



Measurement of charm rare decays

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on behalf of the LHCb collaboration

CHARM 2015, Detroit, USA
18-22 May 2015

Overview

- A brief introduction on charm rare decays and LHCb
- LHCb results:
 - $D^0 \rightarrow \mu^+ \mu^-$
 - $D^0 \rightarrow \pi^+ \pi^- \mu^+ \mu^-$
 - $D_{(s)}^+ \rightarrow \pi^+ \mu^+ \mu^- / D_{(s)}^+ \rightarrow \pi^- \mu^+ \mu^+$
- Future prospects
- Conclusions

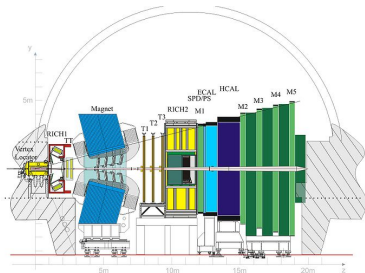
Importance of charm rare decays

Why charm rare decays

- Flavour Changing Neutral Current (FCNC) processes:
 - highly suppressed in the SM
 - only allowed at loop level
 - affected by GIM suppression → D decays more suppressed than B decays, due to the absence of a heavy down-type quark
- Charm → investigate up-type quark FCNCs → studies complementary to those in B and K sectors
- New Physics could enhance SM branching fraction predictions
- Multibody semileptonic decays: angular asymmetries studies
 - A_{CP} and A_{FB} could be enhanced by some NP effects (to $\mathcal{O}(1\%)$ and sometimes $\mathcal{O}(5\%)$)

Large Hadron Collider-beauty

- Single-arm forward spectrometer
- b- and c-hadrons rare decays, CP violation, quark model test
- investigating the physics beyond the Standard Model (matter-antimatter asymmetry)
- reduced luminosity → few p-p interactions per bunch crossing (better reconstructibility of events)



Why LHCb is a very suitable detector:

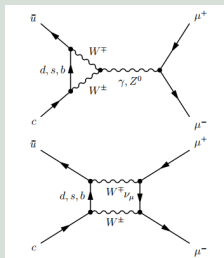
- Excellent muon identification
- $1.9 < \eta < 4.9$
- High momentum resolution: $0.4\% < \frac{\delta p}{p} < 0.6\%$
- Very good performance in reconstruction of vertices
- High performance trigger: flexible and configurable
- $5(2) \cdot 10^{12} D^0(D^+)$ in LHCb acceptance in $3fb^{-1}$ of integrated luminosity at $\sqrt{s} = 7 - 8\text{TeV}$

$$\begin{aligned}
 D^0 &\rightarrow \mu^+ \mu^- \\
 D^0 &\rightarrow \pi^+ \pi^- \mu^+ \mu^- \\
 D^+_{(s)} &\rightarrow \pi^+ \mu^+ \mu^- / D^+_{(s)} \rightarrow \pi^- \mu^+ \mu^+
 \end{aligned}$$

$D^0 \rightarrow \mu^+ \mu^-$: strategy

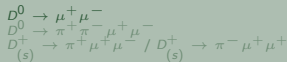
Phys.Lett.B, Vol.725, 2013, 15–24

SM contributions



- SM short distance contribution $\mathcal{O} \sim 10^{-18}$
- SM long distance prediction $\mathcal{O} \sim 10^{-11}$
 (dominated by the two-photon intermediate state)
- Previous limit $\mathcal{O} \sim 10^{-7}$
 Belle Collaboration [PRD 81 (2010) 091102R]

- Signal channel: $D^{*+} \rightarrow D^0(\mu^+ \mu^-) \pi^+$
- Normalization channel: $D^{*+} \rightarrow D^0(\pi^+ \pi^-) \pi^+$
- Control channels: $D^{*+} \rightarrow D^0(K^- \pi^+) \pi^+$, $D^0 \rightarrow K^+ \pi^+$, $J/\psi \rightarrow \mu^+ \mu^-$ (muon identification and trigger efficiency)
- Peaking background (2- or 3-body D^0 decays, hadrons misidentified as muons) \rightarrow tight particle identification criteria
- Combinatorial background \rightarrow multivariate selection (θ_D, χ^2_{IP} of D^0 and muons tracks, minimum muons p_T, \dots)



