## Open charm spectroscopy @ LHCb

## CHARM 2015, Wayne State University, Detroit

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on behalf of the LHCb collaboration

| RW |
| :---: |

## Introduction and outline

- Charm spectra well predicted by HQET
- Important to test the predictions with measurements of mass, width and spin
- Some deviations seen in the $D_{s}^{* *+}$ system - possible exotics?
- First observations of new states
- Reminder of the $D_{s}^{* *+}$ spectroscopy from LHCb in 2014
- Phys. Rev. Lett. 113 (2014) 162001, Phys. Rev. D 90 (2014) 072003
- $D^{* * 0}$ spectroscopy from $B^{-} \rightarrow D^{+} K^{-} \pi^{-}$decays
- Phys. Rev. D 91 (2015) 092002
- $D^{* *+}$ spectroscopy from $B^{0} \rightarrow \bar{D}^{0} \pi^{+} \pi^{-}$and $B^{0} \rightarrow \bar{D}^{0} K^{+} \pi^{-}$
- arXiv:1505.01710 and arXiv:1505.01505
- Both submitted to PRD


## LHCb experiment



## Dalitz plots

- Dalitz plot analysis of $B \rightarrow D h h^{\prime}\left(h, h^{\prime}=K / \pi\right)$ decays
- Kinematics fully constrained by particle masses



## Charm spectroscopy

## - Which charm resonances should we expect to see?

- Only access natural spin-parity states ( $0^{+}, 1^{-}, 2^{+} \ldots$ ) in $B \rightarrow D h h^{\prime}$ decays

Evidence for higher mass states $D(2600)$ and $D(2760)$

Phys. Rev. D82 (2010) 111101 JHEP 09 (2013) 145

1P states measured by B-factories and LHCb

Phys. Rev. D69 (2004) 112002
Phys. Rev. D79 (2009) 112004
Phys. Rev. D82 (2010) 111101
JHEP 09 (2013) 145


## Charmed strange spectroscopy

## - What about the Ds states?

- Full Dalitz plot analysis of $B_{s}^{0} \rightarrow \bar{D}^{0} K^{-} \pi^{+}$decays

Phys. Rev. Lett. 113 (2014) 162001 Phys. Rev. D 90 (2014) 072003

- Resolved the $D_{S J}^{*}(2860)^{-}$state into spin 1 and spin 3 components (>10 sigma)



$$
\left.\begin{array}{rl}
m\left(D_{s 1}^{*}(2860)^{-}\right) & =2859 \pm 12 \pm 6 \pm 23 \mathrm{MeV} / c^{2} \quad m\left(D_{s 3}^{*}(2860)^{-}\right)
\end{array}=2860.5 \pm 2.6 \pm 2.5 \pm 6.0 \mathrm{MeV} / c^{2}\right)
$$

Uncertainties are statistical, experimental systematics and model systematics

Phys. Rev. D 91 (2015) 092002

$$
B^{-} \rightarrow D^{+} K^{-} \pi^{-}
$$

## Analysis details - branching fraction

- Firstly observe the decay!
- Normalise to the similar decay $B^{-} \rightarrow D^{+} \pi^{-} \pi^{-}$
- Roughly signal 2000 candidates in the full $3 \mathrm{fb}^{-1}$ data sample



$$
\mathcal{B}\left(B^{-} \rightarrow D^{+} K^{-} \pi^{-}\right)=(7.31 \pm 0.19 \pm 0.22 \pm 0.39) \times 10^{-5}
$$

- Uncertainties are statistical, systematic and from $\mathcal{B}\left(B^{-} \rightarrow D^{+} \pi^{-} \pi^{-}\right)$(PDG)


## Analysis details - DP fit

- Use mass fit to define a mass window
- Taken as 5239.4 -> 5317.1 MeV
- Purity in the signal region is $\sim 93 \%$


- Building the fit model
- Only expect resonances in $m\left(D^{+} \pi^{-}\right)$
- Angular moments from Legendre polynomials to guide the fit model
- No evidence of structures above spin 2


## Analysis details - DP model

Phys. Rev. D 91 (2015) 092002

| Resonance | Spin | DP axis | Model | Parameters |
| :--- | :---: | :---: | :---: | :---: |
| $D_{0}^{*}(2400)^{0}$ | 0 | $m^{2}(D \pi)$ | RBW | $m=2318 \pm 29 \mathrm{MeV}, \Gamma=267 \pm 40 \mathrm{MeV}$ |
| $D_{2}^{*}(2460)^{0}$ | 2 | $m^{2}(D \pi)$ | RBW | Determined from data |
| $D_{J}^{*}(2760)^{0}$ | 1 | $m^{2}(D \pi)$ | RBW |  |
| Nonresonant | 0 | $m^{2}(D \pi)$ | EFF | Determined from data |
| Nonresonant | 1 | $m^{2}(D \pi)$ | EFF |  |
| $D_{v}^{*}(2007)^{0}$ | 1 | $m^{2}(D \pi)$ | RBW | $m=2006.98 \pm 0.15 \mathrm{MeV}, \Gamma=2.1 \mathrm{MeV}$ |
| $B_{v}^{* 0}$ | 1 | $m^{2}(D K)$ | RBW | $m=5325.2 \pm 0.4 \mathrm{MeV}, \Gamma=0.0 \mathrm{MeV}$ |

