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Outline

- Measurement of the $K \pi \mathcal{S}$-wave amplitude from a Dalitz plot analysis of $\eta_{c} \rightarrow K \bar{K} \pi$ in two-photon interactions ${ }^{(*)}$.
- Dalitz plot analysis of $J / \psi \rightarrow \pi^{+} \pi^{-} \pi^{0}$ and $J / \psi \rightarrow K^{+} K^{-} \pi^{0(* *)}$.

All the results presented here are new and preliminary.
(*) Work done in collaboration with M. Pennington
(**) Work done in collaboration with M. Pennington and A. Szczepaniak
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## Introduction

$\square$ Charmonium decays can be used to obtain new information on light meson spectroscopy.In $e^{+} e^{-}$interactions, samples of charmonium decays can be obtained using different processes.
$\square$ In two-photon interactions we select events in which the $e^{+}$and $e^{-}$beam particles are scattered at small angles and remain undetected.
$\square$ Only resonances with $J^{P C}=0^{ \pm+}, 2^{ \pm+}, 3^{++}, 4^{ \pm+} \ldots$ can be produced.
In the Initial State Radiation (ISR) process, we reconstruct events having a (mostly undetected) fast forward $\gamma_{I S R}$.
$\square$ Only $J^{P C}=1^{--}$states can be produced.


## Previous work

$\square$ The BaBar Dalitz plot analysis of the $\eta_{c} \rightarrow K^{+} K^{-} \eta$ and $\eta_{c} \rightarrow K^{+} K^{-} \pi^{0}$ has provided the unexpected observation of $K_{0}^{*}(1430) \rightarrow K \eta$ (Phys.Rev. D89 (2014) 11, 112004).



$\square$ We measure the $K_{0}^{*}$ (1430) branching ratio

$$
\frac{\mathcal{B}\left(K_{0}^{*}(1430) \rightarrow \eta K\right)}{\mathcal{B}\left(K_{0}^{*}(1430) \rightarrow \pi K\right)}=0.092 \pm 0.025_{-0.025}^{+0.010}
$$

$\square$ We also find that the $\eta_{c}$ three-body hadronic decays proceed almost entirely through:

$$
\eta_{c} \rightarrow \text { pseudoscalar }+ \text { scalar }
$$Therefore three body decays of the $\eta_{c}$ are a unique window to study the properties of the scalar mesons.

## Selection of $\gamma \gamma \rightarrow K \bar{K} \pi$

$\square$ We study the reactions:

$$
\begin{aligned}
\gamma \gamma & \rightarrow K_{S}^{0} K^{+} \pi^{-} \quad(*), \\
\gamma \gamma & \rightarrow K^{+} K^{-} \pi^{0}(* *)
\end{aligned}
$$Select events having only four tracks.

$\square p_{T}$ : transverse momentum of the $K_{S}^{0} K^{+} \pi^{-}$system with respect to the beam axis.
$\square$ The signal at low $p_{T}$ evidences the presence of two-photon events. We require $p_{T}<0.08 \mathrm{GeV} / \mathrm{c}$.
$\square$ We define $M_{\text {rec }}^{2}$ as:

$$
M_{\mathrm{rec}}^{2} \equiv\left(p_{e^{+} e^{-}}-p_{\mathrm{rec}}\right)^{2}
$$


$\square p_{e^{+} e^{-}}$is the four-momentum of the initial state and $p_{\text {rec }}$ is the four-momentum of the $K_{S}^{0} K^{+} \pi^{-}$system.
$\square$ We remove ISR events requiring $M_{\mathrm{rec}}^{2}>10 \mathrm{GeV}^{2} / \mathrm{c}^{4}$.
(*) Charge conjugation is implied through all this work. (**) Details will be given only for the $K_{S}^{0} K^{+} \pi^{-}$final state.

