Charm baryons on the lattice

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- Collaborators : R. G. Edwards, N. Mathur and M. Peardon (For HSC)
- Acknowledgements : TIFR, Mumbai & Austrian Science Fund (FWF)
- Thanks to those who provided me the material

Outline

ABC?

Low lying spectrum from lattice QCD

Excited charm baryon spectrum

Summary

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ABC?

- **ABC** : **A**spiring study of **B**aryons with **C**harm quarks.
- **The heavy flavor tag** :
  Mechanism of confinement and systematics of hadron resonances that are obscure due to the chiral dynamics in light baryons. Particularly Ω_{ccc}.
- **Detection and isolation** : relatively easy.
  Expected to be relatively free of nearby overlapping resonances.
  Production? : No known resonant production mechanism
  Rely on continuum production
- **Spin identification** :
  Most assignments based on quark model expectations!
- **Heavy quark symmetry (HQS)** :
  Qualitative insight into light baryon spectrum (hyperons).
  The quark-diquark picture and the missing baryon resonances.

Shirotori *et al.*, JPCS 569, no. 1, 012085

Baryons with $C = 3, 2$ *and* 1

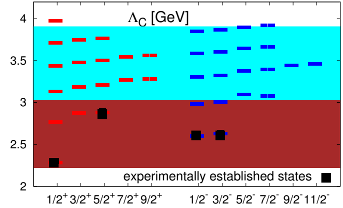
- **Triply charm baryons** :
  Charmonia analogues in baryons.
  Platform to study quark confinement mechanism.
  The triply charmed baryons may provide a new window for understanding the structure of baryons.
  J. D. Bjorken, Report No. FERMILAB-CONF-85/69.

- **Doubly charm baryons** :
  Observations only by SELEX (losing confidence)
  Failed to be observed in FOCUS, Belle, BaBar and LHCb.
  Very large isospin splittings : 9 and 21 MeV.
  HQS : $\lim(m_Q \to \infty)$ J_{light} is a conserved quantum number.

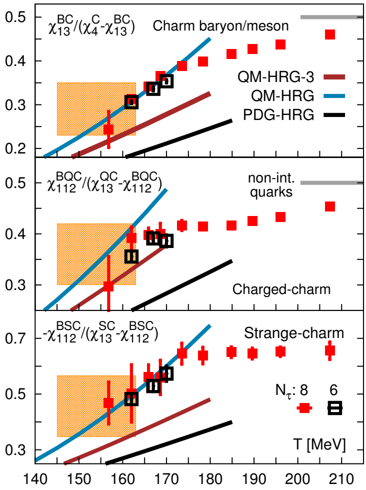
- **Singly charm baryons** :
  20 states with *** or more.
  More levels expected to be observed.
  Interesting indications for the existence of many charm baryons from finite temperature lattice calculations
  HQS : $\lim(m_Q \to \infty)$ J_{light} is a conserved quantum number.
  Light quark dynamics around a static color source.
  Corrections of the $O(\Lambda_{QCD}/m_Q)$.

Indications from finite temperature studies

Ebert *et al.*, PRD 84 014025



Bazavov *et al.*, PLB 737, 210



- **Charm hadron pressure (HRG)** :

$$
\begin{aligned}
P(\hat{\mu}_C, \hat{\mu}_B) &= P_M cosh(\hat{\mu}_C) \\
&\quad + P_{B,C=1} cosh(\hat{\mu}_C + \hat{\mu}_B)
\end{aligned}
$$

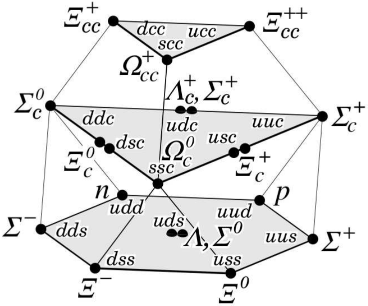
$$
\chi_{kl}^{BC} = \frac{\partial^{(k+l)}[P(\hat{\mu}_C, \hat{\mu}_B)/T^4]}{\partial \hat{\mu}_B^k \partial \hat{\mu}_C^l}
$$

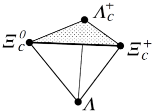
$\Rightarrow$ **Existence of additional charm-light baryons in QGP formed in HIC.**

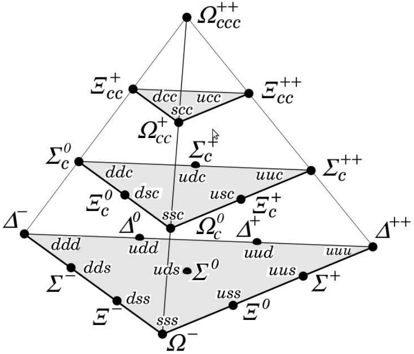
Lattice study of charm baryons

- **Non-perturbative study** : A comprehensive lattice QCD study of spectrum, including excited states, of charm baryons.
- **Predictions and postdictions** : Confirm and guide the experimental searches.
- **Precision Spectroscopy** : Aimed at low lying spectrum.
- **Excited state measurement** : Understanding the spectral patterns. First step towards that goal made. Efforts on the way to 'precision' spectroscopy of excited states.

Charm baryons : SU(4) classifications







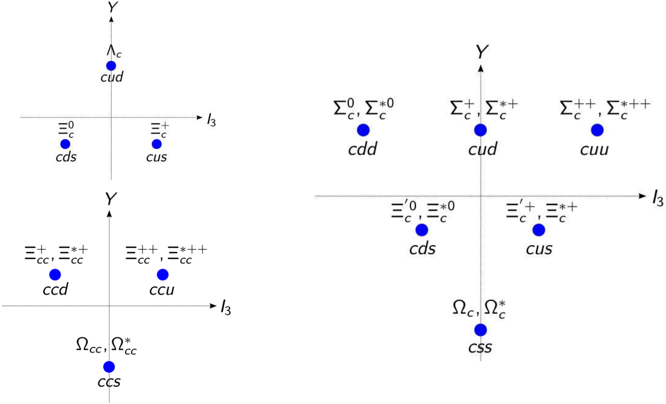
$$4 \otimes 4 \otimes 4 = 20_S \oplus 20_M \oplus 20_M \oplus 4_A$$

Broken flavor symmetry. Classification for enumerating the possible states.
Physical states could be mixture of these multiplets.

Charm baryons : HQET + SU(3)

$$C = 1 \;:\; 3 \otimes 3 = \bar{3}_A \oplus 6_S$$
$$C = 2 \;:\; 3$$

The symmetries are with respect to the light quarks.
The charm quarks are considered as spectators.



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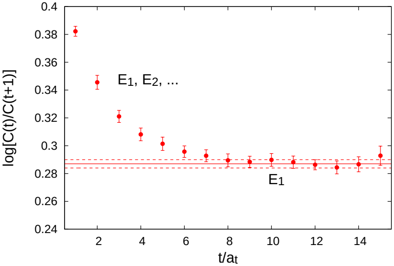
QCD spectrum from Lattice QCD

- Aim : to extract the physical states of QCD.
- Euclidean two point current-current correlation functions

$$C_{ji}(t_f - t_i) = \langle 0|\Phi_j(t_f)\bar{\Phi}_i(t_i)|0\rangle = \sum_n \frac{Z_i^{n*} Z_j^n}{2m_n} e^{-m_n(t_f - t_i)}$$

where $\Phi_j(t_f)$ and $\bar{\Phi}_i(t_i)$ are the desired interpolating operators and $Z_j^n = \langle 0|\Phi_j|n\rangle$.

- Effective mass defined as $\log[\frac{C(t)}{C(t+1)}]$



- The ground states : from the exponential fall off at large times.
  Non-linear fitting techniques.