RBW = Relativistic Breit-Wigner and EFF = Exponential form factor

- Nominal model
- Three resonances, two nonresonant terms and two virtual states
- Efficiencies and backgrounds modelled in the fit
- Fit performed with the Laura++ package using the Isobar model http://laura.hepforge.org


## Results

－$D_{J}^{*}(2760)^{0}$ state favours spin 1
－Other hypotheses rejected with high significance（＞6 sigma）
－Mass and widths reported
－Full results of the amplitude fit in the back－ups


$$
\begin{array}{rlr}
m\left(D_{2}^{*}(2460)^{0}\right) & =(2464.0 \pm 1.4 \pm 0.5 \pm 0.2) \mathrm{MeV} \\
\Gamma\left(D_{2}^{*}(2460)^{0}\right) & =(43.8 \pm 2.9 \pm 1.7 \pm 0.6) \mathrm{MeV} \\
m\left(D_{1}^{*}(2760)^{0}\right) & =(2781 \pm 18 \pm 11 \pm 6) \mathrm{MeV} \\
\Gamma\left(D_{1}^{*}(2760)^{0}\right) & =(177 \pm 32 \pm 20 \pm 7) \mathrm{MeV}
\end{array}
$$

Uncertainties are statistical，experimental systematics and model systematics

| － | Data | －－－－－ | $\mathrm{D}_{2}^{*}(2460)^{0}$ |
| :---: | :---: | :---: | :---: |
|  | Total | ．．．．．．－ | $\mathrm{D}_{1}^{*}(2760)^{0}$ |
|  | Background | －－ | $\mathrm{B}_{\mathrm{v}}{ }^{0}$ |
| $\cdots$ | $\mathrm{D}_{\text {ov }}^{*}(2007)^{0}$ | －．－．＂－－－－－ | Nonresonant S－wave |
| ．．．． | $\mathrm{D}_{0}^{*}(2400)^{0}$ | ーーー | Nonresonant P－wave |

$$
B^{0} \rightarrow \bar{D}^{0} \pi^{+} \pi^{-}
$$

## Analysis details - mass fit

- Full amplitude analysis of $B^{0} \rightarrow \bar{D}^{0} \pi^{+} \pi^{-}$decays
- Expect resonances in $m\left(\pi^{+} \pi^{-}\right)$and $m\left(\bar{D}^{0} \pi^{-}\right)$
- Use only $\bar{D}^{0} \rightarrow K^{+} \pi^{-}$
- Firstly perform a mass fit to select events for DP fit
- Combinatorial background removed with Fisher discriminant MVA
- ~10000 signal candidates (in $3 \mathrm{fb}^{-1}$ )
- Signal region $5250-5310 \mathrm{MeV} / \mathrm{c}^{2}$
- Purity of $\sim 98 \%$ in the signal window




## Analysis details - DP model

## - Two DP fit models to choose from

- Isobar model and K-matrix parameterisations of the $\pi \pi$ S-wave
- Isobar model
- 11 resonances
- Two nonresonant terms
- K-matrix model
- Eight resonances
- 1 non resonant term
- K-matrix term

| Resonance | Spin | Model | $m_{r}\left(\mathrm{MeV} / c^{2}\right)$ | $\Gamma_{0}(\mathrm{MeV})$ |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
| $\overline{D^{0}} \pi^{-}$P-wave | 1 | Eq. 14 | Floated |  |  |
| $D_{0}^{*}(2400)^{-}$ | 0 | RBW | Floated |  |  |
| $D_{2}^{*}(2460)^{-}$ | 2 | RBW | Floated |  |  |
| $D_{J}^{*}(2760)^{-}$ | 3 | RBW | Floated |  |  |
| $\rho(770)$ | 1 | GS | $775.02 \pm 0.35$ |  | $149.59 \pm 0.67$ |
| $\omega(782)$ | 1 | Eq. 13 | $781.91 \pm 0.24$ | $8.13 \pm 0.45$ |  |
| $\rho(1450)$ | 1 | GS | $1493 \pm$ | 15 | $427 \pm$ |
| $\rho(1700)$ | 1 | GS | $1861 \pm$ | 17 | $316 \pm$ |
| $f_{2}(1270)$ | 2 | RBW | $1275.1 \pm$ | 1.2 | $185.1_{-}^{+}$ |
| 2.9 |  |  |  |  |  |
| $\pi \pi$ S-wave | 0 | K-matrix |  |  |  |
| $f_{0}(500)$ | 0 | Eq. 15 |  |  |  |
| $f_{0}(980)$ | 0 | Eq. 18 |  |  |  |
| $f_{0}(2020)$ | 0 | RBW | $1992 \pm$ | 16 | $442 \pm$ |
| Nonresonant | 0 | Eq. 20 |  |  |  |

RBW $=$ Relativistic Breit-Wigner and GS = Gounaris Sakurai. Listed eqs. are in the backup slides

## Analysis details - DP fit

- Plots from the Isobar model fit
- K-matrix plots in the back-ups
- Bump seen in $m\left(\bar{D}^{0} \pi^{-}\right)$at $2760 \mathrm{MeV} / \mathrm{c}^{2}$
- Rich structure also in $m\left(\pi^{+} \pi^{-}\right)$
- Good agreement between models
arXiv:1505.01710




