## Theoretical Aspects Quarkonium Production in Vacuum

Geoffrey Bodwin (ANL)

- Brief Review of NRQCD Factorization
- Status of a Proof of NRQCD Factorization
- Why is large  $p_T$  important?
- What are LP and NLP factorization (fragmentation)?
- What happens in Hadroproduction at LO in  $\alpha_s$ ?
- What happens in Hadroproduction at NLO in  $\alpha_s$ ?
- Why are there so many different NLO predictions?
- What happens to  $J/\psi$  polarization in NLO?
- What do we expect beyond NLO in  $\alpha_s$ ?
- How do we go beyond NLO in  $\alpha_s$ ?
- Outstanding Problems
- Conclusions and Outlook

## **Brief Review of NRQCD Factorization**

• NRQCD Factorization Conjecture (Bodwin, Braaten, Lepage (1995)): The inclusive cross section for producing a quarkonium at large momentum transfer  $(p_T)$  can be written as

$$\sigma(H) = \sum_{n} F_n(\Lambda) \langle 0 | \mathcal{O}_n^H(\Lambda) | 0 \rangle.$$

- The  $F_n(\Lambda)$  are the "short-distance" coefficients (SDCs).
  - The SDCs are essentially the partonic cross sections to make a  $Q\bar{Q}$  pair convolved with the parton distributions.
- The  $\langle 0|\mathcal{O}_n^H(\Lambda)|0\rangle$  are the NRQCD long-distance matrix elements (LDMEs).
  - The LDMEs are the probability for a  $Q\bar{Q}$  pair to evolve into a heavy quarkonium.

- The SDCs depend on the production process. They can be calculated in QCD perturbation theory.
- The LDMEs are nonperturbative, but they are conjectured to be universal (process independent).
- The LDMEs have a known scaling with the heavy-quark velocity v.
  - $v^2 \approx 0.23$  for the  $J/\psi$ .  $v^2 \approx 0.1$  for the  $\Upsilon(1S)$ .
  - The sum in the factorization formula is a v expansion.
- In phenomenology, the *v* expansion in the factorization formula is truncated at a particular order in *v*.
- A key feature of NRQCD factorization: Quarkonium production can occur through color-octet, as well as color-singlet,  $Q\bar{Q}$  states.
  - The color-singlet production LDMEs are simply related to color-singlet decay LDMEs.
  - The color-octet LDMEs must be determined from fits to measured production cross sections.
- If we drop all of the color-octet contributions and retain only the leading colorsinglet contribution, then we have the color-singlet model (CSM).

• The current phenomenology of production of *S*-wave quarkonia  $(J/\psi, \psi(2S), \text{ and } \Upsilon(nS))$  makes use of LDMEs through relative order  $v^4$ :

$$\begin{array}{ll} \langle \mathcal{O}^{H}(^{3}S_{1}^{[1]}) \rangle & (O(v^{0})), \\ \langle \mathcal{O}^{H}(^{1}S_{0}^{[8]}) \rangle & (O(v^{3})), \\ \langle \mathcal{O}^{H}(^{3}S_{1}^{[8]}) \rangle & (O(v^{4})), \\ \langle \mathcal{O}^{H}(^{3}P_{J}^{[8]}) \rangle & (O(v^{4})). \end{array}$$

- Calculations show that the  ${}^3S_1^{[1]}$  contributions are negligible for  $J/\psi$  hadroproduction.
- The  $\langle \mathcal{O}^H({}^3P_J^{[8]}) \rangle$  (J = 0, 1, 2) are related by the heavy-quark spin symmetry.
- Three color-octet LDMEs need to be determined phenomenologically for each state.

## Status of a Proof of NRQCD Factorization

- Nayak, Qiu, Sterman (2005, 2006): Factorization holds through NNLO, up to corrections of relative order  $m_Q^2/p_T^2$ .
- It is not known if this result generalizes to higher orders in  $\alpha_s$ .
- An all-orders proof is essential because soft gluons can violate factorization, and the  $\alpha_s$  that is associated with soft gluons is not small.
- In the absence of further theoretical progress, we must rely on experiment to prove or to disprove NRQCD factorization.

# Why is large $p_T$ important?

- Existing proofs of factorization for light hadrons all require  $p_T$  significantly greater than the hadron masses.
  - Power corrections  $\propto (m_H^2/p_T^2)^n$  get out of control when  $p_T \sim m_H$ .
  - There are known violations of factorization at order  $m_H^4/p_T^4$ .
- Phenomenologically, Drell-Yan factorization doesn't work until  $p_T \geq 3m_H$ .
- Suggests that we should require  $p_T \ge 3m_{\text{quarkonium.}}$

# What are LP and NLP factorization (fragmentation)?

• Leading power (LP) fragmentation: (Collins, Soper (1982))

 $- \, d\sigma/dp_T^2 \sim 1/p_T^4.$ 

 $d\sigma[a+b \rightarrow \text{quarkonium} + X] \sim \underbrace{d\sigma[a+b \rightarrow c+X]}_{\text{hard scattering}} \otimes \underbrace{D[c \rightarrow \text{quarkonium} + X]}_{\text{fragmentation fn.}}.$ 

• Next-to-leading power (NLP) fragmentation:

(Kang, Qiu, and Sterman (2011); Fleming, Leibovich, Mehen, Rothstein (2012)) -  $d\sigma/dp_T^2 \sim m_Q^2/p_T^6$ .

 $d\sigma[a+b \rightarrow \text{quarkonium} + X] \sim \underbrace{d\sigma[a+b \rightarrow Q\bar{Q} + X]}_{\text{hard scattering}} \otimes \underbrace{D[Q\bar{Q} \rightarrow \text{quarkonium} + X]}_{\text{fragmentation fn.}}.$ 

- Believed to hold to all orders in perturbation theory up to corrections of order  $m_Q^4/p_T^8$ .

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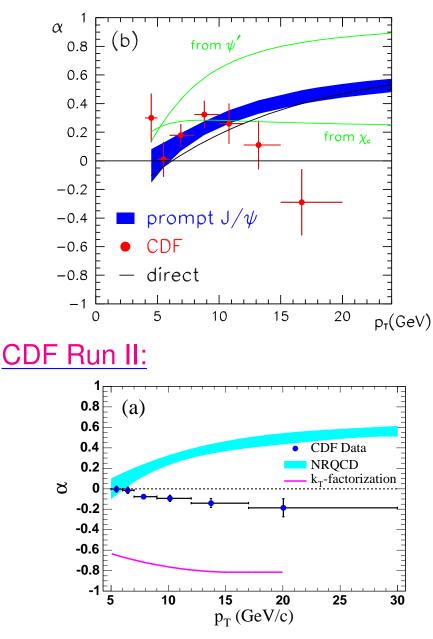
- Not very predictive by itself because the nonperturbative fragmentation functions are unknown.
- If NRQCD factorization holds, then the fragmentation functions can be written as a sum of NRQCD LDMEs times perturbatively calculable short-distance coefficients.
  - Then, the fragmentation approach provides powerful a way to identify and compute the contributions (LP and NLP) that are most important at high  $p_T$ .
  - Much simpler than a full perturbative calculation at any given order in  $\alpha_s$ .
  - Also provides a framework in which to resum logs of  $p_T^2/m_Q^2$ .

# What happens in Hadroproduction at LO in $\alpha_s$ ?

- Only the  ${}^{3}S_{1}^{[8]}$  contribution has LP behavior in LO.
  - Dominates at large  $p_T$ .
- The  ${}^{3}S_{1}^{[8]}$  contribution is transversely polarized.
- LO NRQCD factorization predicts large transverse polarization at large  $p_T$ .
- This prediction is not borne out by the data.