Lattices in use

ID (pub.)	Gluons	$(u, d)_{sea}$	s_{sea}	c_{sea}	$(u, d)_{val}$	s_{val}	c_{val}
Liu(PRD 81 094505)	LW	AsqTad	AsqTad	-	DW	DW	RHQ
Briceño(PRD 86 094504)	LW	HISQ	HISQ	HISQ	clover	clover	RHQ
ILGTI(PoS Lattice2012)	LW	HISQ	HISQ	HISQ	Overlap	Overlap	Overlap
PACS-CS(PRD 87 094512)	Iw	clover	clover	-	clover	clover	RHQ
ETMC(PRD 90 074501)	Iw	TM	TM	TM	TM	OS	OS
Brown(PRD 90 094507)	Iw	DW	DW	-	DW	DW	RHQ
RQCD(arXiv:1503.08440)	LW	clover	clover	-	clover	clover	clover
HSC(arXiv:1502.01845)	SIa	clover	clover	-	clover	clover	clover

Gauge actions

SIa : Symanzik improved anisotropic gluonic action
LW : Lüscher-Weisz gluonic action
Iw : Iwasaki gluonic action

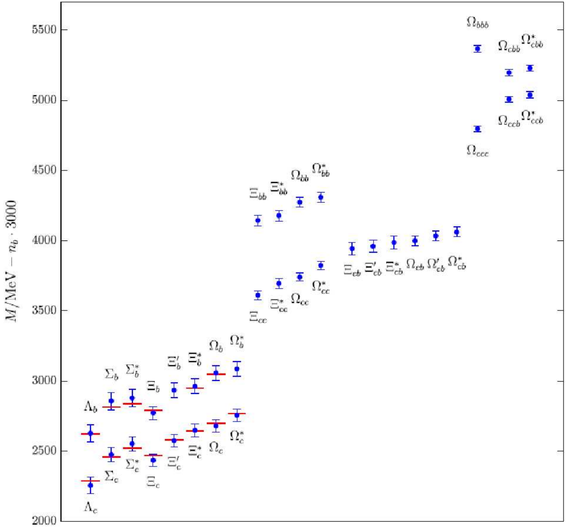
Fermion actions

HISQ : Highly Improved Staggered Quarks
TM : Twisted Mass
DW : Domain Wall
AsqTad: $O(a^2)$, Tadpole improved staggered
OS : Osterwalder-Seiler

All calculations with $m_u = m_d$ and neglect QED $\Rightarrow$ no isospin splittings.
All baryons in the same isospin multiplet appears at same energy.
Dynamical calculations from 2009 onwards.

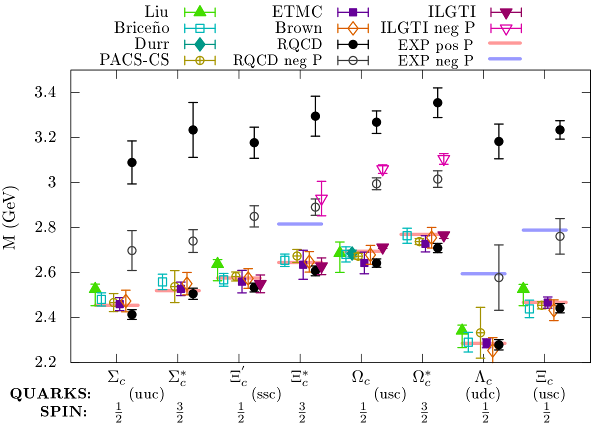
- **Briceño, Brown and ETMC : Chiral (χPT) and continuum extrapolated results**
- **PACS-CS : Measurements at physical point**
- **RQCD : Physical point approached based on Gell-Mann-Okubo relations.**

Low lying heavy baryons



Brown *et al.*, PRD 90 094507.

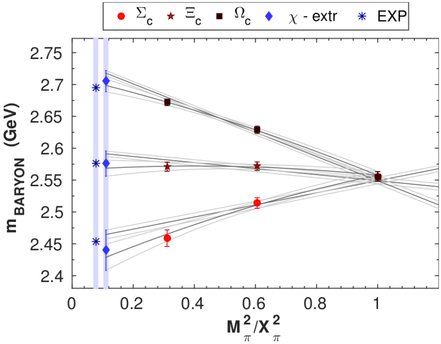
Low lying singly charm baryons



Bali *et. al.*, arXiv:1503.08440[hep-lat].

- **Ground states more or less in agreement between all lattice results and experiments.**
- **Improving control over the systematic and statistical uncertainties.**
- **The excited state determination : challenging!**
- **Systematic spin identification : Even more challenging!!**

Chiral extrapolations



Sextet

$$m_{\Sigma_c^{(*)}} = m_0 - \tfrac{2}{3}A\delta m_\ell + \mathrm{O}(\delta m_\ell^2)$$

$$m_{\Xi_c'^{(*)}} = m_0 + \tfrac{1}{3}A\delta m_\ell + \mathrm{O}(\delta m_\ell^2)$$

$$m_{\Omega_c^{(*)}} = m_0 + \tfrac{4}{3}A\delta m_\ell + \mathrm{O}(\delta m_\ell^2)$$

Anti-triplet

$$m_{\Lambda_c} = m_0 - \tfrac{2}{3}B\delta m_\ell + \mathrm{O}(\delta m_\ell^2)$$

$$m_{\Xi_c} = m_0 + \tfrac{1}{3}B\delta m_\ell + \mathrm{O}(\delta m_\ell^2)$$

Triplet

$$m_{\Xi_{cc}^{(*)}} = m_0 - \tfrac{1}{3}C\delta m_\ell + \mathrm{O}(\delta m_\ell^2)$$

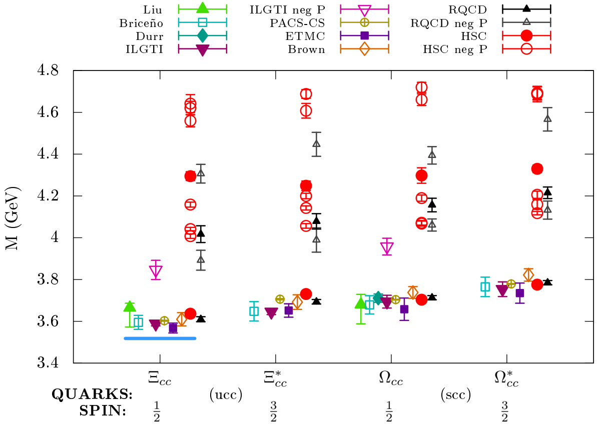
$$m_{\Omega_{cc}^{(*)}} = m_0 + \tfrac{2}{3}C\delta m_\ell + \mathrm{O}(\delta m_\ell^2)$$

$$\delta m_\ell = m_s - m_{u/d} \propto 1 - M_\pi^2/X_\pi^2 + O((\delta m_\ell)^2)$$

Chiral extrapolations based on Gell-Mann-Okubo formulae.

Bali *et. al.*, arXiv:1503.08440[hep-lat].

Low lying doubly charm baryons

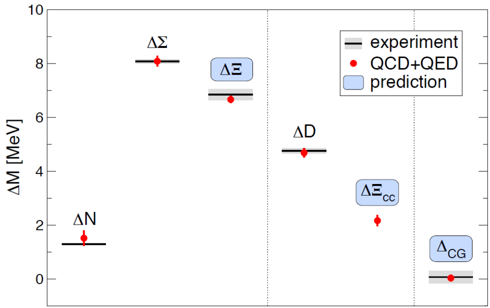


Bali *et. al.*, arXiv:1503.08440[hep-lat].