## Analysis details - results

- $D_{J}^{*}(2760)^{-}$determined to be spin 3
- Other spins ruled out by at least 10 sigma
- No evidence for an additional spin-1 state

| Isobar |  |  |  |  | K-matrix |  |
| :--- | :--- | ---: | ---: | ---: | ---: | :---: |
| $D_{0}^{*}(2400)$ | $m$ | $2349 \pm 6 \pm 1 \pm 4$ | $2354 \pm 7 \pm 11 \pm 2$ |  |  |  |
|  | $\Gamma$ | $217 \pm 13 \pm$ | $5 \pm 12$ | $230 \pm 15 \pm 18 \pm 11$ |  |  |
| $D_{2}^{*}(2460)$ | $m$ | $2468.6 \pm 0.6 \pm 0.0 \pm 0.3$ | $2468.1 \pm 0.6 \pm 0.4 \pm 0.3$ |  |  |  |
|  | $\Gamma$ | $47.3 \pm 1.5 \pm 0.3 \pm 0.6$ | $46.0 \pm 1.4 \pm 1.7 \pm 0.4$ |  |  |  |
| $D_{3}^{*}(2760)$ | $m$ | $2798 \pm 7 \pm 1 \pm 7$ | $2802 \pm 11 \pm 10 \pm$ | 3 |  |  |
|  | $\Gamma$ | $105 \pm 18 \pm$ | $6 \pm 23$ | $154 \pm 27 \pm 13 \pm$ | 9 |  |

Uncertainties are statistical, experimental systematics and model systematics

- See back-ups for further (non-spectroscopy) results
- Branching fraction measurements
- Amplitude fit parameters

$$
B^{0} \rightarrow \bar{D}^{0} K^{+} \pi^{-}
$$

## Analysis details

- Full amplitude analysis of $B^{0} \rightarrow \bar{D}^{0} K^{+} \pi^{-}$decays
- Expect resonances in $m\left(K^{+} \pi^{-}\right)$and $m\left(\bar{D}^{0} \pi^{-}\right)$
- Use only $\bar{D}^{0} \rightarrow K^{+} \pi^{-}$
- Access to the same charm resonances as $B^{0} \rightarrow \bar{D}^{0} \pi^{+} \pi^{-}$but lower statistics
- Most interesting for a future measurement of the CKM angle gamma
- Mass fit to select events for DP fit
- Roughly 2500 signal candidates
- Uses the full $3 \mathrm{fb}^{-1}$ data sample
- Signal window 5248.55-5309.05 MeV
- Purity $\sim 75 \%$ in the signal region



## Analysis details - DP fit

## - Nominal Amplitude model

- Five resonant terms, two nonresonant components and the LASS shape
- Backgrounds and efficiency variation over the DP included
- The Laura++ package was used to perform the amplitude fit http://laura.hepforge.org
- Isobar formalism used

| Resonance | Spin | DP axis | Model | Parameters |
| :--- | :---: | :---: | :---: | :---: |
| $K^{*}(892)^{0}$ | 1 | $m^{2}\left(K^{+} \pi^{-}\right)$ | RBW | $m_{0}=895.81 \pm 0.19 \mathrm{MeV}, \Gamma_{0}=47.4 \pm 0.6 \mathrm{MeV}$ |
| $K^{*}(1410)^{0}$ | 1 | $m^{2}\left(K^{+} \pi^{-}\right)$ | RBW | $m_{0}=1414 \pm 15 \mathrm{MeV}, \Gamma_{0}=232 \pm 21 \mathrm{MeV}$ |
| $K_{0}^{*}(1430)^{0}$ | 0 | $m^{2}\left(K^{+} \pi^{-}\right)$ | LASS | Determined from data |
| $K_{2}^{*}(1430)^{0}$ | 2 | $m^{2}\left(K^{+} \pi^{-}\right)$ | RBW | $m_{0}=1432.4 \pm 1.3 \mathrm{MeV}, \Gamma_{0}=109 \pm 5 \mathrm{MeV}$ |
| $D_{0}^{*}(2400)^{-}$ | 0 | $m^{2}\left(\bar{D}^{0} \pi^{-}\right)$ | RBW | Determined from data |
| $D_{2}^{*}(2460)^{-}$ | 2 | $m^{2}\left(\bar{D}^{0} \pi^{-}\right)$ | RBW |  |
| Nonresonant | 0 | $m^{2}\left(\bar{D}^{0} \pi^{-}\right)$ | dabba | Fixed |
| Nonresonant | 1 | $m^{2}\left(\bar{D}^{0} \pi^{-}\right)$ | EFF | Determined from data |

RBW = Relativistic Breit-Wigner, EFF = Exponential form factor. For dabba and LASS see back-ups

## Analysis details - results

- No evidence of $D_{J}^{*}(2760)^{-}$contribution
- No significant spin 1 or 3 state
- More data required in run 2
- See back-ups for further results

$$
\begin{aligned}
m\left(D_{0}^{*}(2400)^{-}\right) & =(2360 \pm 15 \pm 12 \pm 28) \mathrm{MeV} \\
\Gamma\left(D_{0}^{*}(2400)^{-}\right) & =(255 \pm 26 \pm 20 \pm 47) \mathrm{MeV} \\
m\left(D_{2}^{*}(2460)^{-}\right) & =(2465.6 \pm 1.8 \pm 0.5 \pm 1.2) \mathrm{MeV} \\
\Gamma\left(D_{2}^{*}(2460)^{-}\right) & =(46.0 \pm 3.4 \pm 1.4 \pm 2.9) \mathrm{MeV}
\end{aligned}
$$