### $J/\psi$ Polarization in LO

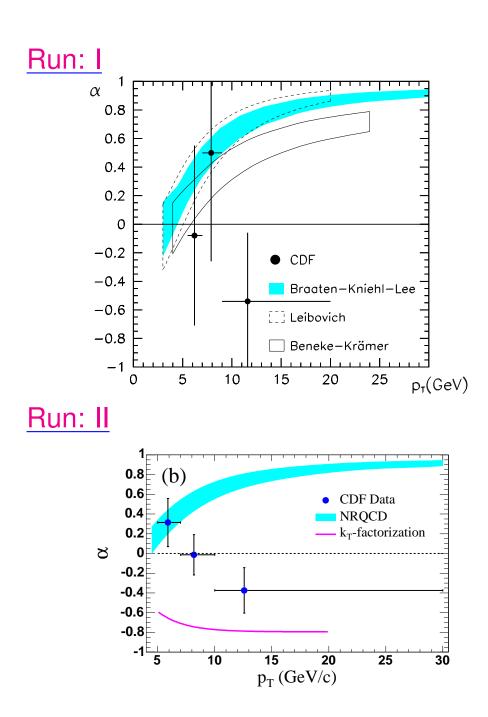
### CDF Run I:



•  $\alpha \equiv \lambda_{\theta} = \begin{cases} 1 \text{ transverse} \\ 0 \text{ unpolarized} \\ -1 \text{ longitudinal} \end{cases}$ 

- LO NRQCD prediction (Braaten, Kniehl, Lee (1999)).
- Run I results are marginally compatible with the LO NRQCD prediction.
- The Run II results are incompatible with the LO NRQCD prediction.
- The Run I and Run II results are inconsistent with each other.

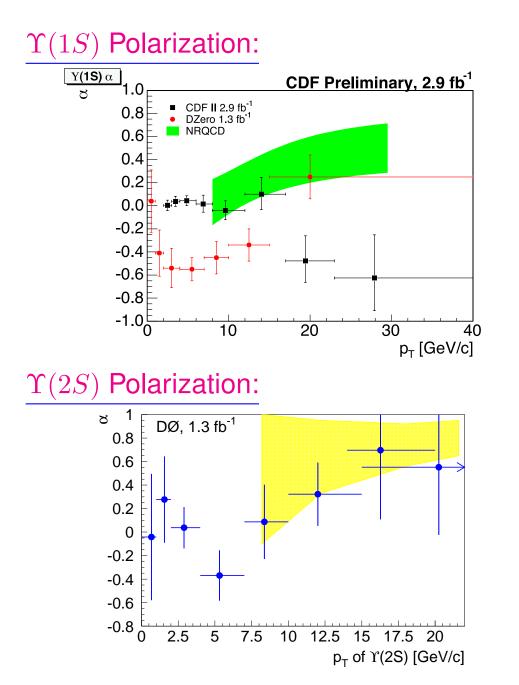
### $\psi(2S)$ Polarization in LO



- The error bars on the Run I data are too large to make a stringent test.
- The Run II data are incompatible with the LO NRQCD prediction.

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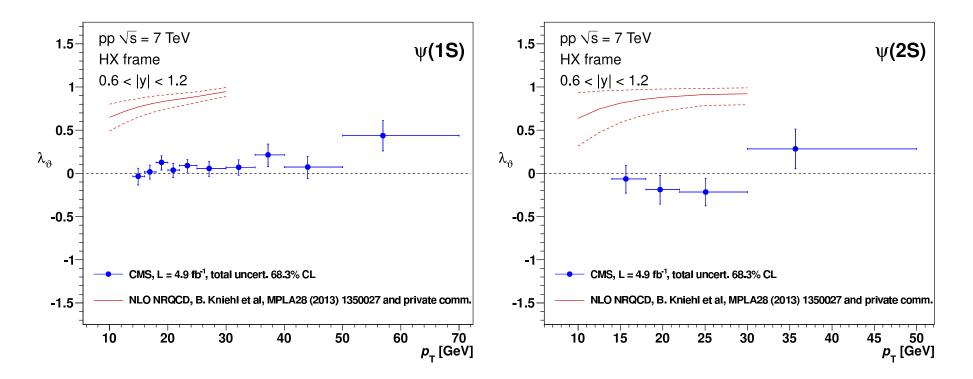
### <u> Υ Polarization in LO</u>



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- In the  $\Upsilon(1S)$  case, the D0 results (red) are incompatible with the CDF results (black).
- Both the CDF and D0 results are incompatible with the LO NRQCD prediction (green) (Braaten and Lee (2000)), but in different regions of  $p_T$ .
- In the  $\Upsilon(2S)$  case, the theoretical and experimental error bars are too large to make a stringent test.

- It was thought that these discrepancies between theory and experiment might not be definitive because
  - there are inconsistencies in the experimental data,
  - $-p_T$  might not be high enough for factorization to work.
- These ideas were laid to rest by the CMS (2013) polarization measurement.



# What happens in Hadroproduction at NLO in $\alpha_s$ ?

- There is a large k factor  $\sim -10$  in the  $^3P_J^{[8]}$  channel.
- On the other hand, NLO corrections to the  ${}^{3}S_{1}^{[8]}$  and  ${}^{1}S_{0}^{[8]}$  channels are small.

### Explanation

- Enhancements at high  $p_T$  from LP behavior can overcome a power of  $\alpha_s$ .
- The  ${}^{3}P_{J}^{[8]}$  channel receives a large (negative) correction in NLO because it first shows LP behavior in NLO (gluon fragmentation).
- The  ${}^{3}S_{1}^{[8]}$  channel receives a small correction in NLO because it already has LP behavior in LO (gluon fragmentation).
- The  ${}^{1}S_{0}^{[8]}$  channel first shows LP behavior in NLO (gluon fragmentation). But the NLO correction happens to be small (no IR enhancement).

## Why are there so many different NLO predictions?

- Three groups have carried out complete NLO calculations.
  - PKU group (Kuang-Ta Chao's group): Ma, Wang, Chao
  - Hamburg group (Bernd Kniehl's group): Butenschön, Kniehl
  - IHEP group (Jianxiong Wang's group): Gong, Wan, Wang, Zhang
- All three groups agree on the SDCs for hadroproduction.
- However, they extract very different NRQCD LDMEs and make different predictions because of different assumptions about the data used in the fits.
- The PKU group (2010) fits the CDF  $J/\psi$  data for  $p_T > 7$  GeV. They were able to determine only 2 linear combinations of LDMEs unambiguously:

$$\begin{split} M_{0,r_0} &= \langle O^{J/\psi} \begin{pmatrix} {}^1S_0^{[8]} \end{pmatrix} \rangle + (r_0/m_c^2) \langle O^{J/\psi} \begin{pmatrix} {}^3P_0^{[8]} \end{pmatrix} \rangle = (7.4 \pm 1.9) \times 10^{-2} \, \text{GeV}^3, \\ M_{1,r_1} &= \langle O^{J/\psi} \begin{pmatrix} {}^3S_1^{[8]} \end{pmatrix} \rangle + (r_1/m_c^2) \langle O^{J/\psi} \begin{pmatrix} {}^3P_0^{[8]} \end{pmatrix} \rangle = (0.05 \pm 0.02) \times 10^{-2} \, \text{GeV}^3. \\ r_0 &= 3.9 \text{ and } r_1 = -0.56. \end{split}$$