- **The only experimental candidate (SELEX) : seems very low.**
- **On average lattice results agree between them.**
- **Improving control over the systematic and statistical uncertainties.**
- **The challenging excited states and spin identification!**

Ξ_{cc} Isospin splittings

- The lowest isospin doublet (SELEX) has splitting 9 MeV.
- The largest isospin splitting ever observed in Ξ_{ssq} : 6.85 ± 0.21 MeV

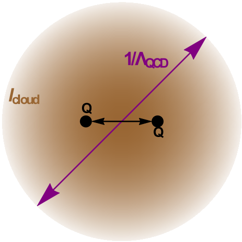


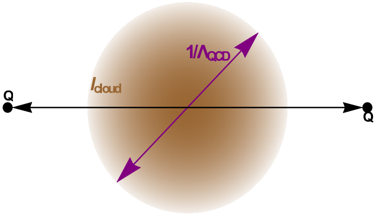
- Fully controlled ab initio calculation with 1+1+1+1 flavor QCD+QED with clover improved Wilson quarks.
- Precision of low energy description is down to per mil level.
- Precision at a level of challenging the experimental numbers.

- Irreducible uncertainties is down to $O(1/N_c/m_b^2, \alpha^2)$.
- Coleman-Glashow relation : $\Delta_{CG} = \Delta M_N - \Delta M_\Sigma + \Delta M_\Xi = 0$.

Borsanyi, *et. al.*, Science Vol. 347 no. 6229 pp. 1452-1455

The doubly heavy picture (B_{QQ})





- **quark-diquark picture** :
  HQET motivated
  Heavy-light meson-like system.
  HQS : $\lim(m_Q \to \infty)$

$$\frac{M(B^*_{QQ}) - M(B_{QQ})}{M(V_Q) - M(PS_Q)} \to \frac{3}{4}$$

  Brambilla *et al.*, hep-ph/0506065
  Low lying levels.

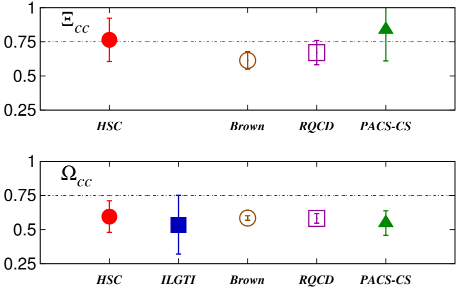
- **Charmonium-like system** :
  Valid for excited states.
  Demands precision measurements of excited levels

HQET expectations

For a heavy-light meson-like system of doubly charm baryons (B_{QQ})

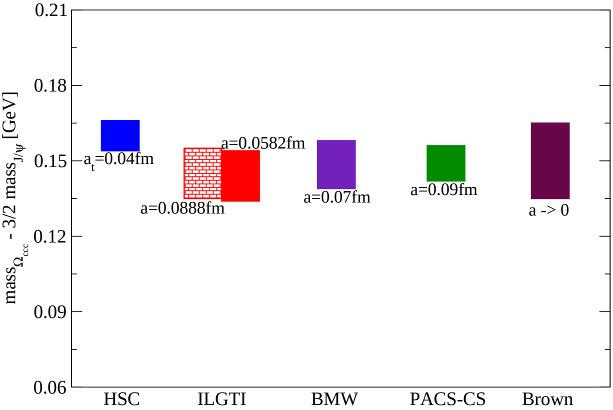
$$\frac{M(B^*_{QQ}) - M(B_{QQ})}{M(V_Q) - M(PS_Q)} \rightarrow \frac{3}{4}$$

Brambilla *et al.*, hep-ph/0506065



- Ω_{cc} consistently below 0.75 in all measurements.
- The study of systematics in 'ILGTI' are in process.
- Systematic uncertainties in 'HSC' and 'PACS-CS'!
- Results from 'Brown' and 'RQCD' indicates similar pattern in Ω_{cc} and Ξ_{cc}.

The triply charm baryon



MP *et al.*, PRD 90 074504.

Outline

Excited charm baryon spectrum

- Prospects include zero to finite temperature physics.
  - Answering fundamental questions
  - Compare, inform and guide the experimental programs
  - Finite temperature prospects
- Challenges include extracting densely populated spectra.
  - Extracting densely populated states
  - Extracting radial and orbital excitations
  - Extracting excitations with spin $> 3/2$
  - Systematic spin identification
  - Multiple scattering channels affecting the single hadron spectra
- Scattering parameters from finite volume energy shifts. Lüscher's formalism and its various generalizations.
- Encouraging achievements in the light and heavy meson spectra

Dudek *et al.* [HSC] PRL 113, 182001

Talk by Sasa Prelovsek (Mon, S1), Daniel Mohler (Mon, S2)

The variational method

Two-point correlator

$$C_{ij}(t) = \langle 0|\Phi_i(t)\Phi_j^\dagger(0)|0\rangle$$

$$C_{ij}(t) = \sum_{\mathfrak{n}} e^{-E_{\mathfrak{n}}t}\langle 0|\Phi_i(0)|\mathfrak{n}\rangle\langle\mathfrak{n}|\Phi_j^\dagger(0)|0\rangle$$

$$Z_i^{\mathfrak{n}} \equiv \langle\mathfrak{n}|\ \Phi_i^\dagger\ |0\rangle$$

Matrix of correlators

$$C(t) = \begin{pmatrix} \langle 0|\Phi_1(t)\Phi_1^\dagger(0)|0\rangle & \langle 0|\Phi_1(t)\Phi_2^\dagger(0)|0\rangle & \cdots \\ \langle 0|\Phi_2(t)\Phi_1^\dagger(0)|0\rangle & \langle 0|\Phi_2(t)\Phi_2^\dagger(0)|0\rangle & \cdots \\ \vdots & & \ddots \end{pmatrix}$$

"Rayleigh-Ritz method"
Diagonalize:
- eigenvalues → spectrum
- eigenvectors → spectral "overlaps" Z_i^n

Each state optimal combination of Φ_i

$$\Omega^{(\mathfrak{n})} = \sum_i v_i^{(\mathfrak{n})}\Phi_i$$

Benefit: orthogonality for near degenerate states

R. G. Edwards slides

Baryon operators

Construction : permutations of 3 objects

- Symmetric:
  - •e.g., uud+udu+duu
- Antisymmetric:
  - •e.g., uud-udu+duu-…
- Mixed: (antisymmetric & symmetric)
  - •e.g., udu - duu & 2duu - udu - uud

Multiplication rules:
- Symmetric Antisymmetric $\rightarrow$ Antisymmetric
- MixedxMixed $\rightarrow$ Symmetric$\oplus$Antisymmetric$\oplus$Mixed
- ….

Color antisymmetric $\rightarrow$ Require **Space x [Flavor x Spin]** symmetric

Space: couple covariant derivatives onto single-site spinors - build any J,M

$$\Phi^{JM} \leftarrow (CGC's)_{i,j,k} \left[\vec{D}\right]_i \left[\vec{D}\right]_j [\Psi]_k$$

$$J \leftarrow \mathbf{1} \otimes \mathbf{1} \otimes S$$

Classify operators by permutation symmetries:
- Leads to rich structure

Edwards *et al.*, PRD 84 074508

R. G. Edwards slides

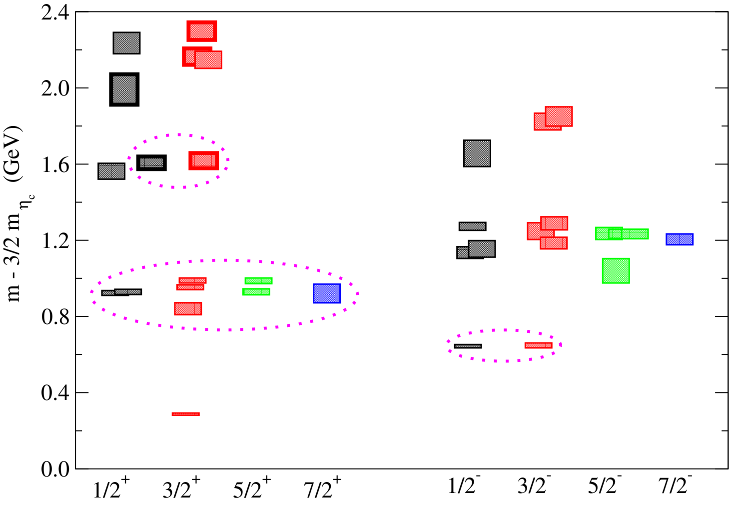
HSC lattices and caveats

- ♣ Anisotropic lattices : $a_s \neq a_t$
  Non-perturbative tuning of action parameters : $\xi = a_s/a_t \sim 3.5$
- ♣ $N_f = 2 + 1$ dynamical configurations
- ♣ $16^3 \times 128$ lattices with $L \sim 2$fm.
- ♣ 96 gauge field configurations.