Uncertainties are statistical, experimental systematics and model systematics



## Summary

- Three amplitude analyses of $B \rightarrow D h h^{\prime}$ decays performed
- $D_{J}^{*}(2760)^{0}$ determined to be spin 1 for the first time
- $D_{J}^{*}(2760)^{-}$determined to be spin 3 for the first time
- Interesting to see how this develops - recall spin 1 and spin 3 in $D_{s}^{* *+}$ system
- Several worlds most precise measurements of masses and widths
- Full results of these analyses in the back-ups
- Branching fraction measurements
- Parameters from the amplitude fits
- Look out for future charm spectroscopy from LHCb


## Back-up

$$
B^{-} \rightarrow D^{+} K^{-} \pi^{-}
$$





$$
B^{-} \rightarrow D^{+} K^{-} \pi^{-}
$$

$$
\text { - Data } \quad-\cdots \quad D_{2}^{*}(2460)^{0}
$$

Total -……....- $D_{1}^{\star}(2760)^{0}$

Background $=-==-=\mathbf{B}_{\mathbf{v}}^{\star 0}$
$\mathrm{D}_{0 \mathrm{v}}^{*}(2007)^{0} \quad$-.........- Nonresonant S-wave
$D_{0}^{\star}(2400)^{0} \quad-\quad-$ Nonresonant P-wave



Table 12: Results for the complex amplitudes and their uncertainties. The three quoted errors are statistical, experimental systematic and model uncertainties, respectively.

|  | Isobar model coefficients |  |
| :--- | :---: | :---: |
| Resonance | Real part | Imaginary part |
| $D_{0}^{*}(2400)^{0}$ | $-0.04 \pm 0.07 \pm 0.03 \pm 0.28$ | $-0.51 \pm 0.07 \pm 0.02 \pm 0.13$ |
| $D_{2}^{*}(2460)^{0}$ | 1.00 | 0.00 |
| $D_{1}^{*}(2760)^{0}$ | $-0.32 \pm 0.06 \pm 0.03 \pm 0.03$ | $-0.23 \pm 0.07 \pm 0.03 \pm 0.03$ |
| S-wave nonresonant | $0.93 \pm 0.09 \pm 0.03 \pm 0.17$ | $-0.58 \pm 0.08 \pm 0.03 \pm 0.15$ |
| P-wave nonresonant | $-0.43 \pm 0.09 \pm 0.03 \pm 0.34$ | $0.75 \pm 0.09 \pm 0.05 \pm 0.68$ |
| $D_{v}^{*}(2007)^{0}$ | $0.16 \pm 0.08 \pm 0.03 \pm 0.56$ | $0.46 \pm 0.09 \pm 0.04 \pm 0.77$ |
| $B_{v}^{*}$ | $-0.07 \pm 0.08 \pm 0.22 \pm 0.09$ | $0.33 \pm 0.07 \pm 0.02 \pm 0.08$ |

Table 13: Results for the complex amplitudes and their uncertainties. The three quoted errors are statistical, experimental systematic and model uncertainties, respectively.

|  | Isobar model coefficients |  |
| :--- | :---: | :---: |
| Resonance | Magnitude | Phase |
| $D_{0}^{*}(2400)^{0}$ | $0.51 \pm 0.09 \pm 0.02 \pm 0.15$ | $-1.65 \pm 0.16 \pm 0.06 \pm 0.50$ |
| $D_{2}^{*}(2460)^{0}$ | 1.00 | 0.00 |
| $D_{1}^{*}(2760)^{0}$ | $0.39 \pm 0.05 \pm 0.01 \pm 0.03$ | $-2.53 \pm 0.24 \pm 0.08 \pm 0.08$ |
| S-wave nonresonant | $1.09 \pm 0.09 \pm 0.02 \pm 0.20$ | $-0.56 \pm 0.09 \pm 0.04 \pm 0.11$ |
| P-wave nonresonant | $0.87 \pm 0.09 \pm 0.03 \pm 0.11$ | $2.09 \pm 0.15 \pm 0.05 \pm 0.95$ |
| $D_{v}^{*}(2007)^{0}$ | $0.49 \pm 0.07 \pm 0.04 \pm 0.05$ | $1.24 \pm 0.17 \pm 0.07 \pm 0.60$ |
| $B_{v}^{*}$ | $0.34 \pm 0.06 \pm 0.03 \pm 0.07$ | $1.78 \pm 0.23 \pm 0.11 \pm 0.27$ |

$$
B^{-} \rightarrow D^{+} K^{-} \pi^{-}
$$

Table 15: Results for the product branching fractions $\mathcal{B}\left(B^{-} \rightarrow R K^{-}\right) \times \mathcal{B}\left(R \rightarrow D^{+} \pi^{-}\right)\left(10^{-4}\right)$. The four quoted errors are statistical, experimental systematic, model and inclusive branching fraction uncertainties, respectively.