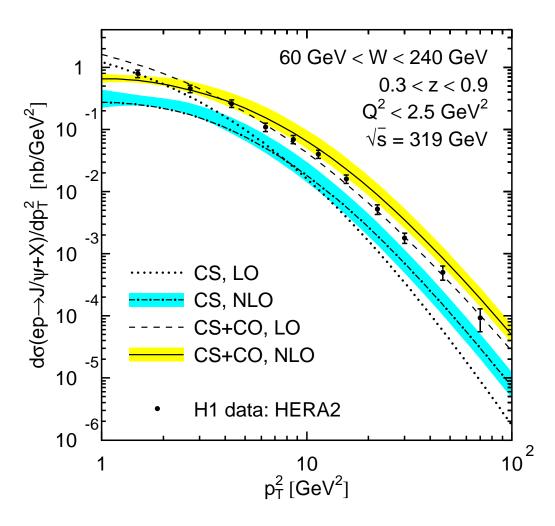
- The Hamburg group (2011) determined all 3 color-octet LDMEs by making a global fit to data with  $p_T > 3$  GeV from the Tevatron, LHC, RHIC, HERA, LEP II, KEKB.
  - They made use of their computations of NLO corrections to  $p\bar{p}$ , pp, ep,  $\gamma\gamma$ , and  $e^+e^-$  production.
  - Their LDMEs are very different from those of the PKU group:

 $M_{0,r_0} = (2.17 \pm 0.56) \times 10^{-2} \text{ GeV}^3,$  $M_{1,r_1} = (0.62 \pm 0.08) \times 10^{-2} \text{ GeV}^3.$ 

- The IHEP group (2012) fit the CDF  $J/\psi$ ,  $\psi(2S)$ , and  $\chi_{cJ}$  data for  $p_T > 7$  GeV.
  - They included NLO feeddown contributions from  $\psi(2S)$  and  $\chi_{cJ}$  in their fit.
  - They were able to determine all 3 color-octet LDMEs.
  - They obtained a quality of fit and a result for the LDME linear combinations that is similar to that of the PKU group:

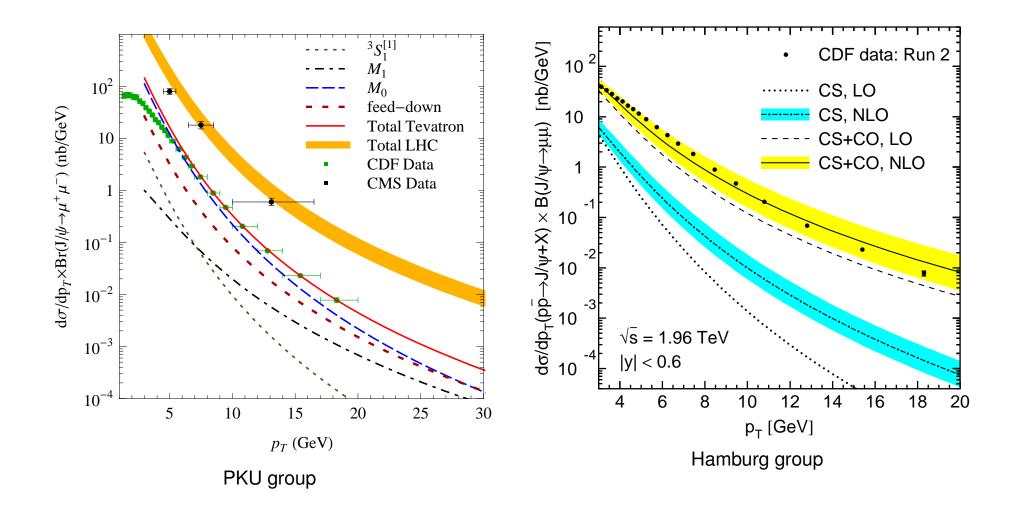
$$M_{0,r_0} = (6.00 \pm 0.98) \times 10^{-2} \text{ GeV}^3,$$
  
$$M_{1,r_1} = (0.07 \pm 0.02) \times 10^{-2} \text{ GeV}^3.$$

• Most of the difference between the Hamburg-group fit and the others comes from the use of HERA (H1 (2002, 2005)) data.



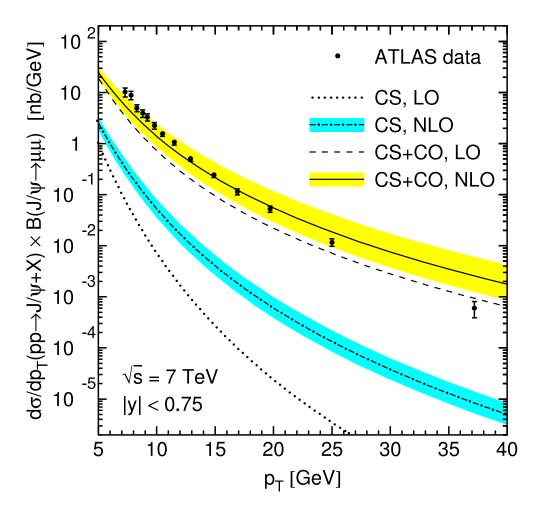
- The HERA data lie at  $p_T \lesssim 8$  GeV.
- Does NRQCD factorization hold at such low values of  $p_T$ ?

- Although the Hamburg-group fits agree with the data, within uncertainties, there are tensions in the shapes.
- The shape of the PKU-group fit agrees with the CDF data better than the shape of the Hamburg-group (2011) global fit.



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• The shape discrepancy between the Hamburg-group prediction and the data becomes more apparent at high  $p_T$ .



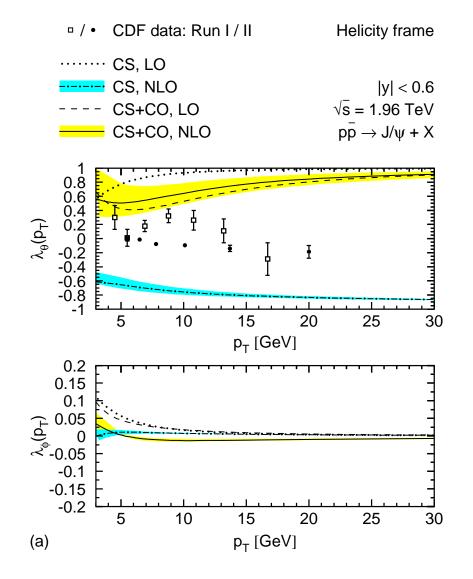
- ATLAS (2011) data.
- Not included in the Hamburg-group global fit.

• All of this suggests that NRQCD factorization may not work until  $p_T$  is much greater than the quarkonium mass.

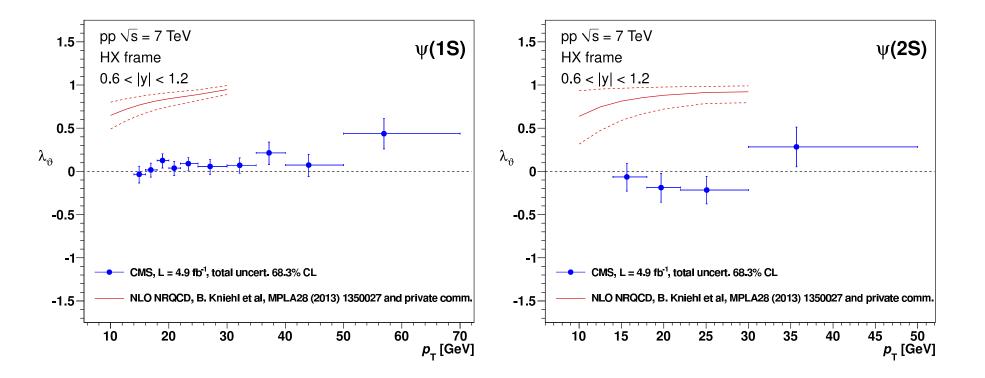
# What happens to $J/\psi$ polarization in NLO?

