The SU(3) framework in $D \to PP$ decays.

Ayan Paul

ERC Ideas: NPFlavour

INFN, Sezione di Roma. Roma, Italy.

European Research Council

Supporting top researchers from anywhere in the world

Istituto Nazionale di Fisica Nucleare

CHARM 2015, Detroit. May 19th 2015.

the importance of $\eta - \eta'$ mixing

- ✓ Complete set of measurements of branching fraction available.
- ✓ The mixing angle is well measured.
- ✓ Previously left unconsidered in analyses based on the complete SU(3) framework.
- ✓ The singlet-octet mixing is a consequence of broken SU(3)
- ✓ There are convincing theoretical arguments and experimental hints that the states have not only quark content but also a gluonic component.

$$\begin{pmatrix} \eta \\ \eta' \end{pmatrix} = \begin{pmatrix} -\cos\theta & +\sin\theta \\ -\sin\theta & -\cos\theta \end{pmatrix} \begin{pmatrix} \eta_8 \\ \eta_1 \end{pmatrix}$$

C. Di Donato et al., Phys. Rev. D **85**, 013016 (2012).

S. V. Donskov, V. N. Kolosov, A. A. Lednev, Yu. V. Mikhailov, V. A. Polyakov, V. D. Samoylenko, G. V. Khaustov, IHEP 2012-22, arXiv:1301.6987

the analysis

1. Choose you favourite set of singlet/octet final states and triplet initial state.
2. Construct your Hamiltonian from the triplet of quarks.

$$
\begin{pmatrix}
\mathcal{A}(D_s^+ \to \pi^+\pi^0) \\
\mathcal{A}(D^+ \to \pi^+\bar{K}^0) \\
\mathcal{A}(D^0 \to \pi^+K^-) \\
\mathcal{A}(D_s^+ \to \pi^+\eta_8) \\
\mathcal{A}(D^0 \to \pi^0\bar{K}^0) \\
\mathcal{A}(D_s^+ \to K^+\bar{K}^0) \\
\mathcal{A}(D^0 \to \bar{K}^0\eta_8)
\end{pmatrix}
=
\begin{pmatrix}
0 & 0 & 0 & \sqrt{\frac{2}{3}} & 0 & 0 & \frac{1}{\sqrt{3}} \\
0 & 0 & -\frac{\sqrt{5}}{6} & -\frac{1}{2\sqrt{3}} & \frac{\sqrt{5}}{3} & -\frac{1}{3\sqrt{2}} & \frac{1}{\sqrt{6}} \\
\sqrt{\frac{3}{10}} & -\sqrt{\frac{3}{10}} & \frac{1}{6\sqrt{5}} & \frac{1}{2\sqrt{3}} & \frac{2}{3\sqrt{5}} & -\frac{1}{3\sqrt{2}} & -\frac{1}{\sqrt{6}} \\
\frac{1}{\sqrt{5}} & \frac{1}{\sqrt{5}} & \sqrt{\frac{2}{15}} & 0 & \sqrt{\frac{2}{15}} & \frac{1}{\sqrt{3}} & 0 \\
-\frac{\sqrt{\frac{3}{5}}}{2} & \frac{\sqrt{\frac{3}{5}}}{2} & -\frac{1}{\sqrt{10}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{10}} & 0 & -\frac{1}{\sqrt{3}} \\
-\sqrt{\frac{3}{10}} & -\sqrt{\frac{3}{10}} & \frac{2}{3\sqrt{5}} & 0 & \frac{2}{3\sqrt{5}} & \frac{\sqrt{2}}{3} & 0 \\
-\frac{1}{2\sqrt{5}} & \frac{1}{2\sqrt{5}} & 2\sqrt{\frac{2}{15}} & 0 & \frac{1}{\sqrt{30}} & -\frac{1}{\sqrt{3}} & 0
\end{pmatrix}
\begin{pmatrix}
\langle 8||\bar{6}_1||\bar{3}\rangle \\
\langle 8||15_1||\bar{3}\rangle \\
\langle 27||\overline{24}_1||\bar{3}\rangle \\
\langle 27||\overline{24}_2||\bar{3}\rangle \\
\langle 27||15_1||\bar{3}\rangle \\
\langle 27||42_1||\bar{3}\rangle \\
\langle 27||42_2||\bar{3}\rangle
\end{pmatrix}
$$

