

Lattice perspective on leptonic and semileptonic decays

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1. Introduction

Goals in the study of leptonic and semileptonic D decays

- * Precise determination of CKM matrix elements ($|V_{cd,cs}|$)

$$\text{Experiment} = (\text{known factors}) \times (V_{CKM}) \times \underbrace{(\text{hadronic matrix elements})}_{\text{lattice QCD}}$$

- * Check Standard Model
 - ** Consistency of different determinations of CKM matrix elements
 - ** Test unitarity of CKM matrix.
 - ** Comparison of shape of form factors with experimental data.

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 - ** Comparison of shape of form factors with experimental data.
- * Validate lattice QCD techniques to use in B physics
- * Constraining possible NP models

Fajfer, Nisandzic and Rojec, 1502.07748, Barranco et al., 1303.3896, 1404.0454

- ** Correlated signals of NP in leptonic and semileptonic decays.

1. Introduction: Lattice QCD

Lattice QCD: Numerical evaluation of QCD path integral (rely only on first principles).

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Goal: Precise calculations ($\leq 5\%$ error)

* Control over systematic errors:

** Unquenched calculations: $N_f = 2$, $\mathbf{N}_f = \mathbf{2} + \mathbf{1}$ or $\mathbf{N}_f = \mathbf{2} + \mathbf{1} + \mathbf{1}$.

** Discretization: improved actions + simulations at several a 's
 \rightarrow continuum limit.

** Chiral extrapolation: simulate at several m_π and extrapolate to m_π^{phys}
using ChPT.

** Renormalization: non-perturbative, perturbative.

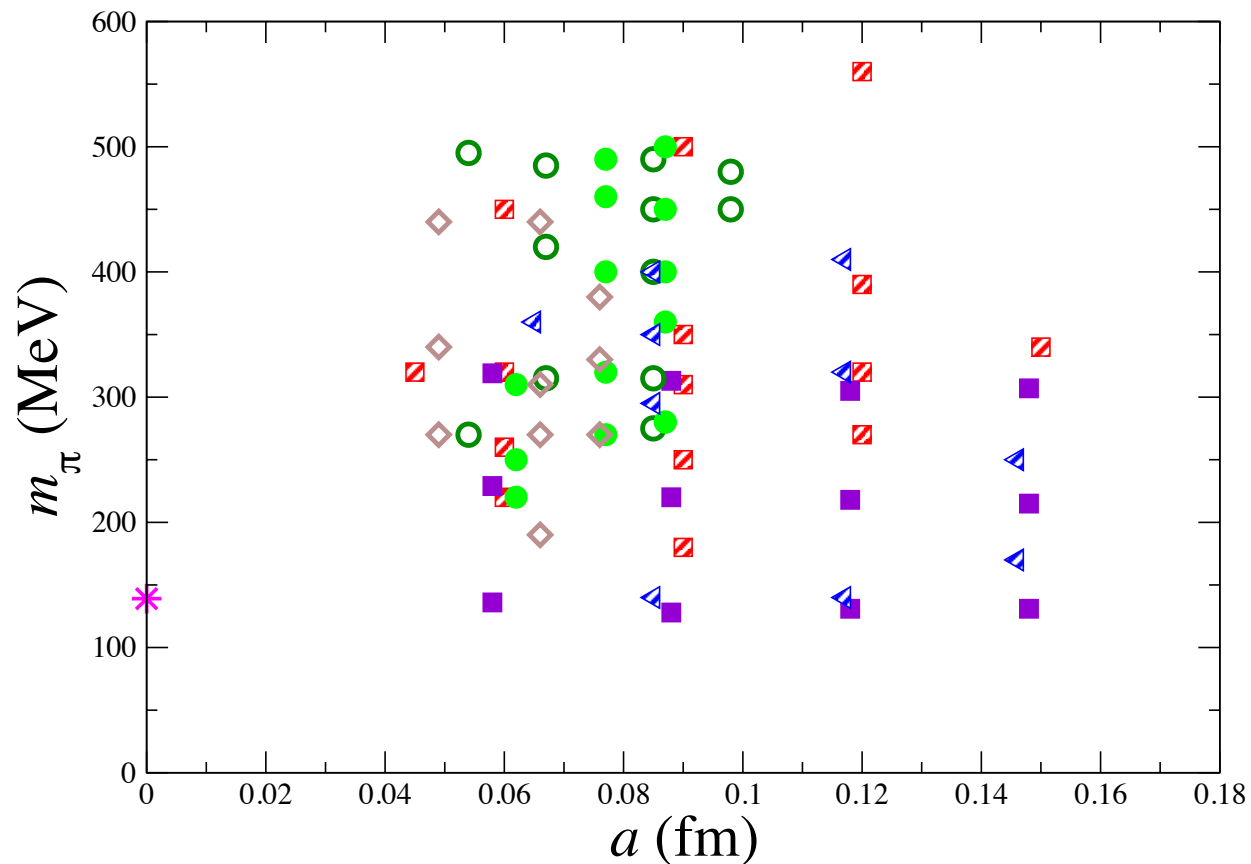
** Tuning lattice scale and masses

** Finite volume, **isospin effects, electromagnetic effects, ...**

Systematically improvable

1. Introduction: Overview of simulations parameters

Several $N_f = 2 + 1$ and even $N_f = 2 + 1 + 1$, and **physical quark masses**.



ETM: Circles (ETM, Orsay)

MILC: Squares

(FNAL/MILC, HPQCD, SWME)

RBC/UKQCD: Triangles (χ QCD)

CLS: Diamonds (ALPHA)

$N_f = 2$: open

$N_f = 2 + 1$: shaded

$N_f = 2 + 1 + 1$: filled

plot by A. El-Khadra,

First results with simulations with physical light quark masses (BMW, PACS-CS, MILC, RBC/UKQCD, ETMC)

1. Introduction: Averaging lattice QCD results

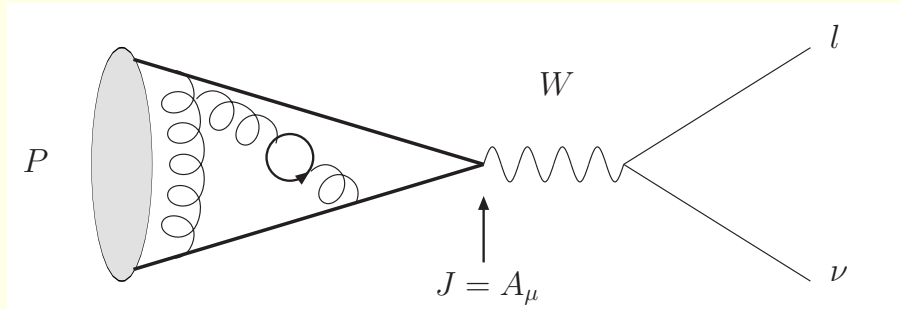
Flavor Lattice Averaging Group (FLAG-2): 28 people representing all big lattice collaborations.

