Charm and Bottom quark masses off the Lattice

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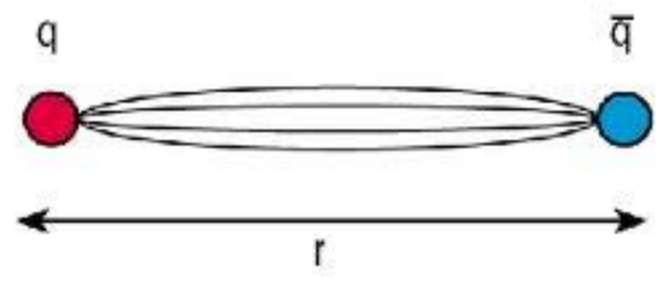
Outline

- Introduction
- Recent determinations
- Theory review
- Sum rule mass determinations
  - charm
  - bottom
- Conclusions and Outlook

Introduction

Theoretical remarks

Confinement: m_q not a physical observable



Parameter in QCD Lagrangian ⟶ formal definition (as for α_s)

$$\mathcal{L}_{\mathrm{QCD}} = \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a + \sum_f \bar{q}_f (\not{D} - m_f) q_f$$

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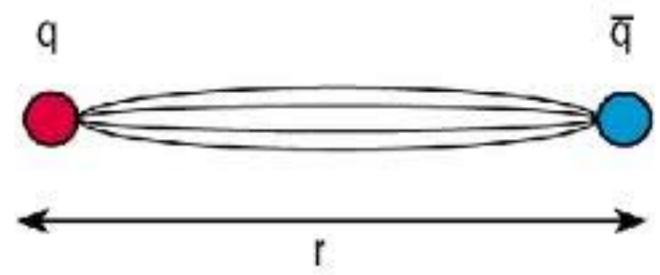

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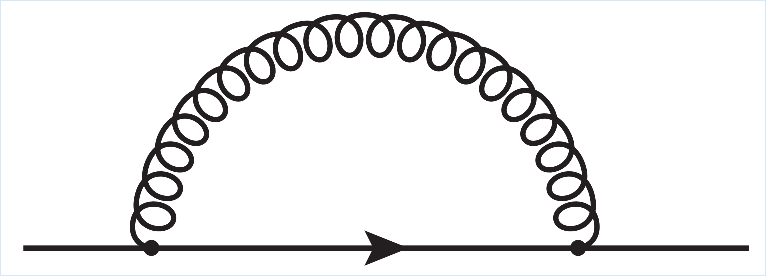
Renormalization and scheme dependent object

In general running mass (RG evolution)

Theoretical remarks

position of pole of propagator $m_{\text{pole}} = m_{\text{short-distance}} + \delta m$



δm defines the scheme and running

mass in short distance scheme

Theoretical remarks

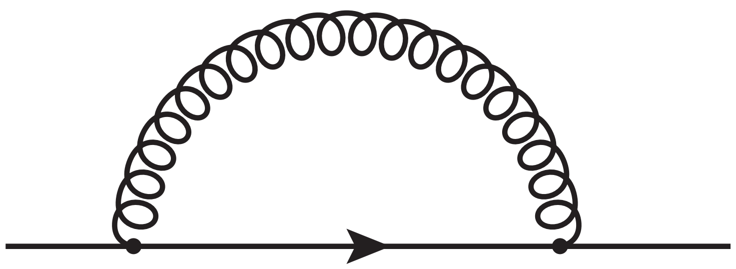
position of pole of propagator

$$m_{\text{pole}} = m_{\text{short-distance}} + \delta m$$

does not suffer from $\mathcal{O}(\Lambda_{\text{QCD}})$ ambiguity

$$\delta m = \mu \sum_{n=1} \alpha_s^{n+1} 2^n \beta_0^n \, n!$$

Contains renormalon



δm defines the scheme and running

infinitely many schemes !!!

Some schemes better than others…

best choice: process dependent

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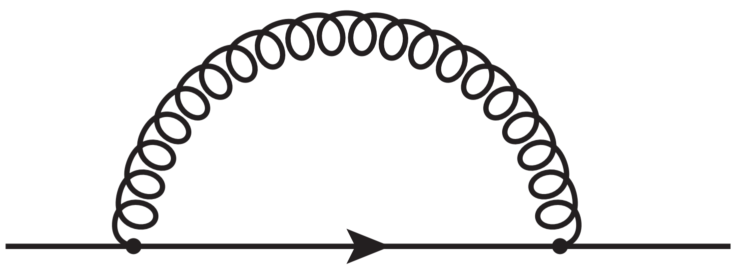

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infinitely many schemes !!!

