

CP violation in $D - \bar{D}$ mixing

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Based on Yuval Grossman, A. K., Zoltan Ligeti, Luca Silvestrini, Gilad Perez, Alexey Petrov,
in preparation

Plan

- Introduction
 - Absorptive and dispersive contributions to mixing and CP violation (CPV)
 - Phenomenology of CPV in mixing
- Today: the “superweak limit” - a constrained fit
 - parametrization of indirect CPV with one universal weak phase
- Future: departure from the superweak limit
 - two universal weak phases (absorptive and dispersive) suffice
- How large can indirect CPV be in the SM?

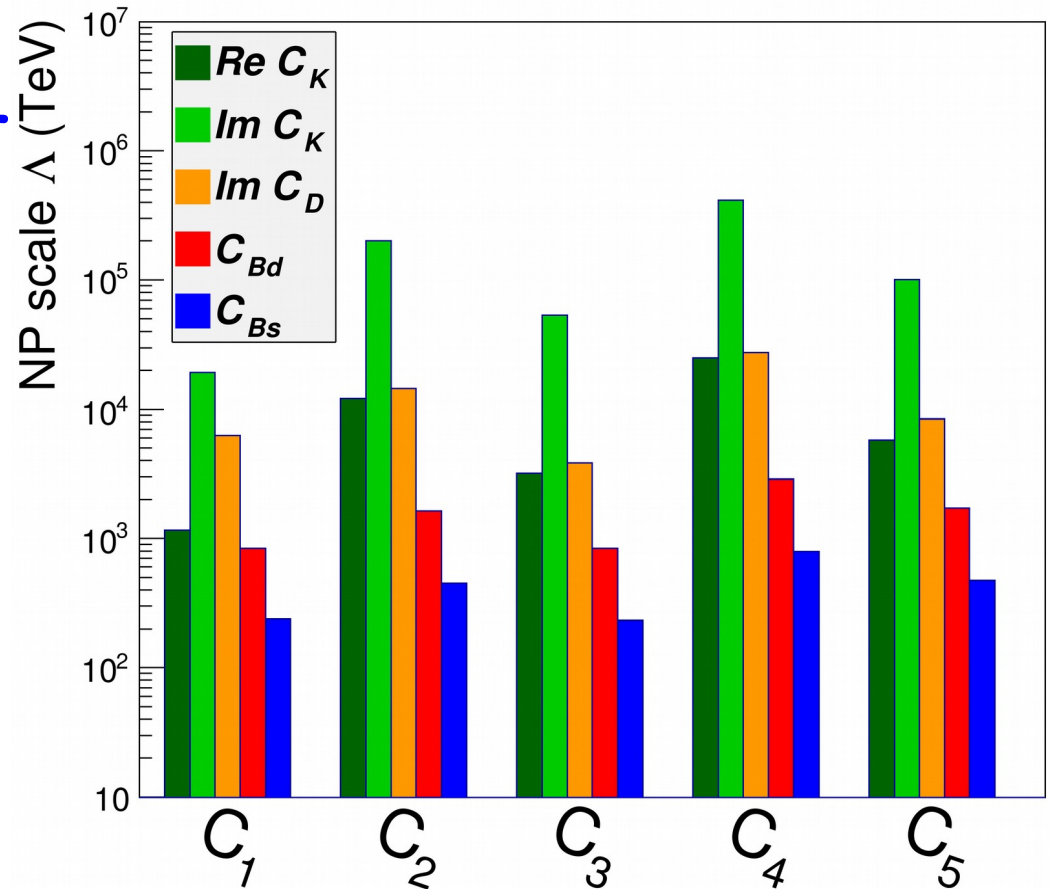
Introduction

Introduction

- In the SM, CP violation (CPV) in mixing enters at $O(V_{cb}V_{ub}/V_{cs}V_{us}) \sim 10^{-3}$, due to weak phase γ
- In view of current and future (LHCb, Belle II; HL-LHC?) improvements in CPV mixing measurements, this statement needs to be sharpened
 - how large is the QCD uncertainty?
 - this has implications for how we should parametrize CPV in mixing
- how large is the current window for New Physics (NP) in mixing CPV?

INTRODUCTION

- CP violation in $\Delta F=2$ processes is the most sensitive probe of NP, reaching scales of $O(10^5)$ TeV
- CPV in D mixing gives best bound after ε_K
- How far can we push it?



Review of formalism for CPV in mixing:

- Mixing of strong interaction eigenstates \bar{D}^0 , D^0 due to transition amplitudes

$$\langle D^0 | H | \bar{D}^0 \rangle = M_{12} - \frac{i}{2} \Gamma_{12}, \quad \langle \bar{D}^0 | H | D^0 \rangle = M_{12}^* - \frac{i}{2} \Gamma_{12}^*$$

- D meson mass eigenstates,

$$|D_{1,2}\rangle = p|D^0\rangle \pm q|\bar{D}^0\rangle$$

- CP conserving observables

$$x = \frac{m_2 - m_1}{\Gamma}, \quad y = \frac{\Gamma_2 - \Gamma_1}{2\Gamma}$$

● M_{12} is **dispersive mixing**: due to long-distance exchange of off-shell intermediate states; and short-distance effects