| Resonance | Branching fraction |
| :--- | ---: |
| $D_{0}^{*}(2400)^{0}$ | $6.1 \pm 1.9 \pm 0.5 \pm 1.4 \pm 0.4$ |
| $D_{2}^{*}(2460)^{0}$ | $23.2 \pm 1.1 \pm 0.6 \pm 1.0 \pm 1.6$ |
| $D_{1}^{*}(2760)^{0}$ | $3.6 \pm 0.9 \pm 0.3 \pm 0.7 \pm 0.2$ |
| S-wave nonresonant | $27.8 \pm 5.4 \pm 1.1 \pm 7.9 \pm 1.9$ |
| P-wave nonresonant | $17.4 \pm 4.1 \pm 1.5 \pm 2.7 \pm 1.2$ |
| $D_{v}^{*}(2007)^{0}$ | $5.6 \pm 1.7 \pm 1.0 \pm 1.1 \pm 0.4$ |
| $B_{v}^{*}$ | $2.6 \pm 1.4 \pm 0.6 \pm 1.2 \pm 0.2$ |

The $\rho-\omega$ interference is taken into account by

$$
R_{\rho-\omega}(s)=\operatorname{GS}_{\rho(770)}(s) \times\left(1+a e^{i \theta} \operatorname{RBW}_{\omega(782)}(s)\right)
$$

where $\Gamma_{0}$ is used, instead of the mass-dependent width $\Gamma^{(L)}(s)$, for $\omega(782)$ [82].

$$
\begin{equation*}
R_{f_{0}(500)}(s)=m_{r} \Gamma_{1}(s) /\left[m_{r}^{2}-s-g_{1}^{2} \frac{s-s_{A}}{m_{r}^{2}-s_{A}} z(s)-i m_{r} \Gamma_{\text {tot }}(s)\right], \tag{15}
\end{equation*}
$$

The Flatté formula [84] is used to describe the $f_{0}(980)$ lineshape,

$$
\begin{equation*}
R_{f_{0}(980)}(s)=\frac{1}{m_{r}^{2}-s-i m_{r}\left(\rho_{\pi \pi}(s) g_{1}+\rho_{K K}(s) g_{2}\right)}, \tag{18}
\end{equation*}
$$

The nonresonant contribution is described by

$$
\begin{equation*}
R_{N R}\left(m^{2}\left(\pi^{+} \pi^{-}\right), m^{2}\left(\bar{D}^{0} \pi^{+}\right)\right)=e^{i \alpha m^{2}\left(\pi^{+} \pi^{-}\right)} \tag{20}
\end{equation*}
$$

Its modulus equals unity, and a slowly varying phase over $m^{2}\left(\pi^{+} \pi^{-}\right)$accounts for rescattering effects of the $\pi^{+} \pi^{-}$final state and $\alpha$ is a free parameter of the model.



## $B^{0} \rightarrow \bar{D}^{0} \pi^{+} \pi^{-}$

Table 7: The moduli of the complex coefficients of the resonant contributions for the Isobar model and the K-matrix model. The first uncertainty is statistical, the second and the third are experimental and model-dependent systematic uncertainties, respectively.

| Resonance | Isobar ( $\left\|c_{i}\right\|$ ) | K-matrix ( $\left\|c_{i}\right\|$ ) |
| :---: | :---: | :---: |
| Nonresonance | $3.43 \pm 0.22 \pm 0.04 \pm 0.51$ | $\mathrm{n} / \mathrm{a}$ |
| $f_{0}(500)$ | $18.7 \pm 0.70 \pm 0.29 \pm 0.80$ | $\mathrm{n} / \mathrm{a}$ |
| $f_{0}(980)$ | $2.62 \pm 0.25 \pm 0.09 \pm 0.46$ | $\mathrm{n} / \mathrm{a}$ |
| $f_{0}(2020)$ | $4.41 \pm 0.51 \pm 0.21 \pm 1.78$ | $\mathrm{n} / \mathrm{a}$ |
| $\rho(770)$ | 1.0 (fixed) | 1.0 (fixed) |
| $\omega(782)$ | $0.30 \pm 0.04 \pm 0.00 \pm 0.01$ | $0.31 \pm 0.04 \pm 0.01 \pm 0.01$ |
| $\rho(1450)$ | $0.23 \pm 0.03 \pm 0.01 \pm 0.02$ | $0.28 \pm 0.03 \pm 0.08 \pm 0.01$ |
| $\rho(1700)$ | $0.078 \pm 0.016 \pm 0.006 \pm 0.008$ | $0.136 \pm 0.020 \pm 0.077 \pm 0.011$ |
| $f_{2}$ (1270) | $0.072 \pm 0.002 \pm 0.000 \pm 0.005$ | $0.073 \pm 0.002 \pm 0.006 \pm 0.003$ |
| $\bar{D}^{0} \pi^{-}$P-wave | $18.8 \pm 0.7 \pm 0.3 \pm 1.9$ | $19.6 \pm 0.7 \pm 0.7 \pm 0.6$ |
| $D_{0}^{*}(2400)^{-}$ | $12.1 \pm 0.8 \pm 0.3 \pm 0.6$ | $13.1 \pm 1.0 \pm 0.8 \pm 0.5$ |
| $D_{2}^{*}(2460)^{-}$ | $1.31 \pm 0.04 \pm 0.02 \pm 0.02$ | $1.31 \pm 0.04 \pm 0.04 \pm 0.00$ |
| $D_{3}^{*}(2760)^{-}$ | $0.053-{ }_{-}^{+0.011} \pm 0.003 \pm 0.008$ | $0.075{ }_{-}^{+}{ }_{0.008}^{0.016} \pm 0.005 \pm 0.003$ |

