Implications of $SU(3)_F$ on Charm CP Violation

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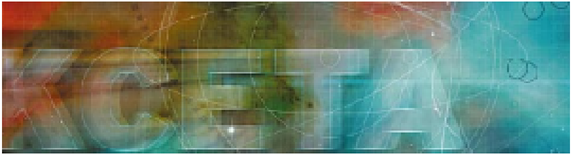


CHARM 2015: The 7th International Workshop on Charm Physics
Detroit, Michigan, USA

based on Müller, Nierste, StS: `arXiv:1503.06759` and in preparation

Can we resolve new physics in charm decays?

Goal: Get the most out of $\mathcal{B}$'s in order to predict CP asymmetries.

Red: Update in 2014

Observable	Measurement
SCS CP asymmetries	
$\Delta a_{CP}^{\mathrm{dir}}(K^+K^-,\pi^+\pi^-)$	-0.00253 ± 0.00104
$\Sigma a_{CP}^{\mathrm{dir}}(K^+K^-,\pi^+\pi^-)$	-0.0011 ± 0.0026
$a_{CP}^{\mathrm{dir}}(D^0 \to K_S K_S)$	-0.23 ± 0.19
$a_{CP}^{\mathrm{dir}}(D^0 \to \pi^0\pi^0)$	-0.0004 ± 0.0064
$a_{CP}^{\mathrm{dir}}(D^+ \to \pi^0\pi^+)$	$+0.029 \pm 0.029$
$a_{CP}^{\mathrm{dir}}(D^+ \to K_S K^+)$	$+0.0011 \pm 0.0017$
$a_{CP}^{\mathrm{dir}}(D_s \to K_S \pi^+)$	$+0.006 \pm 0.005$
$a_{CP}^{\mathrm{dir}}(D_s \to K^+\pi^0)$	$+0.266 \pm 0.228$
Indirect CP violation	
a_{CP}^{ind}	0.00013 ± 0.00052
$\delta_L \equiv 2\mathrm{Re}(\varepsilon)/(1+\lvert\varepsilon\rvert^2)$	$(3.32 \pm 0.06)\cdot 10^{-3}$
$K^+\pi^-$ strong phase difference	
$\delta_{K\pi}$	$(6.45 \pm 10.65)^\circ$

Observable	Measurement
SCS branching ratios	
$\mathcal{B}(D^0 \to K^+K^-)$	$(3.96 \pm 0.08)\cdot 10^{-3}$
$\mathcal{B}(D^0 \to \pi^+\pi^-)$	$(1.402 \pm 0.026)\cdot 10^{-3}$
$\mathcal{B}(D^0 \to K_S K_S)$	$(0.17 \pm 0.04)\cdot 10^{-3}$
$\mathcal{B}(D^0 \to \pi^0\pi^0)$	$(0.820 \pm 0.035)\cdot 10^{-3}$
$\mathcal{B}(D^+ \to \pi^0\pi^+)$	$(1.19 \pm 0.06)\cdot 10^{-3}$
$\mathcal{B}(D^+ \to K_S K^+)$	$(2.83 \pm 0.16)\cdot 10^{-3}$
$\mathcal{B}(D_s \to K_S \pi^+)$	$(1.22 \pm 0.06)\cdot 10^{-3}$
$\mathcal{B}(D_s \to K^+\pi^0)$	$(0.63 \pm 0.21)\cdot 10^{-3}$
CF branching ratios	
$\mathcal{B}(D^0 \to K^-\pi^+)$	$(3.88 \pm 0.05)\cdot 10^{-2}$
$\mathcal{B}(D^0 \to K_S \pi^0)$	$(1.19 \pm 0.04)\cdot 10^{-2}$
$\mathcal{B}(D^0 \to K_L \pi^0)$	$(1.00 \pm 0.07)\cdot 10^{-2}$
$\mathcal{B}(D^+ \to K_S \pi^+)$	$(1.47 \pm 0.07)\cdot 10^{-2}$
$\mathcal{B}(D^+ \to K_L \pi^+)$	$(1.46 \pm 0.05)\cdot 10^{-2}$
DCS branching ratios	
$\mathcal{B}(D^0 \to K^+\pi^-)$	$(1.35 \pm 0.02)\cdot 10^{-4}$
$\mathcal{B}(D^+ \to K^+\pi^0)$	$(1.83 \pm 0.26)\cdot 10^{-4}$

The Problem

Charm is not really heavy compared to $\Lambda_{\rm QCD}$

- $m_c \sim 1.3$ GeV , $m_b \sim 4.2$ GeV.
- Perturbative expansion in $\Lambda_{\rm QCD}/m_c$ will not work.

Available tools

- $\mathrm{SU}(3)_F$ expansion.
- Topological amplitudes.
- $1/N_c$ expansion.
  ['t Hooft 1974, Buras Gerard Rückl 1986]

➥The best we can do at present.