- ♣ Continuum limit not taken.
- ♣ Finite size effects; only one volume, $L \sim 2$fm
- ♣ Heavy pion mass; $m_\pi \sim 400$MeV
- ♣ only single hadron operators $\Rightarrow$ No resonance properties

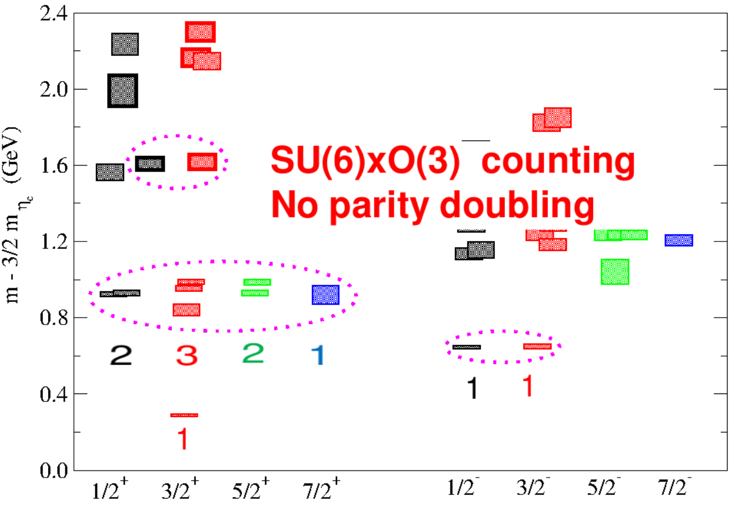
- ♣ Pioneering work on study of charm baryon excited states.
  Precision and systematics : temporarily relaxed (costly).
  Precision determination of excited states : A challenge we are working for.

Ω_{ccc} spectrum



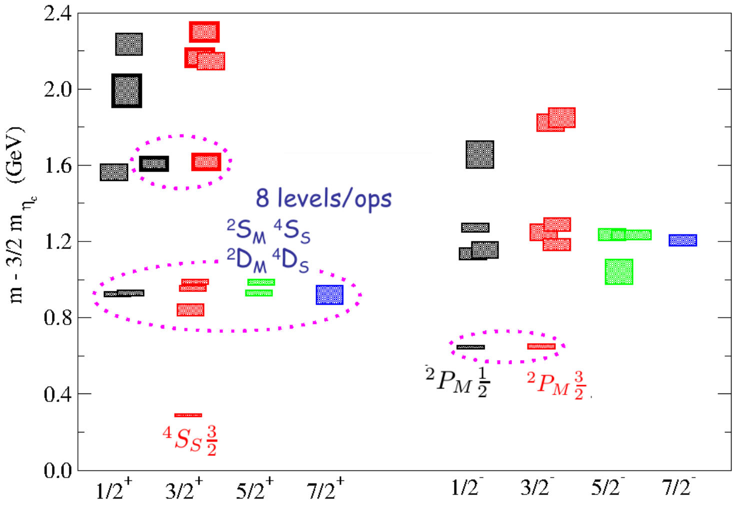
MP *et al.*, PRD 90 074504; $m_\pi = 391$ MeV

Ω_{ccc} spectrum



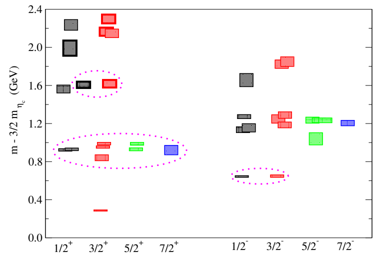
Consistent with $SU(3)_F \otimes SU(2)_S \otimes O(3)$ expectations

Ω_{ccc} spectrum

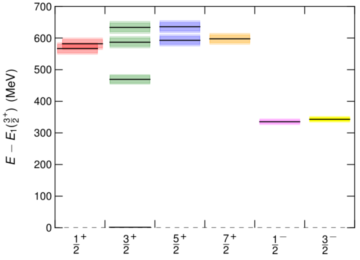


Consistent with $SU(3)_F \otimes SU(2)_S \otimes O(3)$ expectations

A comparison between Ω_{ccc} and Ω_{bbb}



Ω_{ccc} : MP *et al.*, PRD 90 074504

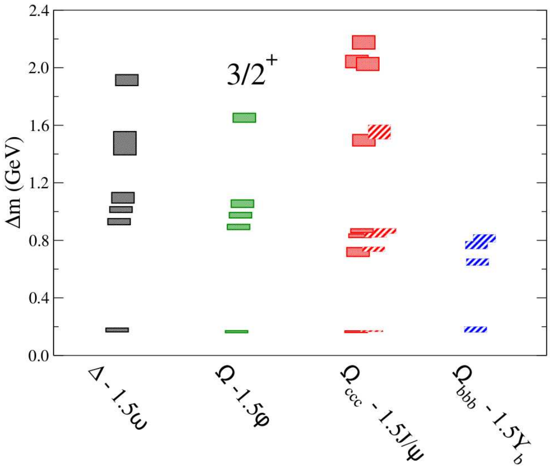


Ω_{bbb} : Meinel, PRD 85 114510

- The spectral pattern remains same up to second excitation band.
- PRD90 with considers relativistic operators also.
  Hence the multitude of states.

Quark mass dependence : Δ-like baryons

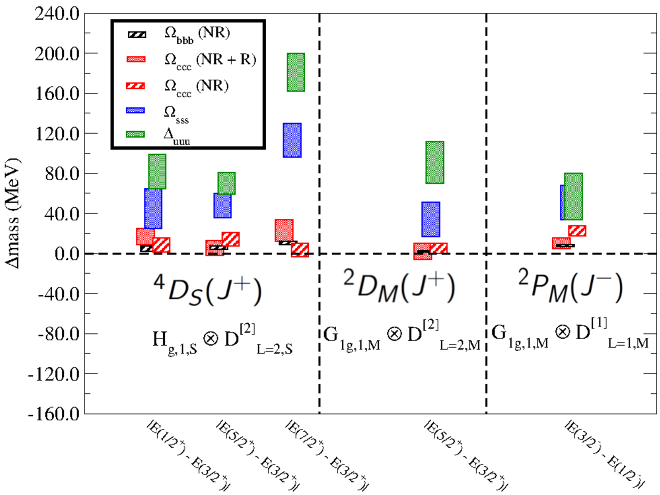
MP *et al.*, PRD 90 074504



- The spectral pattern remain more or less same from light to bottom m_q.
- The binding energy decreases with increasing m_q.