- Advisory Board: S. Aoki, C. Bernard, C. Sachrajda
- Editorial Board: G. Colangelo, H. Leutwyler, T. Vladikas, U. Wenger
- Working Groups:
 - u, d and s quark masses: L. Lellouch, T. Blum, V. Lubicz
 - $|V_{us}|, |V_{ud}|$: A. Jüttner, T. Kaneko, S. Simula
 - LEC's: S. Dürr, H. Fukaya, S. Necco
 - B_K : H. Wittig, J. Laiho, S. Sharpe
 - α_s : R. Sommer, R. Horsley, T. Onogi
 - $f_{B(s)}, f_{D(s)}, \hat{B}_B$: A. El Khadra, Y. Aoki, M. Della Morte
 - B, D semileptonic and radiative decays: R. Van de Water, E. Lunghi, C. Pena

<http://itpwiki.unibe.ch/flag/>

arXiv:1310.8555 (last version: August 2014).

2. Leptonic D decays



$$\mathcal{B}(D_q \rightarrow l\nu) \sim 4.5 - 3.5\%$$

$$\tau_{D_q} < 1.5\%$$

$$f_{D_q}^2 \sim 1\%$$

$$\text{others} < 0.4\%$$

$$\underbrace{\mathcal{B}(D_q \rightarrow l\nu)}_{\text{experiment}} = \frac{G_F^2 \tau_{D_q} m_l^2 m_{D_q}}{8\pi} \left(1 - \frac{m_l^2}{m_{D_q}^2}\right)^2 |V_{cq}|^2 \underbrace{f_{D_q}^2}_{\text{lattice}}$$

(with $q = d, s$ and $D_q = D^+, D_s$)

Simple matrix element $\langle 0 | \bar{c} \gamma_\mu \gamma_5 q | D_q(p) \rangle = i f_{D_q} p_\mu \rightarrow$ precise calculations

or, if using the same action for light and charm valence quarks,

$$(m_c + m_q) \langle 0 | \bar{c} \gamma_5 q | D_{(s)}(p=0) \rangle = f_{D_q} M_{D_q}^2 \quad (\text{no need of renormalization})$$

Reduction of error: use relativistic (improved) formulations for c .

2. Leptonic D decays: New results (> 2013)

$N_f = 2$:

* **TWQCD**, 1404.3648: $a \sim 0.06 fm$ and $m_\pi \leq 260$ MeV

$$f_D = 202.3(3.4) \text{ MeV} \quad f_{D_s} = 258.7(3.1) \text{ MeV} \quad f_{D_s}/f_D = 1.279(26)$$

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$N_f = 2 + 1$:

* **χ QCD**, 1410.3343: Different set of configurations (**RBC/UKQCD**) and valence quark formulation (overlap) than previous calculations: two lattice spacings.

$$f_{D_s} = 254(2)(4) \text{ MeV}$$

2. Leptonic D decays: New results (> 2013)

$N_f = 2 + 1 + 1$:

* **FNAL/MILC**, 1407.3772: highly improved action, **MILC** configurations with **phys. quark masses** and small lattice spacing (4 a 's, smallest $a \approx 0.06$ fm)

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$$f_{D^+} = 212.6_{-1.2}^{+1.1} \text{ MeV} \quad f_{D_s} = 249.0_{-1.5}^{+1.3} \text{ MeV} \quad f_{D_s}/f_{D^+} = 1.1712_{-34}^{+31}$$

** $\sim 0.5\%$ error dominated by **continuum extrapolation** error

** They calculate the difference between f_{D^+} and the isospin limit value, f_D :

$$f_{D^+} - f_D = 0.47_{-5}^{+11} \text{ MeV}$$

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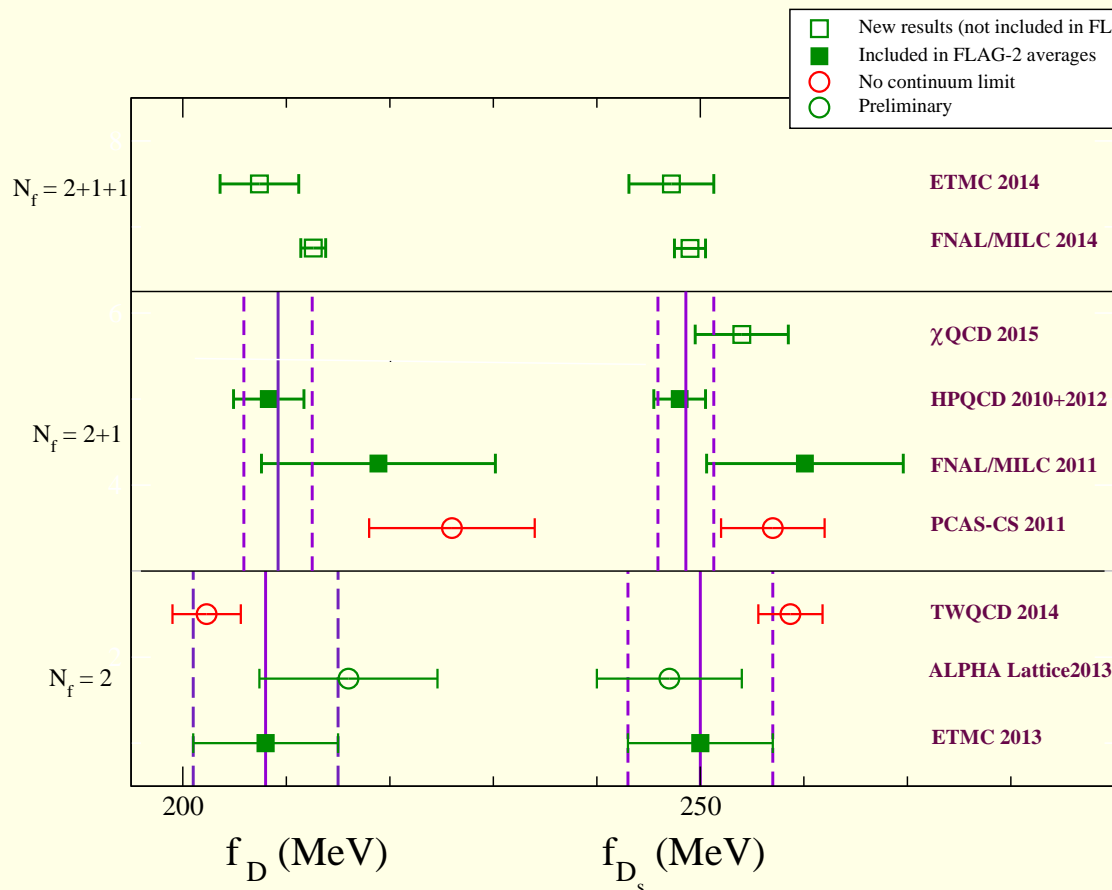
* **ETMC**, 1411.7908: **ETMC** configurations with 3 $a's$ (smallest $a \approx 0.06$ fm),

$$m_\pi \geq 210 \text{ MeV}$$

$$f_D = 207.2(3.8) \text{ MeV} \quad f_{D_s} = 247.2(4.1) \text{ MeV} \quad f_{D_s}/f_D = 1.192(22)$$

