Strategies to determine the X(3872) energy from QCD lattice simulations

R. Molina[1], E. J. Garzon[2], A. Hosaka[3] and E. Oset[2]

[1] George Washington University, [2] IFIC-Valencia University,
[3] RCNP-Osaka University

Contents

Introduction

Search of the X(3872) in lattice QCD:

L. Liu, G. Moir, M. Peardon, S. M. Ryan, C. E. Thomas, P. Vilaseca, J. J. Dudek, R. G. Edwards, B. Joó, and D. G. Richards, J. High Energy Phys. 07 (2012)

G. Bali, S. Collins, and P. Perez-Rubio, J. Phys. Conf. Ser. 426, 012017 (2013).

D. Mohler, S. Prelovsek, and R. M. Woloshyn, Phys. Rev. D87, 034501 (2013).

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S. Prelovsek and L. Leskovec, Phys. Rev. Lett. 111, 192001 (2013): Bound state in dynamical $N_f = 2$ lattice simulation with 11 ± 7 MeV below the $D\bar{D}^*$ threshold and quantum numbers 1^{++}.

- We develop a method to determine accurately the binding energy of the X(3872) from lattice data.
- The analysis of the data requires the use of coupled channels $D^0\bar{D}^{*0}$ and D^+D^{*-}.

The X(3872) in the continuum limit

Hidden gauge Lagrangian

$$\mathcal{L}_{III} = -\frac{1}{4}\langle V_{\mu\nu}V^{\mu\nu}\rangle + \frac{1}{2}M_V^2\langle[V_\mu - \frac{i}{g}\Gamma_\mu]\rangle \quad (1)$$

where $V_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu - ig[V_\mu, V_\nu]$, and $g = \frac{M_V}{2f}$.

$\longrightarrow \mathcal{L}_{PPV} = -ig\langle V^\mu[P, \partial_\mu P]\rangle \longrightarrow \mathcal{L}_{3V} = ig\langle(V^\mu\partial_\nu V_\mu - \partial_\nu V_\mu V^\mu)V^\nu\rangle$

Approximation: $|k_i|^2/M^2 \sim 0$ for external mesons,

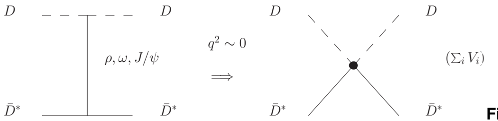

Fig. 1. Pointlike pseudos.-vector interaction.

M. Bando, T. Kugo, S. Uehara, K. Yamawaki and T. Yanagida, Phys. Rev. Lett. **54**, 1215 (1985), M. Bando, T. Kugo and K. Yamawaki, Phys. Rept. **164**, 217 (1988), M. Harada and K. Yamawaki, Phys. Rept. **381**, 1 (2003), U. G. Meissner, Phys. Rept. **161**, 213 (1988).

The X(3872) in the continuum limit

In the approximation $|k_i|^2/M_V^2 \sim 0$, this is equivalent to,

Lagrangian $\mathcal{L}_{PPVV} = -\frac{1}{4f^2} Tr\left(J_\mu \mathcal{J}^\mu\right)$,

Currents $J_\mu = (\partial_\mu P)P - P\partial_\mu P$, $\mathcal{J}_\mu = (\partial_\mu \mathcal{V}_\nu)\mathcal{V}^\nu - \mathcal{V}_\nu \partial_\mu \mathcal{V}^\nu$.