- At high  $p_T$ , the  ${}^{3}P_J^{[8]}$  channel is mostly transversely polarized—just like the  ${}^{3}S_1^{[8]}$  channel.
- Large NLO corrections to the  ${}^{3}P_{J}^{[8]}$  channel give it the same shape as the  ${}^{3}S_{1}^{[8]}$  channel at high  $p_{T}$ .
- The contribution from the  ${}^{3}P_{J}^{[8]}$  channel could cancel the contribution from the  ${}^{3}S_{1}^{[8]}$  channel.
- The resulting  ${}^{1}S_{0}^{[8]}$  dominance would result in near-zero polarization.

#### Hamburg group (2011): NLO Prediction for $J/\psi$ Polarization



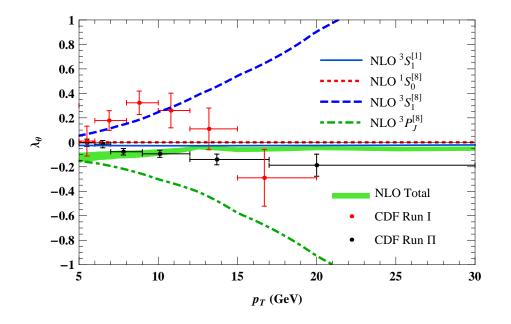
- Uses the Hamburg-group (2011) global fit of the  $J/\psi$  production cross sections.
- The contributions of the  ${}^{3}P_{J}^{[8]}$  and  ${}^{3}S_{1}^{[8]}$  channels add to produce substantial polarization at high  $p_{T}$ .
- The prediction is in disagreement with the CDF data.



• The prediction based on the Hamburg-group (2011) global fit is also in disagreement with the CMS data.

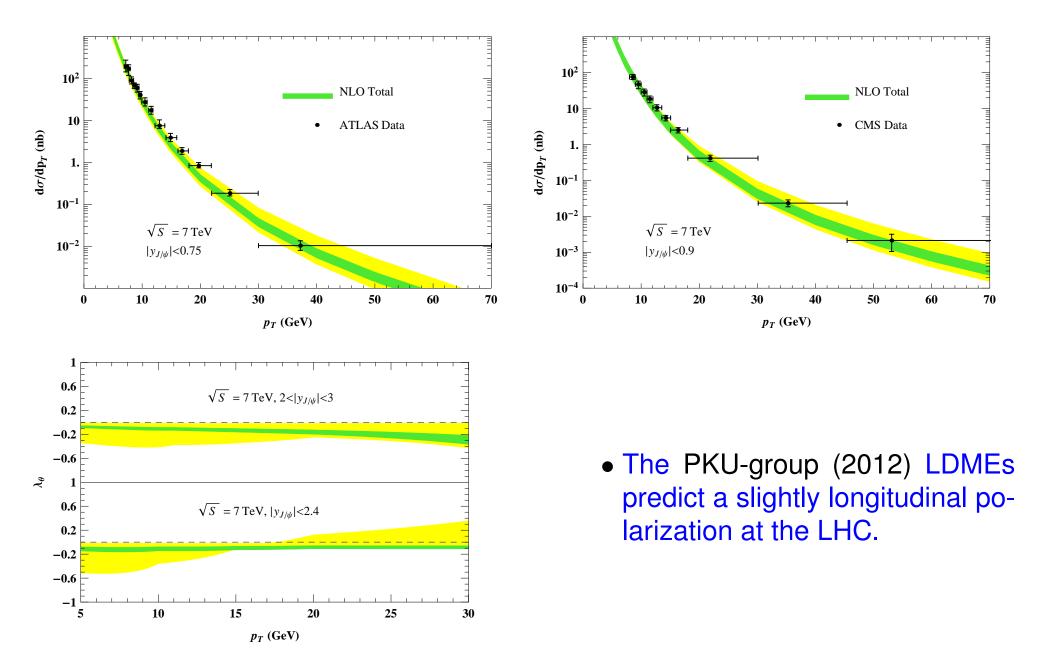
#### Chao, Ma, Shao, Wang, Zhang (PKU Group)(2012): Fit to $J/\psi$ Polarization in NLO

- Two LDME combinations are insufficient to predict the polarization.
- Fix all three LDMEs by including the CDF Run II  $J/\psi$  polarization measurement in the fit, as well as the CDF Run II measurements of  $d\sigma/dp_T$ .



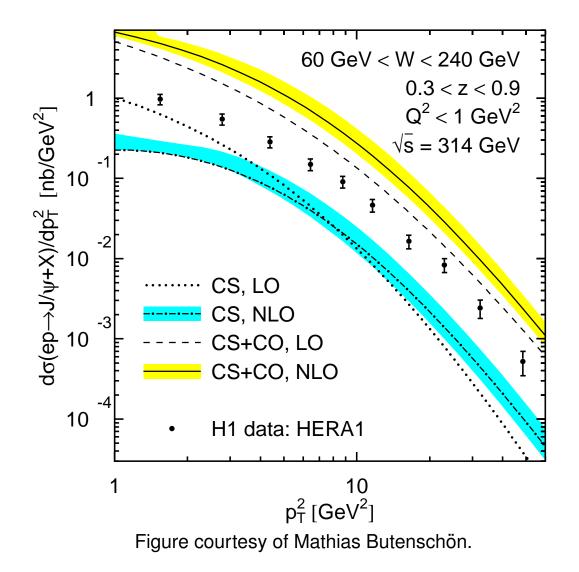
- The  ${}^3S_1^{[8]}$  and  ${}^3P_J^{[8]}$  contributions largely cancel.
- ${}^{1}S_{0}^{[8]}$  dominance  $\Rightarrow$  near-zero polarization.

• The PKU-group (2012) LDMEs still give reasonable predictions for the LHC  $p_T$  spectra.



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• However, the PKU-group LDMEs seem to be incompatible with the HERA data, even at  $p_T \approx 8$  GeV.



• Is higher  $p_T$  needed in order suppress non-factorizing contributions?

### IHEP group (2013): NLO Prediction for $J/\psi$ polarization

- Makes use of the LDMEs from a fit to the CDF (2005, 2009) and LHCb (2011, 2012)  $J/\psi$ ,  $\chi_{cJ}$ , and  $\psi(2S)$  production cross sections.
- Effects of feeddown from  $\chi_{cJ}$  and  $\psi(2S)$  states calculated and included in fits and polarization predictions.

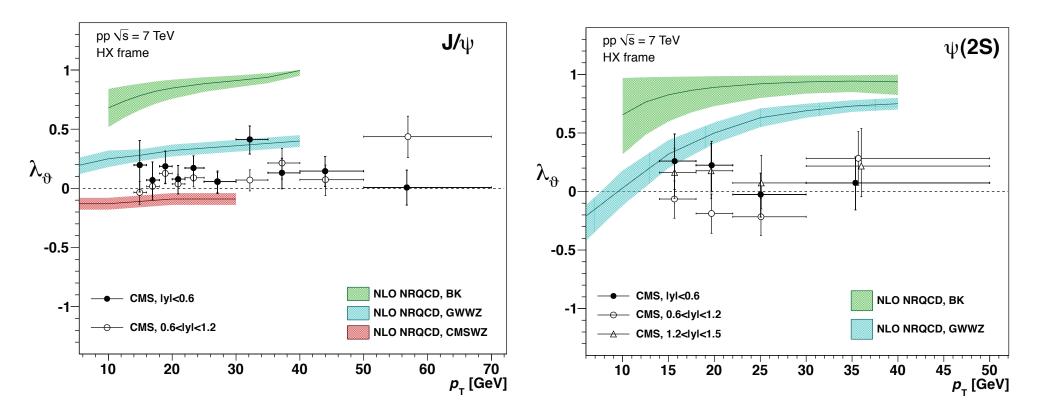
0.8 0.6 0.4 0.2 0  $\sim$ -0.2 feeddown ψ(2S -0.4Tevatron eeddown  $\chi_{cl}$ -0.6|v| < 0.6-0.8- 1 5 10 15 20 25 30 35 40 P<sub>t</sub>(GeV)

- The prediction for CDF shows less transverse polarization than that of the Hamburg group.
- Agrees with the CDF Run I data, but not with the CDF Run II data.