** Error dominated by **stat. + chiral extrapolation** error

2. Leptonic D decays



FLAG - 2, $N_f = 2$

$$f_D = (208 \pm 7) \text{ MeV}$$

$$f_{D_s} = (250 \pm 7) \text{ MeV}$$

FLAG - 2, $N_f = 2 + 1$

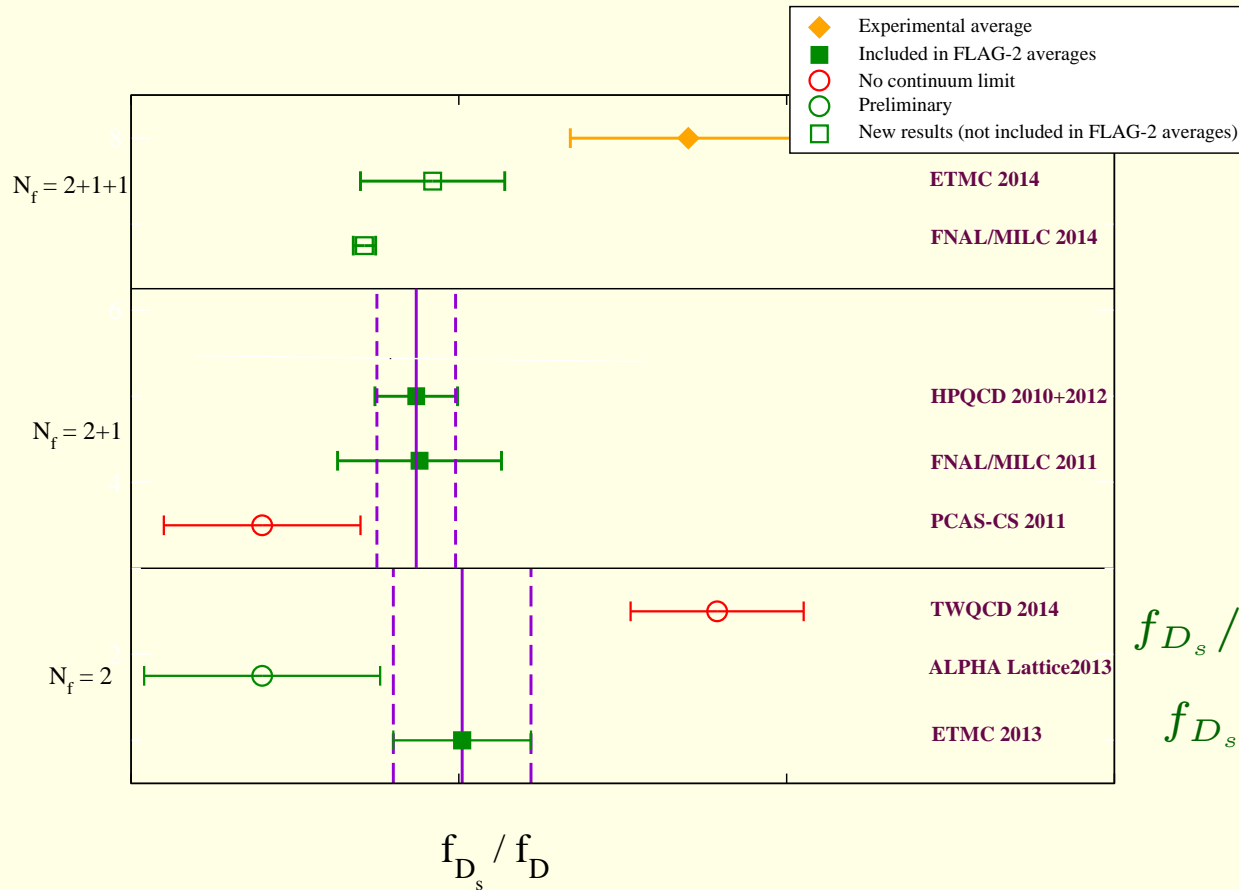
$$f_D = (209.2 \pm 3.3) \text{ MeV}$$

$$f_{D_s} = (248.6 \pm 2.7) \text{ MeV}$$

FNAL/MILC $N_f = 2 + 1 + 1$

$$f_{D^+} = 212.6^{+1.1}_{-1.2} \text{ MeV} \quad f_{D_s} = 249.0^{+1.3}_{-1.5} \text{ MeV}$$