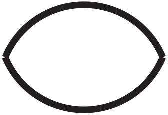
Calculate what is calculable. Fit all other hadronic parameters from $\mathcal{B}$'s.

't Hooft 1974: Study $SU(3)_C \Rightarrow SU(N_c)_C$

N_c = number of colors.

- Asymptotic freedom $\Rightarrow$ Expansion in $\alpha_s(\mu)$ works for high energies.
- Breaks down for low energy QCD $\Rightarrow$ Nonperturbative regime.
- Consider $N_c \to \infty$ and expand in $1/N_c$.
- $g = g_0/\sqrt{N_c}$, $g^2 \sim 1/N_c$.

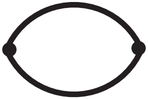
gluon vertex:  $\mathcal{O}(1/\sqrt{N_c})$

closed loop:  $\mathcal{O}(N_c)$

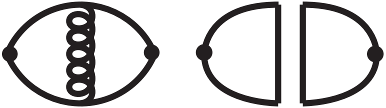
meson vertex:  $\mathcal{O}(1/\sqrt{N_c})$

$1/N_c$ power counting

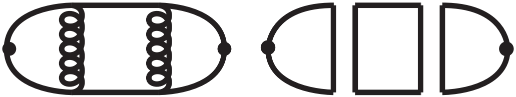
Corrections of the same order:



$\mathcal{O}(\mathbf{N_c})$

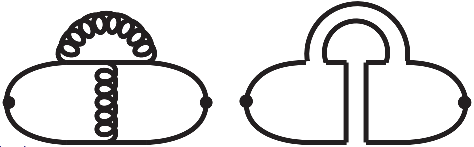


$\mathcal{O}(\mathbf{N_c})$



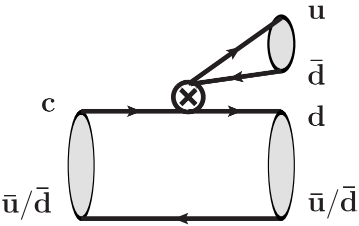
$\mathcal{O}(\mathbf{N_c})$

Suppressed corrections:

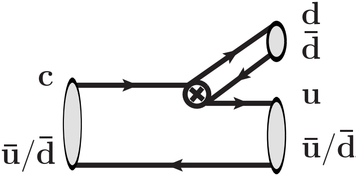


$\mathcal{O}(\mathbf{1/N_c})$

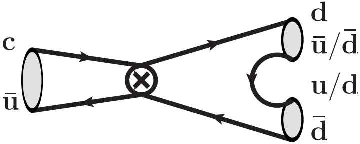
Application to charm decay topologies



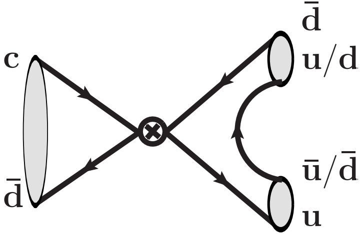
tree (T)



color-suppressed tree (C)



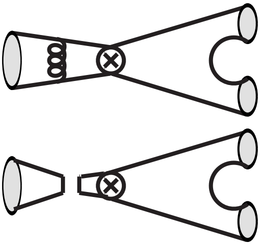
exchange (E)

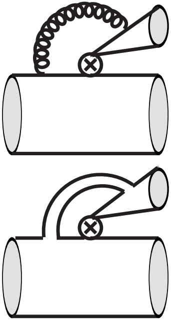


annihilation (A)

[Chau 1980,1982; Zeppenfeld 1981, Buras Silvestrini 1998]

Corrections to T and A diagrams $1/N_c^2$ suppressed





same order in $1/N_c$

$\Rightarrow$ fit E.

$1/N_c^2$-suppressed.

$\Rightarrow$ fit $\delta_T \leq 15\%$ in $T = T^{\rm fac}(1 + \delta_T)$,

analogous: $\Rightarrow$ fit $\delta_A \leq 15\%$ in $A = A^{\rm fac}(1 + \delta_A)$

for example:

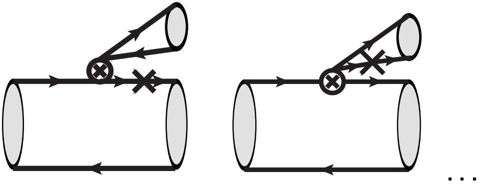
$$T(D^0 \to K^+K^-) = \frac{G_F}{\sqrt{2}} a_1 f_K (m_D^2 - m_K^2) F^{DK}(m_K^2) \left(1 + \mathcal{O}(1/N_c^2)\right)$$

$$A(D_s^+ \to K^0\pi^+) = \frac{G_F}{\sqrt{2}} a_1 f_{D_s} (m_K^2 - m_\pi^2) F^{K\pi}(m_{D_s}^2) \left(1 + \mathcal{O}(1/N_c^2)\right)$$