## $B^{0} \rightarrow \bar{D}^{0} \pi^{+} \pi^{-}$

Table 8: The phase of the complex coefficients of the resonant contributions for the Isobar model and the K-matrix model. The first uncertainty is statistical, the second and the third are experimental and model-dependent systematic uncertainties, respectively.

| Resonance | Isobar $\left(\arg \left(c_{i}\right)^{\circ}\right)$ | K-matrix $\left.\arg \left(c_{i}\right)^{\circ}\right)$ |
| :---: | :---: | :---: |
| Nonresonance | $77.1 \pm 4.5 \pm 2.3 \pm 5.4$ | n/a |
| $f_{0}(500)$ | $38.4 \pm 2.7 \pm 1.3 \pm 3.7$ | $\mathrm{n} / \mathrm{a}$ |
| $f_{0}(980)$ | $138.9 \pm 4.6 \pm 1.5 \pm 10.9$ | n/a |
| $f_{0}(2020)$ | $258.5 \pm 5.0 \pm 1.1 \pm 26.8$ | n/a |
| $\rho(770)$ | 0.0 (fixed) | 0.0 (fixed) |
| $\omega(782)$ | $176.8 \pm 7.8 \pm 0.6 \pm 0.5$ | $174.8 \pm 8.0 \pm 1.5 \pm 0.5$ |
| $\rho(1450)$ | $149.0 \pm 7.5 \pm 4.8 \pm 4.5$ | $132.9 \pm 7.8 \pm 8.5 \pm 5.5$ |
| $\rho(1700)$ | $103.5 \pm 13.1 \pm 4.5 \pm 2.4$ | $77.6 \pm 9.9 \pm 23.1 \pm 4.5$ |
| $f_{2}(1270)$ | $158.1 \pm 3.0 \pm 1.6 \pm 3.8$ | $147.8 \pm 2.5 \pm 8.5 \pm 2.6$ |
| $\bar{D}^{0} \pi^{-}$P-wave | $266.7 \pm 3.7 \pm 0.3 \pm 7.1$ | $261.0 \pm 4.0 \pm 3.3 \pm 6.7$ |
| $D_{0}^{*}(2400)^{-}$ | $83.6 \pm 4.4 \pm 2.8 \pm 4.6$ | $78.4 \pm 4.1 \pm 11.5 \pm 1.7$ |
| $D_{2}^{*}(2460)^{-}$ | $262.9 \pm 2.9 \pm 0.8 \pm 3.0$ | $257.4 \pm 3.4 \pm 0.7 \pm 1.9$ |
| $D_{3}^{*}(2760)^{-}$ | $91.1 \pm 6.7 \pm 1.4 \pm 5.1$ | $92.7 \pm 7.3 \pm 15.2 \pm 2.3$ |

Table 11: Measured branching fractions of $\mathcal{B}\left(B^{0} \rightarrow r h_{3}\right) \times \mathcal{B}\left(r \rightarrow h_{1} h_{2}\right)$ for the Isobar and K-matrix models. The first uncertainty is statistical, the second the experimental systematic, the
 third the model-dependent systematic, and the fourth the uncertainty from the normalisation $B^{0} \rightarrow D^{*}(2010)^{-} \pi^{+}$channel.

| Resonance | Isobar $\left(\times 10^{-5}\right)$ | K-matrix $\left(\times 10^{-5}\right)$ |  |
| :--- | :---: | :---: | :---: |
| $f_{0}(500)$ | $11.2 \pm 0.8 \pm 0.5 \pm 2.1 \pm 0.5$ | $\mathrm{n} / \mathrm{a}$ |  |
| $f_{0}(980)$ | $1.34 \pm 0.25 \pm 0.10 \pm 0.46 \pm 0.06$ | $\mathrm{n} / \mathrm{a}$ |  |
| $f_{0}(2020)$ | $1.35 \pm 0.31 \pm 0.14 \pm 0.85 \pm 0.06$ | $\mathrm{n} / \mathrm{a}$ |  |
| S-wave | $14.1 \pm 0.5 \pm 0.6 \pm 1.3 \pm 0.7$ | $14.2 \pm 0.6 \pm$ | $1.5 \pm 0.9 \pm 0.7$ |
| $\rho(770)$ | $32.1 \pm 1.0 \pm 1.2 \pm 0.9 \pm 1.5$ | $31.0 \pm 1.0 \pm 2.1 \pm 0.7 \pm 1.5$ |  |
| $\omega(782)$ | $0.42 \pm 0.11 \pm 0.02 \pm 0.03 \pm 0.02$ | $0.43 \pm 0.11 \pm 0.02 \pm 0.02 \pm 0.02$ |  |
| $\rho(1450)$ | $1.36 \pm 0.28 \pm 0.08 \pm 0.19 \pm 0.06$ | $1.91 \pm 0.37 \pm 0.73 \pm 0.19 \pm 0.09$ |  |
| $\rho(1700)$ | $0.33 \pm 0.11 \pm 0.06 \pm 0.05 \pm 0.02$ | $0.73 \pm 0.18 \pm 0.53 \pm 0.10 \pm 0.03$ |  |
| $f_{2}(1270)$ | $9.5 \pm 0.5 \pm 0.4 \pm 1.0 \pm 0.4$ | $9.1 \pm 0.6 \pm 0.8 \pm 0.5 \pm 0.4$ |  |
| $D_{0}^{*}(2400)^{-}$ | $7.7 \pm 0.5 \pm 0.3 \pm 0.3 \pm 0.4$ | $8.0 \pm 0.5 \pm 0.8 \pm$ | $0.4 \pm$ |
| $D_{2}^{*}(2460)^{-}$ | $24.4 \pm 0.7 \pm 1.0 \pm 0.4 \pm 1.2$ | $23.8 \pm 0.7 \pm 1.2 \pm 0.5 \pm 1.1$ |  |
| $D_{3}^{*}(2760)^{-}$ | $1.03 \pm 0.16 \pm 0.07 \pm 0.08 \pm 0.05$ | $1.34 \pm 0.19 \pm 0.16 \pm 0.06 \pm 0.06$ |  |