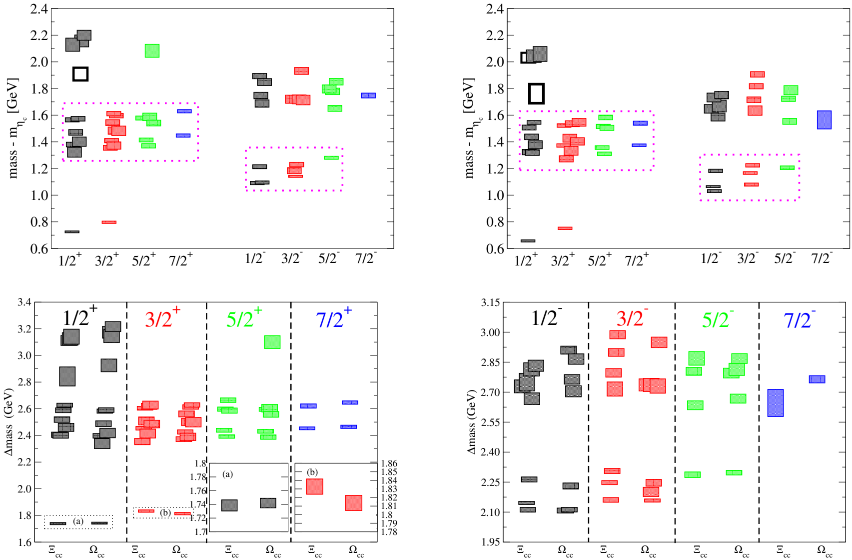
Heaviness of the quark : SO splitting

MP *et al.*, PRD 90 074504



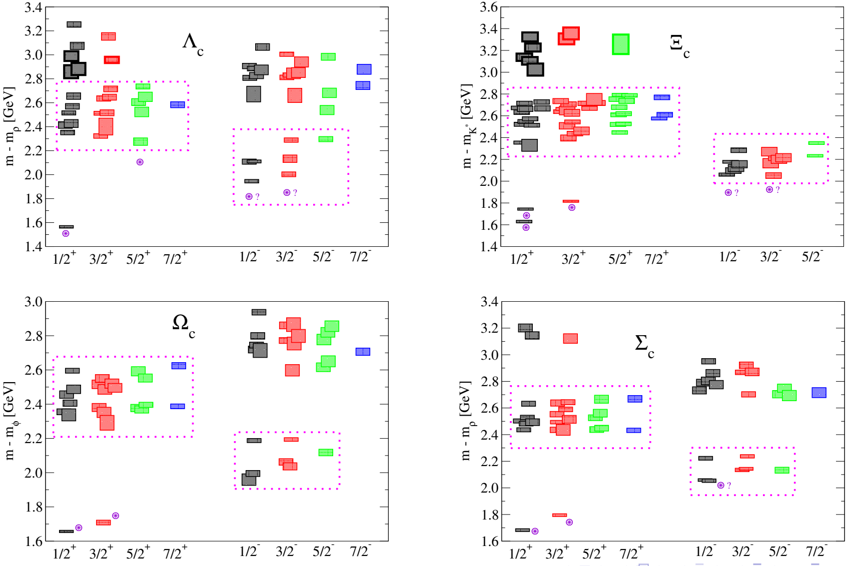
- m_c found to be near to heavy with almost vanishing SO splitting

Doubly charm baryon spectrum



MP *et al.*, PRD 91 094502

Singly charm baryons



Excited state studies

- Systematic extraction of various radial and orbital excitations.
- Systematic methodology for spin identification.
- Broadly consistent non-relativistic quark model.
- No "freezing degrees of freedom" nor parity doubling.
- Yes! There are caveats
  - Continuum limit not taken.
  - Finite size effects; only one volume, $L \sim$ 2fm
  - Heavy pion mass; $m_\pi \sim$ 400MeV
  - only single hadron operators $\Rightarrow$ No resonance properties

  However, a pioneering step towards precision excited state spectroscopy.
- Continuing efforts : Include the effects of baryon-meson interpolators, investigate widths, improving control over systematics.
  Cost of computing increases.

Outline

Summary

- High precision lattice calculations of low lying charm baryons.
- High precision lattice measurement of Ξ_{cc} isospin splitting (2.16(11)(17) MeV). Looking forward to see more results from BMW-c's precision measurements.
- Heavy diquark-antidiquark symmetry : With increasing precision lattice measurements this will be put to test.
- Excited charm baryon spectra extraction using a systematic operator construction procedure.
- First results suggest the spectrum to be broadly consistent with non-relativistic quark model.
- Promising results, however expensive.
  Currently studies made on single L, a and an unphysically heavy m_π.
  Efforts are on the way for calculations with controlled systematics.
- Not covered here : electromagnetic form factors[Can *et al.*, JHEP(2014)125], σ terms, axial charges [Hadjiyiannakou *et al.*, Lattice 2014] and heavy baryon decay widths [Detmold *et al.*, PRL 108 172003].

Thank you...

Backups

HQET expansion for energy splittings

- Consider the energy splittings

$$(\Xi^*_{cc} - D,\ \Omega^*_{cc} - D_s,\ \Omega^*_{ccc} - \eta_c \text{ and } \Omega^*_{ccb} - B_c),$$
$$(\Xi^*_{cc} - D^*,\ \Omega^*_{cc} - D^*_s,\ \Omega^*_{ccc} - J/\psi \text{ and } \Omega^*_{ccb} - B^*_c)$$

- Extrapolation of the fit to these splittings $\rightarrow m_{B^*_c} - m_{B_c}$.
- Heavy Quark Effective Theory (HQET) : Mass of a heavy hadron,

$$m_{H_n\ Q} = n\ m_Q + A + B/m_Q + O(1/m_Q^2).$$

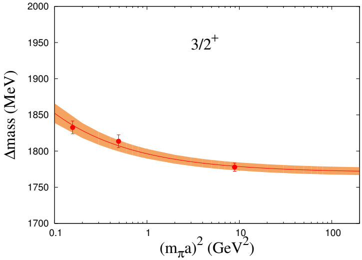
Jenkins, PRD 54, 4515

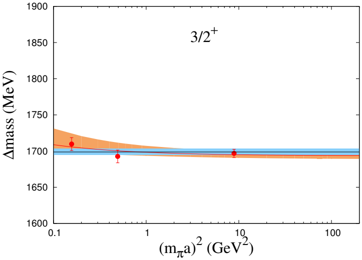
- Splittings : $\Delta m \ \sim\ a_1[(n_1 - n_2)m_Q + A_1 - A_2] + b_1/m_Q + O(1/m_Q^2)$
  $\sim\ a + b/m_{PS} + O(1/m^2_{PS})$.
- Light quark data excluded from the fits.

MP *et al.*, PRD 91 094502

An HQET inspired fit on HSC results

MP *et al.*, PRD 91 094502





$$m_{B_c^*} - m_{B_c} = 80 \ \pm \ 8 \ MeV$$

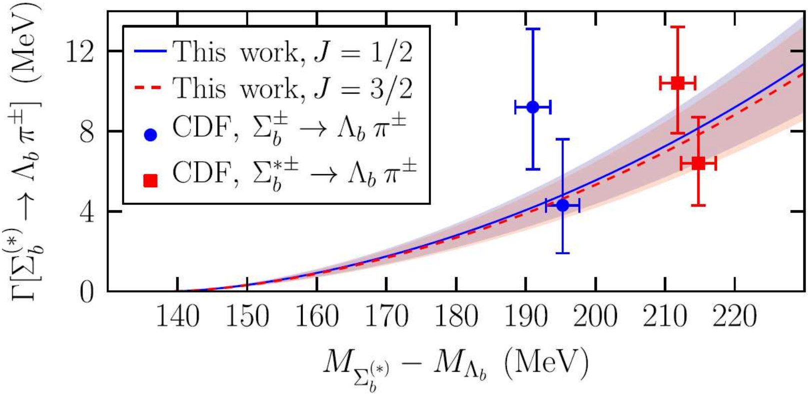
53(7) : Gregory *et al.*, PRL 104 022001