(L-L: J_{88_μ}, H-L: $\mathcal{J}_{83}$, H-H: $\mathcal{J}_{3\bar{3}}$)

Breaking Parameters $m_{8^*} = m_L = 800$ MeV, $m_{3^*} = m_H = 2050$ MeV, $m_{1^*} = m_{J/\psi} = 3097$ MeV, $f_\pi = 93$, $f_D = 165$ MeV.

$$\boxed{\gamma = \left(\frac{m_{8^*}}{m_{3^*}}\right)^2 = \frac{m_L^2}{m_H^2},\ \psi = \left(\frac{m_{8^*}}{m_{1^*}}\right)^2 = \frac{m_L^2}{m_{J/\psi}^2}}$$

$$V \equiv \begin{pmatrix} \frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & \rho^+ & K^{*+} & \bar{D}^{*0} \\ \rho^- & -\frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & K^{*0} & D^{*-} \\ K^{*-} & \bar{K}^{*0} & \phi & D_s^{*-} \\ D^{*0} & D^{*+} & D_s^{*+} & J/\psi \end{pmatrix}_\mu$$

D. Gamermann and E. Oset, Eur. Phys. J. A **36**, 189 (2008)

D. Gamermann and E. Oset, Phys. Rev. D **80**, 014003 (2009)

D. Gamermann, J. Nieves, E. Oset and E. Ruiz Arriola, Phys. Rev. D **81**, 014029 (2010)

The X(3872) in the continuum limit

Bethe-Salpeter Eq.: $T = -[I + VG]^{-1} V \quad V = -\frac{\xi_{ij}}{4f^2}(s-u)\vec{\epsilon}\cdot\vec{\epsilon}'\vec{\epsilon}\cdot\vec{\epsilon}'$

Channels: $\frac{1}{\sqrt{2}}(K^{*-}K^{+} - c.c.)$, $\frac{1}{\sqrt{2}}(\bar{K}^{*0}K^{0} - c.c.)$, $\frac{1}{\sqrt{2}}(D^{*+}D^{-} - c.c.)$, $\frac{1}{\sqrt{2}}(D^{*0}\bar{D}^{0} - c.c.)$, $\frac{1}{\sqrt{2}}(D_s^{*+}D_s^{*-} - c.c.)$.

$$\xi_{ij} \equiv \begin{pmatrix} -3 & -3 & 0 & -\gamma & \gamma \\ -3 & -3 & -\gamma & 0 & \gamma \\ 0 & -\gamma & -(1+\psi) & -1 & -1 \\ -\gamma & 0 & -1 & -(1+\psi) & -1 \\ \gamma & \gamma & -1 & -1 & -(1+\psi) \end{pmatrix}$$

$\gamma = 0.14 \qquad \psi = 0.07$

For G, dim. regularization formula or cutoff method can be used,

$$\begin{aligned} G^{DR}(\sqrt{s}) &= \frac{1}{16\pi^2}\Big\{\alpha(\mu) + \ln\frac{m_1^2}{\mu^2} + \frac{m_2^2 - m_1^2 + s}{2s}\ln\frac{m_2^2}{m_1^2} + \\ &+ \frac{q}{\sqrt{s}}\Big[\ln(s - (m_2^2 - m_1^2) + 2q\sqrt{s}) + \ln(s + (m_2^2 - m_1^2) + 2q\sqrt{s}) \\ &- \ln(-s + (m_2^2 - m_1^2) + 2q\sqrt{s}) - \ln(-s - (m_2^2 - m_1^2) + 2q\sqrt{s})\Big]\Big\}\ , \end{aligned} \tag{2}$$

$$G^{co}(P^0 = \sqrt{s}) = \int_{q<q_{max}} \frac{d^3q}{(2\pi)^3}\,\frac{\omega_1 + \omega_2}{2\omega_1\omega_2}\,\frac{1}{(P^0)^2 - (\omega_1 + \omega_2)^2 + i\epsilon} \tag{3}$$

The X(3872) in the continuum limit

Near to a pole: $T \sim \frac{g_i g_j}{(s-s_p)} \vec{\epsilon} \cdot \vec{\epsilon}'$

$\sqrt{s_0} = (3871.6 - i0.001)$ MeV	
Channel	$\|g_i\|$ [MeV]
$\frac{1}{\sqrt{2}}(K^{*-}K^{+} - c.c)$	53
$\frac{1}{\sqrt{2}}(\bar{K}^{*0}K^{0} - c.c)$	49
$\frac{1}{\sqrt{2}}(D^{*+}D^{-} - c.c)$	3638
$\frac{1}{\sqrt{2}}(D^{*0}\bar{D}^{0} - c.c)$	3663
$\frac{1}{\sqrt{2}}(D_s^{*+}D_s^{-} - c.c)$	3395

Table 1. Couplings of the pole to the channel i with $\alpha_H = -1.265$.

