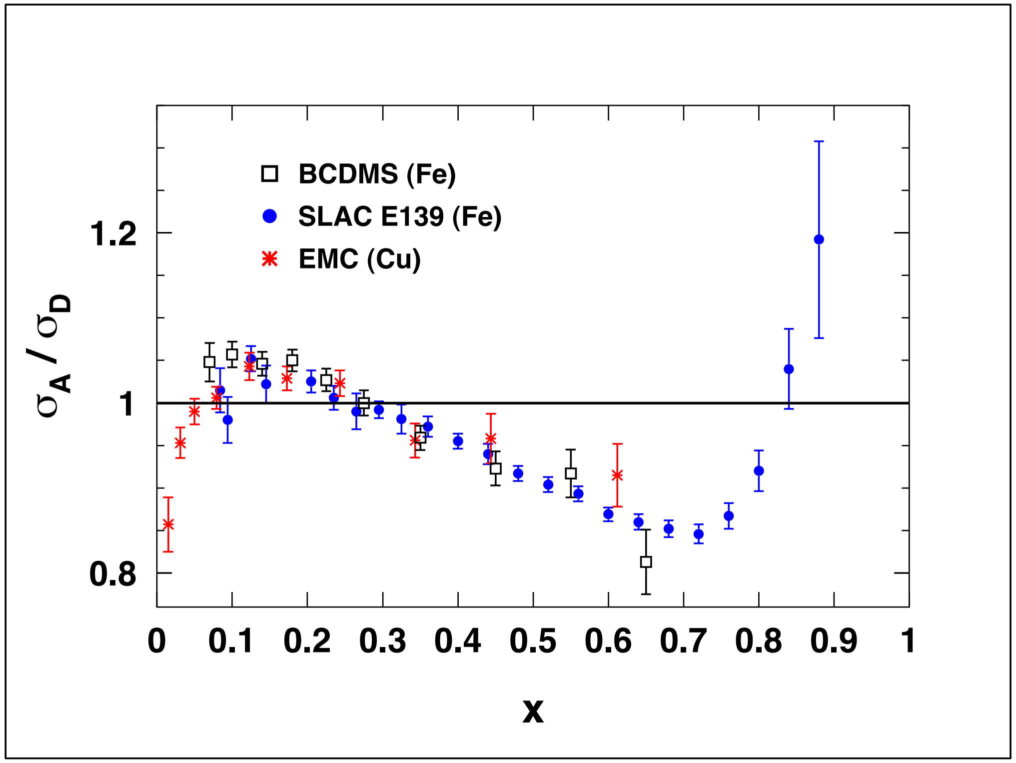
Measurements of F_2^A / F_2^D (EMC, SLAC, BCDMS,...) have shown definitively that quark distributions are modified in nuclei.

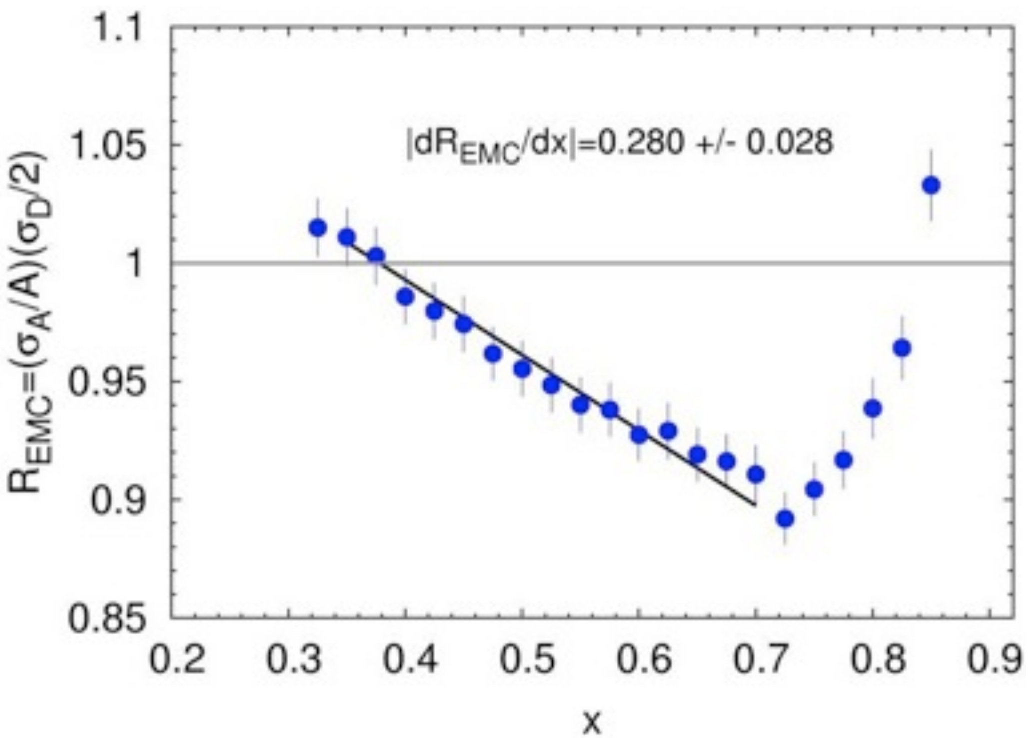
Nucleus is not simply an incoherent sum of protons and neutrons