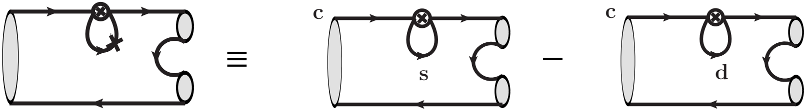
Diagrammatic SU(3)$_F$ breaking

- Feynman rule from $H_{SU(3)_F} = (m_s - m_d)\bar{s}s$: dot on s-quark line. [Gronau 1995]
- Find 14 new topological amplitudes:
  3 diagrams for each T, C, E, A; $P_{\rm break} \equiv P_d - P_s$; $PA_{\rm break} \equiv PA_d - PA_s$.

[Brod Grossman Kagan Zupan 2012]



T$_1$ T$_2$

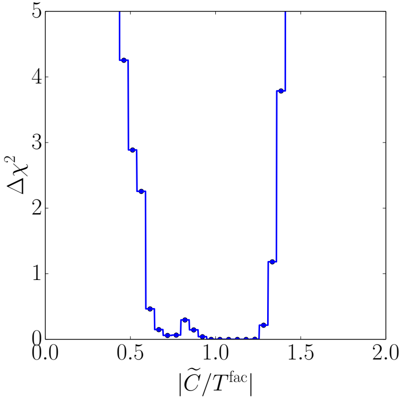


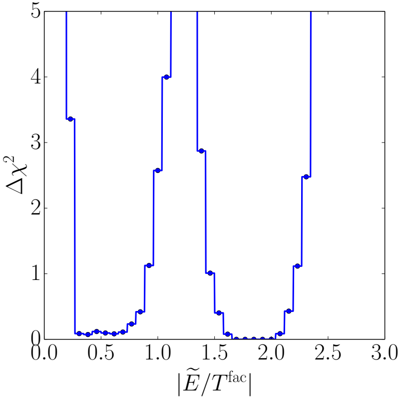
penguin (P$_{\rm break}$)

Diagrammatic Parameterization (excerpt)

Decay d	T	$T_1^{(1)}$	$T_2^{(1)}$	$T_3^{(1)}$	A	$A_1^{(1)}$	$A_2^{(1)}$	$A_3^{(1)}$	C	$C_1^{(1)}$	$C_2^{(1)}$	$C_3^{(1)}$	...
SCS													
$D^0 \to K^+K^-$	1	1	1	0	0	0	0	0	0	0	0	0	...
$D^0 \to \pi^+\pi^-$	-1	0	0	0	0	0	0	0	0	0	0	0	...
$D^0 \to \bar{K}^0K^0$	0	0	0	0	0	0	0	0	0	0	0	0	...
$D^0 \to \pi^0\pi^0$	0	0	0	0	0	0	0	0	$-\frac{1}{\sqrt{2}}$	0	0	0	...
$D^+ \to \pi^0\pi^+$	$-\frac{1}{\sqrt{2}}$	0	0	0	0	0	0	0	$-\frac{1}{\sqrt{2}}$	0	0	0	...
$D^+ \to \bar{K}^0K^+$	1	1	1	0	-1	0	0	-1	0	0	0	0	...
$D_s \to K^0\pi^+$	-1	0	0	-1	1	1	1	0	0	0	0	0	...
$D_s \to K^+\pi^0$	0	0	0	0	$-\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$	0	$-\frac{1}{\sqrt{2}}$	0	0	$-\frac{1}{\sqrt{2}}$	...
CF													
$D^0 \to K^-\pi^+$	1	1	0	0	0	0	0	0	0	0	0	0	...
$D^0 \to \bar{K}^0\pi^0$	0	0	0	0	0	0	0	0	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	0	0	...
$D^+ \to \bar{K}^0\pi^+$	1	1	0	0	0	0	0	0	1	1	0	0	...
$D_s \to \bar{K}^0K^+$	0	0	0	0	1	1	0	1	1	1	0	1	...
DCS													
$D^0 \to K^+\pi^-$	1	0	1	0	0	0	0	0	0	0	0	0	...
$D^0 \to K^0\pi^0$	0	0	0	0	0	0	0	0	$\frac{1}{\sqrt{2}}$	0	$\frac{1}{\sqrt{2}}$	0	...
$D^+ \to K^0\pi^+$	0	0	0	0	1	0	1	0	1	0	1	0	...
$D^+ \to K^+\pi^0$	$\frac{1}{\sqrt{2}}$	0	$\frac{1}{\sqrt{2}}$	0	$-\frac{1}{\sqrt{2}}$	0	$-\frac{1}{\sqrt{2}}$	0	0	0	0	0	...
$D_s \to K^0K^+$	1	0	1	1	0	0	0	0	1	0	1	1	...