$$
\begin{aligned}
& \mathcal{B}\left(B^{0} \rightarrow \bar{D}^{0} \pi^{+} \pi^{-}\right)=(8.46 \pm 0.14 \pm 0.29 \pm 0.40) \times 10^{-4} \\
& \mathcal{B}\left(B^{0} \rightarrow \bar{D}^{0} \omega(782)\right)=\left(2.75 \pm 0.72 \pm 0.13 \pm 0.20 \pm 0.13_{-0.23}^{+0.20}\right) \times 10^{-4} \\
& \mathcal{B}\left(B^{0} \rightarrow \bar{D}^{0} f_{2}(1270)\right)=\left(16.8 \pm 1.1 \pm 0.7 \pm 1.8 \pm 0.7_{-0.2}^{+0.5}\right) \times 10^{-5}
\end{aligned}
$$



The LASS lineshape [51] has been developed to combine these two contributions,

$$
\begin{equation*}
R(m)=\frac{m}{q \cot \delta_{B}-i q}+\exp \left[2 i \delta_{B}\right] \frac{m_{0} \Gamma_{0} \frac{m_{0}}{q_{0}}}{\left(m_{0}^{2}-m^{2}\right)-i m_{0} \Gamma_{0} \frac{q}{m} \frac{m_{0}}{q_{0}}} \tag{8}
\end{equation*}
$$

where $\cot \delta_{B}=\frac{1}{a q}+\frac{1}{2} r q$,
and where $m_{0}$ and $\Gamma_{0}$ are the pole mass and width of the $\bar{K}_{0}^{*}(1430)$ state, and $a$ and $r$ are shape parameters.

## [51] J. Phys. G36 (2009) 075003

The $D \pi$ S-wave nonresonant contribution can be described by the "dabba" lineshape [52], defined as

$$
\begin{equation*}
R(m)=\frac{B^{\prime}\left(m^{2}\right)\left(m^{2}-s_{A}\right) \rho}{1-\beta\left(m^{2}-m_{\min }^{2}\right)-i B^{\prime}\left(m^{2}\right)\left(m^{2}-s_{A}\right) \rho} \tag{10}
\end{equation*}
$$

where

$$
\begin{equation*}
B^{\prime}\left(m^{2}\right)=b \exp \left[-\alpha\left(m^{2}-m_{\min }^{2}\right)\right] . \tag{11}
\end{equation*}
$$

Here $m_{\min }$ is the invariant mass at threshold, $s_{A}=m_{D}^{2}-0.5 m_{\pi}^{2}$ is the Adler zero, $\rho$ is a phase-space factor and $b, \alpha$ and $\beta$ are parameters with values fixed to $24.49 \mathrm{GeV}^{-2}$, $0.1 \mathrm{GeV}^{-2}$ and $0.1 \mathrm{GeV}^{-2}$, respectively, according to Ref. [52].
[52] Nucl. Phys. B296 (1988) 493

$$
B^{0} \rightarrow \bar{D}^{0} K^{+} \pi^{-}
$$



Table 7: Results for the complex amplitudes and their uncertainties presented (top) in terms of real and imaginary parts and (bottom) in terms and magnitudes and phases. The three quoted errors are statistical, experimental systematic and model uncertainties, respectively.