## The $K \bar{K} \pi$ mass spectra in the $\eta_{c}$ region

$\eta_{c} \rightarrow K_{S}^{0} K^{+} \pi^{-}, 12849$ evts with $(64.3 \pm 0.4) \%$ purity.$\eta_{c} \rightarrow K^{+} K^{-} \pi^{0}, 6494$ evts with $(55.2 \pm 0.6) \%$ purity.Residual $J / \psi$ signals from ISR.
Dalitz plots:
$\square$ Dominated by the presence of $K_{0}^{*}(1430)$.

Purity: $P=N_{s i g} /\left(N_{s i g}+N_{b a c k}\right)$



## Efficiency and Background ( $\eta_{c} \rightarrow K_{S}^{0} K^{+} \pi^{-}$)

$\square$ Efficiency evaluated on the $\left(m\left(K^{+} \pi^{-}\right), \cos \theta\right)$ plane, where $\theta$ is the $K^{+}$helicity angle in the $K_{S}^{0} K^{+} \pi^{-}$rest frame.
$\square$ Fitted using Legendre polynomials moments:

$$
\epsilon(\cos \theta)=\sum_{L=0}^{12} a_{L}\left(m_{K^{+} \pi^{-}}\right) Y_{L}^{0}(\cos \theta)
$$

in slices of $m_{K^{+} \pi^{-}}$.
$\square a_{L}\left(m_{K^{+} \pi^{-}}\right)$fitted with seventh-order polynomials.
$\square$ Background estimated from $\eta_{c}$ sidebands.

$\square$ Asymmetric $K^{*}$ 's.Interference between $\mathrm{I}=1$ and $\mathrm{I}=0$ contributions.



## Dalitz plot analysis of $\eta_{c} \rightarrow K \bar{K} \pi$

$\square$ Unbinned maximum likelihood fits.
$\square$ Fits performed using:

- Isobar model: resonances described by Breit-Wigner functions.
(D. Asner, Review of Particle Physics", Phys. Lett. B 592, 1 (2004)).
- Model Independent Partial Wave Analysis (MIPWA) (Phys. Rev. D 73, 032004 (2006)).
$\square$ The total complex amplitude can, in general, be written as:

$$
A=c_{1} A_{1} e^{i \phi_{1}}+c_{2} A_{2} e^{i \phi_{2}}+c_{3} A_{3} e^{i \phi_{3}}+\ldots
$$The $K \pi \mathcal{S}$-wave $\left(A_{1}\right)$ is taken as the reference amplitude, $c_{1}=1$ and $\phi_{1}=0$.

$$
A=A_{1}+c_{2} A_{2} e^{i \phi_{2}}+c_{3} A_{3} e^{i \phi_{3}}+\ldots
$$The $K \pi$ mass spectrum is divided into 30 equally spaced mass intervals 60 MeV wide and for each bin we add to the fit two new free parameters, the amplitude and the phase of the $K \pi \mathcal{S}$-wave (constant inside the bin).We also fix the $A_{1}$ amplitude to 1.0 and its phase to $\pi / 2$ in an arbitrary interval of the mass spectrum (bin 11 which corresponds to a mass of $1.42 \mathrm{GeV} / c^{2}$ ).The number of additional free parameters is therefore 58.

## Model independent Partial Wave Analysis

Interference between the two $K \pi$ modes is determined by G-parity which is positive for $\eta_{c}$ decays.For $\eta_{c} \rightarrow K_{S}^{0} K^{+} \pi^{-}:$$$
A_{S-w a v e}=\frac{1}{\sqrt{2}}\left(a_{j}^{K^{+}} \pi^{-} e^{i \phi_{j}^{K^{+}} \pi^{-}}+a_{j}^{K_{S}^{0} \pi^{-}} e^{i \phi_{j}^{K_{S}^{0}} \pi^{-}}\right)
$$

where $a^{K^{+} \pi^{-}}(m)=a^{K_{S}^{0} \pi^{-}}(m)$ and $\phi^{K^{+} \pi^{-}}(m)=\phi^{K_{S}^{0} \pi^{-}}(m)$For $\eta_{c} \rightarrow K^{+} K^{-} \pi^{0}:$