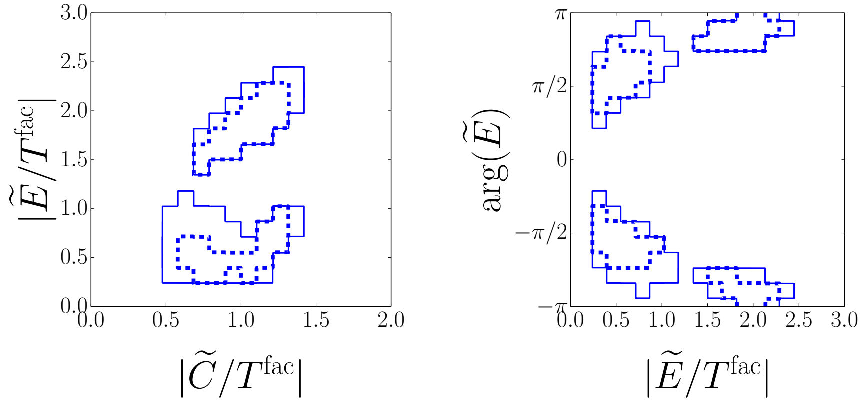
Fit results for SU(3)$_F$-limit parameters





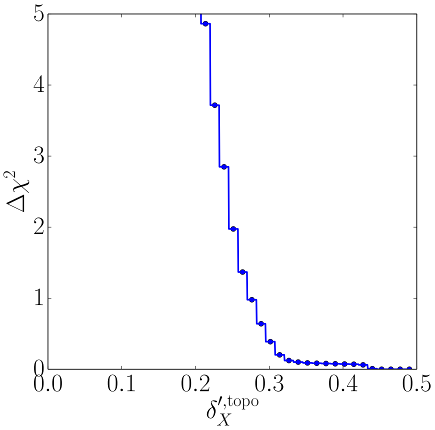
- Perfect fit to branching ratios: $\chi^2 \sim 0$: under-determined problem. But: Nontrivial result due to many parameter constraints: Permit only up to 50% SU(3)$_F$-breaking.

Broad and Multiple Fit Solutions



Example: Quantify SU(3)$_F$-breaking

$\Delta\chi^2$ profile of the parameter $\delta_X^{\prime,\mathrm{topo}}$ which quantifies the overall size of SU(3)$_F$-breaking:



Results:

i) The SU(3)$_F$ limit $\delta_X^{\prime,\mathrm{topo}} = 0$ is ruled out by more than 5σ .

ii) At 68% CL there is at least 28% of SU(3)$_F$ breaking.

Relative Importance of Diagrams: Likelihood Ratio Tests

Hypothesis	Significance of rejection
$P_{\text{break}} = 0$	0.7σ
$P_{\text{break}} = E_i^{(1)} = C_i^{(1)} = 0\ \forall\, i$	$> 5\sigma$
$E_i^{(1)} = 0\ \forall\, i$	3.0σ
$E = E_i^{(1)} = 0\ \forall\, i$	$> 5\sigma$
$C_i^{(1)} = 0\ \forall\, i$	4.3σ
$C = C_i^{(1)} = 0\ \forall\, i$	$> 5\sigma$

- Clear need for $SU(3)_F$ breaking.
- P_{break} allowed to be zero at 0.7σ.

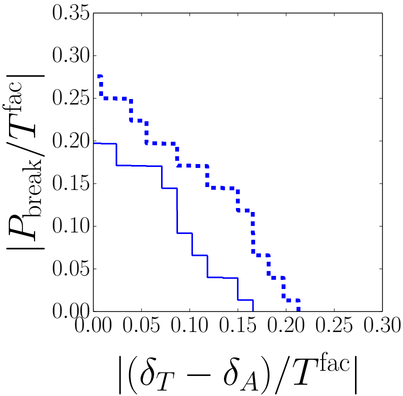
Probe the GIM mechanism in Charm

Complementarity of P_{break} and $1/N_c^2$-corrections:

$$\mathcal{B}(D^+ \to K_{S,L}K^+) = |\lambda_{sd}|^2\,\mathcal{P}(D^+, K^0, K^+)\times \left|\mathcal{A}^{\text{fac}}(D^+ \to \bar{K}^0 K^+) + (\delta_T - \delta_A) + P_{\text{break}}\right|^2 ,$$

$$\mathcal{B}(D_s^+ \to K_{S,L}\pi^+) = |\lambda_{sd}|^2\,\mathcal{P}(D_s^+, K^0, \pi^+)\times \left|\mathcal{A}^{\text{fac}}(D_s^+ \to K^0 \pi^+) - (\delta_T - \delta_A) + P_{\text{break}}\right|^2 ,$$

$$\mathcal{B}(D^+ \to K^+\pi^0) = |V_{cd}^* V_{us}|^2\,\mathcal{P}(D^+, K^+, \pi^0)\times \left|\mathcal{A}^{\text{fac}}(D^+ \to K^+ \pi^0) + (\delta_T - \delta_A)\right|^2 ,$$