Conventional" nuclear physics based explanations (convolution calculations)

Medium Modifications on quark distributions, clusters etc

E03-103 at JLAB Measured EMC ratios for light nuclei. Established new definition of 'size' of EMC effect : Slope of line fit from x=0.35 to 0.7

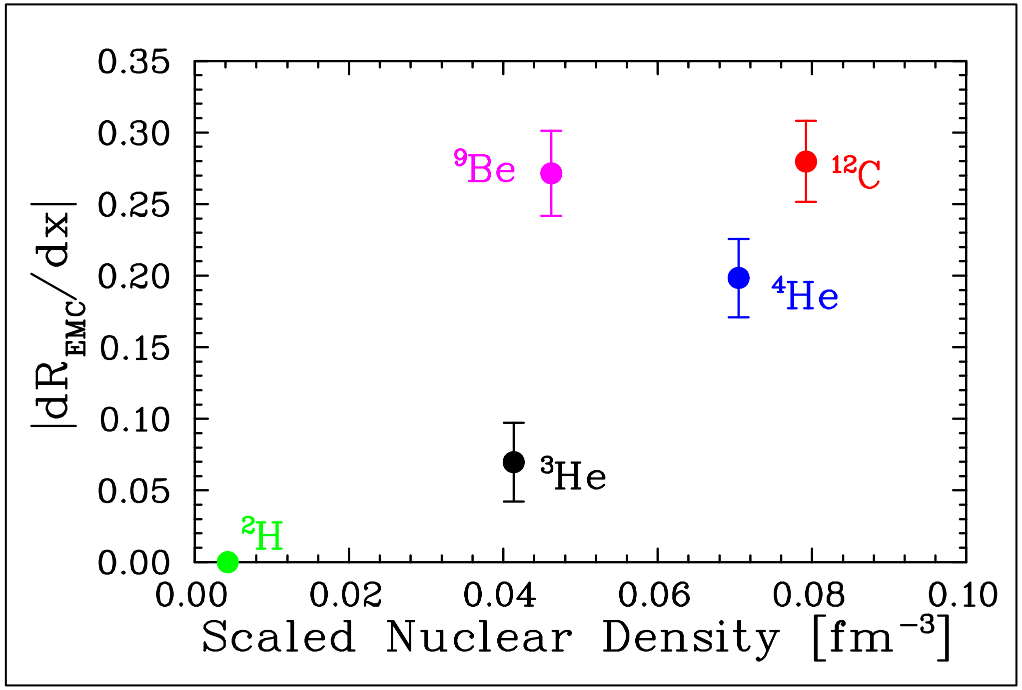


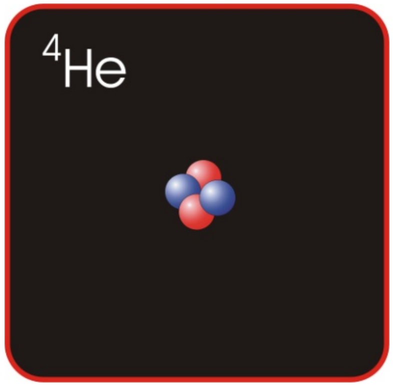


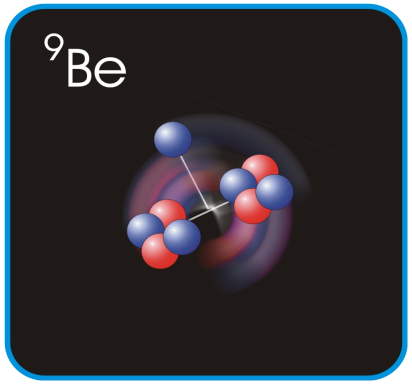
EMC Effect and Local Nuclear Density

^{9}Be has low average density

- Large component of structure is 2α+n
- Most nucleons in tight, α-like configurations
- EMC effect driven by local rather than average nuclear density