Some schemes better than others…

best choice: process dependent

$\overline{\rm MS}$ scheme

- Short-distance scheme
- Standard mass for comparison $\overline{m}_q(\overline{m}_q)$
- And free from renormalon ambiguities

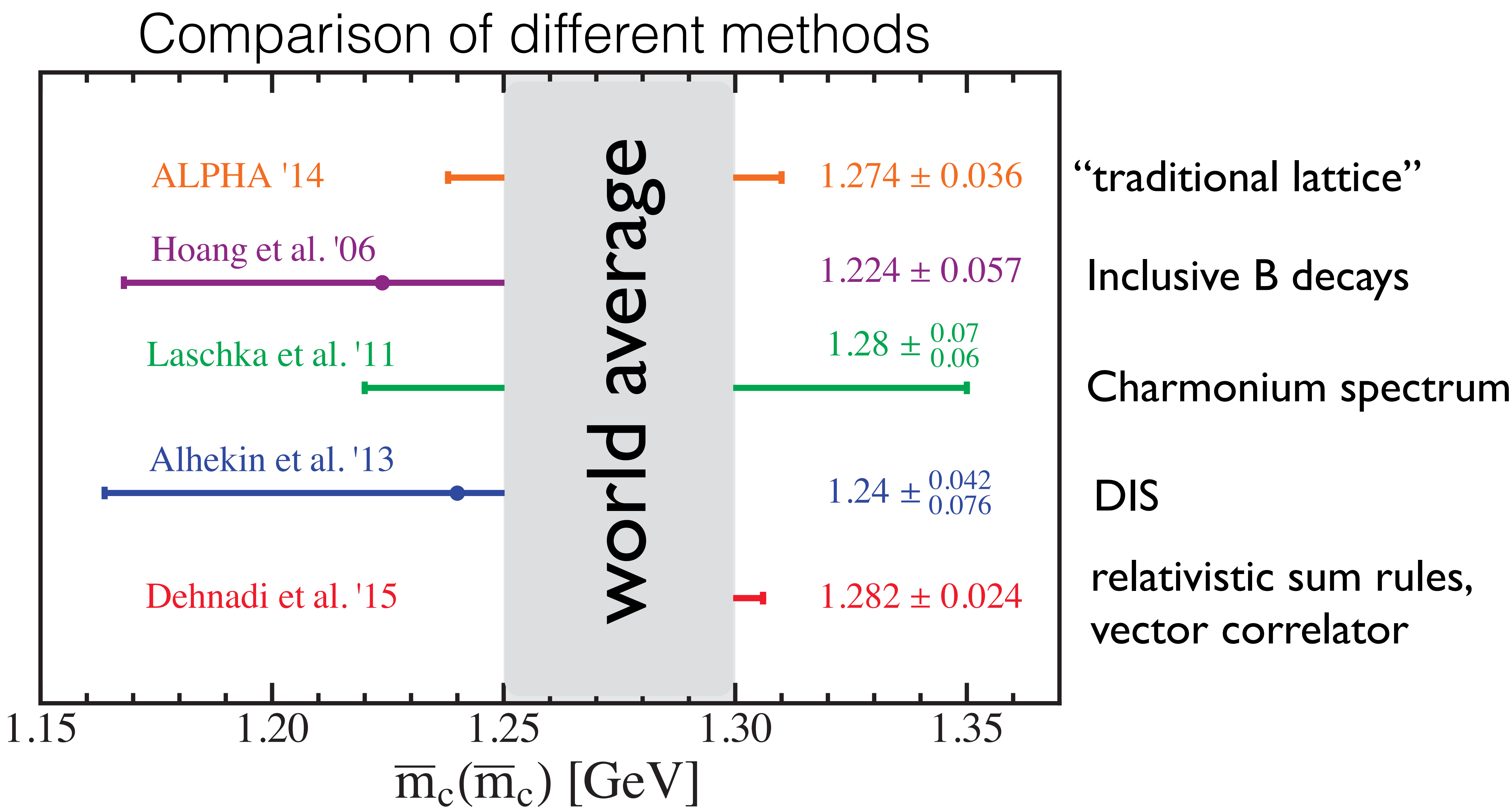
Short-distance masses in general have an ambiguity ~ $\mathcal{O}\left(\frac{\Lambda_{\rm QCD}^2}{m_q}\right)$