$$
A_{S-\text { wave }}=\frac{1}{\sqrt{2}}\left(a_{j}^{K^{+} \pi^{0}} e^{i \phi_{j}^{K^{+} \pi^{0}}}+a_{j}^{K^{-} \pi^{0}} e^{i \phi_{j}^{K^{-} \pi^{0}}}\right)
$$

where $a^{K^{+} \pi^{0}}(m)=a^{K^{-} \pi^{0}}(m)$ and $\phi^{K^{+} \pi^{0}}(m)=\phi^{K^{-} \pi^{0}}(m)$
$\square$ The $K_{2}^{*}(1420), a_{0}(980), a_{0}(1400), a_{2}(1310), \ldots$ contributions are modelled as relativistic Breit-Wigner functions multiplied by the corresponding angular functions.Backgrounds are fitted separately and interpolated into the $\eta_{c}$ signal regions.

## An additional $a_{0}(1950)$ resonance

The fits improves when an additional high mass $a_{0}(1950) \rightarrow K \bar{K} \mathrm{I}=1$ resonance is included with free parameters in both $\eta_{c}$ decay modes.$\square$ The fits return the following parameters:

| Final state | Mass $\left(\mathrm{MeV} / c^{2}\right)$ | Width $(\mathrm{MeV})$ |
| :--- | :---: | :---: |
| $\eta_{c} \rightarrow K_{S}^{0} K^{+} \pi^{-}$ | $1949 \pm 32 \pm 75$ | $265 \pm 36 \pm 110$ |
| $\eta_{c} \rightarrow K^{+} K^{-} \pi^{0}$ | $1927 \pm 15 \pm 23$ | $274 \pm 28 \pm 30$ |
| Mean | $1931 \pm 14 \pm 22$ | $271 \pm 22 \pm 29$ |

BaBar preliminary
red line:no $a_{0}(1950)$

Statistical significances for the $a_{0}(1950)$ effect (including systematics) are $2.5 \sigma$ for $\eta_{c} \rightarrow K_{S}^{0} K^{+} \pi^{-}$and $4.0 \sigma$ for $\eta_{c} \rightarrow K^{+} K^{-} \pi^{0}$.

## Dalitz plots mass projections

$\square$ Dalitz plot projections with fit results for $\eta_{c} \rightarrow K_{S}^{0} K^{+} \pi^{-}$(top) and $\eta_{c} \rightarrow K^{+} K^{-} \pi^{0}$ (bottom)
Shaded is contribution from the interpolated background.$K^{*}(890)$ contributions entirely from background.

## Legendre polynomial moments: $\eta_{c} \rightarrow K_{S}^{0} K^{+} \pi^{-}$

$\square$ Mass projections weighted by $Y_{L}^{0}$ moments and compared with fit results. $m\left(K^{+} \pi^{-}\right)+m\left(K_{S}^{0} \pi^{-}\right)$projections:






$m\left(K_{S}^{0} K^{+}\right)$projections:

$\square$ Good agreement in all the projections.