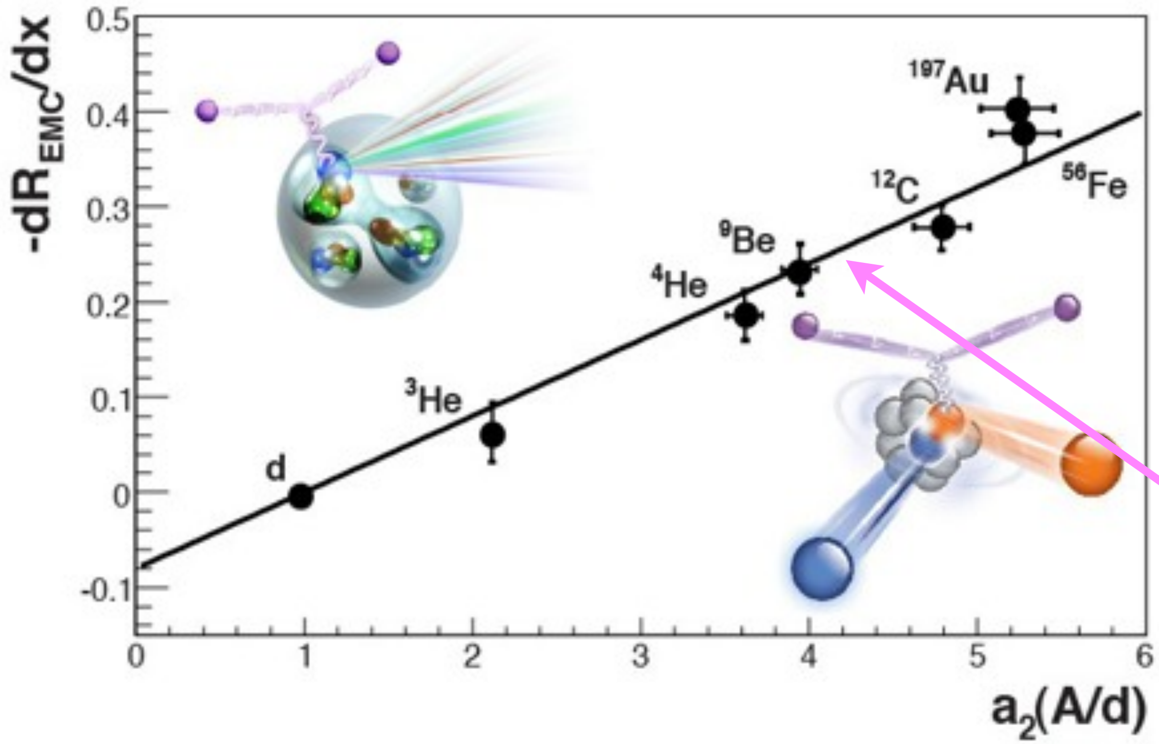




Scaled nuclear density = (A-1)/A <ρ>
--> remove contribution from struck nucleon

<ρ> from ab initio few-body calculations - [S.C. Pieper and R.B. Wiringa, Ann. Rev. Nucl. Part. Sci 51, 53 (2001)]