top	0.5 - 1	MeV
bottom	20 - 50	MeV
charm	60 - 150	MeV

provably better in $\overline{\rm MS}$ scheme

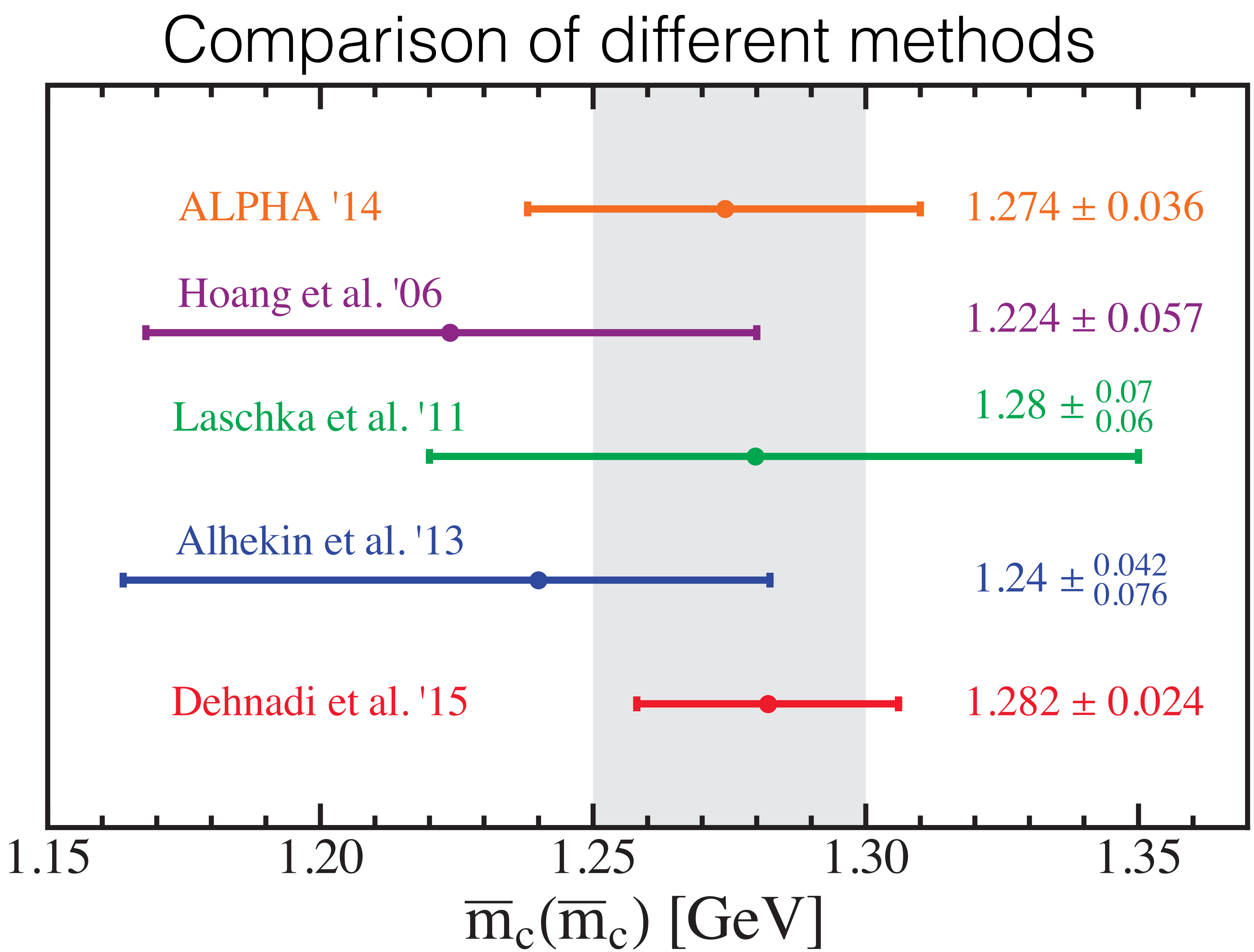
Recent charm and bottom mass determinations