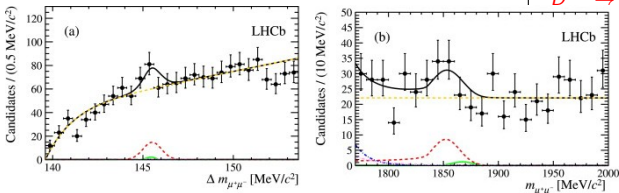
$D^0 \rightarrow \mu^+ \mu^-$: results

Phys.Lett.B, Vol.725, 2013, 15–24

Unbinned maximum likelihood fit of two-dimensional distributions of:

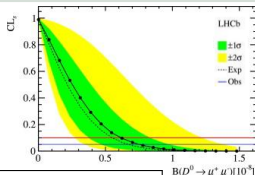
- $\Delta m_{\mu^+ \mu^-} = m_{\mu^+ \mu^- \pi^+} - m_{\mu^+ \mu^-}$
- $m_{\mu^+ \mu^-}$

Total distribution
 Combinatorial background
 Signal
 $D^{*+} \rightarrow D^0(K^- \pi^+) \pi^+$
 $D^{*+} \rightarrow D^0(\pi^- \pi^+) \pi^+$



Upper limit ($\sqrt{s}=7$ TeV, 0.9 fb^{-1}):

$$B(D^0 \rightarrow \mu^+ \mu^-) < 6.2(7.6) \cdot 10^{-9} \text{ at } 90\%(95\%) \text{ CL}$$



2 orders of magnitude above SM predictions

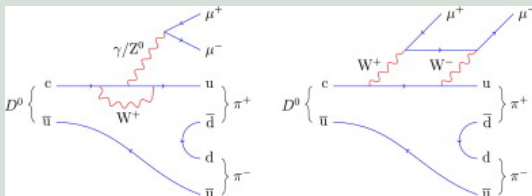
Improved by a factor 20

$$\begin{aligned}
 D^0 &\rightarrow \mu^+ \mu^- \\
 D^0 &\rightarrow \pi^+ \pi^- \mu^+ \mu^- \\
 D^+_{(s)} &\rightarrow \pi^+ \mu^+ \mu^- / D^+_{(s)} \rightarrow \pi^- \mu^+ \mu^+
 \end{aligned}$$

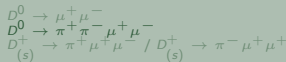
$D^0 \rightarrow \pi^+ \pi^- \mu^+ \mu^-$: strategy

Phys.Lett.B, Vol.728, 2014, 234-243

SM contributions



- SM prediction $\mathcal{O}(\lesssim 10^{-9})$
- Previous limit $\mathcal{O}(\sim 10^{-5})$ E791 Collaboration [PRL 86(2001)3969]
- Signal: $D^{*+} \rightarrow D^0(\pi^+ \pi^- \mu^+ \mu^-) \pi^+$
- Control leakage from resonant regions into low and high dimuon mass
- Signal regions away from η , ρ^0 and ϕ resonances ($250 < m_{\mu\mu} < 525 \text{ MeV}/c^2$, $m_{\mu\mu} > 1100 \text{ MeV}/c^2$)
- Peaking background $D^0 \rightarrow \pi^+ \pi^- \pi^+ \pi^-$ + combinatorial
- Reference sample: $D^0 \rightarrow \pi^+ \pi^- \phi (\rightarrow \mu^+ \mu^-)$
- Combined multivariate analysis (θ_D, χ^2 of D^0 decay vertex and flight distance, p and p_T of all tracks, ...) and muon particle identification

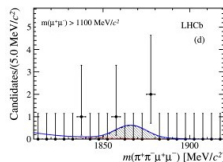
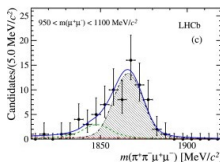
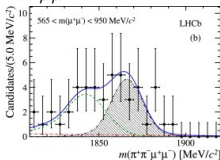
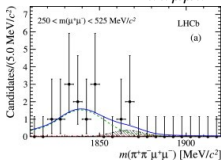