2. Leptonic D decays



FLAG - 2, $N_f = 2$

$$f_{D_s} / f_D = 1.20 \pm 0.020$$

FLAG - 2, $N_f = 2 + 1$

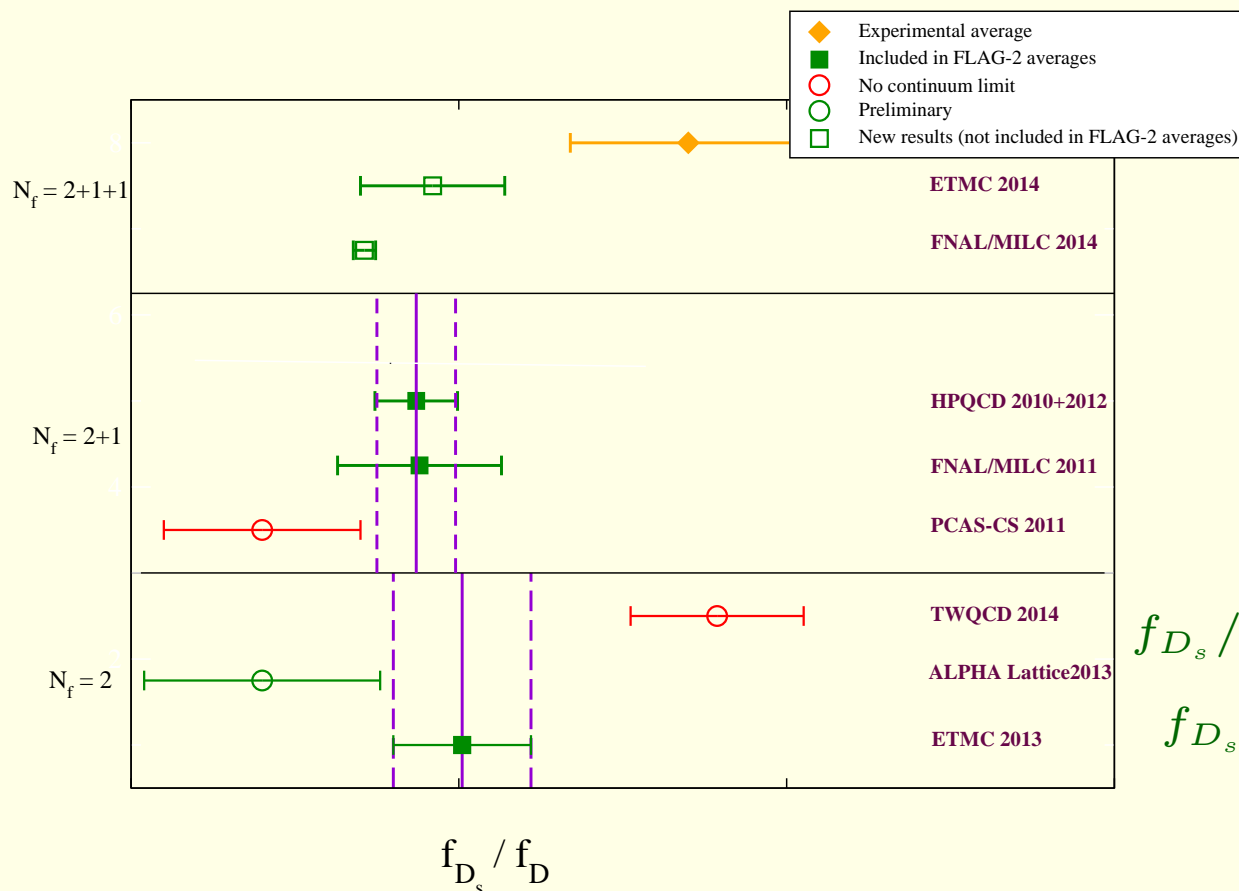
$$f_{D_s} / f_D = 1.187 \pm 0.012$$

$N_f = 2 + 1 + 1$:

$$f_{D_s} / f_{D^+} \Big|_{\text{FNAL/MILC}} = 1.1712^{+31}_{-34}$$

$$f_{D_s} / f_D \Big|_{\text{ETMC}} = 1.192 \pm 0.022$$

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$$f_{D_s}/f_D \Big|_{\text{ETMC}} = 1.192 \pm 0.022$$

Experiment: Average from **G. Rong**, CKM2014, 1411.3868 and unitarity values $|V_{cs}| = 0.97343 \pm 0.00015$, $|V_{cd}| = 0.22522 \pm 0.00061$ from **PDG2014**:

$$f_{D_s}/f_{D^+} \Big|_{\text{exp.}} = 1.270 \pm 0.036$$

2.7σ larger than $N_f = 2 + 1 + 1$ **FNAL/MILC** result
and **2.3σ** larger than $N_f = 2 + 1$ **FLAG-2** average

2. Leptonic D decays: $N_f = 2 + 1$ calculations in progress

- * **RBC/UKCD**, 1502.00845: plans to calculate several charm physics observables (and extrapolate to am_b).

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- * **RBC/UKCD**, 1502.00845: plans to calculate several charm physics observables (and extrapolate to am_b).
- * **FNAL/MILC**, 1501.01991: Fermilab $c + \text{asqtad}$ (relativistic) $s, u = d$ calculation with 5 a 's and high statistics
- ** Estimated total error: $f_D \sim 2.1\%$, $f_{D_s} \sim 1.8\%$, $f_{D_s}/f_D \sim 1\%$
(larger error is heavy-quark mass tuning for $f_{D_{(s)}}$)
- ** Errors reduced by a factor of ~ 2.5 , now comparable to **HPQCD**
- ** Use same action for b and $c \rightarrow$ precise calculations of ratios
 f_B/f_D and f_{B_s}/f_{D_s} (many systematics cancel)

2. Leptonic decays: Charm-light and charm-charm vector mesons

$$\langle 0 | V_i | M^* (\vec{0}, \lambda) \rangle = f_{M^*} m_{M^*} e_i^\lambda$$

where e_i^λ is the polarization vector of the meson M^* .

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Predictions for f_{D^*} and $f_{D_s^*}$ (using relativistic action for c)

* $N_f = 2$ on **ETM** configurations

$$f_{D_s^*} / f_{D_s} = 1.26(3) \quad f_{D^*} / f_D = 1.208(27)$$

Becirevic et al, 1201.4039

Becirevic et al, 1407.1019

* $N_f = 2 + 1$ calculation by **HPQCD**, 1312.5264

$$f_{D_s^*} / f_{D_s} = 1.10(2) \quad \rightarrow \quad f_{D_s^*} = 274(6) \text{ MeV}$$

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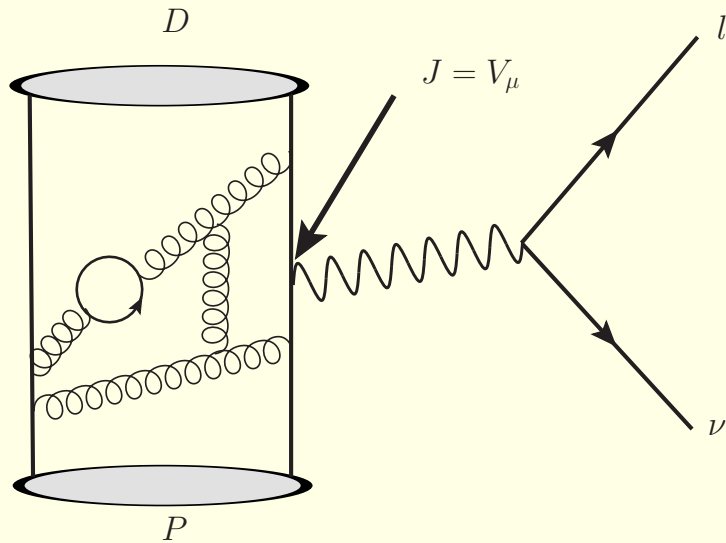
Calculations of $f_{J/\Psi}$ (experimental value $f_{J/\Psi}^{exp} = 407(5) \text{ MeV}$)

* Calculation on $N_f = 2$ **ETM** configurations by **Becirevic and Sanfilippo**, 1206.1445 :

$$f_{J/\Psi} = 414 \pm 8_{-0}^{+9} \text{ MeV}$$

* $N_f = 2 + 1$ calculation by **HPQCD**, 1208.2855: $f_{J/\Psi} = 405(6)(2) \text{ MeV}$

3. Semileptonic D decays



$$P = \pi, K$$

$x = d, s$ daughter light quark

$q = (p_D - p_P)$ (momentum of lepton pair)

$$\underbrace{\frac{d\Gamma(D \rightarrow Pl\nu)}{dq^2}}_{\text{experimental}} = \frac{G_F^2}{24\pi^3} \frac{(q^2 - m_l^2)^2 \sqrt{E_P^2 - m_P^2}}{q^4 m_D^2} |V_{cx}|^2$$

$$\left[\left(1 + \frac{m_l^2}{2q^2}\right) m_D^2 (E_P^2 - m_P^2) \underbrace{|f_+(q^2)|^2}_{\text{lattice QCD}} + \frac{3m_l^2}{8q^2} (m_D^2 - m_P^2)^2 \underbrace{|f_0(q^2)|^2}_{\text{lattice QCD}} \right]$$