● long distance dominates in SM

● significant short distance effect would be new physics (NP)

● Γ_{12} is **absorptive mixing**: due to long distance exchange of on-shell intermediate states

$$\Gamma_{12} \cong \sum_f \langle D^0 | H_W | f \rangle \langle f | H_W | \bar{D}^0 \rangle$$

● The “theoretical” mixing parameters

$$x_{12} \equiv 2|M_{12}|/\Gamma, \quad y_{12} \equiv |\Gamma_{12}|/\Gamma, \quad \phi_{12} \equiv \arg(M_{12}/\Gamma_{12})$$

● Relations to CP conserving observables:

$$|x| = x_{12} + O(\text{CPV}^2), \quad |y| = y_{12} + O(\text{CPV}^2)$$

- CPV in mixing via Γ_{12} and via **long distance** part of M_{12} requires **subleading decay amplitudes containing weak phases**:
 - SM: $V_{cb}V_{ub}^*$ suppressed amplitudes containing $e^{i\gamma}$
 - NP: subleading decay amplitudes with new weak phases
- assume only **singly Cabibbo suppressed (SCS) decays contribute**
 - CF/DCS contributions negligible in SM
 - NP with non-negligible DCPV in DCS/CF decays, which evades ϵ_K bounds, must be extremely exotic **Bergmann, Nir**

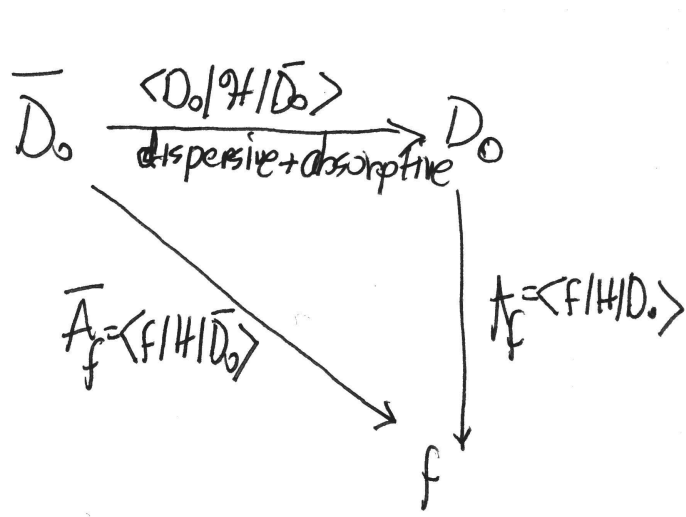
Two kinds of indirect CPV

- **CPVMIX**: CPV in pure mixing due to $\phi_{12} \neq 0 \Rightarrow$ **interference** between the dispersive and absorptive mixing amplitudes

$$\phi_{12} \neq 0 \quad \Rightarrow \quad \left| \frac{q}{p} \right| \neq 1$$

e.g., a non-vanishing **semileptonic CP asymmetry**, $A_{\text{SL}} \propto \sin \theta_{12}$

- **CPVINT**: CP violation in the interference of decays with and without mixing



- CPVINT observable for **CP eigenstate** final state, $f = \bar{f}$:

$$\phi_{\lambda_f} = \arg \left(\frac{q}{p} \frac{\bar{A}_f}{A_f} \right)$$

- CPVINT observable pairs for **non-CP eigenstate** final states, $f \neq \bar{f}$:

$$\phi_{\lambda_f} = \arg \left(\frac{q}{p} \frac{\bar{A}_f}{A_f} \right), \quad \phi_{\lambda_{\bar{f}}} = \arg \left(\frac{q}{p} \frac{\bar{A}_{\bar{f}}}{A_{\bar{f}}} \right)$$

- $\phi_{\lambda_f}, \phi_{\lambda_{\bar{f}}} \neq 0 \Rightarrow$ CPVINT time-dependent CP asymmetries, e.g. in

- SCS decays (A_Γ): $D^0(t) \rightarrow K^+ K^-, \pi^+ \pi^- \neq \bar{D}^0(t) \rightarrow K^+ K^-, \pi^+ \pi^-$

- DCS decays: $D^0(t) \rightarrow K^+ \pi^- \neq \bar{D}^0(t) \rightarrow K^- \pi^+$

- these asymmetries contain both CPVMIX and CPVINT contributions

The superweak limit

CPVINT in the “superweak” limit

- neglect effects of subleading decay weak phases in indirect CPV:
 $O(x A_{\text{CP}}^{\text{d}}, y A_{\text{CP}}^{\text{d}})$, where A_{CP}^{d} is the direct CP asymmetry
 - suppressed by x, y
 - A_{CP}^{d} CKM suppressed in SM
 - allowing for NP in SCS decays, A_{CP}^{d} bounds \Rightarrow this is still an excellent approximation compared to current experimental indirect CPV errors
- in **time-integrated** CP asymmetries both indirect and direct CPV contribute
 - keep leading direct CPV contribution A_{CP}^{d}
 - neglect subleading direct CPV effects entering the indirect CPV contribution
- in the superweak limit $\phi_{12} \neq 0$ is dispersive, entirely due to short-distance NP in M_{12} (SM short-distance is negligible)