## Fit fractions from the MIPWA. Comparison with the Isobar Model

| Amplitude | $\eta_{\mathrm{c}} \rightarrow \mathrm{K}_{\mathrm{S}}^{0} \mathrm{~K}^{+}{ }_{\pi}^{-}$ |  | $\eta_{\mathbf{c}} \rightarrow \mathrm{K}^{+} \mathrm{K}^{-} \pi^{\mathbf{0}}$ |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Fraction (\%) | Phase | Fraction (\%) | Phase |
| ( $K \pi \boldsymbol{S}$-wave) $K$ | $107.3 \pm 2.6 \pm 17.9$ | 0. | $125.5 \pm 2.4 \pm 4.2$ | 0 . |
| $a_{0}(980) \pi$ | $0.83 \pm 0.46 \pm 0.80$ | $1.08 \pm 0.18 \pm 0.18$ | $0.00 \pm 0.03 \pm 1.7$ | $\ldots$ |
| $a_{0}(1450) \pi$ | $0.7 \pm 0.2 \pm 1.4$ | $2.63 \pm 0.13 \pm 0.17$ | $1.2 \pm 0.4 \pm 0.7$ | $2.90 \pm 0.12 \pm 0.25$ |
| $a_{0}(1950) \pi$ | $3.1 \pm 0.4 \pm 1.2$ | $-1.04 \pm 0.08 \pm 0.77$ | $4.4 \pm 0.8 \pm 0.7$ | $-1.45 \pm 0.08 \pm 0.27$ |
| $a_{2}(1320) \pi$ | $0.15 \pm 0.06 \pm 0.08$ | $1.85 \pm 0.20 \pm 0.23$ | $0.61 \pm 0.23 \pm 0.3$ | $1.75 \pm 0.23 \pm 0.42$ |
| $K_{2}^{*}(1430)^{0} K$ | $4.7 \pm 0.9 \pm 1.4$ | $4.92 \pm 0.05 \pm 0.1$ | $3.0 \pm 0.8 \pm 4.4$ | $5.07 \pm 0.09 \pm 0.3$ |
| Total | $116.8 \pm 2.8$ |  | $134.8 \pm 2.7$ |  |
| $\chi_{2} / N_{\text {cells }}$ | $301 / 254=1.17$ |  | $283.2 / 233=1.22$ |  |
| Isobar Model |  |  |  |  |
| $\left(K_{0}^{*}(1430) K\right)+$ | $73.6 \pm 3.7$ |  | $63.6 \pm 5.6$ |  |
| $\left(K_{0}^{*}(1950) K\right)+$ |  |  |  |  |
| Nonresonant |  |  |  |  |
| ...... | ...... | $\ldots$ | ...... | $\ldots$ |
| $\chi_{2} / N_{\text {cells }}$ | $457 / 254=1.82$ |  | $383 / 233=1.63$ |  |

$\square$ For MIPWA, good agreement between the two $\eta_{c}$ decay modes.
$\square\left(K \pi \mathcal{S}\right.$-wave) $K$ amplitude dominant with small contributions from $K_{2}^{*}(1430)^{0} K$ and $a_{0}(1950) \pi$ amplitudes.
$\square$ Spin-1 resonances consistent to come entirely from background.
$\square$ Good description of the data with MIPWA.
$\square$ Worse description of the data with the Isobar Model.

## Test for multiple solutions and Systematic uncertainties

$\square$ We have generated and fitted MC simulations with different mixtures of amplitudes.
$\square$ We started the fits from random values for the $K \pi \mathcal{S}$-wave amplitude and phase.
$\square$ We have evaluated the following systematic uncertainties.

- Fit bias. We generate MC simulations according to the fit results and re-fit. The distribution of the absolute value of the fractional residuals is fit with a Gaussian having zero mean and take the $\sigma$ as systematic uncertainty.
- The amplitude and phase are constant within the mass bins in the reference fit. We replace the representation using a cubic spline.
- We remove low significances amplitudes such as $a_{0}(980)$ and $a_{2}(1310)$ resonances.
- We vary up and down the purity of the signal.
- The effect of the efficiency variation as a function of the $K \bar{K} \pi$ mass is evaluated by computing separate efficiencies in the regions below and above the $\eta_{c}$ mass.Total average systematic uncertainty is of the order of $16 \%$.


## New measurement of the $K \pi$ S-wave

$\square$ Fitted amplitude and phase.Red: $\eta_{c} \rightarrow K^{+} K^{-} \pi^{0}$. Black: $\eta_{c} \rightarrow K_{S}^{0} K^{+} \pi^{-}$.Clear $K_{0}^{*}(1430)$ resonance and corresponding phase motion.
$\square$ At high mass broad $K_{0}^{*}(1950)$ contribution.


$\square$ Dashed lines are $K \eta$ and $K \eta^{\prime}$ thresholds.Good agreement between the two $\eta_{c}$ decay modes.