With vector and scalar form factors $f_+(q^2)$ and $f_0(q^2)$ defined by

$$\langle P(p_P) | V_\mu | D(p_D) \rangle = \left(p_{P\mu} + p_{D\mu} - \frac{m_D^2 - m_P^2}{q^2} q_\mu \right) f_+(q^2) + \frac{m_D^2 - m_P^2}{q^2} q_\mu f_0(q^2)$$

3. Semileptonic D decays

For $l = e, \mu$ the contribution from $f_0(q^2)$ can be neglected and

$$\underbrace{\frac{d\Gamma(D \rightarrow Pl\nu)}{dq^2}}_{\text{experimental}} = \frac{G_F^2}{24\pi^3} |\vec{p}_P|^3 |V_{cx}|^2 \underbrace{|f_+^{DP}(q^2)|^2}_{\text{lattice QCD}}$$

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The errors on those studies are still dominated by errors in the calculation of the relevant form factors.

$$\frac{d}{dq^2} \Gamma(D \rightarrow K(\pi)l\nu) \quad \propto \quad |V_{cs(cd)}|^2 \quad |f_+^{D \rightarrow K(\pi)}(q^2)|^2$$

1.1(2.8)% error 5(8.7)% error

3. Semileptonic D decays: $q^2 = 0$

Two main strategies to eliminate the need of renormalize the lattice currents

- # Double ratios of 3-point correlators [Becirevic, Haas, Mescia 0710.1741](#)
(get the form factors from linear combinations of the double ratios)

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Two main strategies to eliminate the need of renormalize the lattice currents

- # Double ratios of 3-point correlators **Becirevic, Haas, Mescia 0710.1741**
(get the form factors from linear combinations of the double ratios)
- # Use the Ward identity ($S = \bar{x}c$) **HPQCD, Phys.Rev.D82:114506(2010)**

$$q^\mu \langle P | V_\mu^{cont.} | D \rangle = (m_c - m_x) \langle P | S^{cont} | D \rangle$$

that relates matrix elements of vector and scalar currents. In the lattice

$$q^\mu \langle P | V_\mu^{lat.} | D \rangle Z = (m_c - m_x) \langle P | S^{lat.} | D \rangle$$

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$$q^\mu \langle P | V_\mu^{lat.} | D \rangle Z = (m_c - m_x) \langle P | S^{lat.} | D \rangle$$

→ replace the V_μ with an S current in the 3-point function

$$f_0^{DP}(q^2) = \frac{m_c - m_x}{m_D^2 - m_P^2} \langle P | S | D \rangle_{q^2} \implies \boxed{f_+^{PD}(0) = f_0^{PD}(0) = \frac{m_c - m_x}{m_D^2 - m_P^2} \langle S \rangle_{q^2=0}}$$

3. Semileptonic D decays: $q^2 = 0$

Important reduction of errors in the lattice determination of the form factors $f_+^{D\pi(K)}(0)$ by the **HPQCD Collaboration**, Phys.Rev.D82:114506(2010), due mainly to

* Use a relativistic action, **HISQ**, to describe light and charm quarks.

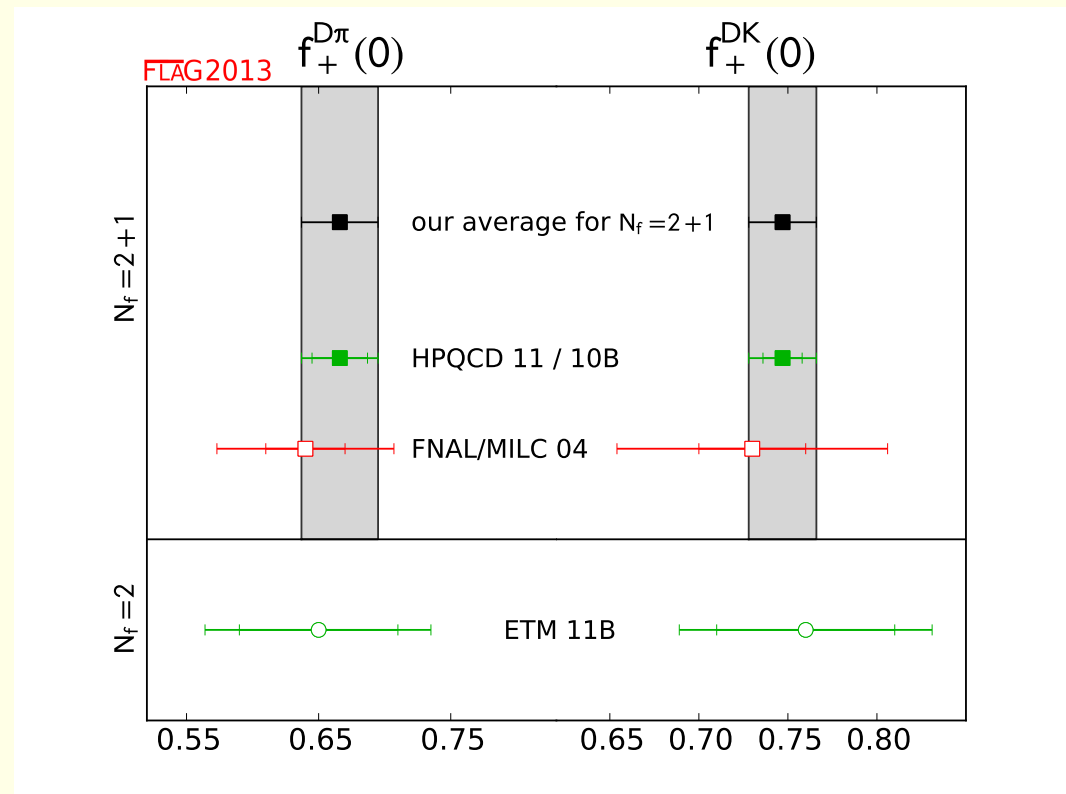
* Absolutely normalized current

HPQCD, 1008.4562, 1109.1501

$$f_+^{D\pi}(0) = 0.666(29)$$

$$f_+^{DK}(0) = 0.747(19)$$

Work in progress: $N_f = 2 + 1 + 1$ **FNAL/MILC**, 1411.1651 with physical quark masses.



3. Semileptonic D decays: $q^2 \neq 0$

Determination of $|V_{cs}|$ from $D \rightarrow Kl\nu$ at non-zero momentum transfer

HPQCD, 1305.1462

Calculation of $f_0^{DK}(q^2)$ (using Ward identity method) and $f_+^{DK}(q^2)$ (using its definition)

- * Global fit to available experimental data (using z -expansion) \rightarrow extraction of $|V_{cs}|$ using all experimental q^2 bins.