$$\phi_{12} = \phi_{12}^M, \quad \phi_{12}^\Gamma = 0$$

● ϕ_{12} is only source of CPVINT, which is universal

● $\phi_{\lambda_f} \rightarrow \phi$, universal CPVINT

● CPVINT and CPVMIX related: Ciuchini et al '07; Grossman, Perez, Nir '09; A.K., Sokoloff '09

$$\tan 2\phi \approx -\frac{x_{12}^2}{x_{12}^2 + y_{12}^2} \sin 2\phi_{12}$$

$$\tan \phi \approx \left(1 - \left|\frac{q}{p}\right|\right) \frac{x}{y}$$

e.g., $\Delta Y_f = -A_\Gamma = -x_{12} \sin \phi_{12}$

● with only one CPV phase ϕ_{12} controlling all indirect CPV, superweak fits to CPV data are much more constrained than fits in which ϕ and $|q/p|$ are independent

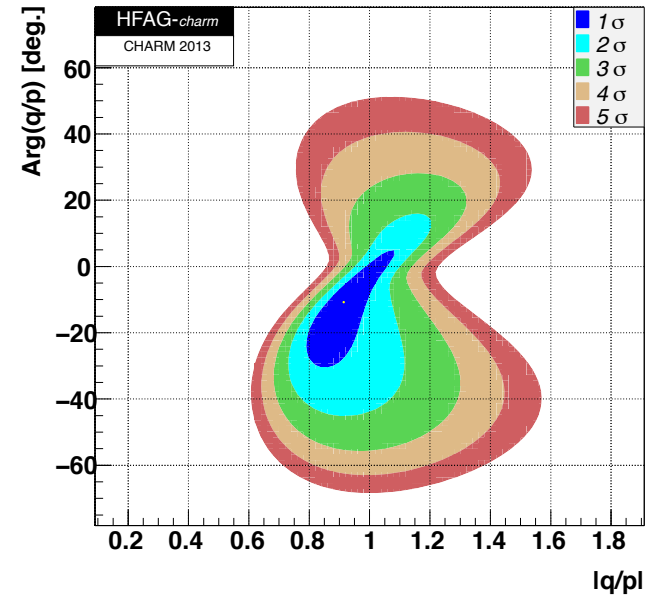
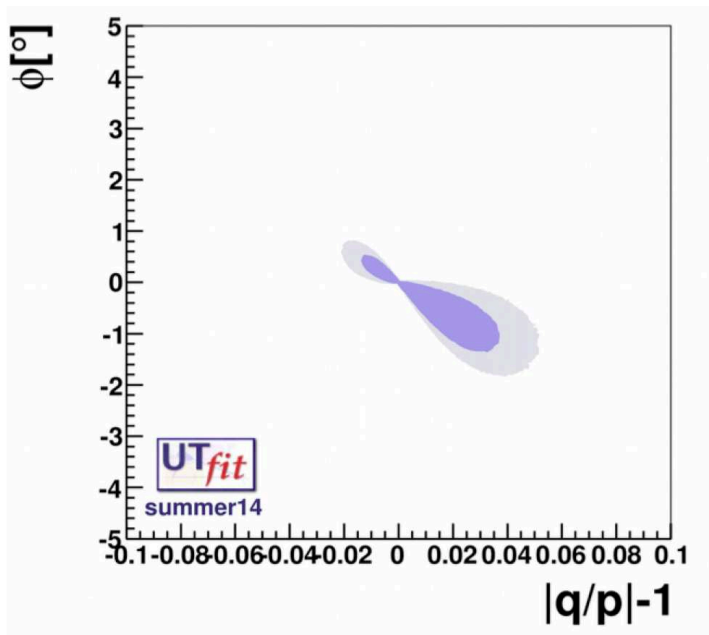
● fits assume no direct CPV in doubly Cabibbo suppressed (DCS) decays
 $D^0 \rightarrow K^+ \pi^-$

● Fit results:

HFAG, NEW : ϕ_{12} [rad] = $0.017_{-0.03}^{+0.035}$ (1σ); $[-0.05, +0.14]$ 95% cl

UTfit : ϕ_{12} [rad] = 0.003 ± 0.03 (1σ); $[-0.07, +0.21]$ 95% cl

● ϕ vs $|q/p|$ in superweak fit vs. fit with independent $|q/p|$, ϕ



Departure from the superweak limit

Departure from the superweak limit

- we are transitioning to the $< 10\%$ era on ϕ_{12}
- at the coming level of precision, will the superweak limit continue to be a good approximation?
- what is the best way to parametrize deviations from the superweak limit?
- how large can the SM contribution to indirect CPV be?