54(3) : Dowdall *et al.*, PRD 86 094510

$$m_{\Omega^*_{ccb}} = 8050 \ \pm \ 10 \ MeV$$

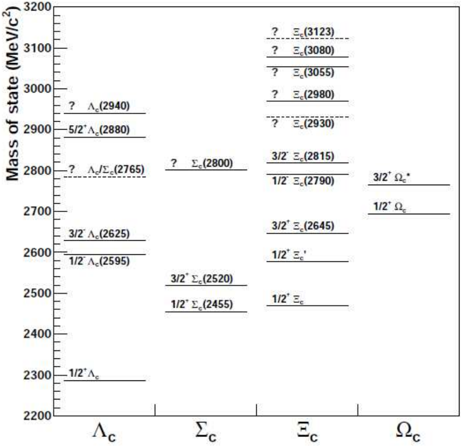
8037(9)(20) : Brown *et al.*, PRD 90 094507

Σ_b decay width



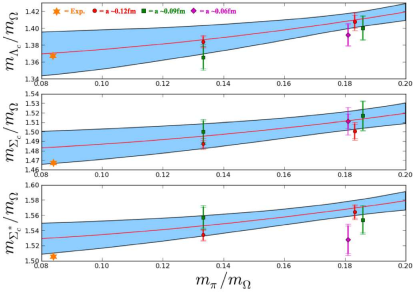
Detmold *et al.*, PRL 108 172003

Known charm baryons

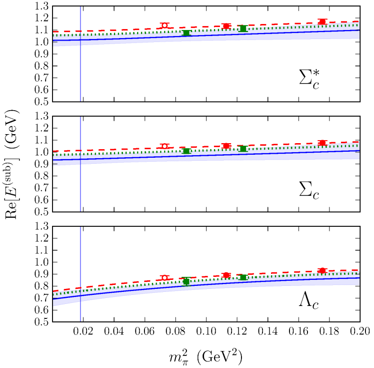


Chiral extrapolations

Briceño et. al, PRD 86 094504



Brown et. al, PRD 90 094507



Chiral extrapolations based on Heavy hadron chiral perturbation theory.

Savage, PLB 359, 189; Mehen and Tiburzi, hep-lat/0607023.

Spectroscopy : baryon operator construction

- Aim : Extraction of highly excited states.
  Local operators $\rightarrow$ low lying states.
  Extended operators $\rightarrow$ States with radial and orbital excitations.

- Proceeds in two steps
  Construct continuum operators with well defined quantum nos.
  Reduce/subduce into the irreps of the reduced symmetry.

- Used set of baryon continuum operators of the form
  $\Gamma^{\alpha\beta\gamma} q^\alpha q^\beta q^\gamma$, $\Gamma^{\alpha\beta\gamma} q^\alpha q^\beta (D_i q^\gamma)$ and $\Gamma^{\alpha\beta\gamma} q^\alpha q^\beta (D_i D_j q^\gamma)$

- Excluding the color part, the flavor-spin-spatial structure
$$O^{[J^P]} = [\mathcal{F}_{\Sigma_F} \otimes \mathcal{S}_{\Sigma_S} \otimes \mathcal{D}_{\Sigma_D}]^{J^P}.$$

- γ-matrix convention : $\gamma_4 = \text{diag}[1,1,-1,-1]$;
  Non-relativistic $\rightarrow$ purely based on the upper two component of q.
  Relativistic $\rightarrow$ All operators except non-relativistic ones.

- Subset of $D_i D_j$ operators that include $[D_i, D_j] \sim F_{ij} \rightarrow$ hybrid.

Charm baryon : Flavor symmetry structures (1)

20_M					
	I	I_z	S	$\mathcal{F}_{MS}$	$\mathcal{F}_{MA}$
Λ_c^+	0	0	0	$\frac{1}{\sqrt{2}}(\|cud\rangle_{MS} - \|udc\rangle_{MS})$	$\frac{1}{\sqrt{2}}(\|cud\rangle_{MA} - \|udc\rangle_{MA})$
Σ_c^{++}	1	$+1$	0	$\|uuc\rangle_{MS}$	$\|uuc\rangle_{MA}$
Σ_c^+	1	0	0	$\|ucd\rangle_{MS}$	$\|ucd\rangle_{MA}$
Σ_c^0	1	-1	0	$\|ddc\rangle_{MS}$	$\|ddc\rangle_{MA}$
$\Xi_c^{\prime +}$	$\frac{1}{2}$	$+\frac{1}{2}$	-1	$\|ucs\rangle_{MS}$	$\|ucs\rangle_{MA}$
$\Xi_c^{\prime 0}$	$\frac{1}{2}$	$-\frac{1}{2}$	-1	$\|dcs\rangle_{MS}$	$\|dcs\rangle_{MA}$
Ξ_c^+	$\frac{1}{2}$	$+\frac{1}{2}$	-1	$\frac{1}{\sqrt{2}}(\|cus\rangle_{MS} - \|usc\rangle_{MS})$	$\frac{1}{\sqrt{2}}(\|cus\rangle_{MA} - \|usc\rangle_{MA})$
Ξ_c^0	$\frac{1}{2}$	$-\frac{1}{2}$	-1	$\frac{1}{\sqrt{2}}(\|cds\rangle_{MS} - \|dsc\rangle_{MS})$	$\frac{1}{\sqrt{2}}(\|cds\rangle_{MA} - \|dsc\rangle_{MA})$
Ω_c^0	0	0	-2	$\|scs\rangle_{MS}$	$\|scs\rangle_{MA}$
Ξ_{cc}^{++}	$\frac{1}{2}$	$+\frac{1}{2}$	0	$\|ccu\rangle_{MS}$	$\|ccu\rangle_{MA}$
Ξ_{cc}^+	$\frac{1}{2}$	$-\frac{1}{2}$	0	$\|ccd\rangle_{MS}$	$\|ccd\rangle_{MA}$
Ω_{cc}^+	0	0	-1	$\|ccs\rangle_{MS}$	$\|ccs\rangle_{MA}$

$\mathcal{F}_{MS}$ $\rightarrow$ Mixed Symmetric flavor structure

$\mathcal{F}_{MA}$ $\rightarrow$ Mixed Antisymmetric flavor structure

Charm baryon : Flavor symmetry structures (2)

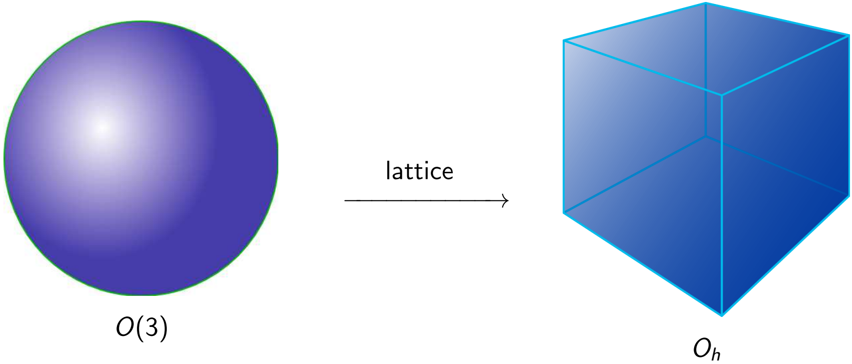
20_S				
	I	I_z	S	$\mathcal{F}_S$
Σ_c^{++}	1	$+1$	0	$\|uuc\rangle_S$
Σ_c^{+}	1	0	0	$\|ucd\rangle_S$
Σ_c^{0}	1	-1	0	$\|ddc\rangle_S$
Ξ_c^{+}	$\frac{1}{2}$	$+\frac{1}{2}$	-1	$\|ucs\rangle_S$
Ξ_c^{0}	$\frac{1}{2}$	$-\frac{1}{2}$	-1	$\|dcs\rangle_S$
Ω_c^{0}	0	0	-2	$\|ssc\rangle_S$
Ξ_{cc}^{++}	$\frac{1}{2}$	$+\frac{1}{2}$	0	$\|ccu\rangle_S$
Ξ_{cc}^{+}	$\frac{1}{2}$	$-\frac{1}{2}$	0	$\|ccd\rangle_S$
Ω_{cc}^{+}	0	0	-1	$\|ccs\rangle_S$
Ω_{ccc}^{++}	0	0	0	$\|ccc\rangle_S$