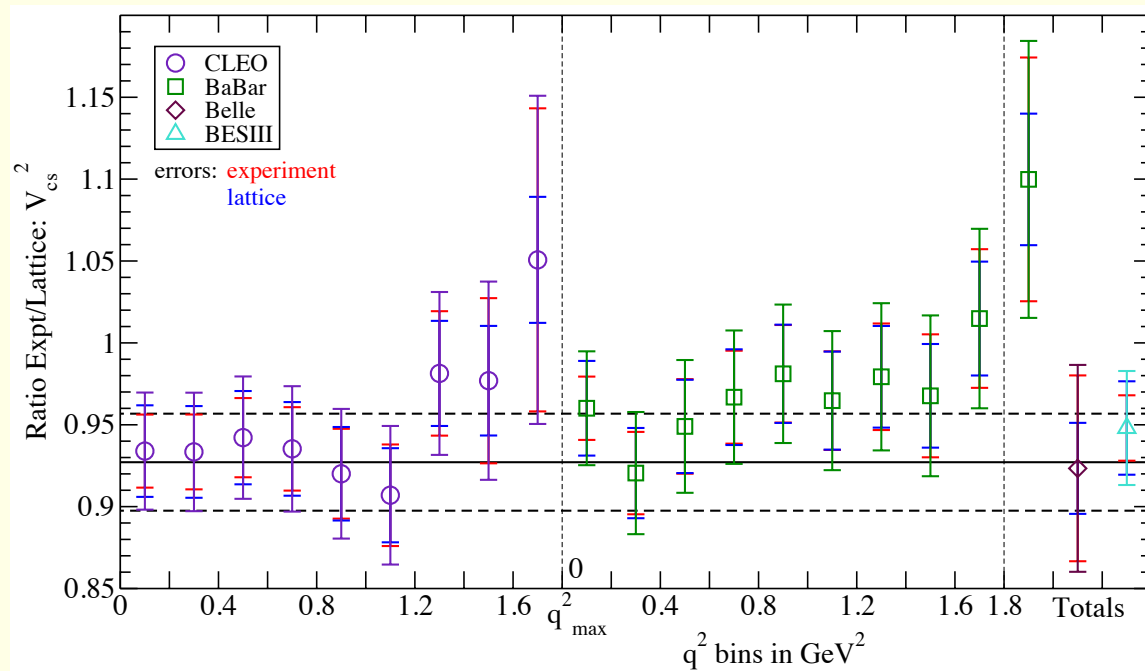
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$$|V_{cs}| = 0.963(5)_{exp}(14)_{lat}$$

Unitarity value PDG2014: $|V_{cs}| = 0.97343 \pm 0.00015$

3. Semileptonic D decays: $q^2 \neq 0$

Work in progress ($N_f = 2$):

* **ETM**: Preliminary results in **PoS Lattice 2013, 391 (2013)**

Work in progress ($N_f = 2 + 1$):

* **FNAL/MILC**: Preliminary results in 1211.4964. Fermilab charm and staggered light, four lattice spacings, $m_\pi \geq 180$ MeV.

** Same actions used for $B \rightarrow \pi l \nu$ form factors calculation \rightarrow can calculate accurately $\left| \frac{f_+^{B\pi}}{f_+^{D\pi}} \right| \rightarrow$ alternative calculation of $|V_{ub}|$ (see **A. Oyanguren** talk)

Work in progress ($N_f = 2 + 1 + 1$):

* **ETM, Lattice2014**: Twisted mass, three lattice spacings, $m_\pi \geq 210$ MeV

* **FNAL/MILC**: relativistic action for c , physical quark masses, four lattice spacings.

3. Semileptonic D decays: beyond gold-plated quantities

Alternative determination of $|V_{cs}|$: $D_s \rightarrow \phi l \nu$ HPQCD, 1311.6669

More challenging: five form factors (vector meson), unstable meson ...

* Treat ϕ as stable and estimate the error.

3. Semileptonic D decays: beyond gold-plated quantities

Alternative determination of $|V_{cs}|$: $D_s \rightarrow \phi l \nu$ HPQCD, 1311.6669

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* q^2 and angular distributions agree with BaBar data.

$$|V_{cs}| = 1.017(44)_{lat}(35)_{exp}(30)_{K\bar{K}}$$

* Expected reduction of exper. errors at BESIII \rightarrow need improvement of theor. calculation (lattice error dominated by statistical error)

* Are the heavy meson form factors at a given q^2 insensitive to the spectator m_q ? (compare $D_s \rightarrow \phi$ and $D \rightarrow K^*$).

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Exploratory $N_f = 2 + 1$ calculation of $D \rightarrow \eta^{(\prime)} l \nu$ **G. Bali et al**, 1406.5449

* Calculate $\eta - \eta'$ mixing angles and disconnected contributions

3. Semileptonic D decays: correlations with leptonic decays

Cancel CKM matrix elements building ratios of semileptonic and leptonic decay widths

* $N_f = 2 + 1$ HPQCD calculation

$$\left[\frac{f_+^{D\pi}(0)}{f_D} \right]_{lat} = (3.20 \pm 0.15) \text{ GeV}^{-1}$$

* Using HPQCD $f_+^{D\pi}(0)$ and $N_f = 2 + 1 + 1$ FNAL/MILC f_{D^+}

$$\left[\frac{f_+^{D\pi}(0)}{f_{D^+}} \right]_{lat} = (3.13 \pm 0.14) \text{ GeV}^{-1}$$

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$$\left[\frac{f_+^{D\pi}(0)}{f_{D^+}} \right]_{exp} = (3.11 \pm 0.08) \text{ GeV}^{-1} \quad \text{G. Rong et al, 1410.3232}$$

Good agreement experiment-theory

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* $N_f = 2 + 1$ HPQCD calculation

$$\left[\frac{f_+^{D\pi}(0)}{f_D} \right]_{lat} = (3.20 \pm 0.15) \text{ GeV}^{-1}$$

* Using HPQCD $f_+^{D\pi}(0)$ and $N_f = 2 + 1 + 1$ FNAL/MILC f_{D+}

$$\left[\frac{f_+^{D\pi}(0)}{f_{D+}} \right]_{lat} = (3.13 \pm 0.14) \text{ GeV}^{-1}$$

$$\left[\frac{f_+^{D\pi}(0)}{f_{D+}} \right]_{exp} = (3.11 \pm 0.08) \text{ GeV}^{-1} \quad \text{G. Rong et al, 1410.3232}$$

Good agreement experiment-theory

Several $N_f = 2 + 1$ and $N_f = 2 + 1 + 1$ calculations in progress.