● most general final-state specific parametrization: physical absorptive and dispersive contributions to ϕ_{12}

● $f = CP$ eigenstate:

$$\phi_{12 f}^M \equiv \frac{1}{2} \arg \left[\frac{M_{12}}{M_{12}^*} \left(\frac{A_f}{\bar{A}_f} \right)^2 \right], \quad \phi_{12 f}^\Gamma \equiv \frac{1}{2} \arg \left[\frac{\Gamma_{12}}{\Gamma_{12}^*} \left(\frac{A_f}{\bar{A}_f} \right)^2 \right]$$

$$\text{CPVMIX } \phi_{12} = \phi_{12 f}^M - \phi_{12 f}^\Gamma$$

● non-CP eigenstates f, \bar{f} :

$$\phi_{12 f}^M \equiv \frac{1}{2} \arg \left[\frac{M_{12}}{M_{12}^*} \frac{A_f A_{\bar{f}}}{\bar{A}_f \bar{A}_{\bar{f}}} \right], \quad \phi_{12 f}^\Gamma \equiv \frac{1}{2} \arg \left[\frac{\Gamma_{12}}{\Gamma_{12}^*} \frac{A_f A_{\bar{f}}}{\bar{A}_f \bar{A}_{\bar{f}}} \right]$$

$$\text{CPVMIX } \phi_{12} = \phi_{12 f}^M - \phi_{12 f}^\Gamma$$

Beyond superweak with a universal parametrization

- in general,

$$M_{12} = M_{12}^0 + \delta M_{12,\text{SM}} + \delta M_{12,\text{NP}}, \quad \Gamma_{12} = \Gamma_{12}^0 + \delta \Gamma_{12,\text{SM}} + \delta \Gamma_{12,\text{NP}}$$

$$M_{12}^0, \Gamma_{12}^0 \propto (\lambda_s - \lambda_d)^2; \quad \delta M_{12,\text{SM}}, \delta \Gamma_{12,\text{SM}} \propto (\lambda_s - \lambda_d)\lambda_b$$

(in superweak $\delta M_{12,\text{SM}} = \delta \Gamma_{12} = 0$)

- define “theoretical” phase convention independent universal CPV phases

$$\phi_{12}^\Gamma \equiv \arg\left(\frac{\Gamma_{12}}{\Gamma_{12}^0}\right), \quad \phi_{12}^M \equiv \arg\left(\frac{M_{12}}{M_{12}^0}\right), \quad \phi \equiv \arg\left(\frac{q}{p} \frac{1}{\Gamma_{12}^0}\right)$$

- $\arg(\Gamma_{12}^0) = \arg(M_{12}^0) = \arg[(\lambda_s - \lambda_d)^2]$ provides a “reference ruler” for the no CPV direction in the complex plane
- ϕ_{12}^Γ takes into account the weak phases of the subleading amplitudes in all decays

Define the misalignment between the general parametrization and the “theoretical” universal phases

$$\delta\phi_f \equiv \phi_{12f}^\Gamma - \phi_{12}^\Gamma = \phi_{12f}^M - \phi_{12}^M = \phi - \phi_{\lambda_f}$$

- in CF/DCS decays with no NP, the misalignment is known and negligible, e.g., in CPVINT in $D^0 \rightarrow K^\pm \pi^\mp$, $D^0 \rightarrow K_S \pi^+ \pi^-$

$$\phi_{\lambda_f} = \phi, \quad \phi_{12f}^\Gamma = \phi_{12}^\Gamma, \quad \phi_{12f}^M = \phi_{12}^M$$

- $\delta\phi_f$ is related to direct CPV: $\delta\phi_f = A_{CP}^{\text{dir}}(D \rightarrow f) \cot \delta$, δ is a strong phase

- $D^0 \rightarrow K^+ K^-, \pi^+ \pi^-$: $A_{CP}^{\text{dir}} \lesssim \text{few} \times 10^{-3} \Rightarrow \delta\phi_f \lesssim \text{few} \times 10^{-3}$

\Rightarrow small misalignment compared to expected BelleII/LHCb sensitivity:

$$\delta\phi \approx 3^\circ \approx 0.05 \text{ [rad]}$$

- In general, in SM: $\phi_{12}^\Gamma = O(1/\epsilon)$, $\epsilon \sim 0.2$ characterizes nominal U-spin breaking

$$\Rightarrow \frac{\delta\phi_f}{\phi_{12}^\Gamma} = O(\epsilon) \text{ in SCS } D^0 \text{ decays}$$

yielding parametric suppression of misalignment relative to ϕ_{12}^Γ

- therefore, for expected Belle/LHCb sensitivity, can account for deviation from superweak limit with only one additional universal CPV phase beyond ϕ_{12} , e.g. ϕ_{12}^Γ
- fit mixing data to ϕ_{12}^Γ and ϕ_{12} or, equivalently, ϕ_{12}^M and ϕ_{12}^Γ
 - in practice, equivalent to “traditional” two parameter fit for ϕ , $|q/p|$
 - back to a less constrained fit, but Belle/LHCb improved sensitivity will overcome this

Examples

- Time-dependent CPV in $D^0 \rightarrow K_S \pi^+ \pi^-$, assuming no NP in CF/DCS, yields a measurement of $x, y, |q/p|, \phi$: ($|x| = x_{12}, |y| = y_{12}$)

- use the relations

$$\left| \frac{q}{p} \right| - 1 \approx \frac{|x||y|}{x^2 + y^2} \sin \phi_{12}$$

$$\tan 2(\phi + \phi_{12}^\Gamma) \approx -\frac{x^2}{x^2 + y^2} \sin 2\phi_{12}$$

$$\phi_{12} = \phi_{12}^M - \phi_{12}^\Gamma$$

to obtain the fundamental dispersive and absorptive universal phases $\phi_{12}^M, \phi_{12}^\Gamma$