D. Gamermann, J. Nieves, E. Oset and E. R. Arriola, PRD 81, 014029

Generalized compositeness condition:

$$-\sum_i g_i^2 \frac{\partial G}{\partial s} = 1$$

Probability of finding the i ch. in the wave func.,

0.86 for $D^{*0}\bar{D}^{0} - c.c$,
0.124 for $D^{*+}D^{-} - c.c$
and 0.016 for $D_s^{*+}D_s^{-} - c.c$.

However $(2\pi)^{3/2}\psi(0)_i = g_i G_i$ (wave function at the origin) are nearly equal, and this usually enters the evaluation of observables.

Formalism in finite volume

$$G \to \tilde{G} : \quad \tilde{G}(P^0) = \frac{1}{L^3} \sum_{\vec{q}_i} \frac{\omega_1(\vec{q}_i)+\omega_2(\vec{q}_i)}{2\omega_1(\vec{q}_i)\omega_2(\vec{q}_i)} \frac{1}{(P^0)^2-(\omega_1(\vec{q}_i)+\omega_2(\vec{q}_i))^2}$$
$$= \frac{1}{L^3} \sum_{\vec{q}_i} I(P^0, \vec{q})$$

where $\omega_i = \sqrt{m_i^2 + |\vec{q}_i|^2}$ and the momentum $\vec{q}$ is quantized as $\vec{q}_i = \frac{2\pi}{L}\vec{n}_i$, $|\vec{q}_i| = \frac{2\pi}{L}\sqrt{m_i}$, $n_{x,i}^2 + n_{y,i}^2 + n_{z,i}^2 = m_i$ and $n_{max} = \frac{q_{max}L}{2\pi}$ (symmetric box).

A. M. Torres, L. R. Dai, C. Koren, D. Jido, and E. Oset, PRD 85, 014027

$$\begin{aligned} \tilde{G} &= G^{DR} + \lim_{q_{max}\to\infty} \left(\frac{1}{L^3} \sum_{q<q_{max}} I(P^0, \vec{q}) - \int_{q<q_{max}} \frac{d^3q}{(2\pi)^3} I(P^0, \vec{q}) \right) \\ &\equiv G^{DR} + \lim_{q_{max}\to\infty} \delta G \end{aligned} \quad (4)$$

$$T \to \tilde{T} : \quad \tilde{T} = (I - V\tilde{G})^{-1} V$$

Energy levels in the box:

$$\boxed{det(I - V\tilde{G}) = 0}$$

Formalism in finite volume

- δG converges as $q_{\text{max}} \to \infty$. We take and average for different values between $q_{\text{max}} = 1500 - 2500$ MeV.

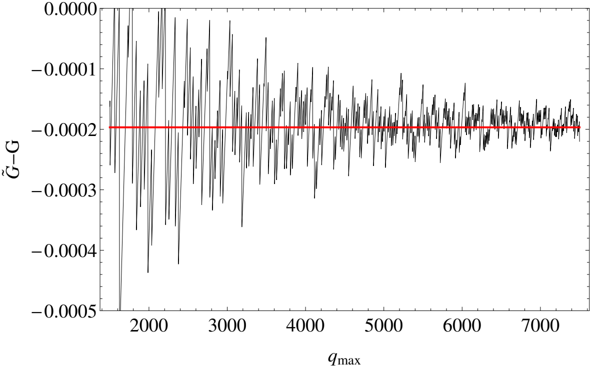

Fig. 2. Representation of $\delta G = \tilde{G} - G$ for $D^+ D^{*-}$ in function of q_{max} for $\sqrt{s} = 3850$ MeV. The thick line represents the average.