$D^0 \rightarrow \pi^+ \pi^- \mu^+ \mu^-$: results

Phys.Lett.B, Vol.728, 2014, 234-243

Unbinned maximum likelihood of two-dimensional distributions of $m_{\pi\pi\mu\mu}$ and

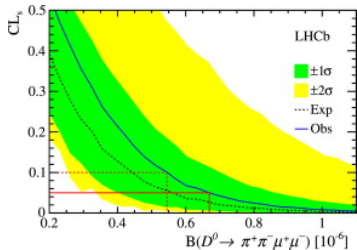
$$\Delta m = m_{\pi\pi\mu\mu\pi} - m_{\pi\pi\mu\mu}$$



Total distribution
 Filled area - Signal
 $D^0 \rightarrow \pi^+ \pi^- \pi^+ \pi^-$
 Non peaking background

Upper limit ($\sqrt{s} = 7 \text{ TeV}, 1 \text{ fb}^{-1}$):

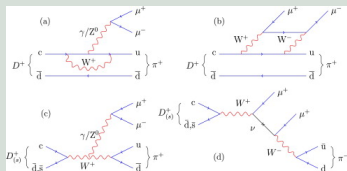
$$B(D^0 \rightarrow \pi^+ \pi^- \mu^+ \mu^-) < 5.5(6.7) \cdot 10^{-7} \text{ at } 90\%(95\%) \text{ CL}$$



x70 improvement — 2 orders of magnitude above SM predictions

$D_{(s)}^+ \rightarrow \pi^+ \mu^+ \mu^- / D_{(s)}^+ \rightarrow \pi^- \mu^+ \mu^+$: strategy

SM contributions



- Previous limits: $D^+ \rightarrow \pi^- \mu^+ \mu^+$ $\mathcal{O} \sim (10^{-6})$ Babar Collaboration [PRD 84(2011)072006]
 $D_s^+ \rightarrow \pi^- \mu^+ \mu^+$ $\mathcal{O} \sim (10^{-5})$ Babar Collaboration [PRD 84(2011)072006]
 $D^+ \rightarrow \pi^+ \mu^+ \mu^-$ $\mathcal{O} \sim (10^{-6})$ D0 Collaboration [PRL 100(2008)101801]
 $D_s^+ \rightarrow \pi^+ \mu^+ \mu^-$ $\mathcal{O} \sim (10^{-5})$ FOCUS Collaboration [PRB 572 (2003)21]

- Control leakage from resonant regions
- Control channel $D_{(s)}^+ \rightarrow \pi^+(\phi \rightarrow \mu^+ \mu^-)$
- Peaking background $D_{(s)}^+ \rightarrow \pi^+ \pi^+ \pi^-$
- Multivariate analysis (θ_D, χ^2 of $D_{(s)}^+$ decay vertex and flight distance, p and p_T of all tracks,...) + particle identification selection

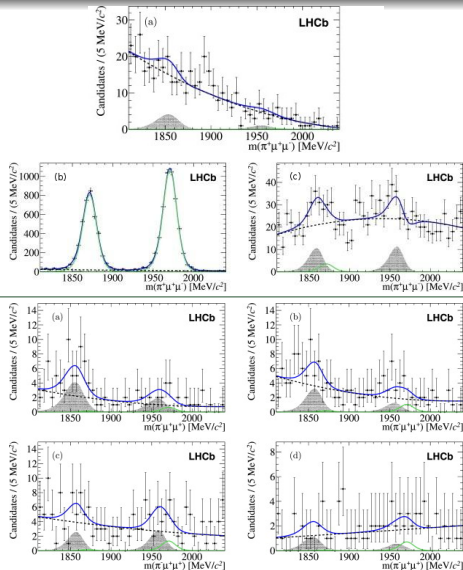
$$D^0 \rightarrow \mu^+ \mu^-$$

$$D^0 \rightarrow \pi^+ \pi^-$$

$$D^+ \rightarrow \pi^+ \mu^+ \mu^- / D^+_{(s)} \rightarrow \pi^- \mu^+ \mu^+$$

$D^+_{(s)} \rightarrow \pi^+ \mu^+ \mu^- / D^+_{(s)} \rightarrow \pi^- \mu^+ \mu^+$: results I