- Time-dependent CP asymmetry in SCS decays to a CP eigenstate,

$$A_\Gamma = -\Delta Y_f \approx |x| \sin \phi_{12}^M,$$

analogous relation holds for $D^0 \rightarrow K_S \pi^+ \pi^-$ time-integrated, time-dependent CP asymmetries: $\propto \sin \phi_{12}^M$

CHARM CPV @ LHCb UPGRADE

- Expected errors w. LHCb upgrade:
 - $\delta x = 1.5 \cdot 10^{-4}$, $\delta y = 10^{-4}$, $\delta |q/p| = 10^{-2}$, $\delta \phi = 3^\circ$ (from $K_s \pi \pi$); $\delta \gamma_{CP} = \delta A_\Gamma = 4 \cdot 10^{-5}$ (from $K^+ K^-$)
- Allows to experimentally determine $\phi_{\Gamma 12}$ with a reach on CPV @ the degree level:
 - $\delta \phi_{M12} = \pm 1^\circ$ (17 mrad) and $\delta \phi_{\Gamma 12} = \pm 2^\circ$ (34 mrad) @ 95% prob.
 - $\Lambda > 10^5$ TeV

- Another example: time-integrated tagged, untagged CP asymmetries for DCS/CF decays, e.g. $f = K^+ \pi^-$, $\bar{f} = K^- \pi^+$

$$A_{\text{CP}}^{\text{tag, DCS (CF)}} \equiv \frac{\int dt (\Gamma[D^0(t) \rightarrow \bar{f}(f)] - \Gamma[\bar{D}^0(t) \rightarrow f(\bar{f})])}{\int dt (\Gamma[D^0(t) \rightarrow \bar{f}(f)] + \Gamma[\bar{D}^0(t) \rightarrow f(\bar{f})])}$$

$$A_{\text{CP}}^{\text{untag}} \equiv \frac{\int dt (\Gamma[D^0(t) \rightarrow \bar{f}] + \Gamma[D^0(t) \rightarrow f] - \Gamma[\bar{D}^0(t) \rightarrow \bar{f}] - \Gamma[\bar{D}^0(t) \rightarrow f])}{\int dt (\Gamma[D^0(t) \rightarrow \bar{f}] + \Gamma[D^0(t) \rightarrow f] + \Gamma[\bar{D}^0(t) \rightarrow \bar{f}] + \Gamma[\bar{D}^0(t) \rightarrow f])}$$

For $R_f \equiv |\bar{A}_f^{\text{DCS}}/A_f^{\text{CF}}|$, and Δ_f the strong phase between DCS and CF, obtain

$$\frac{A_{\text{CP}}^{\text{tag, CF}}}{R_f} + R_f A_{\text{CP}}^{\text{tag, DCS}} = -2|x| \sin \phi_{12}^M \cos \Delta_f$$

$$\frac{1 + R_f^2}{R_f} A_{\text{CP}}^{\text{untag}} = \frac{A_{\text{CP}}^{\text{tag, CF}}}{R_f} - R_f A_{\text{CP}}^{\text{tag, DCS}} = -2|y| \sin \phi_{12}^\Gamma \sin \Delta_f$$

- analogous relations hold for time-dependent CP asymmetries in SCS decays to non-CP eigenstates, e.g. $D^0 \rightarrow \rho\pi$, K^*K , with $R_f \equiv |A_{\bar{f}}/A_f| = O(1)$ and $A_{\text{CP}}^{\text{tag, CF (DCS)}} \rightarrow \Delta Y_f(\bar{f})$

How large can indirect CPV be in the SM?

U-spin decomposition of Γ_{12} and M_{12} in the SM

- using CKM unitarity,

$$\Gamma_{12} = \frac{(\lambda_s - \lambda_d)^2}{4} \Gamma_5 + \frac{(\lambda_s - \lambda_d)\lambda_b}{2} \Gamma_3 + \frac{\lambda_b^2}{4} \Gamma_1$$

$$M_{12} = \frac{(\lambda_s - \lambda_d)^2}{4} M_5 + \frac{(\lambda_s - \lambda_d)\lambda_b}{2} M_3 + \frac{\lambda_b^2}{4} M_1$$

- $\Gamma_{5,3,1}$, $M_{5,3,1}$ are $\Delta U_3 = 0$ elements of U-spin multiplets, e.g.

$$\Gamma_5 = \Gamma_{ss} + \Gamma_{dd} - 2\Gamma_{sd} \sim (\bar{s}s - \bar{d}d)^2 \Rightarrow \Delta U = 2 \text{ (5 plet)} \Rightarrow O(\epsilon^2), \text{ CF/DCS/SCS}$$

$$\Gamma_3 = \Gamma_{ss} - \Gamma_{dd} \sim (\bar{s}s - \bar{d}d)(\bar{s}s + \bar{d}d) \Rightarrow \Delta U = 1 \text{ (3 plet)} \Rightarrow O(\epsilon), \text{ SCS}$$