Probe the quality of the $1/N_c$ expansion

Sum rules between $D^+ \to K_S K^+$, $D_s^+ \to K_S \pi^+$ and $D^+ \to K^+ \pi^0$

$$\widetilde{\mathcal{A}}(D^+ \to \bar{K}^0 K^+) - \widetilde{A}(D_s^+ \to K^0 \pi^+) = 2\,(\delta_T - \delta_A)$$

$$\widetilde{\mathcal{A}}(D^+ \to K^+ \pi^0) = \frac{1}{\sqrt{2}}\,(\delta_T - \delta_A)$$

Combination of both:

$$\widetilde{\mathcal{A}}(D^+ \to \bar{K}^0 K^+) - \widetilde{\mathcal{A}}(D_s^+ \to K^0 \pi^+) - 2\sqrt{2}\widetilde{\mathcal{A}}(D^+ \to K^+ \pi^0) = 0$$

Definition

$$\widetilde{\mathcal{A}}(d) \equiv \mathcal{A}(d) - \mathcal{A}^{\mathrm{fac}}(d)\,,$$

$$\mathcal{A}^{\mathrm{fac}}(d) \equiv T^{\mathrm{fac}}(d) + A^{\mathrm{fac}}(d)\,.$$

Predictions for Branching Ratios

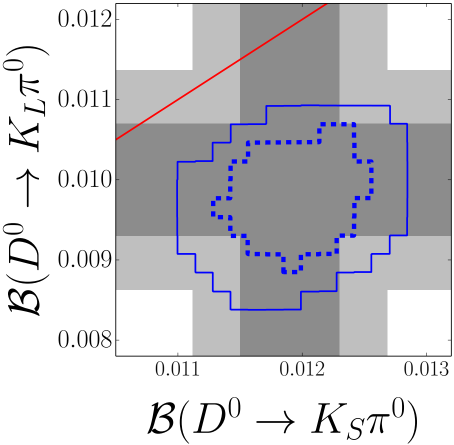
- In general, individual branching ratios predicted from global fit not more precise than current measurements.
- Exception: Probe DCS amplitudes.
- $A(D^0 \to \overline{K}^0\pi^0)$ (CF) and $A(D^0 \to K^0\pi^0)$ (DCS) interfere.
  $\Rightarrow A(D^0 \to K_S\pi^0)$ and $A(D^0 \to K_L\pi^0)$
- $SU(3)_F$ limit:

$$\mathcal{B}(D^0 \to K_S\pi^0) \propto |E - C|^2 + 2\lambda^2|E - C|^2$$

$$\mathcal{B}(D^0 \to K_L\pi^0) \propto |E - C|^2 - 2\lambda^2|E - C|^2$$

Can $SU(3)_F$ breaking change the $SU(3)_F$-limit prediction
$\mathcal{B}(D^0 \to K_L\pi^0) < \mathcal{B}(D^0 \to K_S\pi^0)$?

Probe of DCS amplitudes I



Gray:
68% CL and 95% CL measurements

Blue:
68% CL and 95% CL fit regions

Red line:
$\mathcal{B}(D^0 \to K_L\pi^0) = \mathcal{B}(D^0 \to K_S\pi^0)$

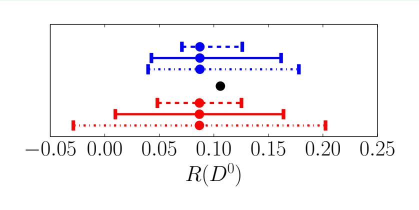
While SU(3)$_F$ breaking can be sizable, $\mathcal{B}(D^0 \to K_L\pi^0) < \mathcal{B}(D^0 \to K_S\pi^0)$ holds with a significance of more than 4σ.

Probe of DCS amplitudes II

Formulated differently

Asymmetry

$$R(D^0) \equiv \frac{\mathcal{B}(D^0 \to K_S\pi^0) - \mathcal{B}(D^0 \to K_L\pi^0)}{\mathcal{B}(D^0 \to K_S\pi^0) + \mathcal{B}(D^0 \to K_L\pi^0)}$$

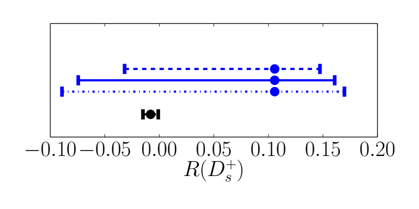


Blue: $1, 2, 3\sigma$. Black: SU(3)$_F$-limit.
[Bigi Yamamoto 1994, Rosner 2006, Gao 2006]

$\mathcal{B}(D_s^+ \to K_L K^+)$ not measured yet.