Phys.Lett.B, Vol.724, 2013, 203-212



Binned maximum likelihood fit

- o $D^+_{(s)} \rightarrow \pi^+ \mu^+ \mu^-$ in $m(\mu^+ \mu^-)$ bins:
 - o a) low- $m(\mu^+ \mu^-)$ 250-525 MeV/c^2
 - o b) ϕ 850-1850 MeV/c^2
 - o c) high- $m(\mu^+ \mu^-)$ 1250-2000 MeV/c^2

Total distribution
Signal
 Solid area-Peaking background
 Dashed line-Non peaking background

- o $D^+_{(s)} \rightarrow \pi^- \mu^+ \mu^+$ in $m(\mu^+ \pi^-)$ bins:
 - o a) 250-1140 MeV/c^2
 - o b) 1140-1340 MeV/c^2
 - o c) 1340-1550 MeV/c^2
 - o d) 1540-2000 MeV/c^2

$D_{(s)}^+ \rightarrow \pi^+ \mu^+ \mu^- / D_{(s)}^+ \rightarrow \pi^- \mu^+ \mu^+$: results II

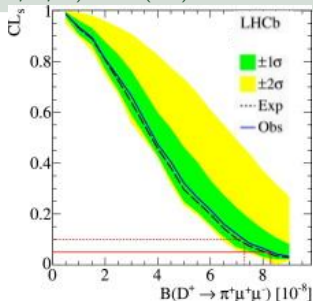
Upper limits ($\sqrt{s} = 7 \text{ TeV}, 1 \text{ fb}^{-1}$):

$$\mathcal{B}(D^+ \rightarrow \pi^+ \mu^+ \mu^-) < 7.3(8.3) \cdot 10^{-8} \text{ at } 90\%(95\%) \text{ CL}$$

$$\mathcal{B}(D_s^+ \rightarrow \pi^+ \mu^+ \mu^-) < 4.1(4.8) \cdot 10^{-7} \text{ at } 90\%(95\%) \text{ CL}$$

$$\mathcal{B}(D^+ \rightarrow \pi^- \mu^+ \mu^+) < 2.2(2.5) \cdot 10^{-8} \text{ at } 90\%(95\%) \text{ CL}$$

$$\mathcal{B}(D_s^+ \rightarrow \pi^- \mu^+ \mu^+) < 1.2(1.4) \cdot 10^{-7} \text{ at } 90\%(95\%) \text{ CL}$$



Improved by a factor 50

1 order of magnitude above largest NP predictions for $D_s^+ \rightarrow \pi^+ \mu^+ \mu^-$

Future prospects: ongoing

Analyses in progress or planned

- Lepton Flavour Violation $D^0 \rightarrow e^\pm \mu^\mp$
- $D^0 \rightarrow K^\mp \pi^\pm \mu^+ \mu^-$
- Update of $D^0 \rightarrow \mu^+ \mu^-$
- $\Lambda \rightarrow p \mu \mu$
- $D^0 \rightarrow \phi \gamma$
- ...and some others planned

Future prospects: run II and upgrade

- LHCb Run II: $8fb^{-1}$, $\sqrt{s} = 13TeV$
- LHCb Upgrade: $50fb^{-1}$, $\sqrt{s} = 14TeV$

Predictions on branching fractions's upper limits:

Assuming the same efficiency and signal-to-background ratio:

Mode	Run I	Run II	Upgrade
$D^0 \rightarrow hh' \mu^+ \mu^-$	few 10^{-7}	fewer 10^{-7}	10^{-8}
$D^0 \rightarrow \mu^+ \mu^-$	few 10^{-9}	fewer 10^{-9}	10^{-10}
$D^+ \rightarrow \pi^+ \mu^+ \mu^-$	few 10^{-8}	fewer 10^{-8}	10^{-9}
$D_s^+ \rightarrow K^+ \mu^+ \mu^-$	few 10^{-7}	fewer 10^{-7}	10^{-8}
$\Lambda \rightarrow p \mu \mu$	few 10^{-7}	fewer 10^{-7}	10^{-8}
$D^0 \rightarrow e \mu$	few 10^{-8}	fewer 10^{-8}	10^{-9}
$\sigma_{ACP}(D^0 \rightarrow \phi \gamma)$	10%	5%	?