- $\Gamma_{12}^0 \propto \Gamma_5$, $M_{12}^0 \propto M_5$ are CP conserving $\Rightarrow y_{12}$, x_{12} or y , x
- $\delta\Gamma_{12} \propto \Gamma_3$, $\delta M_{12} \propto M_3 \Rightarrow \text{CPV via } \gamma = \arg(\lambda_b)$
- neglect $O(\lambda_b^2)$ effects of Γ_1 , M_1

- the U-spin decomposition yields the rough estimate

$$\phi_{12}^{\Gamma} \equiv \arg \left(\frac{\Gamma_{12}}{\Gamma_{12}^0} \right) \approx \text{Im} \left(\frac{2\lambda_b}{\lambda_s - \lambda_d} \frac{\Gamma_3}{\Gamma_5} \right) \sim \left| \frac{\lambda_b}{\theta_c} \right| \sin \gamma \times \frac{1}{\epsilon}$$

and similarly for ϕ_{12}^M

- “nominal” U-spin breaking,

$$\epsilon \sim 0.2 \quad \Rightarrow \quad \phi_{12}^{\Gamma} \sim \phi_{12}^M \sim 3 \times 10^{-3}$$

compared to $\phi_{12} \in [-0.07, +0.08]$ (HFAG), $[-0.07, +0.21]$ (UTfit) at 95% c.l.
from “superweak” fit

- allowing for large uncertainty in this estimate, current CPV measurements
 \Rightarrow $O(10)$ window for NP

A more refined analysis of ϕ_{12}^Γ in the SM

- in ϕ_{12}^Γ trade $\Gamma_5 \cong \Gamma_{12}^0$ for $y \times \Gamma$
- shifts explicit ϵ dependence from $1/\epsilon \rightarrow \epsilon$, because $y = O(1/\epsilon^2)$, $\Gamma_3 = O(\epsilon)$

$$|\phi_{12}^\Gamma| = \left| \frac{\sin \gamma \lambda_b (\lambda_s - \lambda_d)}{2y} \right| \frac{|\Gamma_3|}{\Gamma} \approx 0.005 \frac{|\Gamma_3|}{\Gamma}$$

where $\Gamma_3 = O(\epsilon)$, and is due to SCS decays:

$$\Gamma_3 = \frac{2}{(\lambda_s - \lambda_d)\lambda_b} \sum_f A(\bar{D}^0 \rightarrow f)_{\text{SCS}} A^*(D^0 \rightarrow f)_{\text{SCS}}$$

- consider U-spin decomposition of the SCS and CF decay amplitudes
 - two-body decays account for $\approx 75\%$ of all hadronic D^0 decays, with $D^0 \rightarrow VP, VV, PP, AP$ accounting for $\approx 33\%, 12\%, 12\%, 12\%$, respectively (Cheng, Chiang)
 - comparison of $D^0 \rightarrow VP, VV, PP, AP$ branching ratios, direct CP asymmetries with U-spin decompositions could tell us how large a $|\Gamma_3|/\Gamma$ ratio is plausible

● currently,

$$\frac{|\Gamma_3|}{\Gamma} \sim 1 \Rightarrow \phi_{12}^\Gamma \sim 0.005$$

is plausible, consistent with our more naive estimate

● for $\delta\phi_f$ in SCS decays, this yields

$$\left| \frac{\phi_{12}^\Gamma}{\delta\phi_f} \right| = \left| \frac{\lambda_s^2 \sin \gamma}{2y} \right| \frac{1}{\text{Re}(r_f)} \frac{|\Gamma_3|}{\Gamma} \sim \frac{\lambda_s^2}{2y} \approx 4,$$

consistent with the $\delta\phi_f/\phi_{12}^\Gamma = O(\epsilon)$ parametric suppression

- r_f is the ratio of subleading to leading $D^0 \rightarrow f$ decay amplitudes, $r_f \sim P/T \sim 1$

● improved precision, particularly for $A_{\text{CP}}^d(D^0 \rightarrow VP)$ and $\text{Br}(D^0 \rightarrow VP)$ modes will be most welcome, since VP modes are expected to contribute substantially to Γ_3 (based on their relative importance in Γ)

this would allow a sharper comparison of prominent U-spin amplitudes in Γ_3 and Γ

Conclusion

- we are transitioning to a very exciting period for CPV in $D - \bar{D}$ mixing
- currently we have an $O(10)$ window to NP
- we have introduced a new universal parametrization that captures the departure from the superweak limit at a level of precision that is appropriate for the sensitivity expected in the next generation experiments at LHCb, Belle II
 - it requires one additional universal phase, e.g. ϕ_{12}^Γ
 - final state specific phases associated with direct CPV are not required, in the absence of a surprisingly large $A_{\text{CP}}^{\text{dir}}$ measurement
- If there is NP in CPV, it is almost certainly short distance in ϕ_{12}^M
 - the parametrization allows separate measurements of ϕ_{12}^M and ϕ_{12}^Γ

- mapping out the branching ratios and direct CP asymmetries in a large number of D^0 decay modes is important
 - this will directly impact our understanding of how large absorptive CPV in mixing can be in the SM, with our current estimate being $\phi_{12}^\Gamma = O(0.005)$
 - it could also help us further understand how large the dispersive SM contribution (ϕ_{12}^M) could be, by relating it to the absorptive one using dispersion relations - more challenging
 - a simple U-spin based estimate yields $\phi_{12}^M \sim \phi_{12}^\Gamma$

● In the Belle II / LHCb era we roughly expect

$$\delta\phi_{12}^M \approx 0.017, \quad \delta\phi_{12}^\Gamma \approx 0.034 \quad @95\%c.l.$$

to be compared with the current window

$\phi_{12} \in [-0.07, +0.08]$ (HFAG, new); $[-0.07, +0.21]$ (UTfit) 95% c.l.