Charm mass determinations



Relativist sum rules from the vector correlator give the most accurate results

Charm mass determinations



"traditional lattice"

Inclusive B decays

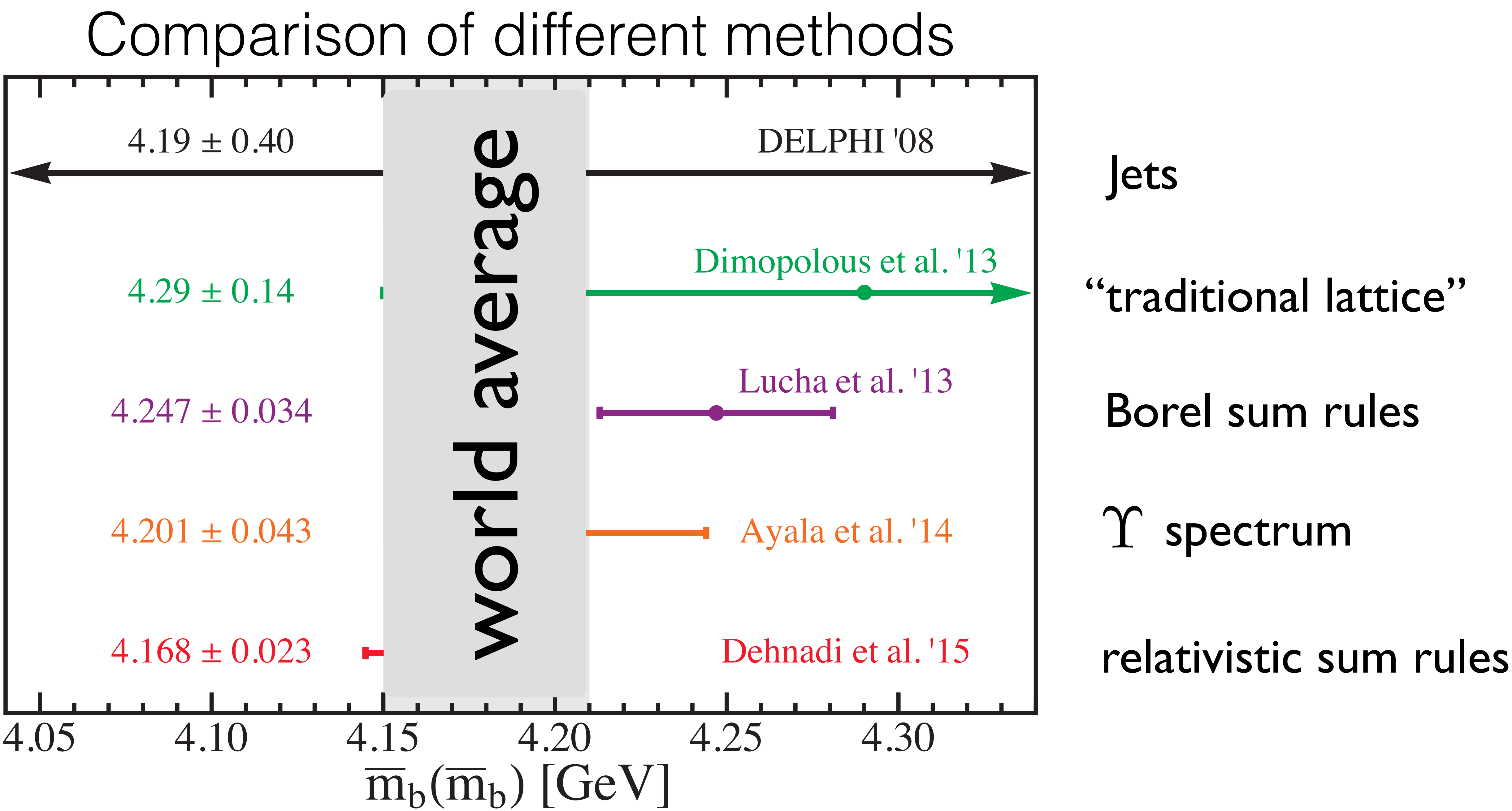
Charmonium spectrum

DIS

relativistic sum rules, vector correlator