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 $J/\psi$  Polarization at CDF:

## $J/\psi$ Polarization at CMS:



- The IHEP-group prediction (blue band) shows less transverse polarization than the Hamburg-group prediction (green band).
- Still in disagreement with the CMS data.

#### **Conclusion about NLO Polarization**

• NLO calculations either fail to make a polarization prediction (PKU) or make predictions that disagree with the data (Hamburg, IHEP).

## What do we expect beyond NLO in $\alpha_s$ ?

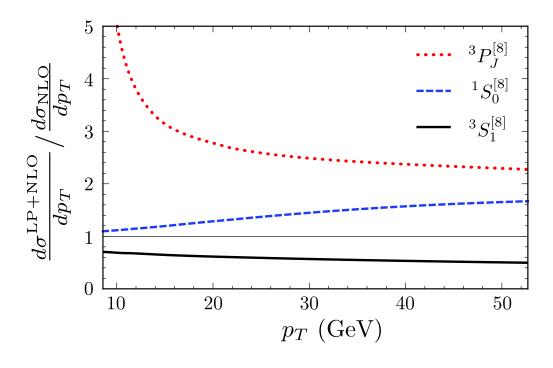
- Since all three color-octet channels have LP behavior at NLO, we expect further corrections to have "normal" *k* factors: factor of two or less.
- Exception: The LO and NLO corrections in the  ${}^3P_J$  channel have opposite signs and cancel completely at  $p_T \approx 7$  GeV.
  - NNLO corrections could be very important in this channel.

How do we go beyond NLO in  $\alpha_s$ ?

- Full NNLO calculations are probably not feasible at present.
- Use LP and NLP factorization to simplify the calculation.
- LP and NLP factorization are only valid for  $p_T$  significantly larger than the quarkonium mass.

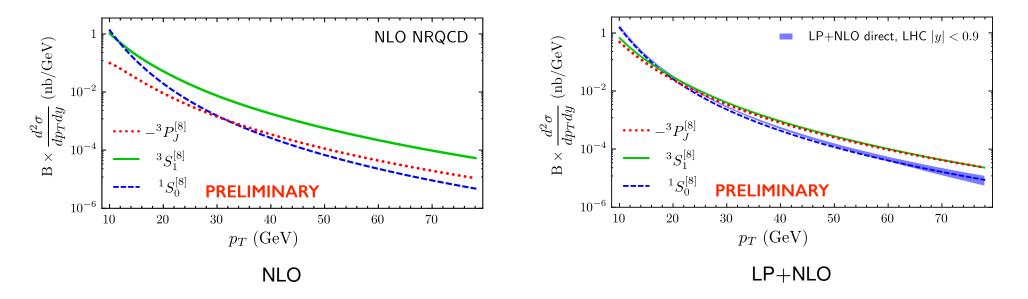
LP-Fragmentation Corrections to  $J/\psi$  Hadroproduction

- Bodwin, Chung, Kim, Lee (2014):
  - Fragmentation functions through  $\alpha_s^2$ .
  - Parton-production cross sections through NLO ( $\alpha_s^3$ ).
  - Resummation of leading logs of  $p_T^2/m_c^2$ .

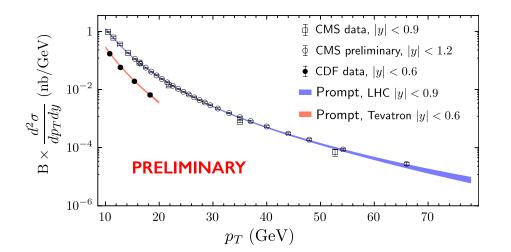


• As expected, the  ${}^{3}P_{J}^{[8]}$  channel receives large corrections.

# • The ${}^{3}P_{J}^{[8]}$ contribution now has almost the same shape as the ${}^{3}S_{1}^{[8]}$ contribution.

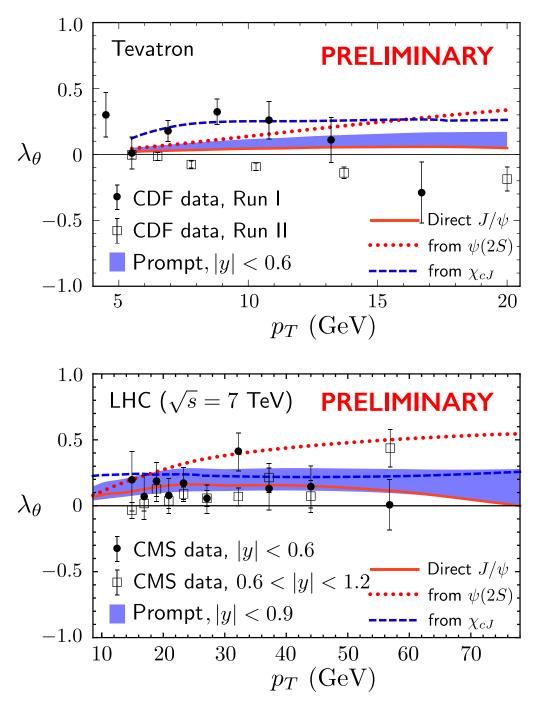


• Bodwin, Chao, Chung, Kim, Lee, Ma (in progress): Include feeddown from  $\chi_{cJ}$  and  $\psi(2S)$  states.



• Fit to the CDF and CMS hadroproduction cross sections at  $p_T > 10 \text{ GeV} \approx 3m_{J/\psi}$  fixes the LDMEs.

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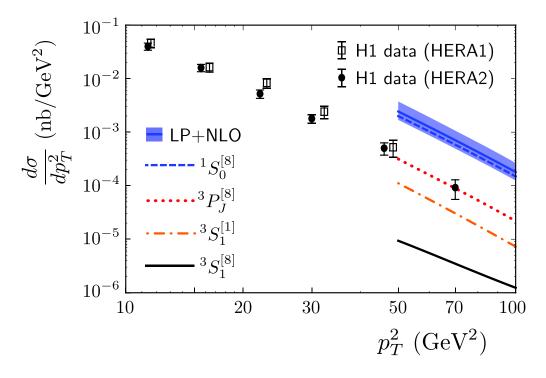


- The  ${}^3S_1^{[8]}$  and  ${}^3P_J^{[8]}$  contributions largely cancel:  ${}^1S_0^{[8]}$  dominance.
- Prediction of near-zero polarization.

# **Outstanding Problems**

 $J/\psi$  Photoproduction at HERA

• Bodwin, Chung, Kim, Lee (2015): LP-fragmentation corrections do not resolve the discrepancy between theory and experiment for photoproduction.

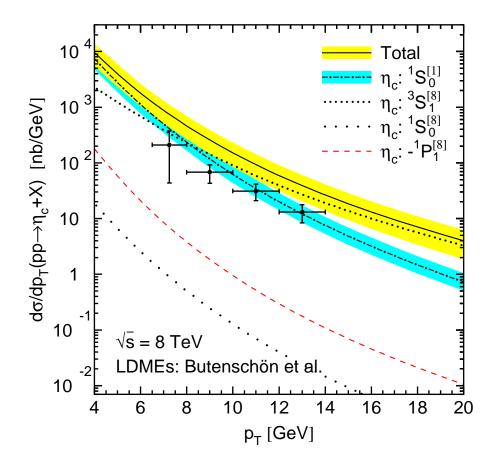


- Additional LP-fragmentation corrections beyond NLO are small.
- The p<sub>T</sub> of the highest measured point is only about 8 GeV. Maybe too small for NRQCD factorization to hold.
- But theory and data are not trending toward each other as  $p_T$  increases.