reduced matrix elements

decay amplitudes

Clebsch-Gordon coefficients

$$
\begin{pmatrix}
\mathcal{A}(D_s^+ \to \pi^+\eta_1) \\
\mathcal{A}(D^0 \to \bar{K}^0\eta_1)
\end{pmatrix}
=
\begin{pmatrix}
\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\
\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}}
\end{pmatrix}
\begin{pmatrix}
\langle 8'||\bar{6}_1||\bar{3}\rangle \\
\langle 8'||15_1||\bar{3}\rangle
\end{pmatrix}
$$

singlet-octet final state different from octet-octet final state

Now you are ready to do some serious fitting...!!

the problem with not considering $\eta - \eta'$ mixing

- ✓ The transfer matrix between the amplitudes and the reduced matrix elements are square.
- ✓ Hence there are as many complex amplitudes as reduced matrix elements. (before making the Hamiltonian association)
- ✓ Not considering all *PP* channels makes the transfer matrix non-square.
- ✓ This leads to un-physical combinations of the reduced matrix elements when the number of free parameters are reduced by Gaussian reduction.
- ✓ While SU(3) breaking can possibly be inferred from a judicious combination of these transformed reduced matrix elements, they lack the full information that could be carried by taking all channels into consideration.

Caveat: Considering $\eta - \eta'$ mixing increases the number of parameters because then one has to distinguish between the singlet and octet reduced matrix elements. However, the increase in the number of branching fractions is greater than the increase in the number of parameters.

in the limit of SU(3) conservation

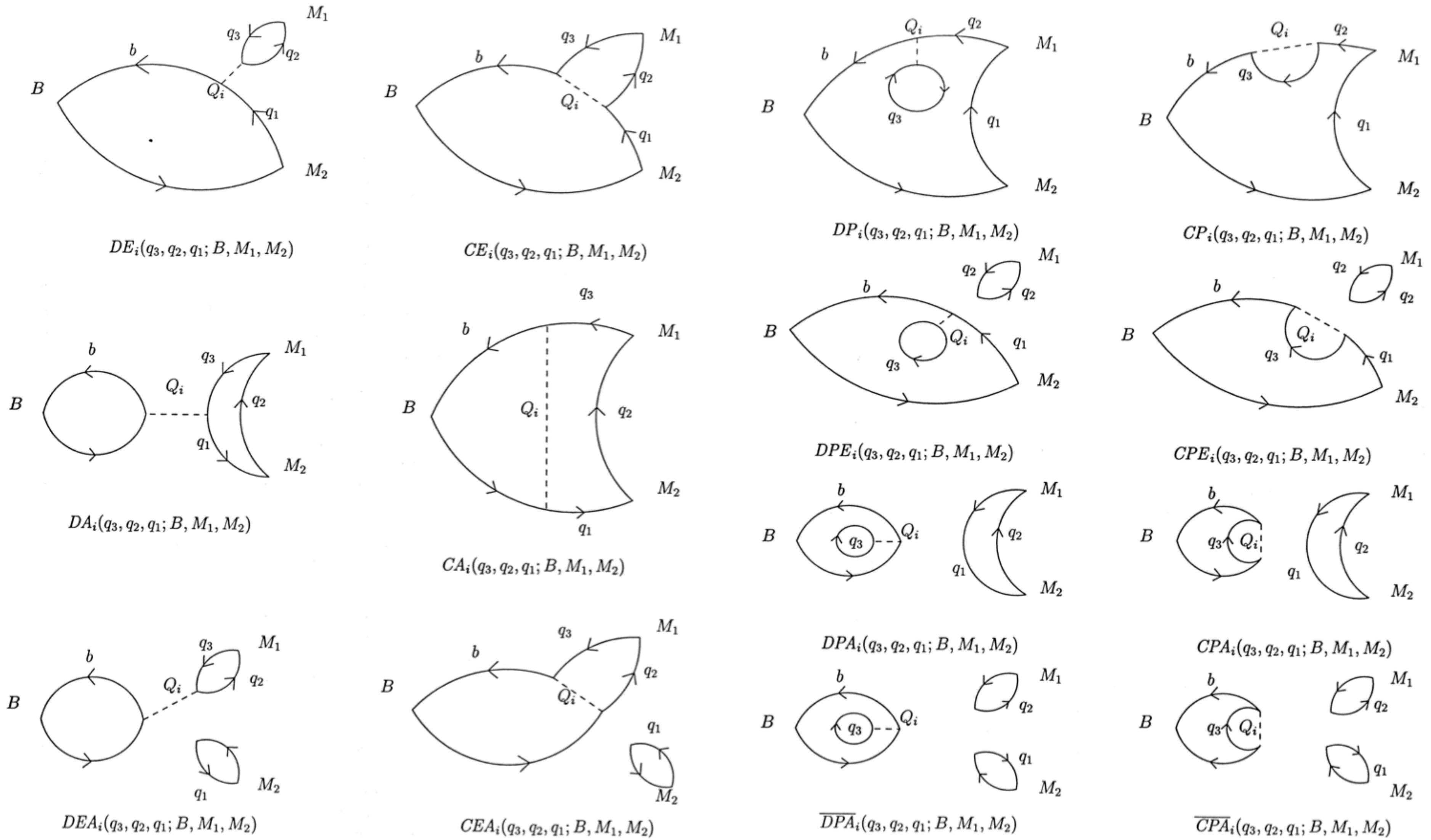
$$\mathcal{H}=\frac{V_{cd}^*V_{us}}{\sqrt{2}}\mathcal{H}_0^6+\frac{(V_{cd}^*V_{ud}-V_{cs}^*V_{us})}{2}\mathcal{H}_{1/2}^6-\frac{V_{cs}^*V_{ud}}{\sqrt{2}}\mathcal{H}_1^6-\frac{(V_{cd}^*V_{ud}-3V_{cs}^*V_{us})}{2\sqrt{6}}\mathcal{H}_{1/2}^{15}$$
$$+\frac{(V_{cd}^*V_{us}+V_{cs}^*V_{ud})}{\sqrt{2}}\mathcal{H}_1^{15}+\frac{V_{cd}^*V_{ud}}{\sqrt{3}}\mathcal{H}_{3/2}^{15}$$