| Resonance | Real part | Imaginary part |
| :--- | ---: | ---: |
| $K^{*}(892)^{0}$ | $-0.00 \pm 0.15 \pm 0.24 \pm 0.34$ | $-1.27 \pm 0.06 \pm 0.03 \pm 0.06$ |
| $K^{*}(1410)^{0}$ | $0.15 \pm 0.06 \pm 0.04 \pm 0.09$ | $-0.09 \pm 0.09 \pm 0.18 \pm 0.18$ |
| $K_{0}^{*}(1430)^{0}$ | $0.14 \pm 0.38 \pm 0.48 \pm 0.38$ | $0.45 \pm 0.15 \pm 0.37 \pm 0.17$ |
| LASS nonresonant | $-0.10 \pm 0.24 \pm 0.16 \pm 0.42$ | $0.44 \pm 0.14 \pm 0.17 \pm 0.23$ |
| $K_{2}^{*}(1430)^{0}$ | $-0.32 \pm 0.09 \pm 0.15 \pm 0.23$ | $-0.47 \pm 0.07 \pm 0.14 \pm 0.15$ |
| $D_{0}^{*}(2400)^{-}$ | $-0.80 \pm 0.08 \pm 0.07 \pm 0.22$ | $-0.44 \pm 0.14 \pm 0.12 \pm 0.18$ |
| $D_{2}^{*}(2460)^{-}$ | 1.00 | 0.00 |
| $D \pi$ S-wave (dabba) | $-0.39 \pm 0.09 \pm 0.09 \pm 0.14$ | $0.36 \pm 0.17 \pm 0.14 \pm 0.23$ |
| $D \pi$ P-wave (EFF) | $-0.62 \pm 0.06 \pm 0.03 \pm 0.11$ | $-0.03 \pm 0.06 \pm 0.05 \pm 0.10$ |
| Resonance | Magnitude | Phase |
| $K^{*}(892)^{0}$ | $1.27 \pm 0.06 \pm 0.03 \pm 0.05$ | $-1.57 \pm 0.11 \pm 0.16 \pm 0.27$ |
| $K^{*}(1410)^{0}$ | $0.18 \pm 0.07 \pm 0.10 \pm 0.11$ | $-0.54 \pm 0.21 \pm 0.55 \pm 1.04$ |
| $K_{0}^{*}(1430)^{0}$ | $0.47 \pm 0.09 \pm 0.10 \pm 0.14$ | $1.27 \pm 0.95 \pm 1.04 \pm 0.81$ |
| LASS nonresonant | $0.46 \pm 0.14 \pm 0.16 \pm 0.29$ | $1.79 \pm 0.65 \pm 0.35 \pm 0.69$ |
| $K_{2}^{*}(1430)^{0}$ | $0.57 \pm 0.05 \pm 0.04 \pm 0.08$ | $-2.16 \pm 0.19 \pm 0.43 \pm 0.43$ |
| $D_{0}^{*}(2400)^{-}$ | $0.91 \pm 0.07 \pm 0.06 \pm 0.17$ | $-2.64 \pm 0.15 \pm 0.14 \pm 0.23$ |
| $D_{2}^{*}(2460)^{-}$ | 1.00 | 0.00 |
| $D \pi$ S-wave (dabba) | $0.53 \pm 0.07 \pm 0.04 \pm 0.14$ | $2.40 \pm 0.27 \pm 0.24 \pm 0.44$ |
| $D \pi$ P-wave (EFF) | $0.62 \pm 0.06 \pm 0.04 \pm 0.11$ | $-3.09 \pm 0.10 \pm 0.07 \pm 0.17$ |

Table 9: Results for the product branching fractions. The four quoted errors are statistical, experimental systematic, model and PDG uncertainties, respectively. Upper limits are given at $90 \%$ (95\%) confidence level.

| Resonance | Product branching fraction $\left(10^{-5}\right)$ | Upper limit $\left(10^{-5}\right)$ |
| :--- | :--- | :---: |
| $K^{*}(892)^{0}$ | $3.42 \pm 0.13 \pm 0.10 \pm 0.16 \pm 0.40$ |  |
| $K^{*}(1410)^{0}$ | $0.07 \pm 0.03 \pm 0.08 \pm 0.07 \pm 0.01$ | $<0.29(0.34)$ |
| $K_{0}^{*}(1430)^{0}$ | $0.47 \pm 0.18 \pm 0.22 \pm 0.31 \pm 0.05$ |  |
| LASS nonresonant | $0.44 \pm 0.34 \pm 0.34 \pm 0.61 \pm 0.05$ |  |
| $\quad$ LASS total | $0.61 \pm 0.25 \pm 0.25 \pm 0.49 \pm 0.07$ |  |
| $K_{2}^{*}(1430)^{0}$ | $0.68 \pm 0.15 \pm 0.10 \pm 0.18 \pm 0.08$ |  |
| $D_{0}^{*}(2400)^{-}$ | $1.77 \pm 0.26 \pm 0.19 \pm 0.67 \pm 0.20$ |  |
| $D_{2}^{*}(2460)^{-}$ | $2.12 \pm 0.10 \pm 0.11 \pm 0.11 \pm 0.25$ |  |
| $D_{3}^{*}(2760)^{-}$ |  |  |
| $D \pi$ S-wave (dabba) | $0.60 \pm 0.13 \pm 0.11 \pm 0.34 \pm 0.07$ |  |
| $D \pi$ P-wave (EFF) | $0.81 \pm 0.15 \pm 0.20 \pm 0.27 \pm 0.09$ |  |

Table 10: Results for the branching fractions. The four quoted errors are statistical, experimental systematic, model and PDG uncertainties, respectively. Upper limits are given at $90 \%$ ( $95 \%$ ) confidence level.

| Resonance | Branching fraction $\left(10^{-5}\right)$ | Upper limit $\left(10^{-5}\right)$ |
| :--- | :---: | :---: |
| $K^{*}(892)^{0}$ | $5.13 \pm 0.20 \pm 0.15 \pm 0.24 \pm 0.60$ |  |
| $K^{*}(1410)^{0}$ | $1.59 \pm 0.68 \pm 1.81 \pm 1.59 \pm 0.36$ | $<6.7(7.8)$ |
| $K_{0}^{*}(1430)^{0}$ | $0.71 \pm 0.27 \pm 0.33 \pm 0.47 \pm 0.08$ |  |
| LASS nonresonant | $0.66 \pm 0.51 \pm 0.51 \pm 0.92 \pm 0.08$ |  |
| $\quad$ LASS total | $0.92 \pm 0.38 \pm 0.38 \pm 0.74 \pm 0.11$ |  |
| $K_{2}^{*}(1430)^{0}$ | $2.04 \pm 0.45 \pm 0.30 \pm 0.54 \pm 0.25$ |  |