(current errors on ϕ_{12}^M , ϕ_{12}^Γ are much larger) and $\phi_{12}^M \sim \phi_{12}^\Gamma = O(0.005)$ in SM

● at HI- LUMI (LHCb $\times 100$) would be sensitive to SM indirect CPV

Backup slides on U-spin decomposition

● U-spin structure of $\Delta C = 1$ Hamiltonian

$$H_1 : \Delta U = 1 \text{ triplet} \propto \bar{c}u (\bar{d}s, \bar{s}s - \bar{d}d, \bar{s}d)$$

$$H_0 : \Delta U = 0 \text{ singlet} \propto \bar{c}u (ss + \bar{d}d)$$

● Possible final state U -spin quantum numbers

$$\text{triplet } f_1 (U = 1, U_3 = 0, \pm 1), \quad \text{singlet } f_0 (U = 0, U_3 = 0)$$

● $\bar{D}^0 \rightarrow PP$ example, with CP eigenstates:

$$f_1 = \frac{K^+K^- - \pi^+\pi^-}{\sqrt{2}}, \quad K^+\pi^-, \quad K^-\pi^+; \quad f_0 = \frac{K^+K^- + \pi^+\pi^-}{\sqrt{2}}$$

● $\bar{D}^0 \rightarrow VP$ example, non-CP eigenstates ($\bar{D}^0 \rightarrow f_1, f_0; \bar{f}_1, \bar{f}_0$):

$$f_1 = \frac{K^{*+}K^- - \rho^+\pi^-}{\sqrt{2}}, \quad K^{*+}\pi^-, \quad K^-\rho^+; \quad f_0 = \frac{K^{*+}K^- + \rho^+\pi^-}{\sqrt{2}}$$

$$\bar{f}_1 = \frac{K^{*-}K^+ - \rho^-\pi^+}{\sqrt{2}}, \quad K^+\rho^-, \quad K^{*-}\pi^+; \quad \bar{f}_0 = \frac{K^{*-}K^+ + \rho^-\pi^+}{\sqrt{2}}$$

- there are two decay amplitudes at 0'th order in $SU(3)$ breaking, where $|0\rangle$ is U-spin singlet D^0 :

$$t_0[f_1] \propto \langle f_1 | H_1 | 0 \rangle, \quad p_0[f_0] \propto \langle f_0 | H_0 | 0 \rangle$$

- there are three decay amplitudes at 1st order in $SU(3)$ breaking, $O(\epsilon)$:

$$s_1[f_0] \propto \langle f_0 | (H_1 \times M_\epsilon)_0 | 0 \rangle, \quad t_1[f_1] \propto \langle f_1 | (H_1 \times M_\epsilon)_1 | 0 \rangle, \quad p_1[f_1] \propto \langle (f_1 \times M_\epsilon)_0 | H_0 | 0 \rangle$$

M_ϵ is the U-spin breaking "spurion"

- M_ϵ connects $\Delta U = 1$ operator H_1 with singlet f_0 final state, and $\Delta U = 0$ operator H_0 with triplet final state f_1

- amplitudes for CP conjugate final states (non-CP eigenstates):

$$t_0[\bar{f}_1], p_0[\bar{f}_0]; \quad s_1[\bar{f}_0]\epsilon, t_1[\bar{f}_1], p_1[\bar{f}_1]$$

- The SCS decay amplitudes to $O(\epsilon)$, for f_1, f_0 final states ($U_3 = 0$),

$$\sqrt{2}A(\bar{D}^0 \rightarrow f_0) = (\lambda_s - \lambda_d) s_1[f_0] \epsilon - \lambda_b 2 p_0[f_0] + O(\epsilon^2)$$

$$\sqrt{2}A(\bar{D}^0 \rightarrow f_1) = (\lambda_s - \lambda_d) t_0[f_1] - \lambda_b p_1[f_1] \epsilon + O(\epsilon^2)$$

and similarly for $\bar{D}^0 \rightarrow \bar{f}_0, \bar{f}_1$

- The CF/DCS decay amplitudes, for f_1 final states ($U_3 = \pm 1$)

$$A_{\text{CF}}(\bar{D}^0 \rightarrow f_1) = V_{cs} V_{ud}^* (t_0[f_1] - \frac{1}{2} t_1[f_1] \epsilon + O(\epsilon^2))$$

$$A_{\text{DCS}}(\bar{D}^0 \rightarrow f_1) = V_{cd} V_{us}^* (t_0[f_1] + \frac{1}{2} t_1[f_1] \epsilon + O(\epsilon^2))$$

and similarly for $\bar{D}^0 \rightarrow \bar{f}_1$

- the ϵ 's are “factored out” to keep track of orders in U-spin breaking. Thus nominally