Formalism in finite volume

- **One channel case**: $T = (\tilde{G}(E_i) - G(E_i))^{-1}$

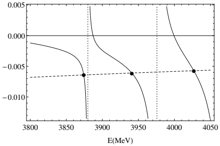

Fig. 3. $\tilde{G}$(solid) and V^{-1}(dashed) energy dependence of D^+D^{*-} for $Lm_\pi = 2.0$. Dotted lines are the free energies.

- **Energy levels in a cubic box for one channel**:

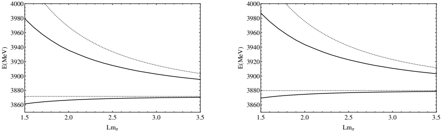

Fig. 4. L dependence of the energies for a single channel, $D^0\bar{D}^{*0}$ and D^+D^{*-} respectively.

Formalism in finite volume

- At $Lm_\pi = 1.4$ ($L = 2$ fm), $\Delta E = E_2(L) - E_1(L) = 137$ MeV (if $V = 0$ $\Delta E^0 = 194$ MeV). While in the approach of S. Prelovsek and L. Leskovec, it is 161 MeV: Both approaches have an attractive interaction with similar strength, $E_1 = 3860$ vs. (3853 ± 8) MeV, $E_2 = 3997$ vs. (4014 ± 11) MeV.

S. Prelovsek and L. Leskovec, Phys. Rev. Lett. **111**, 192001 (2013)

- **Two channel case**: For simplicity, we redone the calculation of Table 1 with $\alpha = -1.153$ in order to get the same position of the pole of the X(3872) with two channels instead of five.

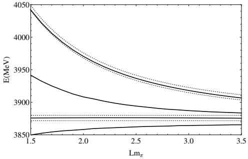

Fig. 5. L dependence of the energies for the two first levels of $D^0\bar{D}^{*0}$ and D^+D^{*-}. Dotted lines correspond to the free energies.

The inverse problem

QCD lattice data can be used to determine bound states of the $D\bar{D}^*$ system,

- We assume that the lattice data are some discrete points on the energy trajectories (synthetic data).
- We want to determine the potential and evaluate the pole position of the X(3872) in infinite volume.
- A set of data of 5 points in a range of $Lm_\pi = [1.5,\ 3.5]$ for each level (four levels with $n = 0$ and 1) with uncertainties, moving randomly by 1 MeV the centroid assigning an error of 2 MeV, are generated.

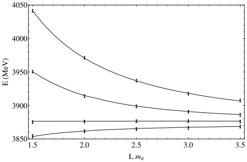

Fig. 6. Fit to the data. Dots: synthetic data. Solid lines: energy levels using the potential fitted to the data.

The inverse problem

Potential (six parameters to fit) : $V_i = a_i + b_i\left(\sqrt{s} - \sqrt{s^{th}}\right)$

$i = 1: D^+D^{*-}, \; i = 2: D^0\bar{D}^{*0} : i = 3:$ nondiagonal

The χ^2 function is **minimized**. The binding energy is essentially **independent** of the choice of α (one channel: $T = (\tilde{G}(E_i) - G(E_i))^{-1}$).

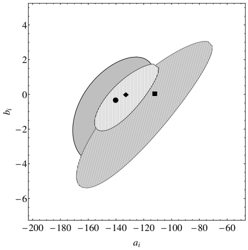

Fig. 6. Contour plot for the χ^2 representing $\chi^2 \leq \chi^2_{min} + 1$. Points correspond to values of the parameters in the χ^2 minimum. (Circle and grey area: a_1 and b_1, Square and diagonal lined area: a_2 and b_2 and Diamond and vertical lined area: a_3 and b_3.)