Grinstein-Lebed Relations

$$\frac{\langle f||6_{I=1/2}||i\rangle}{\langle f||6_{I=0}||i\rangle}=\frac{V_{cd}^*V_{ud}-V_{cs}^*V_{us}}{\sqrt{2}V_{cd}^*V_{us}},\quad\frac{\langle f||6_{I=1}||i\rangle}{\langle f||6_{I=0}||i\rangle}=-\frac{V_{cs}^*V_{ud}}{V_{cd}^*V_{us}}$$
$$\frac{\langle f||6_{I=1}||i\rangle}{\langle f||6_{I=1/2}||i\rangle}=-\frac{\sqrt{2}V_{cs}^*V_{ud}}{V_{cd}^*V_{ud}-V_{cs}^*V_{us}},\quad\frac{\langle f||15_{I=1}||i\rangle}{\langle f||15_{I=1/2}||i\rangle}=-\frac{2\sqrt{3}(V_{cs}^*V_{ud}+V_{cd}^*V_{us})}{V_{cd}^*V_{ud}-3V_{cs}^*V_{us}}$$
$$\frac{\langle f||15_{I=3/2}||i\rangle}{\langle f||15_{I=1/2}||i\rangle}=-\frac{2\sqrt{2}V_{cd}^*V_{ud}}{V_{cd}^*V_{ud}-3V_{cs}^*V_{us}},\quad\frac{\langle f||15_{I=3/2}||i\rangle}{\langle f||15_{I=1}||i\rangle}=\sqrt{\frac{2}{3}}\frac{V_{cd}^*V_{ud}}{V_{cs}^*V_{ud}+V_{cd}^*V_{us}}$$

"The ratios of pieces of the Hamiltonian of the same representation are independent of isospin and depend only on the Clebsch-Gordon coefficients and CKM elements for a fixed pair of initial and final state representations."