#### $\eta_c$ Production at LHCb

Butenschön and Kniehl (2014)

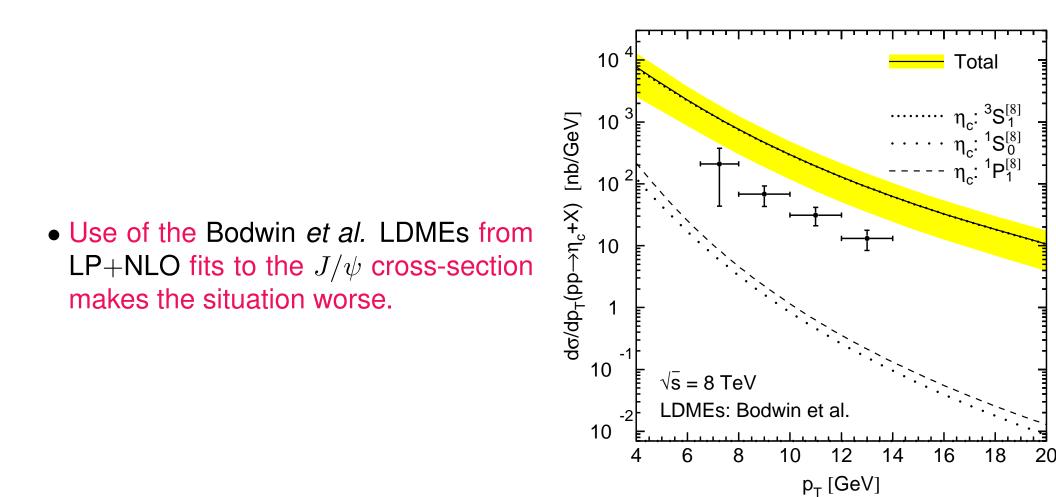
• The NLO prediction for the  $\eta_c$  cross section overshoots the LHCb (2014) measurement by a factor of about 6.



• The  $\eta_c$  LDMEs are fixed by using the heavy-quark spin symmetry of NRQCD to relate them to the  $J/\psi$  LDMEs.

Good up to corrections of relative order  $v^2$ .

• The color-singlet contribution alone accounts for the measured cross section.

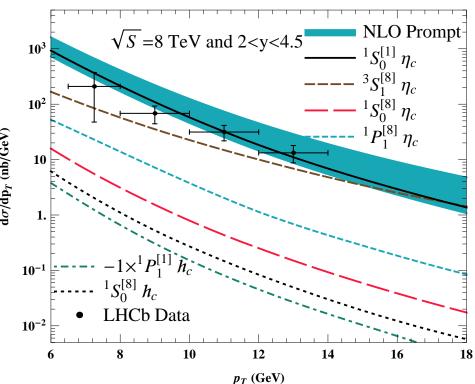


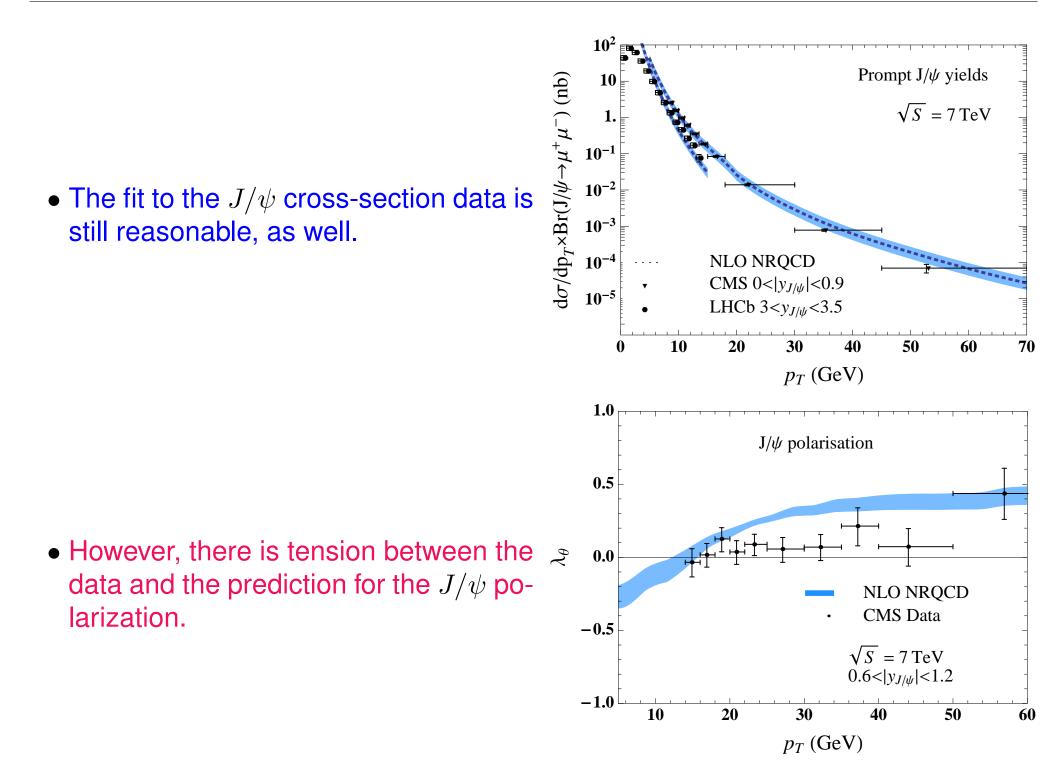
Han, Ma, Meng, Shao, Chao (PKU Group) (2014)

• Apply an additional constraint to the PKU 2010 LDME fit:

 $0 < \langle \mathcal{O}^{\eta_c}({}^3S_1^{[8]}\rangle < 0.0146 \text{ GeV}^3 \implies 0 < \langle \mathcal{O}^{J/\psi}({}^1S_0^{[8]}\rangle < 0.0146 \text{ GeV}^3$ 

• They obtain reasonable agreement with the  $\eta_c$  cross-section data.





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### A Possible Weakness in the Measurement

- LHCb measures the relative rates of the  $\eta_c$  and  $J/\psi$  in the  $p\bar{p}$  channel.
- BF( $\eta_c \to p\bar{p}$ ) is determined from a global fit to BFs that has a very poor  $\chi^2/d.o.f.$
- Direct measurements of  ${\rm BF}(\eta_c \to p\bar{p})$  have large uncertainties.
- A  $2\sigma$  deviation to the low side would boost the cross section by a factor 3.

### **Conclusions and Outlook**

- Theoretical predictions for quarkonium production have undergone a major transformation in recent years.
  LO ⇒ NLO ⇒ LP+NLO ⇒ LP+NLP+NLO ??
- Much of this transformation has been driven by high-quality collider measurements.
- The  $J/\psi$  hadroproduction cross sections and polarization are now well described by theory.
- However, important discrepancies between theory and experiment remain.
  - $J/\psi$  photoproduction
  - $\eta_c$  hadroproduction

#### What do we need from theory?

- NLO calculations for more processes
  - double-charmonium production
  - $J/\psi + Z$ ,  $J/\psi + W^{\pm}$
  - $J/\psi + \mathrm{jet}$
- LP and NLP calculations for more processes
- New ideas for additional experimental tests of theory
- A proof or disproof of NRQCD factorization

#### What do we need from experiment?

- Check of the  $\eta_c$  cross-section measurement
- Measurements at the highest accessible values of  $p_T$  in order to minimize effects of non-factorizing contributions
- Measurements of direct-production cross sections and polarizations in order avoid confusion from feeddown effects
- Measurements of  $\chi_{cJ}$  cross sections and polarizations
  - Particularly interesting because only one LDME must be determined from phenomenology.
  - Initial cross-section measurements show good agreement between theory and experiment, but uncertainties are still large.
  - The color-singlet LDME from fits to experiment agrees well with potentialmodel values:

Suggests that NRQCD factorization is working.