Future prospects: run II and upgrade

Predictions on asymmetries sensitivity:

Assuming the same efficiency and signal-to-background ratio:

Mode	Run II	Upgrade
$D^+ \rightarrow \pi^+ \mu^+ \mu^-$	0.6%(30000 events)	0.2%(300000 events)
$D^0 \rightarrow \pi^+ \pi^- \mu^+ \mu^-$	3%(1500 events)	1%(15000 events)
$D^0 \rightarrow K^- \pi^+ \mu^+ \mu^-$	1%(10000 events)	0.3%(100000 events)
$D^0 \rightarrow K^+ \pi^- \mu^+ \mu^-$	40%(30 events)	12%(300 events)
$D^0 \rightarrow K^+ K^- \mu^+ \mu^-$	11%(150 events)	4%(1500 events)

These predictions could improve under the upgrade conditions:

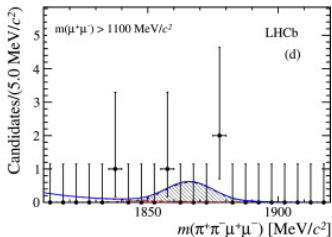
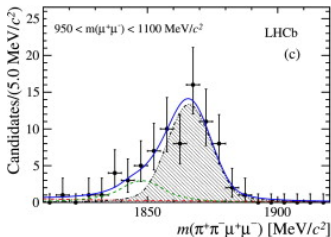
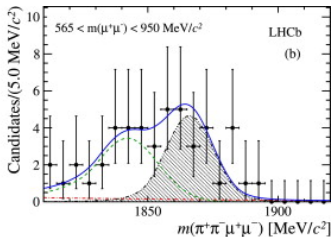
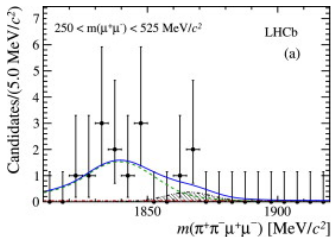
- offline reconstruction quality available in a fully software trigger ($\epsilon \sim \times 3$)
- other improvements in the analyses
- combinations of modes might matter more than individual sensitivities

Conclusions

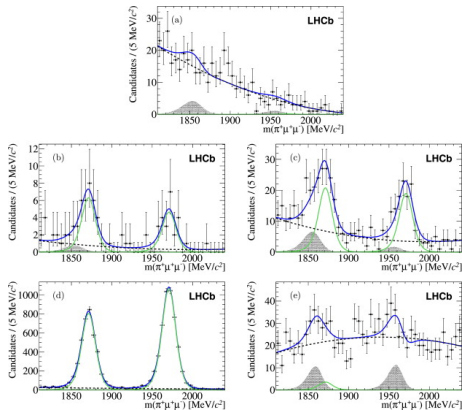
- The results shown are all best world limits
- Results on $D^0 \rightarrow e\mu$ and $D^0 \rightarrow K\pi\mu\mu$ will become public very soon
- Upgrades ongoing or planned
- Wait for Run II data

Backup Slides

$$D^0 \rightarrow \pi^+ \pi^- \mu^+ \mu^-$$



$$D_{(s)}^+ \rightarrow \pi^+ \mu^+ \mu^-$$



Trigger conditions	Bin description	$m(\mu^+\mu^-)$ range [MeV/c ²]	D^+ yield	D_s^+ yield
	low- $m(\mu^+\mu^-)$	250 – 525	-3 ± 11	1 ± 6
Triggers without $m(\mu^+\mu^-) > 1.0$ GeV/c ²	η	525 – 565	29 ± 7	22 ± 5
	ρ/ω	565 – 850	96 ± 15	87 ± 12
	ϕ	850 – 1250	2745 ± 67	3855 ± 86
All triggers	ϕ	850 – 1250	3683 ± 90	4857 ± 90
	high- $m(\mu^+\mu^-)$	1250 – 2000	16 ± 16	-17 ± 16