4. $|V_{cd}|$, $|V_{cs}|$: CKM unitarity in the second row

Extracting CKM matrix elements $|V_{cd(cs)}|$ from leptonic decays

Experimental averages: BaBar, Belle, CLEO-c, BESIII

$$f_{D_s} |V_{cs}| = (252.0 \pm 3.7 \pm 1.8) \text{ MeV} \quad f_{D^+} |V_{cd}| = (45.92 \pm 1.04 \pm 0.15) \text{ MeV}$$

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- * Universal long-distance EM: $\downarrow \sim 2.5\%$ Kinoshita, PRL2, 1959
- * Universal short-distance EM: $\uparrow \sim 1.8\%$ Sirlin, NPB196, 1982
- * Hadronic structure dependent EM effects: **rough estimate** $\sim 0.6\%$.

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$$|V_{cd}| = 0.220 \pm 0.004_{lat} \pm 0.005_{exp} \pm 0.001_{EM}$$

$$|V_{cs}| = 1.017 \pm 0.011_{lat} \pm 0.017_{exp} \pm 0.006_{EM} \quad \text{FLAG-2 } N_f = 2 + 1$$

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(phenomological estimates available only in the K sector)

$$|V_{cd}| = 0.217 \pm 0.001_{lat} \pm 0.005_{exp} \pm 0.001_{EM}$$

$$|V_{cs}| = 1.016 \pm 0.005_{lat} \pm 0.017_{exp} \pm 0.006_{EM} \quad \text{FNAL/MILC, } N_f = 2 + 1 + 1$$

4. $|V_{cd}|$, $|V_{cs}|$: CKM unitarity in the second row

Extracting CKM matrix elements $|V_{cd(cs)}|$ from semileptonic decays

Experimental averages:

$$f_+^{DK}(0)|V_{cs}| = 0.717 \pm 0.004$$

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$$f_+^{D\pi}(0)|V_{cd}| = 0.143 \pm 0.002$$

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(not included Babar, Phys.Rev.D 91 052022 (2015))

$|V_{cd}|f_+^{D\pi}(0) = 0.1374 \pm 0.0038 \pm 0.0022 \pm 0.0009$, talk by A. Oyanguren)

* Experimental averages for neutral and charged D do not remove corrections from Coulomb attraction between charged FS particles in neutral mode $\sim 1\%$

→ Needed when lattice errors are reduced (forthcoming calculations from FNAL/MILC, ETM, RBC/UKQCD, ...)

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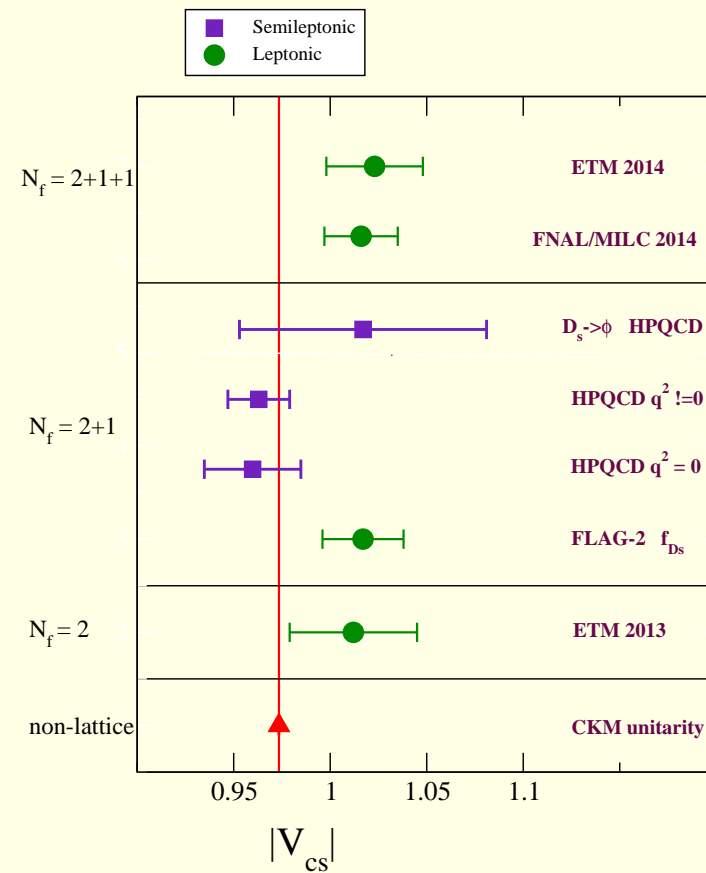
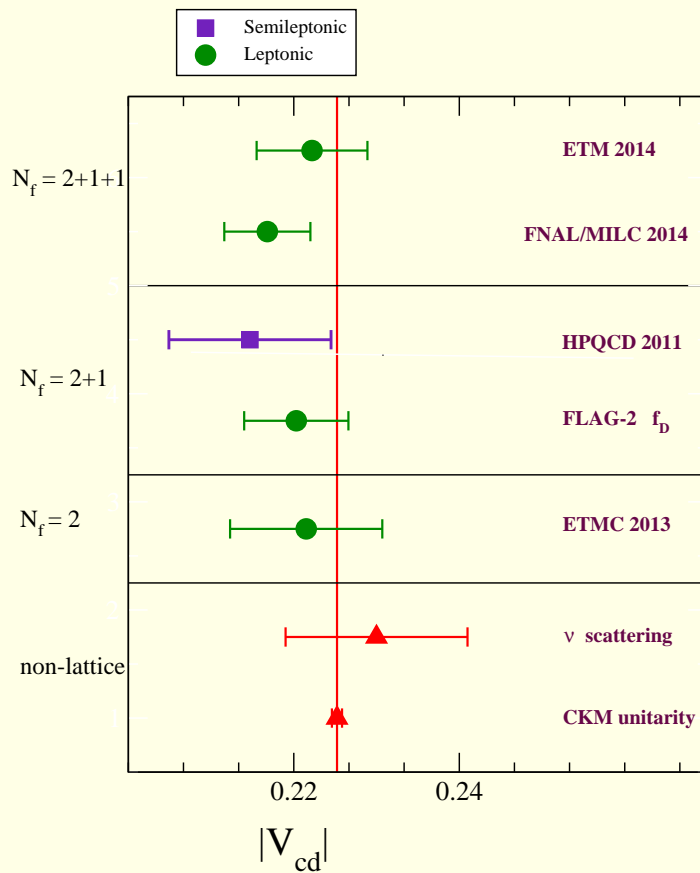
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$$|V_{cd}| = 0.215 \pm 0.009_{lat} \pm 0.003_{exp} \quad |V_{cs}| = 0.960 \pm 0.024_{lat} \pm 0.005_{exp}$$

(with HPQCD, $N_f = 2 + 1$ $f_+^{D\pi}(0) = 0.666(29)$ and $f_+^{DK}(0) = 0.747(19)$)