- Measurements of  $\Upsilon(nS)$  and  $\chi_b(nP)$  cross sections and polarizations at higher precision and higher  $p_T$  (some measurements already exist) Tests NRQCD in a new regime in which  $v^2$  is much smaller than for charmonium systems.
  - The v expansion should work much better for  $\Upsilon(nS)$  than for  $J/\psi$ .
  - Different  $v^2$  may mean different relative sizes of LDMEs.

Non-factorizing contributions may be suppressed only at  $p_T \gg m_{\Upsilon}$ .

- Measurements of additional production processes
  - double-charmonium production
  - $J/\psi + Z$ ,  $J/\psi + W^{\pm}$
  - $J/\psi$  + jet

### <u>Outlook</u>

- Thanks to the interplay between theory and experiment, we have come a long way in understanding quarkonium production.
- We are not yet at the end of the story.

# Backup Slides

• The current phenomenology of production of *P*-wave quarkonia ( $\chi_{cJ}$ ) makes use of LDMEs through relative order  $v^4$ :

$$\langle \mathcal{O}^{H}({}^{3}P_{J}^{[1]}) \rangle$$
 ( $O(v^{4})$ ),  
 $\langle \mathcal{O}^{H}({}^{3}S_{1}^{[8]}) \rangle$  ( $O(v^{4})$ ).

- The  $\langle \mathcal{O}^H({}^3P_J^{[1]}) \rangle$  (J = 0, 1, 2) are related by the heavy-quark spin symmetry. They can be determined from potential models.
- Only one LDME ( $\langle \mathcal{O}^H({}^3S_1^{[8]}) \rangle$ ) has to be determined from phenomenology.

### Why isn't the CSM a viable description of production?

- The CSM is theoretically inconsistent.
  - Uncanceled infrared divergences at leading order in v for production of P-wave states and at higher orders in v for other states.
- The CSM predictions in NLO fall well below the observed cross sections.

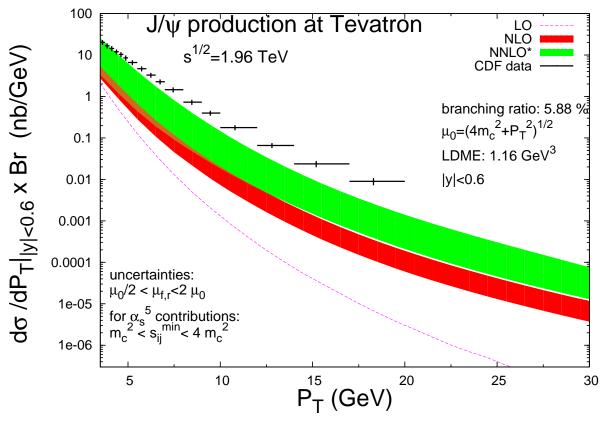
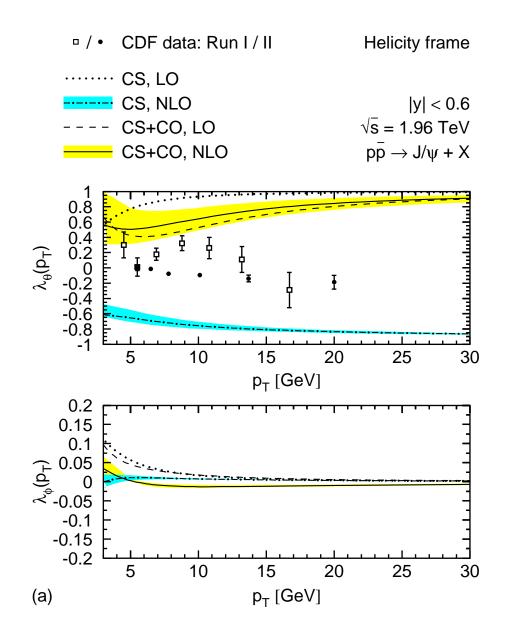


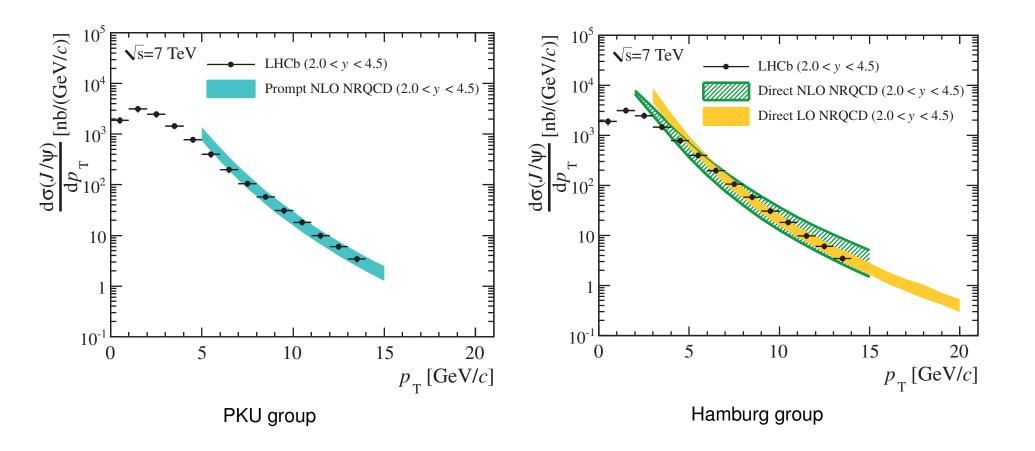
Figure courtesy of Pierre Artoisenet.

- The NNLO\* calculation is an estimate based on real-emission contributions only.
- When the virtual contributions are added, the true NNLO contribution will likely be smaller.

#### • The CSM predictions in NLO do not describe the polarization data.



• There is also a slight discrepancy in shape between the LHCb data and the Hamburg-group (2010) NLO fit to the Tevatron and HERA data.



### Why isn't the CEM viable as a description of production?

- The Color Evaporation Model (CEM) says that rate to produce a quarkonium is proportional to the rate to produce a  $Q\bar{Q}$  pair, regardless of the quantum numbers of the  $Q\bar{Q}$  pair or the quarkonium.
  - Not plausible in quantum field theory: Different  $Q\bar{Q}$  states will have different overlaps with a given quarkonium state.
- The CEM requires an *ad hoc* modification,  $k_T$  smearing, in order to describe the data reasonably well.
- Nevertheless, because of its simplicity, the CEM is a useful way to describe production when a fundamental theory is not necessary, *e.g.* in studies of production in media.

### Why isn't the $k_T$ -Factorization Approach getting more attention?

- The  $k_T$ -Factorization Approach could, in principle, yield valid results. But...
- it relies on  $k_T$ -dependent parton distributions, which are poorly determined;
- calculations are usually carried out within the CSM;
- calculations are usually carried out only in LO.

### $e^+e^- \rightarrow J/\psi + X(\text{non-}c\bar{c})$

• Belle (2009):

$$\sigma(e^+e^- \to J/\psi + X(\text{non-}c\bar{c})) = 0.43 \pm 0.09 \pm 0.09 \text{ pb}.$$

• NLO calculation (Zhang, Ma, Wang, Chao (2009), Butenschön and Kniehl (2011)):

$$\sigma(e^+e^- \to J/\psi + X(\text{non-}c\bar{c})) = 0.99^{+0.35}_{-0.17} \text{ pb} \qquad (\mu = \sqrt{s}/2).$$

- NRQCD LDMEs from the Butenschön-Kniehl (2011) global fit.
- Includes feeddown estimate of 0.29 pb from Zhang, Ma, Wang, Chao (2009).
- The comparison with the Belle data favors the Butenschön-Kniehl value of  $M_{0,r_0}$ .