Results

(B,P,ΔE,ΔC)	(a_1 , a_2 , a_3)	(b_1 , b_2 , b_3)	χ^2	Pole	Mean Pole	σ
(4,5,2,1)	(-140.2,-112.1,-132.8)	(-0.31, 0.074, 0.012)	2.32	3871.51	3871.49	0.07
(4,5,5,2)	(-140.2,-112.1,-132.8)	(-0.31, 0.074, 0.012)	0.79	3871.51	3871.25	0.38
(4,3,2,1)	(-133.0,-131.9,-124.6)	(-0.24, 0.048,-0.075)	1.02	3871.44	3871.49	0.18
(4,3,5,2)	(-120.1, -98.2,-150.9)	(-0.38,-0.075, 0.102)	0.28	3871.41	3871.15	0.49
(2,5,2,1)	(-176.1,-154.1, -89.3)	(9.92, 7.01, -8.72)	0.259	3871.70	3871.47	0.30
(2,5,5,2)	(-158.5,-152.2,-103.2)	(4.56, 6.58, -6.74)	0.982	3871.34	3871.30	0.43
(2,3,2,1)	(-132.7,-176.6,-105.5)	(3.23, 0.84, -3.36)	0.074	3870.51	3870.48	0.61
(2,3,5,2)	(-226.6,-194.5, -32.7)	(31.81,13.28,-18.89)	0.942	3869.49	3870.37	1.06

Table 2. All possible set up changing number of branches (B), number of points (P), energy error bar (ΔE) and centroid of the energies (ΔC) and their set of parameters fitted. The columns denoted as Results are the χ^2 obtained in the fit, the pole is determined with the parameters, and the mean pole and dispersion.

- The use of different values of α change the potential but not the binding energy.
- With errors in the data of 5 MeV, one can obtain the binding energy with 1 MeV precision, and two levels are enough to have an accurate value.
- To have a very high precision in the binding energy ($\sim$ 0.2 MeV), requires high precision in the data.
- It is necessary to distinguish between the two channels.

Results

- Considering the channel $J/\psi\omega$ does not change the results since the coupling between $D\bar{D}^*$ goes through anomalous couplings VVP together with the exchange of the heavy meson. This agrees with S. Prelovsek and L. Leskovec, PRL 111, 192001: 'the $J/\psi\omega$ is not significantly coupled to the rest of the system'.

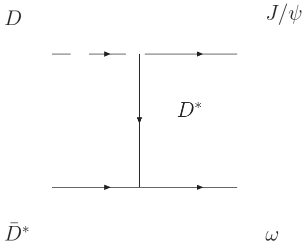

Fig. 7. Mechanism for the transition from $D\bar{D}^*$ to $J/\psi\omega$.

Results

In addition, we can know about the nature of the X(3872),

- If the X(3872) was genuine, we can generate it using a potential containing a CDD pole

$$V = V_M + \frac{g^2_{\rm CDD}}{s - s_{\rm CDD}} \quad (Castillejo, Dalitz, Dyson) \tag{5}$$

  with $V_M = 1/10\, V'_M$. Taking $\sqrt{s_{\rm CDD}}$ as 20 MeV above the threshold, we obtain $g_{\rm CDD} = 4620$ MeV.

- With the two lower levels, we get $-\sum_{i=1}^{2} g_i^2 \frac{\partial G_i}{\partial s} = 1 - Z = 0.51$. This tell us that the state has a large genuine component $Z \simeq 0.5$, or $Z \simeq 0.63$ if we consider the shape of Eq. (5).

- On the contrary, if $V_M = V'_M$, $-\sum_{i=1}^{2} g_i^2 \frac{\partial G_i}{\partial s} = 1 - Z = 0.97$.

E. J. Garzon, R. Molina, A. Hosaka and E. Oset, Phys. Rev. D **89**, 014504 (2014).

Conclusions

We have studied the X(3872) state using coupled channels D^+D^{*-} and $D^0\bar{D}^0$ in a finite box simulating lattice data and showing some strategies to extract the binding energy.

- We obtain two energy curves for each level corresponding to the neutral and charged channels. It is necessary to differentiate them to have an accurate value of the binding energy.
- Even with errors in the data points of $\simeq 5$ MeV and two levels, one can obtain the binding energy with $\simeq 1$ MeV precision.
- Having precise data allows us to get some information on the nature through the generalized compositeness condition.