Short Range Correlations and the EMC Effect

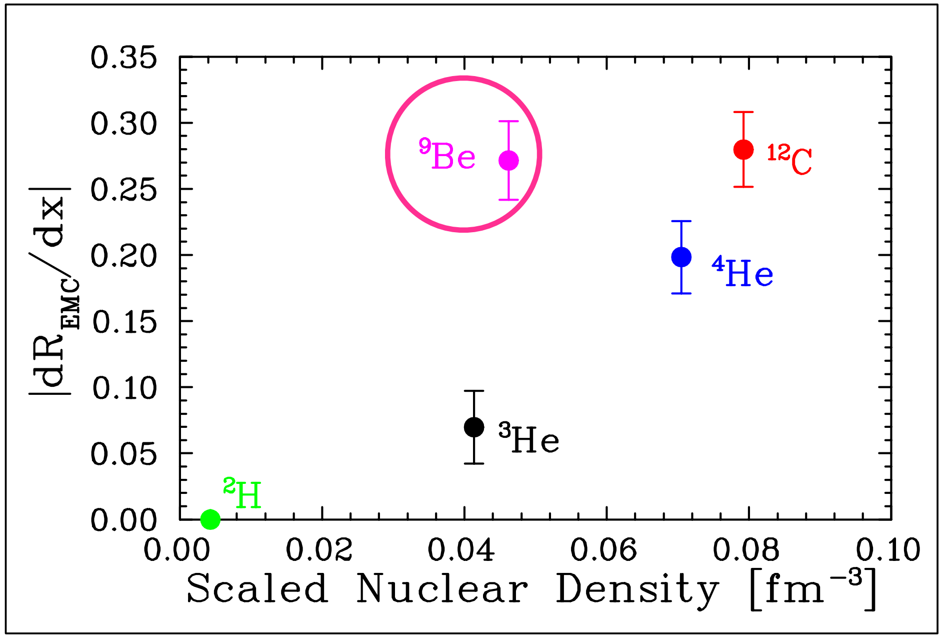


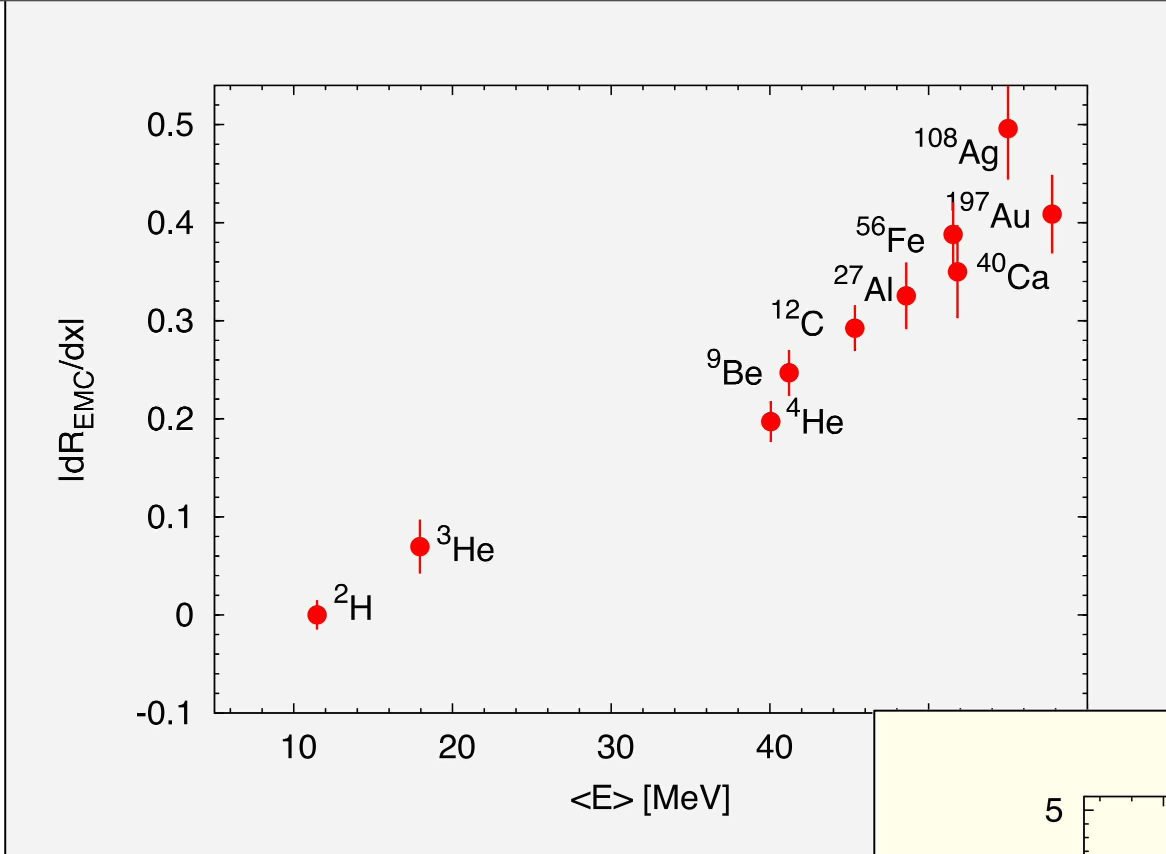
L. B. Weinstein, E. Piasetzky, D. W. Higinbotham, J. Gomez, O. Hen, and R. Shneor Phys. Rev. Lett. 106, 052301 (2011)

O. Hen, E. Piasetzky, and L. B. Weinstein, Phys. Rev. C 85, 047301 (2012)

Arrington, Daniel, Day, Gaskell and Fomin,, nucl-ex arXiv:1206.6343.

Given the fact that the reaction dynamics very different – DIS vs. QE scattering, why the same nuclear dependence? The two regions integrate over very different parts of the spectral function this probably deserves more study.