Prediction:
$\mathcal{B}(D_s^+ \to K_L K^+) = 0.012^{+0.007}_{-0.002}$ at 3σ

$$R(D_s^+) \equiv \frac{\mathcal{B}(D_s^+ \to K_S K^+) - \mathcal{B}(D_s^+ \to K_L K^+)}{\mathcal{B}(D_s^+ \to K_S K^+) + \mathcal{B}(D_s^+ \to K_L K^+)}$$



Black: QCDF@1σ [Gao 2014]

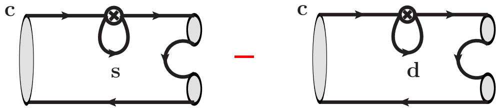
CP Asymmetries: Theory service for Experiment

- Every measurement hinting for $a_{CP}^{\mathrm{dir}} \neq 0$ was successfully postdicted...
  - ...in the Standard Model.
  - ...or New Physics Models.

- **Why is that?**

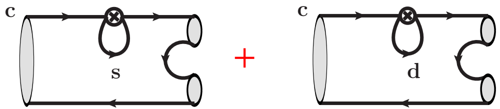
Problem of CP Asymmetry Predictions:

- New hadronic quantities appear which cannot be extracted from $\mathcal{B}$ measurements.
- $\mathcal{B}$'s involve only



$\Rightarrow$ Difference can be extracted. ✓

- A_{CP}'s involve also



The sum is **unknown.** ✗

[Brod, Grossman, Kagan, Zupan 2012]

Solution: CP asymmetry sum rules

Strategy: Sum rules among CP asymmetries.

- Build combinations out of several CP asymmetries...
- ...containing only those topological amplitudes in coefficients which can be extracted from the global fit to the branching ratios.

Extent known $SU(3)_F$ limit sum rules

[see, *e.g.*, Grossman Kagan Nir 2006, Hiller Jung Schacht 2012, Grossman Ligeti Robinson 2014]

$$a_{CP}^{\mathrm{dir}}(D^0 \to K^+K^-) + a_{CP}^{\mathrm{dir}}(D^0 \to \pi^+\pi^-) = 0,$$

$$a_{CP}^{\mathrm{dir}}(D^+ \to \bar{K}^0K^+) + a_{CP}^{\mathrm{dir}}(D_s^+ \to K^0\pi^+) = 0,$$

valid at zeroth order $SU(3)_F$ breaking.

- Include corrections of sum rules due to $SU(3)_F$ breaking in the CKM-leading part of the amplitude...

Result

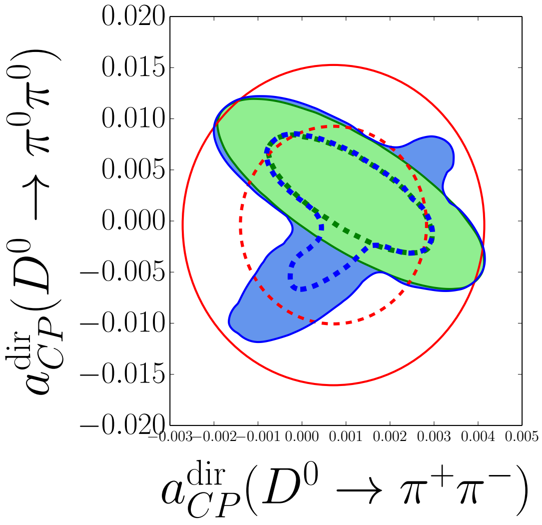
Two sum rules each correlating three direct CP asymmetries

I $D^0 \to K^+K^-$, $D^0 \to \pi^+\pi^-$, and $D^0 \to \pi^0\pi^0$,
and
II $D^+ \to \overline{K}^0K^+$, $D_s^+ \to K^0\pi^+$, and $D_s^+ \to K^+\pi^0$.

- Note: Still works to zeroth order in SU(3)$_F$ breaking only, as SU(3)$_F$ breaking in CKM-subleading part of amplitudes is not taken into account, e.g. SU(3)$_F$ breaking of $P_s + P_d$.
- Still: theoretical accuracy of new-physics tests only limited by the assumed size of SU(3)$_F$ breaking, i.e. generically $\mathcal{O}(30\%)$.
- Great progress compared to $\mathcal{O}(1000\%)$ spread of past predictions.

➥Look at phenomenological implications.