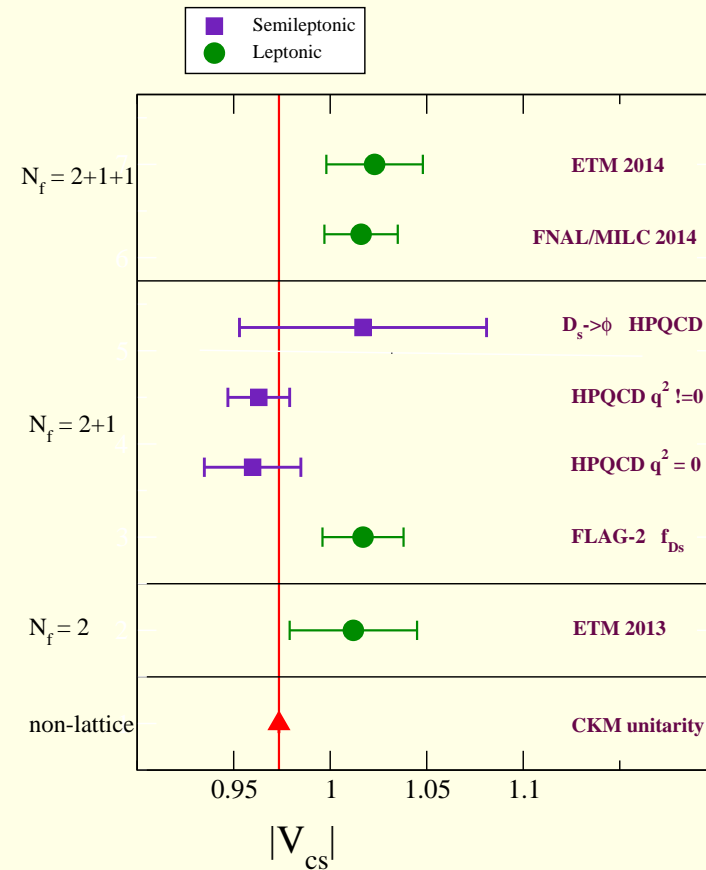
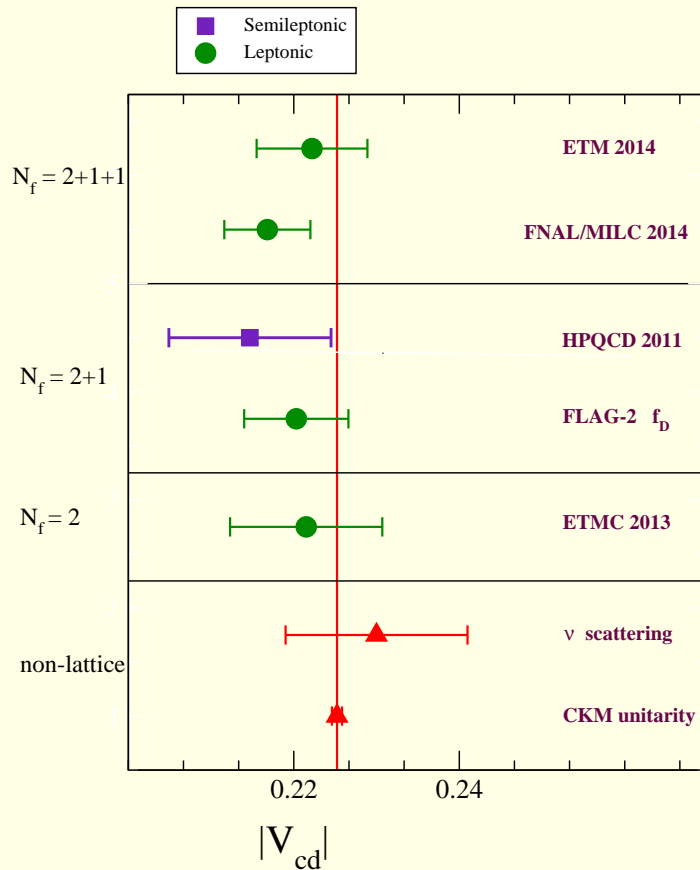
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* $|V_{cd}|$: Pretty good agreement between different determinations, but some tension $N_f = 2 + 1 + 1$ FNAL/MILC leptonic-unitarity.

* $|V_{cs}|$: Slight tensions leptonic-semileptonic ($D \rightarrow Kl\nu$) and leptonic-unitarity.

4. $|V_{cd}|$, $|V_{cs}|$: CKM unitarity in the second row



Using the most precise leptonic numbers ($N_f = 2 + 1 + 1$ **FNAL/MILC**)

$$1 - |V_{cd}|^2 - |V_{cs}|^2 - |V_{cb}|^2 = -0.07(4)$$

Using $N_f = 2 + 1$ **FLAG-2 averages** for decay constants

$$1 - |V_{cd}|^2 - |V_{cs}|^2 - |V_{cb}|^2 = -0.08(4)$$

4. Conclusions and outlook

Relativistic description of charm → important reduction of lattice QCD errors in decay constants and semileptonic form factors ...

$$\text{Error } f_{D_{(s)}} \sim 0.5\% \quad \text{Error } f_{+}^{DK(\pi)} \sim 2.5 - 4.3\%$$

... still theory errors are dominant in $|V_{cd(cs)}|$ extractions from semileptonic decays.

- * Several on-going calculations of the shape of $f_{+(0)}(q^2)$ will further reduce errors **FNAL/MILC** (with two different descriptions of the c), **ETM**, **HPQCD** ...
- ** Need experimental results reported in a model independent way, i.e., in q^2 bins (including full covariance matrix).
- * Physical quark masses also important in the reduction of errors, especially for D^+ quantities.

4. Conclusions and outlook

At current level of precision we need to include subdominant effects:

* **EM effects** → Eventually will do QCD+QED simulations.

N. Tantalo, 1311.2797, Divitiis et al, 1303.4896, A. Portelli, plenary talk at Lattice 2014,
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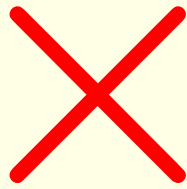
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Extend the same techniques to **B** physics



2. Leptonic D decays: New results (> 2013)

Reduction of errors in f_D and f_{D_s} mainly due to the use of relativistic actions, and using the same action for light and charm quarks.

$N_f = 2 + 1 + 1$:

* **FNAL/MILC**, 1407.3772: highly improved action, **MILC** configurations with **phys. quark masses** and small lattice spacing (smallest $a \approx 0.06$ fm)

$$f_{D_s} = 249.0 \pm 0.3_{stat} \left. \begin{array}{l} +0.1 \\ -1.4 \end{array} \right|_{a^2 extr.} \pm 0.2_{FV} \pm 0.1_{em} \pm 0.4_{f_\pi} \text{ MeV}$$
$$f_{D^+} = 212.6 \pm 0.4_{stat} \left. \begin{array}{l} +0.9 \\ -1.1 \end{array} \right|_{a^2 extr.} \pm 0.3_{FV} \pm 0.1_{em} \pm 0.3_{f_\pi} \text{ MeV}$$