4_A				
	I	I_z	S	ϕ_A
Λ_c^{+}	0	0	0	$\|udc\rangle_A$
Ξ_c^{+}	$\frac{1}{2}$	$+\frac{1}{2}$	-1	$\|ucs\rangle_A$
Ξ_c^{0}	$\frac{1}{2}$	$-\frac{1}{2}$	-1	$\|dcs\rangle_A$

20_S $\rightarrow$ Symmetric flavor structure

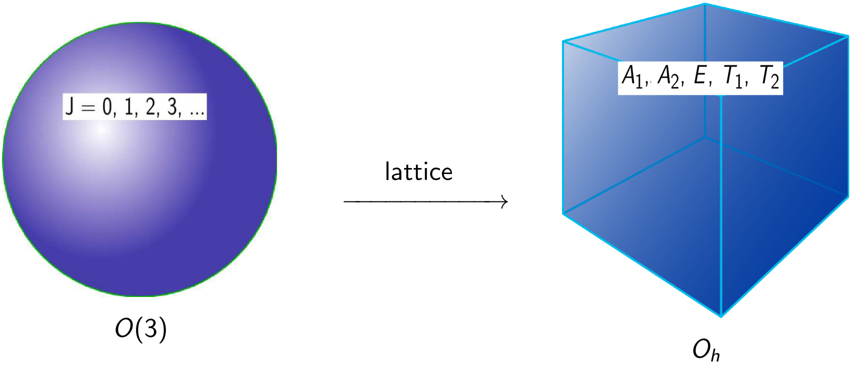
20_A $\rightarrow$ Antisymmetric flavor structure

Continuum $\rightarrow$ Lattice : Symmetries



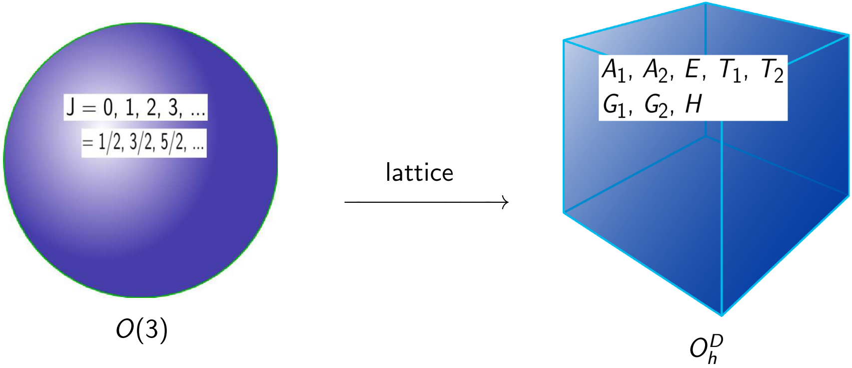
- Eigenstates of lattice Hamiltonian transform under irreps, Λ^n, of O_h.
- Continuum states with same J^P but different J_z : separated across different lattice irreps.
- Subduce the continuum operators into the irreps of O_h.

Continuum → Lattice : Irreps (1)



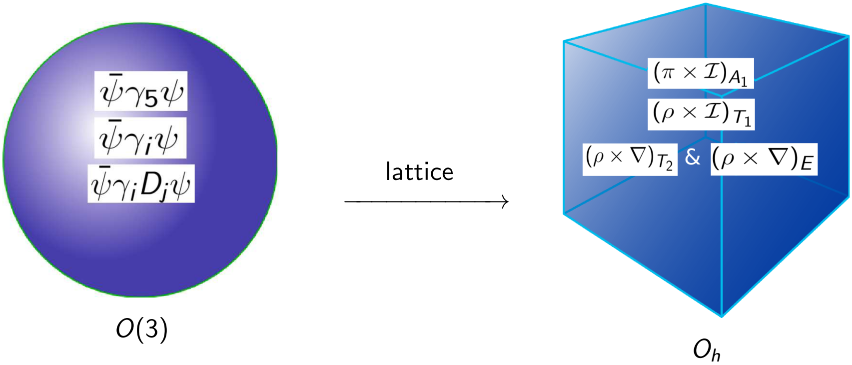
- Integer spin objects see an O_h symmetry on lattice.

Continuum $\rightarrow$ Lattice : Irreps (2)



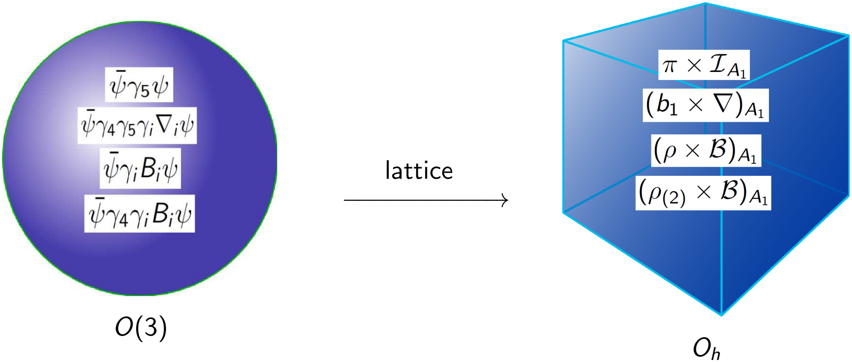
- Half-integer spin objects see an O_h^D symmetry on lattice.

Continuum $\rightarrow$ Lattice : Operators (1)



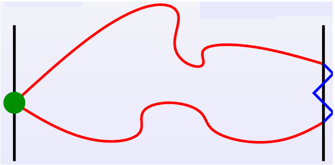
- Operators in the continuum get distributed over the lattice irreps.

Continuum $\rightarrow$ Lattice : Operators (2)

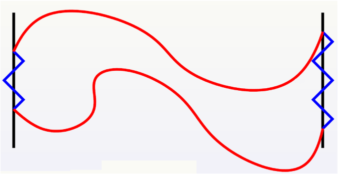


- Multiple continuum operators with various spin-spatial structures reducing onto same lattice irreps with varying lattice extensions : Excited states.

Local and extended operators



Meson two point correlators using local source operators



Meson two point correlators using extended source operators

We used a technique called "Distillation".

Aids in computing the correlation functions with much less computational requirements.

M. Peardon *et al.*, PRD **80**, 054506, 2009

No. of interpolating operators

Ω_{ccc}

	G_1		H		G_2	
	g	u	g	u	g	u
Total	20	20	33	33	12	12
Hybrid	4	4	5	5	1	1
NR	4	1	8	1	3	0

Λ_{cdu}

	G_1		H		G_2	
	g	u	g	u	g	u
Total	53	53	86	86	33	33
Hybrid	12	12	16	16	4	4
NR	10	3	17	4	7	1

Ω_{ccs}, Ξ_{ccu}, Ω_{css} and Σ_{cuu}.

	G_1		H		G_2	
	g	u	g	u	g	u
Total	55	55	90	90	35	35
Hybrid	12	12	16	16	4	4
NR	11	3	19	4	8	1

Ξ_{csu}

	G_1		H		G_2	
	g	u	g	u	g	u
Total	116	116	180	180	68	68
Hybrid	24	24	32	32	8	8
NR	23	6	37	10	15	2

Generalized eigenvalue problem

Solving the generalized eigenvalue problem for $C_{ij}(t)$.

$$C_{ij}(t)v_j^{(n)}(t,t_0) = \lambda^{(n)}(t,t_0)C_{ij}(t_0)v_j^{(n)}(t,t_0)$$

Solve for many t_0's.