$$D_{(s)}^+ \rightarrow \pi^+ \mu^+ \mu^-$$

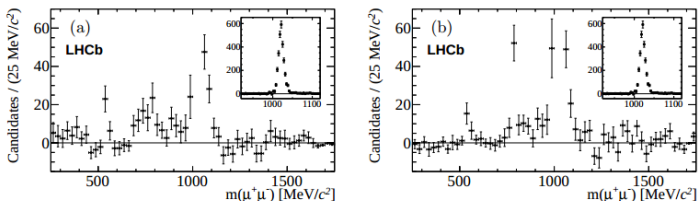
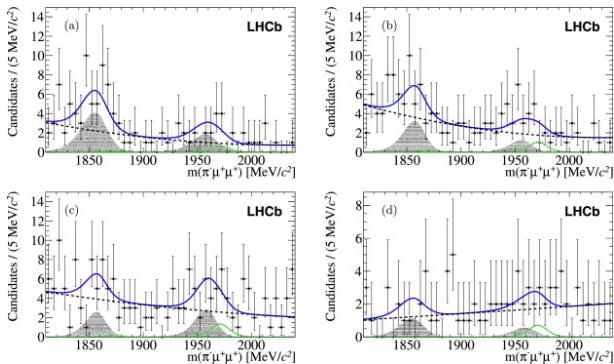


Figure 4: Background-subtracted $m(\mu^+\mu^-)$ spectrum of (a) $D^+ \rightarrow \pi^+ \mu^+ \mu^-$ and (b) $D_s^+ \rightarrow \pi^+ \mu^+ \mu^-$ candidates that pass the final selection. The inset shows the ϕ contribution, and the main figure shows the η and the ρ/ω contributions. The non-peaking structure of the low and high- $m(\mu^+\mu^-)$ regions is also visible.

$$D_s^+ \rightarrow \pi^- \mu^+ \mu^+$$



Bin description	$m(\mu^+x^-)$ range [MeV/ c^2]	D^+ yield	D_s^+ yield
ϕ	850 – 1250	2771 ± 65	3885 ± 85
bin 1	250 – 1140	7 ± 6	4 ± 4
bin 2	1140 – 1340	-3 ± 6	3 ± 5
bin 3	1340 – 1550	-1 ± 6	6 ± 6
bin 4	1540 – 2000	0 ± 4	4 ± 5

Model name	Main characteristics	Affected observables
Minimal Supersymmetric Model with R-parity violation (MSSM $\#$)	$c \rightarrow u\mu^+\mu^-$ current is possible at tree level via down-type squark. It has a very large impact on C_9 and C_{10} , quantified in [49, 50], and updated in [52] in the light of the constraints brought in 2007 by the discovery of the D^0 mixing. In the light of recent constraints from $K \rightarrow \pi\nu\nu$ decays and charm decays, there is now little hope to get sizeable contributions from this kind of NP to the decays we are interested in. See for instance [54].	For $D^+ \rightarrow \pi^+\mu^+\mu^-$: $\mathcal{B} = 6.5 \times 10^{-6}$ [52], recently measured $\mathcal{B} < 7.3 \times 10^{-8}$ @90%CL [13]. New constraints from $D^0 \rightarrow \mu^+\mu^-$: $\mathcal{B} = 2 \times 10^{-8}$ [55]
Extra up-like quark singlet	New quark doublet or singlet, extended CKM matrix. The FCNC possible at tree level with cuZ coupling. A study of their impact can be found in [56], with an update in [52]. Large effects on C_9 and C_{10} were predicted there.	For $D^+ \rightarrow \pi^+\mu^+\mu^-$: $\mathcal{B} = 1.6 \times 10^{-9}$ [52]. For $D^0 \rightarrow \rho^0\mu^+\mu^-$: $\mathcal{A}_{FB} \sim \text{few } \%$
Littlest Higgs Model	Particular version of models with cuZ coupling, where the Higgs boson is a pseudo-Nambu-Goldstone boson of spontaneously broken global symmetry. It contains a new massive gauge boson and a new up-like quark t . Weak currents are modified, CKM is extended to be 4×3 [56, 57]. The model modifies coefficients C_9 and C_{10} . In particular, C_{10} , while $\simeq 0$ in the SM becomes of the order of C_9 [56].	For $D^+ \rightarrow \pi^+\mu^+\mu^-$: $\mathcal{B} = 8.0 \times 10^{-11}$ [57]; For $D^0 \rightarrow \rho^0\mu^+\mu^-$: $\mathcal{A}_{FB} \sim \mathcal{O}(10^{-3})$ [57]
Leptoquark model	Carrying both lepton and baryon numbers, new bosons can couple to a lepton and a quark [58].	For $D^+ \rightarrow \pi^+\mu^+\mu^-$: $\mathcal{B} = 9.4 \times 10^{-8}$ [58]
Randall-Sundrum model with a warped extra dimension	New gauge bosons appear, that mediate flavour violation. It brings a small contribution to C_9 , which at the most could be comparable to the SM value (for some marginal values of the models parameters). On the other hand, as in other models, the tiny C_{10} is enhanced by several orders of magnitude [51].	For $D \rightarrow X_c\mu^+\mu^-$: \mathcal{A}_{FB} , $\mathcal{A}_{CP} \sim \text{few } \%$ $\mathcal{A}_{FB}^{CP} > \mathcal{O}(10\%)$ [51]