Implications of sum rule I [preliminary]



Red solid:
95% CL measurement

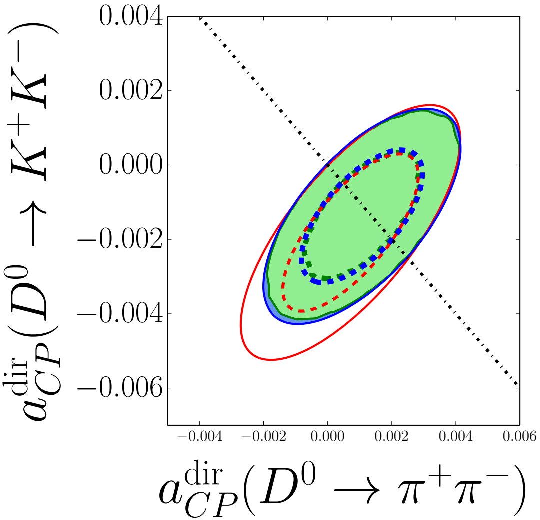
Red dashed:
68% CL measurement

Present data:
Light blue:
95% CL from global fit
Dark blue dashed:
68% CL from global fit

Future scenario:
assume $\sqrt{50}$ better branching ratios, but $a_{CP}^{\rm dir}(D^0 \to K^+K^-)$ as today.

Light green:
95% CL from global fit
Dark green dashed:
68% CL from global fit

Implications of sum rule I, contd. [preliminary]



Black dashed: SU(3)$_F$ limit

Red solid:
95% CL measurement

Red dashed:
68% CL measurement

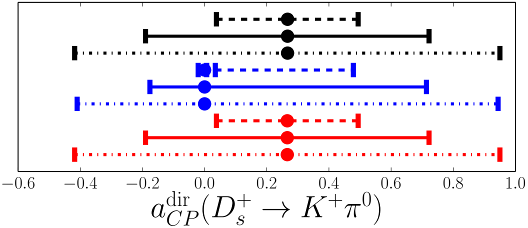
Present data:
Light blue:
95% CL from global fit
Dark blue dashed:
68% CL from global fit

Future scenario:
assume $\sqrt{50}$ better branching ratios, but $a_{CP}^{\text{dir}}(D^0 \to K^+K^-)$ as today.

Light green:
95% CL from global fit
Dark green dashed:
68% CL from global fit

Implications of sum rule II [preliminary]

Use measured values of $D^+ \to \overline{K}^0 K^+$ and $D_s^+ \to K^0 \pi^+$ to predict $a_{CP}^{\mathrm{dir}}(D_s^+ \to K^+ \pi^0)$:



Blue: prediction from $a_{CP}^{\mathrm{dir}}(D^+ \to \overline{K}^0 K^+)$, $a_{CP}^{\mathrm{dir}}(D_s^+ \to K^0 \pi^+)$, and global fit to branching ratios.

Black: same as blue, but without $1/N_c$ constraints.

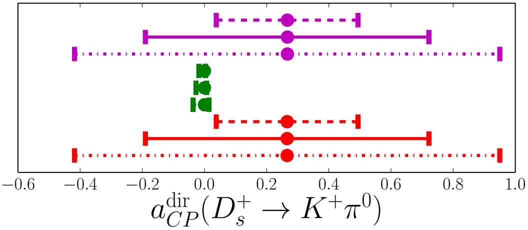
Red: measurement. Dashed: 1σ, solid: 2σ, dot-dashed: 3σ.

Not shown: error from $\mathrm{SU}(3)_F$ breaking in $P_s + P_d$.

$\Rightarrow$ yet another successful postdiction.

Implications of sum rule II, future scenario [preliminary]

But: Assuming better measurements of the branching ratios by a factor of $\sqrt{50}$ changes the picture:



Green: prediction from $a_{CP}^{\mathrm{dir}}(D^+ \to \overline{K}^0 K^+)$, $a_{CP}^{\mathrm{dir}}(D_s^+ \to K^0\pi^+)$, and global fit to branching ratios.

Magenta: same as blue, but without $1/N_c$ constraints.

Red: measurement. Dotted: 1σ, solid: 2σ, dot-dashed: 3σ.

Not shown: error from $SU(3)_F$ breaking in $P_s + P_d$.

Conclusion

- Global fit of $D \to PP'$ branching ratios to topological amplitudes including linear $\mathrm{SU}(3)_F$ breaking and $1/N_c$-counting gives multiple degenerate best-fit solutions.
- The method permits likelihood ratio test to quantify e.g. the size of $\mathrm{SU}(3)_F$ breaking and $P_{\text{break}} \neq 0$ (GIM).
- We predict:

$$\mathcal{B}(D_s^+ \to K_L K^+) = 0.012^{+0.007}_{-0.002} \qquad \text{at } 3\sigma$$
$$\mathcal{B}(D^0 \to K_L \pi^0) < \mathcal{B}(D^0 \to K_S \pi^0) \qquad \text{at } 4\sigma$$

- CP asymmetries involve topological amplitudes not constrained by the fit. These can be eliminated by forming judicious combinations of several CP asymmetries $\to$ sum rules .
- The sum rules test the quality of $\mathrm{SU}(3)_F$ in penguin amplitudes and/or new physics.