Relativist sum rules from the vector correlator give the most accurate results

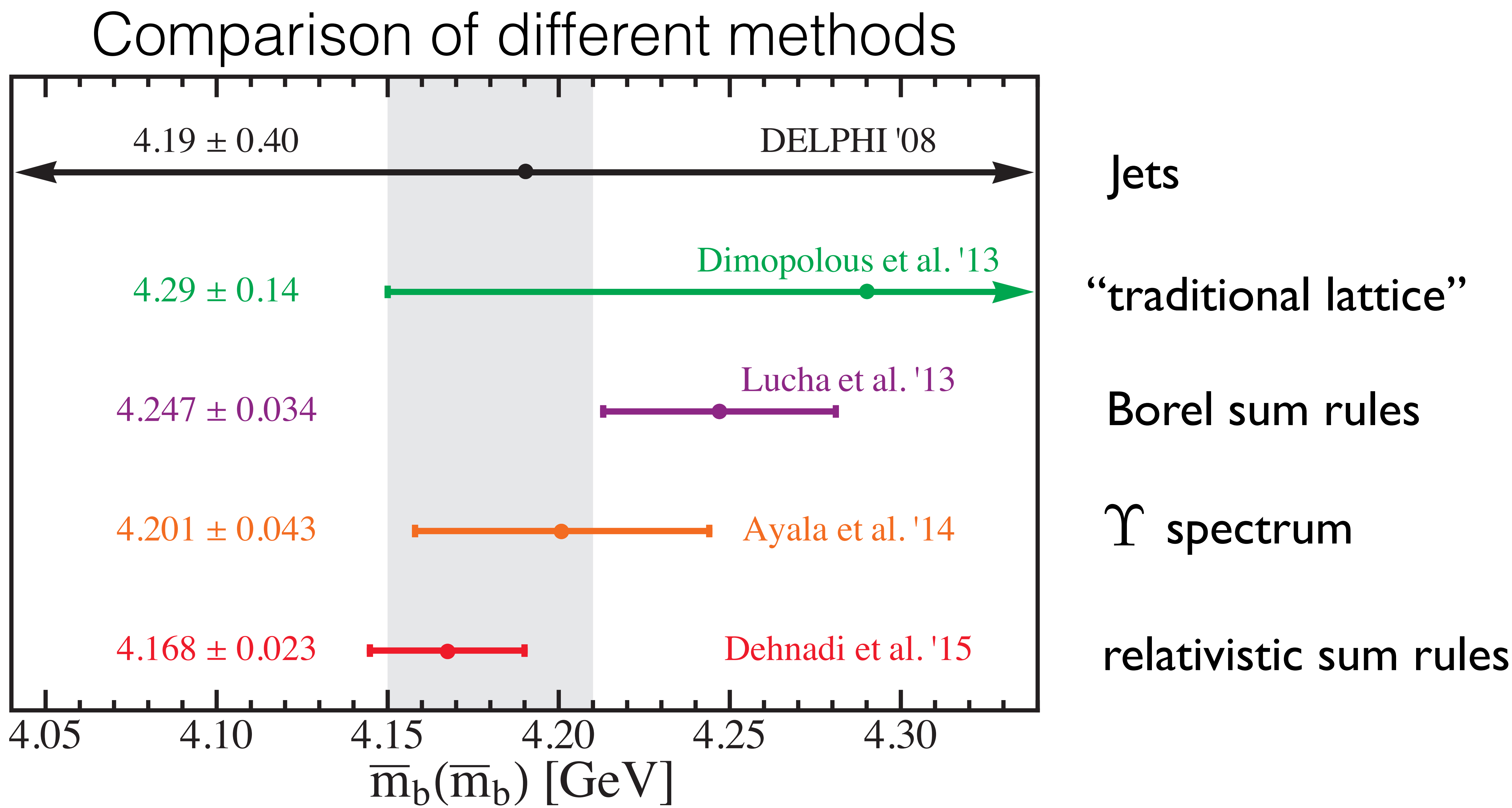
Bottom mass determinations



Relativist sum rules give the most accurate results

There seems to be a tension with Borel determination (heavy to light)