Model name	Main characteristics	Affected observables
Minimal Supersymmetric Model with R-parity conservation (MSSM R)	New sources of flavour symmetry breaking. In the mass insertion approach, off-diagonal elements in the squark mass matrix yield flavour changing couplings $(\delta_{ij}^N)_{LL,RR}$. They allow squarks of flavour i , of helicity H and type q (up or down), to turn spontaneously into a squarks of flavour j , of helicity H' and type q . Loop amplitudes as that in Figure 1.19(a) are then possible. They enhance C_7 , C_9 and C_{10} . This is discussed in [40, 59–61].	For $D^0 \rightarrow \rho^0 \mu^+ \mu^-$: $\mathcal{B} \simeq 1.3 \times 10^{-6}$ [49]
Littlest Higgs Model with T-parity (LHT)	LH Model with additional T-parity. Enhancement of the C_9 and C_7 is very small. The main effect is in fact on C_{10} , which is enhanced by orders of magnitude.	For $D \rightarrow X_s \mu^+ \mu^-$: $\mathcal{A}_{FB} \sim \mathcal{O}(0.5\%)$, \mathcal{A}_{CP}^{FB} up to $\mathcal{O}(10\%)$ [47]
Generic models with generated weak phases	Models that generate weak phases acquired by C_7 and C_9 without sensitive impact on C_{10} [46].	For $D^* \rightarrow \pi(\mu^+ \mu^-)_S$: $\mathcal{A}_{CP} \sim \mathcal{O}(1\% - 10\%)$ [46]
Generic Z-mediated models	Loop amplitudes with an internal Z^0 and an internal top quark (Figure 1.19(b)) can bring C_9 and C_{10} up to $\mathcal{O}(1)$, if the couplings they involve are tuned to reproduced the measured value of ΔA_{CP} [18].	For $D^* \rightarrow h^+ h^0 \mu^+ \mu^-$ [18]: \mathcal{A}_{FB} up to 8%, \mathcal{A}_{CP} up to 3%

Table 1.6: Overview of the NP theoretical models that have FCNC at loop level. The estimates of affected observables are presented as well

- [18] L. Cappiello, O. Cata, and G. D'Ambrosio, *Standard Model prediction and new physics tests for $D^0 \rightarrow h^+ h^- l^+ l^-$ ($h = \pi, K; l = e, \mu$)*, JHEP **1304** (2013) 135, arXiv:1209.4235.
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- [60] J. Lyon and R. Zwicky, *Anomalously large O_8 and long-distance chirality from $A_{CP}[D^0 \rightarrow (\rho^0, \omega)\gamma](t)$* , arXiv:1210.6546.
- [61] G. F. Giudice, G. Isidori, and P. Paradisi, *Direct CP violation in charm and flavor mixing beyond the SM*, JHEP 1204 (2012) 060, arXiv:1201.6204.