BACK-UP

Equivalence to SU(3)$_F$

- Diagrammatic parameterization ⇔ matrix which expresses decay amplitudes in terms of SU(3)$_F$ matrix elements.
- Same rank. Same 6 sum rules. Explicit matching (excerpt):

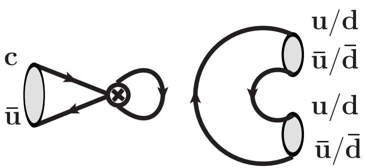
SU(3)$_F$ ME	...	E	$E_1^{(1)}$	$E_2^{(1)}$	$E_3^{(1)}$	P^{break}
A_{27}^{15}	...	0	0	0	0	0
A_8^{15}	...	$-\frac{5}{2\sqrt{2}}$	$-\frac{5}{3\sqrt{2}}$	$-\frac{5}{6\sqrt{2}}$	0	0
$A_8^{\bar{6}}$	...	$\frac{\sqrt{5}}{2}$	0	$\frac{\sqrt{5}}{2}$	0	0
B_1^3	...	0	$\frac{32\sqrt{\frac{35}{421}}}{3}$	$\frac{32\sqrt{\frac{35}{421}}}{3}$	$-\frac{16\sqrt{\frac{35}{421}}}{3}$	$\frac{32\sqrt{\frac{35}{421}}}{3}$
B_8^3	...	0	$-\frac{20\sqrt{\frac{7}{3937}}}{3}$	$-\frac{20\sqrt{\frac{7}{3937}}}{3}$	$\frac{40\sqrt{\frac{7}{3937}}}{3}$	$\frac{160\sqrt{\frac{7}{3937}}}{3}$
$B_8^{\bar{6}_1}$	...	0	$20\sqrt{\frac{7}{2869}}$	$-20\sqrt{\frac{7}{2869}}$	0	0
$B_8^{15_1}$	...	0	$460\sqrt{\frac{7}{1330969}}$	$20\sqrt{\frac{133}{70051}}$	$-840\sqrt{\frac{7}{1330969}}$	0
$B_8^{15_2}$	...	0	$-20\sqrt{\frac{6}{871}}$	$10\sqrt{\frac{6}{871}}$	$10\sqrt{\frac{6}{871}}$	0
$B_{27}^{15_1}$	...	0	0	0	0	0
$B_{27}^{15_2}$	...	0	0	0	0	0
$B_{27}^{24_1}$	...	0	0	0	0	0

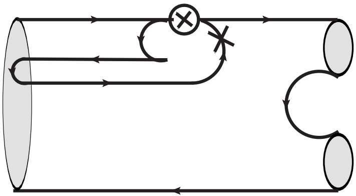
Consequences of Equivalence to $SU(3)_F$

- Topological amplitude parameterization is algebraically complete.
  ↳All further diagrammatic contributions can be absorbed.

- Example 1: $PA_{\rm break} \equiv PA_s - PA_d$ can be absorbed into exchange diagrams.
- Example 2: Contributions from higher Fock states:

$$\left|K^0\right\rangle = |d\bar{s}\rangle + |d\bar{s}g\rangle + |d\bar{s}u\bar{u}\rangle + \ldots .$$





Consistency Constraints on $SU(3)_F$ breaking: Validity of perturbative expansion

Diagrammatic measures of $SU(3)_F$ breaking $\leq 50\%$

1. $\delta_X^{\prime,\mathcal{T}} \equiv \max_d \left|\mathcal{A}_X^{\mathcal{T}}(d)/\mathcal{A}(d)\right|, \quad \mathcal{T} = C, E, P_{\text{break}}$

   individual amount of $\cancel{SU(3)_F}$ by topology $\mathcal{T}$.

2. $\delta_X^{\prime,\text{topo}} \equiv \max_d \left|\sum_{\mathcal{T}} A_X^{\mathcal{T}}(d)/\mathcal{A}(d)\right|$

   overall amount of $\cancel{SU(3)_F}$.

3. $\delta_X^{C_i/C} \equiv |C_i/C|$ $\quad \cancel{SU(3)_F}$ in C-parameters.

4. $\delta_X^{E_i/E} \equiv |E_i/E|$ $\quad \cancel{SU(3)_F}$ in E-parameters.

$\mathcal{A}_X^{\mathcal{T}}(d)$: $\cancel{SU(3)_F}$ part of the amplitude of decay d stemming from topology $\mathcal{T}$.