an insight into isospin universality



A. J. Buras and L. Silvestrini, Nucl. Phys. B **569** (2000) 3 [hep-ph/9812392].

an insight into isospin universality

$$\mathrm{R}_8^6 = \frac{\sqrt{5}}{2}\left(-A_1 + A_2 + E_1 - E_2\right)$$
$$\mathrm{R}_8^{15} = \frac{1}{2\sqrt{2}}\left(5A_1 + 5A_2 + E_1 + E_2\right)$$
$$\mathrm{R}_{27}^{15} = \sqrt{2}\left(E_1 + E_2\right)$$

$$\mathrm{R}_{8,s}^6 = -\frac{1}{2}\left(A'_{1,s} + A_{1,s} - A'_{2,s} - A_{2,s} + E_{1,s} - E_{2,s} - 3EA_{1,s} + 3EA_{2,s}\right)$$
$$\mathrm{R}_{8,s}^{15} = \frac{1}{2}\sqrt{\frac{5}{2}}\left(A'_{1,s} + A_{1,s} + A'_{2,s} + A_{2,s} + E_{1,s} + E_{2,s} + 3EA_{1,s} + 3EA_{2,s}\right)$$

- ✓ In the exact SU(3) limit the reduced matrix elements are isospin universal. Hence the Grinstein-Lebed relations hold good.
- ✓ These reduced matrix elements can be extracted from CF and DCS branching fractions.
- ✓ Note that penguin contributions are absent.

in the limit of SU(3) conservation

Parameter	Re()	Im()
6^8	-0.050 $\pm$ 0.010	0.031 $\pm$ 0.039
15^8	-1.052 $\pm$ 0.013	
15^{27}	-0.038 $\pm$ 0.013	-0.245 $\pm$ 0.004
6^8_s	-0.270 $\pm$ 0.021	0.374 $\pm$ 0.043
15^8_s	0.287 $\pm$ 0.055	0.186 $\pm$ 0.075

Channel	Fit	Experimental
CF		
BR($D^+ \to \pi^+ \bar{K}_S$)	14.6 $\pm$ 0.4	14.7 $\pm$ 0.7
BR($D^+ \to \pi^+ \bar{K}_L$)	14.6 $\pm$ 0.4	14.6 $\pm$ 0.5
BR($D^0 \to \pi^+ K^-$)	38.8 $\pm$ 0.5	38.8 $\pm$ 0.5
BR($D_s^+ \to \pi^+ \eta$)	17.3 $\pm$ 1.0	16.9 $\pm$ 1.0
BR($D^0 \to \pi^0 \bar{K}_S$)	11.4 $\pm$ 0.4	11.9 $\pm$ 0.4
BR($D^0 \to \pi^0 \bar{K}_L$)	11.4 $\pm$ 0.4	10.0 $\pm$ 0.7
BR($D_s^+ \to K^+ \bar{K}_S$)	14.9 $\pm$ 0.6	14.9 $\pm$ 0.6
BR($D^0 \to \bar{K}_S \eta$)	4.78 $\pm$ 0.30	4.79 $\pm$ 0.3
BR($D_s^+ \to \pi^+ \eta'$)	42.3 $\pm$ 2.3	39.4 $\pm$ 2.5
BR($D^0 \to \bar{K}_S \eta'$)	9.39 $\pm$ 0.5	9.4 $\pm$ 0.5
DCS		
BR($D^+ \to \pi^0 K^+$)	0.135 $\pm$ 0.0142	0.183 $\pm$ 0.026
BR($D^0 \to \pi^- K^+$)	0.117 $\pm$ 0.0013	0.138 $\pm$ 0.0028
BR($D^+ \to K^+ \eta$)	0.0931$\pm$ 0.0097	0.108 $\pm$ 0.017
BR($D^+ \to K^+ \eta'$)	0.129 $\pm$ 0.017	0.176 $\pm$ 0.022

the black sheep in the family

notes on SU(3) breaking

- ✓ To implement SU(3) breaking one continues on this procedure.

- ✓ The number of reduced matrix elements are much larger hence one has to make assumptions:

  - ➢ Isopsin universality can be assumed. This leads to a new set of Grinstein-Lebed relationships

  - ➢ One can assume the reduced matrix elements from the conserving and breaking part of the Hamiltonian are not independent.