## Comparison with the LASS and E791 experiments

$\square$ Black is $\eta_{c} \rightarrow K_{S}^{0} K^{+} \pi^{-}$.
$\operatorname{LASS}\left(K^{-} p\right)$
$\operatorname{E791}\left(D^{+} \rightarrow K^{-} \pi^{+} \pi^{+}\right)$Normalization is arbitrary.LASS analysis has two solutions above 1.9 GeV .Phases before the $K \eta^{\prime}$

 threshold are similar, as expected from Watson theoren $\frac{\tilde{\circ}}{\text { ² }}$.
$\square$ Amplitudes are very different.


(LASS: Nucl. Phys. B 296, 493 (1988)), (E791: Phys. Rev. D 73, 032004 (2006)), (K.M. Watson, Phys. Rev. 88, 1163 (1952))

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Dalitz plot analysis of }J/\psi->\mp@subsup{\pi}{}{+}\mp@subsup{\pi}{}{-}\mp@subsup{\pi}{}{0}\mathrm{ and }J/\psi->\mp@subsup{K}{}{+}\mp@subsup{K}{}{-}\mp@subsup{\pi}{}{0
```

$\square$ Only a preliminary result exists, to date, on a Dalitz-plot analysis of $J / \psi$ decays to $\pi^{+} \pi^{-} \pi^{0}$ (SLAC-PUB-5674, (1991)).
$\square$ While large samples of $J / \psi$ decays exist, some branching fractions remain poorly measured. In particular the $J / \psi \rightarrow K^{+} K^{-} \pi^{0}$ branching fraction has been measured by MarkII using only 25 events.
$\square$ The BES III experiment has performed an angular analysis of $J / \psi \rightarrow K^{+} K^{-} \pi^{0}$. The analysis requires the presence of a broad $J^{P C}=1^{--}$state in the $K^{+} K^{-}$ threshold region, which is interpreted as a multiquark state (Phys. Rev. Lett. 97, 142002 (2006)).

## Data selection

$\square$ We study the following reactions:

$$
e^{+} e^{-} \rightarrow \gamma_{\mathrm{ISR}} \pi^{+} \pi^{-} \pi^{0}, \quad e^{+} e^{-} \rightarrow \gamma_{\mathrm{ISR}} K^{+} K^{-} \pi^{0}
$$

where $\gamma_{\mathrm{ISR}}$ indicate the (undetected) ISR photon.Select events having only two tracks and one (mass constrained) $\pi^{0}$.$\square$ We compute $M_{\mathrm{rec}}^{2} \equiv\left(p_{e^{-}}+p_{e^{+}}-p_{h^{+}}-p_{h^{-}}-p_{\pi^{0}}\right)^{2}$, where $h=\pi / K$.$\square$ This quantity should peak near zero for ISR events.Plot of $M_{\text {rec }}^{2}$ in the $J / \psi$ signal region. In red are Monte Carlo simulations.



## $J / \psi$ signals and yields

$\square$ We select events in the ISR region by requiring $\left|M_{\mathrm{rec}}^{2}\right|<2 \mathrm{GeV}^{2} / \mathrm{c}^{4}$ and obtain the $J / \psi$ signals.

$\square$ We fit the mass spectra using the Monte Carlo resolution functions described by a Crystal Ball+Gaussian functions and obtain the yields:

| $J / \psi$ decay mode | $\chi^{2} / N D F$ | $J / \psi$ mass $(\mathrm{MeV})$ | Signal region events | Purity |
| :---: | :---: | :---: | :---: | :---: |
| $J / \psi \rightarrow \pi^{+} \pi^{-} \pi^{0}$ | $84 / 115$ | $3099.8 \pm 0.2$ | 21974 | $(86.1 \pm 1.3) \%$ |