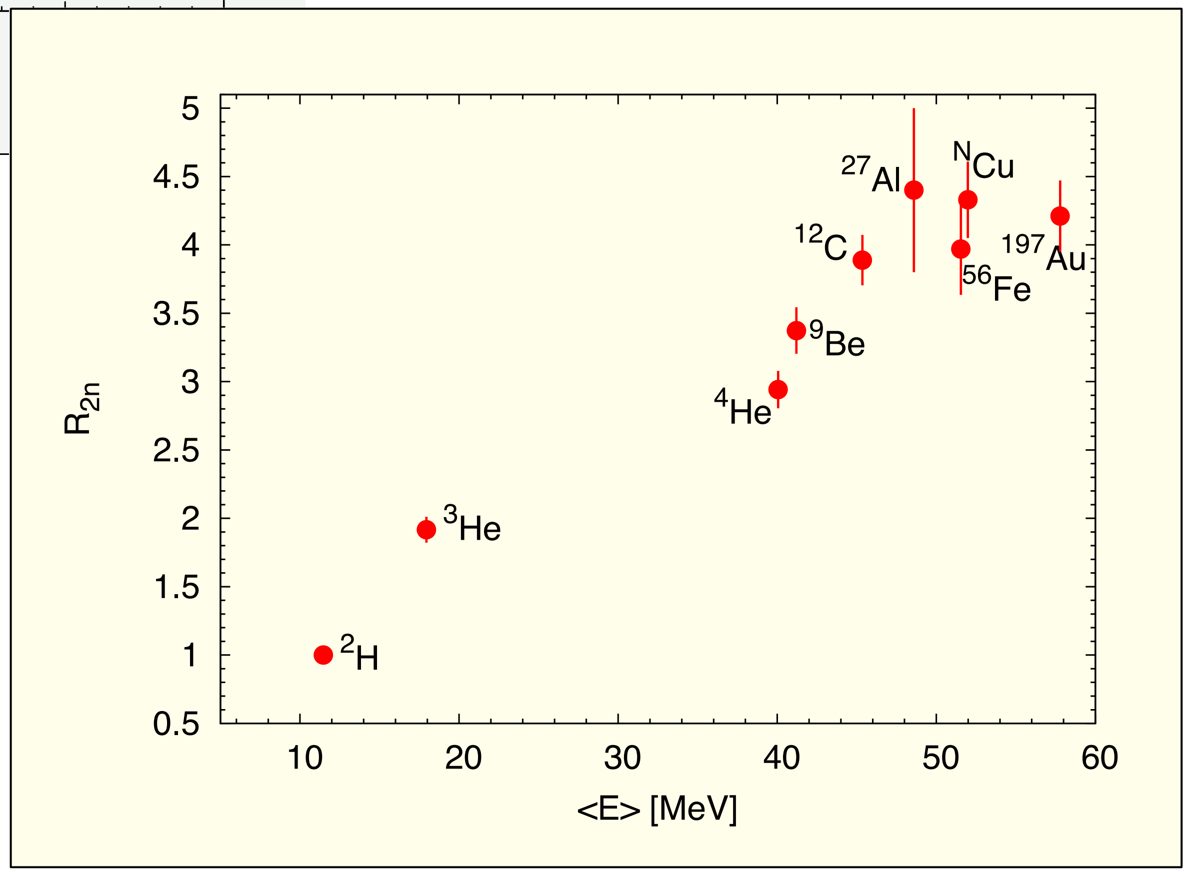




Back to the spectral function

EMC effect and the SRC measure both correlate nicely with <SE>

The x > 1 and the x < 1 regions integrate over very different parts of the spectral function – this deserves more study.



Separation energies calculated using spectral functions including contributions from mean-field, correlated part of wave function. Courtesy S. Kulagin see PRC 82, 054614 (2010) and NPA 765, 126 (2006)

Finish

- Inclusive electron scattering in the QES region is a rich source of information about the gs properties of nuclei; significant data set already exists and easily accessible.
- Different Q^2 dependences allow the QES and DIS regimes to be, in principal, separated.
- Scaling in terms of scattering from nucleons and partons is demonstrated
- SRC are a significant element in the gs – they appear to scale with local ρ^A and, surprisingly, are correlated with the EMC effect and <SE> is indicated.
- Did not mention: extrapolation to NM, separation of responses, other forms of scaling, medium modifications, duality, SF Q^2 dependence (from DIS)
- Continued collaboration between electron scattering and neutrino communities should prove productive

"Inclusive quasi-elastic electron-nucleus scattering", O. Benhar, D. Day and I. Sick, Rev. Mod. Phys. 80, 189-224, 2008, arXiv:nucl-ex/0603029