  - ➢ One can further assume that the breaking is determined by a single parameter representative of the strange quark mass and independent of the representation of the reduced matrix element.

- ✓ These assumptions allow the fitting of all reduced matrix elements since these assumptions lead to 22 free parameters that can be fit with 28 branching fractions.

what the assumptions really mean

- ✓ The number of reduced matrix elements are much larger hence one has to make assumptions:

  - ➢ Isospin universality can be assumed. This leads to a new set of Grinstein-Lebed relationships

this assumption is taken to hold good as there is no way to check it within the SU(3) framework

  - ➢ One can assume the reduced matrix elements from the conserving and breaking part of the Hamiltonian are not independent.

this assumption along with the previous one but without the next one is sufficient to allow a fit if one includes both branching fractions and CP violation data

  - ➢ One can further assume that the breaking is determined by a single parameter representative of the strange quark mass and independent of the representation of the reduced matrix element.

This assumption allows the fitting of all reduced matrix elements with just branching fractions data

an insight into isospin universality

$$
\begin{aligned}
{}^{0}\mathrm{R}_8^6 = \frac{1}{2\sqrt{5}}(&-A_1^{uss} - 4A_1^{uus} + A_2^{uss} + 4A_2^{uus} + 5E_1^{usu} - 5E_2^{usu} - EA_1^{uss} + EA_1^{usu} + EA_2^{uss} \\
&-EA_2^{usu})
\end{aligned}
$$

$$
\begin{aligned}
{}^{\frac{1}{2}}\mathrm{R}_8^6 = \frac{1}{4\sqrt{5}}(&-A_1^{uss} - 3A_1^{usu} - 4A_1^{uus} - 2A_1^{uuu} + 2A_2^{sss} + 3A_2^{sus} + 5A_2^{uuu} + 5E_1^{uss} + 5E_1^{uuu} \\
&-5E_2^{ssu} - 5E_2^{uuu} - EA_1^{uss} + EA_1^{usu} + 2EA_1^{uus} - 2EA_1^{uuu} + 2EA_2^{sss} - 2EA_2^{ssu} + 5P_1^{us} \\
&-5P_1^{uu} - 5P_2^{us} + 5P_2^{uu} + P_3^{ss} + 3P_3^{us} - 4P_3^{uu} + P_4^{ss} - 2P_4^{us} + P_4^{uu})
\end{aligned}
$$

$$
\begin{aligned}
{}^{1}\mathrm{R}_8^6 = \frac{1}{2\sqrt{5}}(&3A_1^{usu} + 2A_1^{uuu} - A_2^{ssu} - 4A_2^{suu} - 5E_1^{uus} + 5E_2^{suu} - 2EA_1^{uus} + 2EA_1^{uuu} - EA_2^{sus} \\
&+EA_2^{suu})
\end{aligned}
$$

$$
\begin{aligned}
{}^{\frac{1}{2}}\mathrm{R}_1^3 = \frac{1}{6}(&-2A_2^{sss} - 6A_2^{sus} + 3A_2^{usu} + 5A_2^{uuu} - 3E_1^{uss} + 3E_1^{uuu} + E_2^{ssu} - E_2^{uuu} - 2EA_2^{sss} \\
&+2EA_2^{ssu} - EA_2^{uus} + EA_2^{uuu} - 3P_1^{us} - 5P_1^{uu} + P_2^{us} - P_2^{uu} - P_3^{ss} - 6P_3^{us} - 5P_3^{uu} \\
&-P_4^{ss} + 2P_4^{us} - P_4^{uu})
\end{aligned}
$$