| $J / \psi \rightarrow K^{+} K^{-} \pi^{0}$ | $111 / 95$ | $3101.0 \pm 0.2$ | 2393 | $(87.8 \pm 0.7) \%$ |

## Efficiency and Branching fraction

$\square$ The efficiency is mapped and fitted on the $\left(m\left(h^{+} h^{-}\right), \cos \theta_{h}\right)$ plane, where $\theta_{h}$ is the $h^{+}$helicity angle in the $J / \psi$ rest frame

where negative weights $f_{i}$ are assigned to sidebands events.We obtain the following preliminary result:

$$
\mathcal{R}=\frac{\mathcal{B}\left(J / \psi \rightarrow K^{+} K^{-} \pi^{0}\right)}{\mathcal{B}\left(J / \psi \rightarrow \pi^{+} \pi^{-} \pi^{0}\right)}=0.0929 \pm 0.002 \pm 0.002
$$

$\square$ The PDG reports $\mathcal{B}\left(J / \psi \rightarrow K^{+} K^{-} \pi^{0}\right)=55.2 \pm 0.12 \times 10^{-4}$, based on 25 events, and $\mathcal{B}\left(J / \psi \rightarrow \pi^{+} \pi^{-} \pi^{0}\right)=2.11 \pm 0.07 \times 10^{-2}$.
$\square$ These values give a ratio $\mathcal{R}=0.262 \pm 0.057$, which differs from our result by $3 \sigma$.

$$
J / \psi \rightarrow \pi^{+} \pi^{-} \pi^{0} \text { Dalitz plot and projections }
$$Dominated by three $\rho(770) \pi$ contributions.Dalitz plot analysis performed using:

- Isobar model using Zemach tensors;
C. Zemach, Phys Rev. 133, B1201 (1964),
C. Dionisi et. al., Nucl. Phys. B169, 1 (1980).
- Veneziano model.
(A. P. Szczepaniak, M.R. Pennington, Phys. Lett. B737, 283 (2014)).
$\square$ Dalitz plot projections.


$\square$ Shaded is the background interpolated by sidebands.

$$
J / \psi \rightarrow \pi^{+} \pi^{-} \pi^{0} \text { Dalitz plot analysis }
$$

$\square$ The Veneziano model deals with trajectories rather than single resonances.
$\square$ The complexity of the model is related to $n$, the number of Regge trajectories included in the fit.
$\square$ The fit requires $\mathrm{n}=5$. 路 $_{\text {º }}$
$\square$ Combinatorial $\pi$ helicity angle vs. $m(\pi \pi)$.
$\square m(\pi \pi)$ mass projection for $\left|\cos \theta_{\pi}\right|<0.2$.



| Final state | Isobar fraction $\%$ | Phase (radians) | Veneziano fraction $\%$ |  |
| :--- | ---: | :---: | :---: | :---: |
| $\rho(770) \pi$ | $119.0 \pm$ | $1.1 \pm$ | 3.3 | 0. |
| $\rho(1460) \pi$ | $16.9 \pm$ | $2.0 \pm$ | 3.1 | $3.92 \pm 0.05 \pm 0.11$ |
| $\rho(1700) \pi$ | $0.1 \pm$ | $0.1 \pm$ | 0.2 | $1.01 \pm 0.35 \pm 0.79$ |
| $\rho(2150) \pi$ | $0.04 \pm$ | $0.05 \pm$ | 0.02 | $1.89 \pm 0.30 \pm 0.48$ |
| $\rho 3(1690) \pi$ |  |  |  | $120.0 \pm 1.9$ |
| Sum | $136.0 \pm$ | $2.3 \pm$ | 4.3 | $0.84 \pm 0.08$ |
| $\chi^{2} / \nu$ |  | $764 / 552$ |  | $0.09 \pm 0.02$ |

$\square$ The two models give almost similar data representation, but different fractions.

## $J / \psi \rightarrow K^{+} K^{-} \pi^{0}$ Dalitz plot analysis

$\square$ Clear $K^{*+}$ and $K^{*}$ bands.