Choice of t_0's crucial $\Rightarrow$ Determine quality of extractions.

- Principal correlators given by eigenvalues

  $$\lambda_n(t,t_0) \sim (1-A_n)\exp^{-m_n(t-t_0)} + A_n \exp^{-m'_n(t-t_0)}$$

  Extraction of a tower of states.

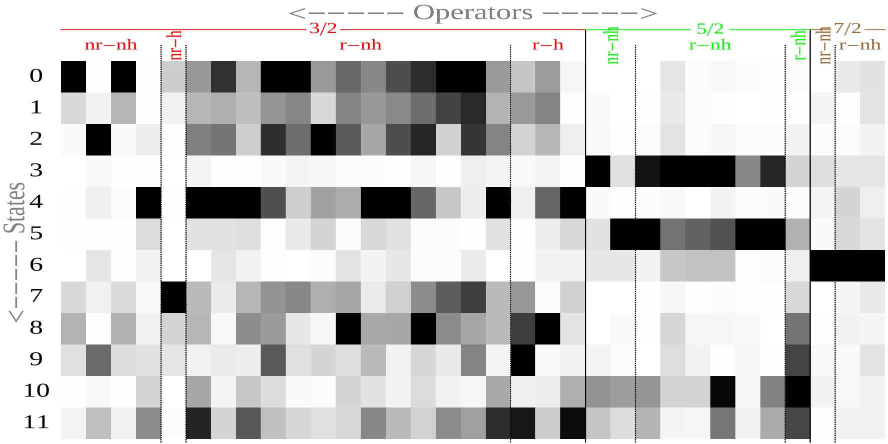
- Eigenvectors related to the overlap factors

  $$Z_i^{(n)} = \langle 0|\mathcal{O}_i|n\rangle = \sqrt{2E_n}\exp^{E_n t_0/2} v_j^{(n)\dagger} C_{ji}(t_0)$$

  Spin identification.

C. Michael, Nucl. Phys. B 259, 58, (1985).
M. Lüscher and U. Wolff, Nucl. Phys. B 339, 222 (1990).

Spin identification using overlap factors : (Ω_{ccc}, H_g)



$nr - nh \quad = \quad non - relativistic \ \& \ non - hybrid$

$nr - h \quad = \quad non - relativistic \ \& \ hybrid$

$r - nh \quad = \quad relativistic \ \& \ non - hybrid$

$r - h \quad = \quad relativistic \ \& \ hybrid$

Spin identification from overlap factors

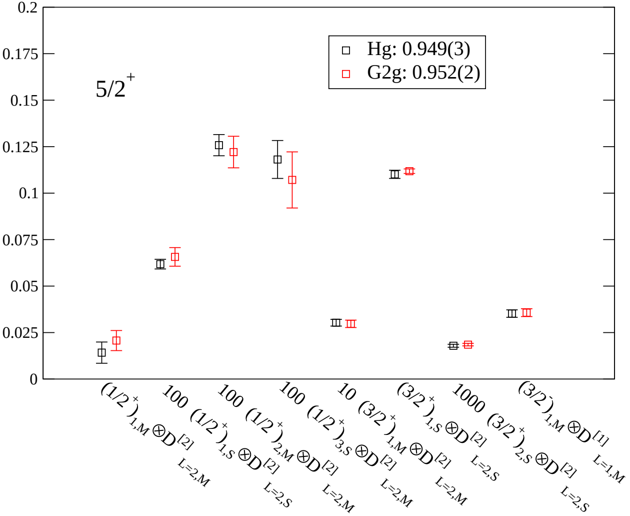
- For example, a continuum operator $O_{jk} = \bar{\psi}\gamma_j D_k \psi$.
  Projects on to 2^{++}.
- In the continuum, $\langle 0|O_{jk}|2^{++}\rangle = Z\epsilon_{jk}$.
- On lattice, O_{jk} gets subduced over two lattice irreps $(\rho \times \nabla)_{T_2}$ and $(\rho \times \nabla)_E$.
- Then

$$\langle 0|(\rho \times \nabla)^i_{T_2})|2^{++}\rangle = \alpha_{ijk}\langle 0|O_{jk}|2^{++}\rangle = Z_1\alpha_{ijk}\epsilon_{jk}$$

$$\langle 0|(\rho \times \nabla)^i_{E})|2^{++}\rangle = \beta_{ijk}\langle 0|O_{jk}|2^{++}\rangle = Z_2\beta_{ijk}\epsilon_{jk}$$

  where α_{ijk} and β_{ijk} are the Clebsch-Gordan coefficients.
- If "close" to the continnum, then $Z \sim Z_1 \sim Z_2$.

Overlap factors (Z) across multiple irreps : $5/2^+$



Connecting lattice to continnum irreps

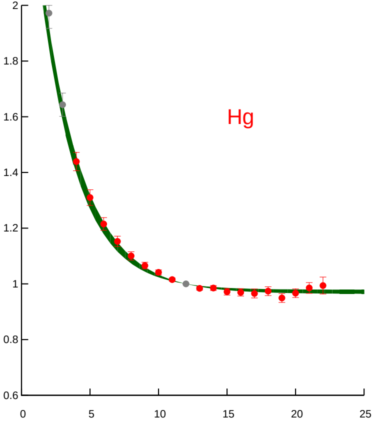
Lattice irrep, Λ	Dimension	Continuum irrep, J
A_1	1	0,4,...
A_2	1	3,5,...
E	2	2,4,...
T_1	3	1,3,...
T_2	3	2,3,...
G_1	2	$\frac{1}{2},\frac{7}{2},\frac{9}{2},\ldots$
G_2	2	$\frac{5}{2},\frac{7}{2},\frac{9}{2},\ldots$
H	4	$\frac{3}{2},\frac{5}{2},\frac{7}{2},\ldots$

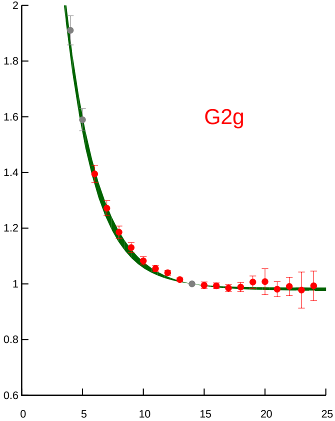
Including the spatial inversions : doubles the group elements.

A_{1g}, A_{1u}, A_{2g}, A_{2u}, E_g, E_u, T_{1g}, T_{1u}, T_{2g}, T_{2u},
G_{1g}, G_{1u}, G_{2g}, G_{2u}, H_g and H_u;
$g \to +$ and $u \to -$.

Joint fitting principal correlators for $J = 5/2^+$

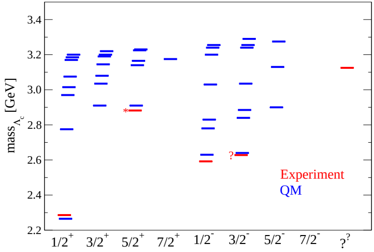
$a_t E = 0.770(3)$ $a_t E = 0.775(1)$

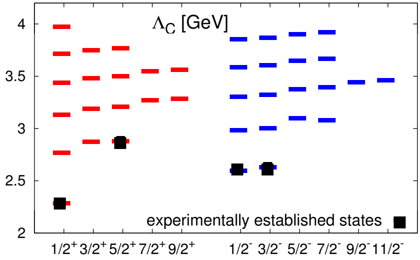




$\lambda_n(t, t_0) \exp^{m_n(t-t_0)}$ Vs t/a_t; $a_t E = 0.771(3)$

Λ_c (uuc) baryon spectrum in potential model





Capstick and Isgur, PRD 34 2809

Ebert *et al.*, PRD 84 014025