$$t_0 \sim p_0 \sim s_1 \sim p_1 \sim t_1$$

- Expressed as exclusive sums over all decays, obtain

$$\frac{\Gamma_3}{\Gamma} = - \frac{\sum_{f_{\text{CP}}} \Gamma_3(f_{\text{CP}}) + \sum_{f, \bar{f}} \Gamma_3(f, \bar{f})}{\sum_{f_1, \text{CP}} |t_0[f_1]|^2 + \sum_{f_1, \bar{f}_1} (|t_0[f_1]|^2 + |t_0[\bar{f}_1]|^2) + O(\epsilon)}$$

where

$$\Gamma_3(f_{\text{CP}}) = 4 \text{Re}(p_0^*[f_0] s_1[f_0]\epsilon) + 2 \text{Re}(t_0^*[f_1] p_1[f_1]\epsilon)$$

$$\Gamma_3(f, \bar{f}) = 4 \text{Re}(p_0^*[f_0] s_1[\bar{f}_0]\epsilon) + 4 \text{Re}(p_0^*[\bar{f}_0] s_1[f_0]\epsilon) + 2 \text{Re}(t_0^*[f_1] p_1[\bar{f}_1]\epsilon) + 2 \text{Re}(t_0^*[\bar{f}_1] p_1[f_1]\epsilon)$$

- information about the amplitude ratios

$$\frac{s_0[f_0]\epsilon}{t_0[f_1]}, \quad \frac{p_0[f_0]}{t_0[f_1]}$$

follows from branching ratio and direct CP asymmetry measurements

- as more of these ratios are constrained, knowledge of how large $|\Gamma_3|/\Gamma$ can reasonably be improved

- for the branching ratios

$$\left| \frac{A(D^0 \rightarrow \pi^+ \pi^-)}{A(D^0 \rightarrow K^+ K^-)} \right| = (1.82 \pm 0.02)^{-1} \sim 1 + 2\text{Re} \left(\frac{s_1 \epsilon}{t_0} \right) + O(\epsilon^2)$$

and similarly for (Grossman, Robinson '12)

$$\left| \frac{A(D^0 \rightarrow \pi^+ \rho^-)}{A(D^0 \rightarrow K^+ K^{*-})} \right| = 1.59 \pm 0.10, \quad \left| \frac{A(D^0 \rightarrow \pi^- \rho^+)}{A(D^0 \rightarrow K^- K^{*+})} \right| = 1.33 \pm 0.05$$

- above suggests that $s_1 \epsilon / t_0 \sim 0.25 - 1$ in PP , and is smaller in VP than PP , but precise statements are difficult due to unknown strong phases

- for SCS PP direct CP asymmetries

$$A_{\text{CP}}^{\text{dir}}(D^0 \rightarrow \pi^+ \pi^-, K^+ K^-) \leq O(\text{few} \times 0.1\%) \sim \pm 2 \left| \frac{\lambda_b}{\lambda_s} \right| \text{Im} \left(\frac{p_0}{t_0} \right) \sin \gamma + O(\epsilon).$$

- for SCS VP direct CP asymmetries, have an **HFAG** bound

$$A_{\text{CP}}(D^0 \rightarrow \pi^+ \pi^- \pi^0) < -0.0023 \pm 0.0042$$

and a new LHCb result

- $\Delta A_{\text{CP}} \Rightarrow p_0 \lesssim t_0$ in PP modes

Examples of CPVINT

- SCS decays to CP eigenstates, e.g. $D^0 \rightarrow K^+ K^-$, $\pi^+ \pi^-$

$$\Gamma(D^0(t) \rightarrow f) \propto \exp[-\hat{\Gamma}_{D^0 \rightarrow f} t], \quad \Gamma(\bar{D}^0(t) \rightarrow f) \propto \exp[-\hat{\Gamma}_{\bar{D}^0 \rightarrow f} t]$$

Time-dependent CP asymmetry: $A_\Gamma \equiv (\hat{\Gamma}_{D^0 \rightarrow f} - \hat{\Gamma}_{\bar{D}^0 \rightarrow f})/2\Gamma \neq 0$

$$A_\Gamma \text{ from CPVINT} \propto \sin \phi_{\lambda_f}, \quad A_\Gamma \text{ from CPVMIX} \propto |q/p| - |p/q|$$

- DCS decays to non-CP eigenstates, e.g. wrong sign $D^0 \rightarrow K^+ \pi^-$ vs $\bar{D}^0 \rightarrow K^- \pi^+$

$$\Gamma(D^0(t) \rightarrow K^+ \pi^-) \propto e^{-\Gamma_D t} (a^+ + b^+ t + c^+ t^2), \quad \Gamma(\bar{D}^0(t) \rightarrow K^- \pi^+) \propto e^{-\Gamma_D t} (a^- + b^- t + c^- t^2)$$

Time-dependent CP asymmetries: $b^+ - b^- \neq 0$, $c^+ - c^- \neq 0$

$$\phi_{\lambda_f} + \phi_{\lambda_{\bar{f}}} \neq 0 \Rightarrow b^+ - b^- \neq 0 \text{ from CPVINT}$$

$$|q/p| - |p/q| \neq 0 \Rightarrow b^+ - b^- \neq 0 \text{ from CPVMIX}$$