$\square$ Broad structure in the low $K^{+} K^{-}$mass region.
$\square$ We make use of the Isobar model only.

| Final state | fraction $\%$ | phase |
| :--- | :---: | :---: |
| $K^{*}(892) K$ | $87.8 \pm 2.0 \pm 1.7$ | 0. |
| $\rho(1450)^{0} \pi^{0}$ | $11.5 \pm 2.1 \pm 2.1$ | $-2.81 \pm 0.25 \pm 0.36$ |
| $K^{*}(1410) K$ | $1.7 \pm 0.7 \pm 1.1$ | $2.89 \pm 0.35 \pm 0.08$ |
| $K_{2}^{*}(1430) K$ | $3.8 \pm 1.4 \pm 0.5$ | $-2.42 \pm 0.22 \pm 0.07$ |
| $\rho(1700)^{0} \pi^{0}$ | $0.9 \pm 1.0 \pm 0.6$ | $1.06 \pm 0.20 \pm 0.7$ |
| Total | $105.6 \pm 3.4 \pm 3.0$ |  |
|  | $\chi^{2} / \nu=94 / 92$ |  |

$\square$ Leaving free the $\rho(1450)$ parameters:


$$
\begin{aligned}
& m(\rho(1450))=1361 \pm 43 \mathrm{MeV} / c^{2} \\
& \Gamma(\rho(1450))=479 \pm 63 \mathrm{MeV}
\end{aligned}
$$

in the range of other $\rho(1450)$ measurements.

Dalitz plot projections:

Shaded is the background.

## $\rho(1450)$ branching fraction

$\square$ We find the parameters of the low mass $K^{+} K^{-}$structure consistent for being associated to $\rho(1450)$.
$\square$ We have measured the ratio

$$
\mathcal{R}=\mathcal{B}\left(J / \psi \rightarrow K^{+} K^{-} \pi^{0}\right) / \mathcal{B}\left(J / \psi \rightarrow \pi^{+} \pi^{-} \pi^{0}\right)=0.0929 \pm 0.002 \pm 0.002
$$

$\square$ From the Dalitz-plot analysis of $J / \psi \rightarrow \pi^{+} \pi^{-} \pi^{0}$ we obtain:

$$
\mathcal{B}_{1}=\mathcal{B}\left(J / \psi \rightarrow \rho(1450)^{0} \pi^{0}\right)=[(16.9 \pm 2.0 \pm 3.1) / 3 .] \%=(5.63 \pm 0.67 \pm 1.03) \%
$$

$\square$ From the Dalitz-plot analysis of $J / \psi \rightarrow K^{+} K^{-} \pi^{0}$ we obtain:

$$
\mathcal{B}_{2}=\mathcal{B}\left(J / \psi \rightarrow \rho(1450)^{0} \pi^{0}\right)=(11.5 \pm 2.1 \pm 2.1) \%
$$

$\square$ We therefore obtain:

$$
\begin{array}{|l|l|}
\hline \mathcal{B}\left(\rho(1450)^{0} \rightarrow K^{+} K^{-}\right) \\
\mathcal{B}\left(\rho(1450)^{0} \rightarrow \pi^{+} \pi^{-}\right) & =\frac{\mathcal{B}_{2}}{\mathcal{B}_{1}} \cdot \mathcal{R}=0.190 \pm 0.042 \pm 0.049 \\
\hline
\end{array}
$$

## Summary

- We show preliminary results on the Dalitz plot analyses of $\eta_{c} \rightarrow K_{S}^{0} K^{+} \pi^{-}$and $\eta_{c} \rightarrow K^{+} K^{-} \pi^{0}$ produced in two-photon interactions.
- We extract for the first time the $K \pi \mathcal{S}$-wave amplitude and phase using the MIPWA method. We find a very different amplitude with respect to that measured by previous experiments in different processes.
- We show preliminary results on Dalitz plot analyses of $J / \psi \rightarrow \pi^{+} \pi^{-} \pi^{0}$ and $J / \psi \rightarrow K^{+} K^{-} \pi^{0}$ produced in Initial State Radiation events using the isobar and Veneziano models.