#### <u>Comments</u>

• The most recent Belle (2009) measurements give

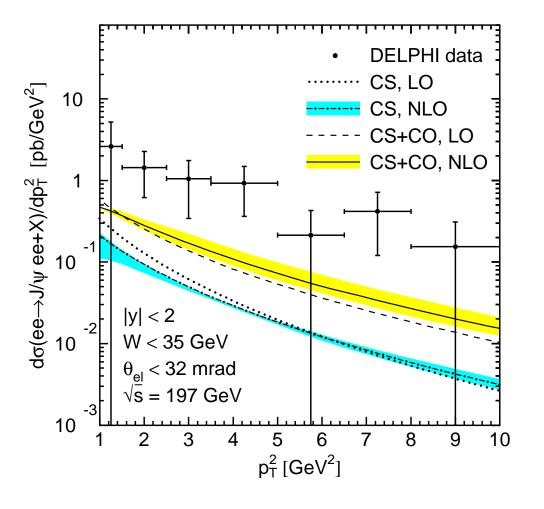
$$\sigma(e^+e^- \to J/\psi + X) = \sigma(e^+e^- \to J/\psi + c\bar{c} + X) + \sigma(e^+e^- \to J/\psi + X(\mathsf{non-}c\bar{c}))$$
  
= 1.17 ± 0.12<sup>+0.13</sup><sub>-0.12</sub> pb.

• However, BaBar (2001) obtained

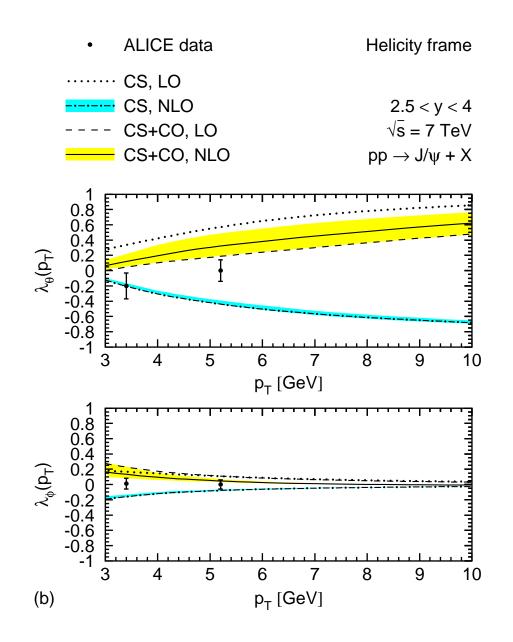
$$\sigma(e^+e^- \to J/\psi + X) = 2.52 \pm 0.21 \pm 0.21 \text{ pb}$$

• Most of the data are at  $p_T \lesssim 3~{\rm GeV}.$  Does factorization hold at such small values of  $p_T$ ?

## $J/\psi$ Production in $\gamma\gamma$ Scattering at LEP II

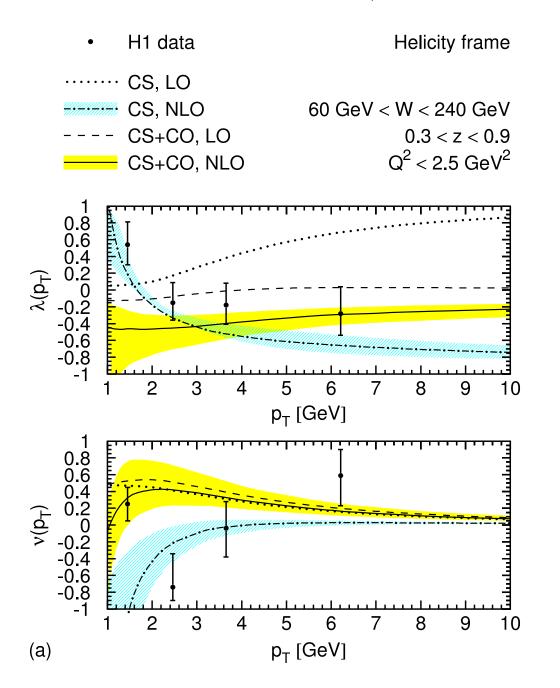


- The DELPHI (2003) data are slightly incompatible with the prediction of the Butenschön and Kniehl (2011) global fit.
- The error bars are large, especially at high  $p_T$ .
- Factorization may not hold at low values of  $p_T$ .



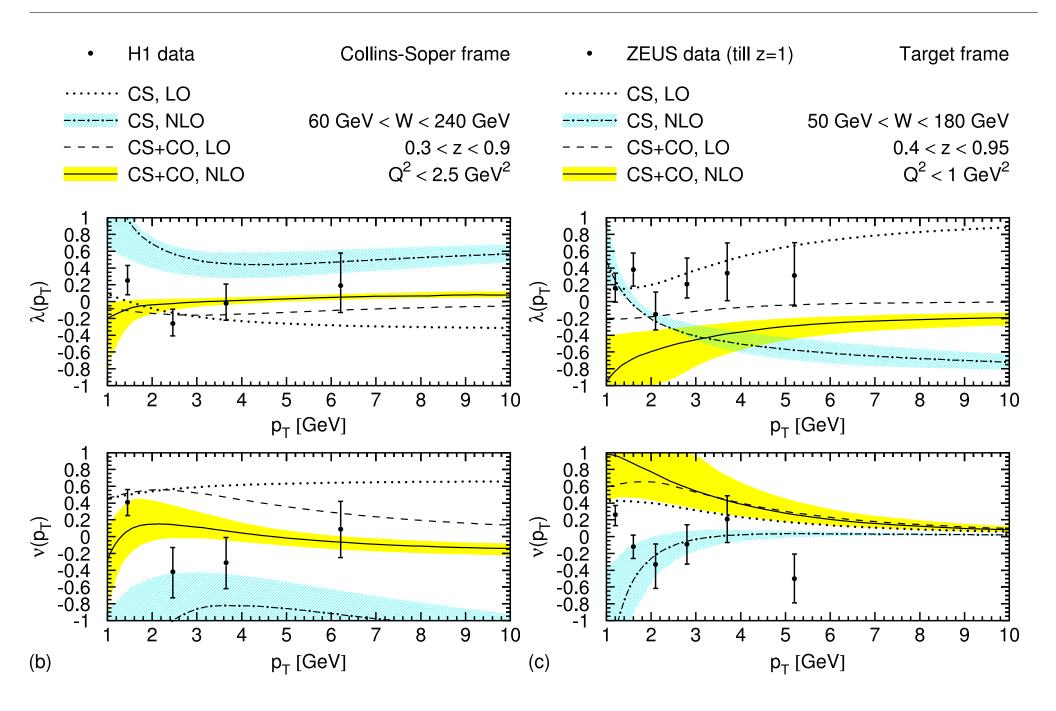
- The prediction from the Butenschön and Kniehl (2011) global fit is in agreement with the ALICE (2012) data.
- But the theory is for direct production, while the ALICE data includes production in *B*-meson decays and feeddown from  $\chi_{cJ}$  states and the  $\psi(2S)$ .

• The Butenschön and Kniehl (2011) global fit can also be used to predict the polarization in inelastic  $J/\psi$  photoproduction at HERA.



• The data are roughly compatible with the theory at large  $p_T$ , but the error bars are large.

Theoretical Aspects Quarkonium Production in Vacuum



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