$$
\begin{aligned}
{}^{\frac{1}{2}}\mathrm{R}_8^3 = \frac{1}{6\sqrt{10}}(&-3A_1^{uss} + 9A_1^{usu} - 12A_1^{uus} + 6A_1^{uuu} + 2A_2^{sss} + 3A_2^{sus} - 6A_2^{usu} + A_2^{uuu} - 15E_1^{uss} \\
&+15E_1^{uuu} + 5E_2^{ssu} - 5E_2^{uuu} - 3EA_1^{uss} + 3EA_1^{usu} - 6EA_1^{uus} + 6EA_1^{uuu} + 2EA_2^{sss} \\
&-2EA_2^{ssu} + 4EA_2^{uus} - 4EA_2^{uuu} - 15P_1^{us} - 25P_1^{uu} + 5P_2^{us} - 5P_2^{uu,} + P_3^{ss} + 3P_3^{us} \\
&-4P_3^{uu} + P_4^{ss} - 2P_4^{us} + P_4^{uu})
\end{aligned}
$$

the validity of the SU(3) formalism

- ✓ The number of reduced matrix elements are much larger hence one has to make assumptions:

  - ➢ Isospin universality can be assumed. This leads to a new set of Grinstein-Lebed relationships

this assumption seems to break down when one take a careful look at it

  - ➢ One can assume the reduced matrix elements from the conserving and breaking part of the Hamiltonian are not independent.

the number of reduced matrix elements can not be sufficiently reduced with just the last two assumptions

  - ➢ One can further assume that the breaking is determined by a single parameter representative of the strange quark mass and independent of the representation of the reduced matrix element.

the question of final states

- ✓ Identifying the charm meson as a triplet of SU(3) and the pseudoscalars as an octet of SU(3) allows for the set up of the SU(3) framework.
- ✓ The SU(3) framework requires that the hadronic final states be identified and there be no variation in the reduced matrix elements due to just the final states.
- ✓ Not making this assumption renders the SU(3) framework completely useless.
- ✓ However, the strength of the SU(3) framework lies in its dynamical constructions, i.e. , the Hamiltonian.
- ✓ While we know how to deal with the weak part of the Hamiltonian, we depend solely on motivated arguments and data for the QCD part.

what can sum rules say?

- Sum rules come from two sources:

1. From the zeroes in the reduced matrix elements.
2. From the isospin association of the reduced matrix elements in the SU(3) limit. (another manifestation of the Grinstein-Lebed relations)

- The first is well documented. (Grossman-Robinson arXiv:1211:3361)

- The second gives additional relations.

- These relations are amplitude relations that are broken when SU(3) is broken.

- Branching fractions relations are more important as they can be directly measured. (Grossman-Robinson)

- Gronau presented a branching fraction relation that holds till second order in SU(3) breaking – a very precise test. (arXiv:1501.03272)

- There are two more such relations but they involves $\eta - \eta'$ mixing.

diagrammatic approach + factorization

- The big question is whether factorization can be applied to charmed meson decays…

- If we assume it is possible then the machinery from B physics can be used

- A recent approach made by Müller, Nierste and Schacht. (talk tomorrow) (arXiv:1503:06759)
- Another approach made by Biswas, Sinha and Abbas which includes: (arXiv:1503.08176)
  - An estimation of the topologies within the factorization ansatz
  - A parameterization of the non-factorizable part fit to data
  - A data-driven parameterization of the final state interactions through a scattering matrix.
  - All $\eta-\eta'$ channels considered.

- Fits prove to be quite agreeable except in some channels.

- Singlet contribution in $\eta-\eta'$ channels not considered. (left as future work)

Man goes to doctor.
Says he's depressed.
Says life seems harsh and cruel.
Says he feels all alone in threatening world where what lies ahead is vague and uncertain.

The Watchmen.

Man goes to doctor.
Says he's depressed.
Says life seems harsh and cruel.
Says he feels all alone in threatening world where what lies ahead is vague and uncertain.

Doctor says, “Treatment is simple. Great clown Pagliacci is in town tonight. Go and see him. That should pick you up."

The Watchmen.

Man goes to doctor.
Says he's depressed.
Says life seems harsh and cruel.
Says he feels all alone in threatening world where what lies ahead is vague and uncertain.

Doctor says, “Treatment is simple. Great clown Pagliacci is in town tonight. Go and see him. That should pick you up."

Man bursts into tears. Says “But Doctor... I am Pagliacci.”

The Watchmen.

Thank you…!!

To my Mother and Father, who showed me what I could do,

and to Ikaros, who showed me what I could not.

"To know what no one else does, what a pleasure it can be!"

– adopted from the words of

Eugene Wigner.