Bottom mass determinations



Relativist sum rules give the most accurate results

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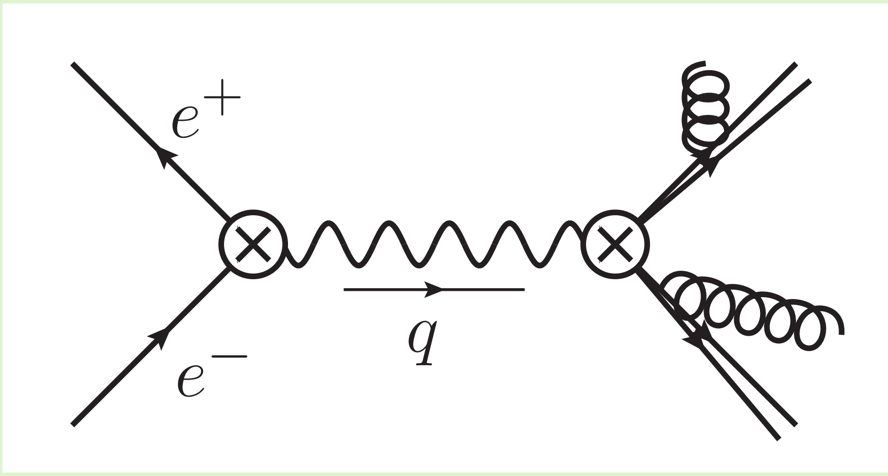
Sum rules:
Theoretical framework

QCD sum rules

Total hadronic cross section

$$R(s) = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$



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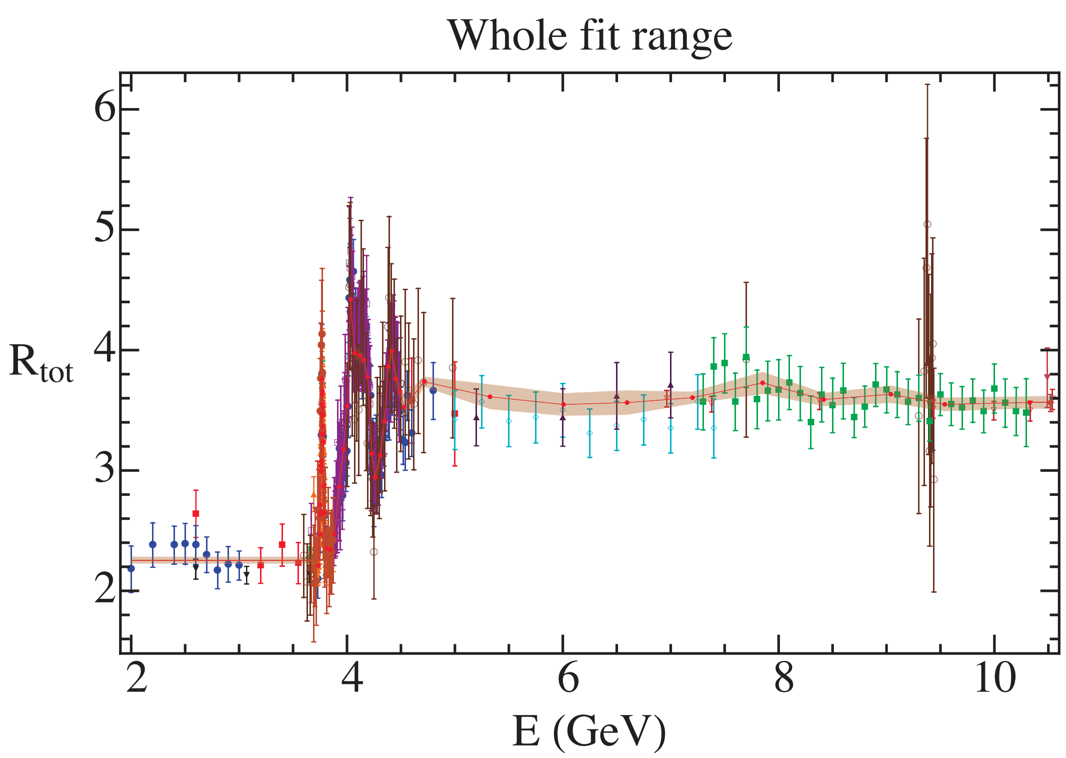
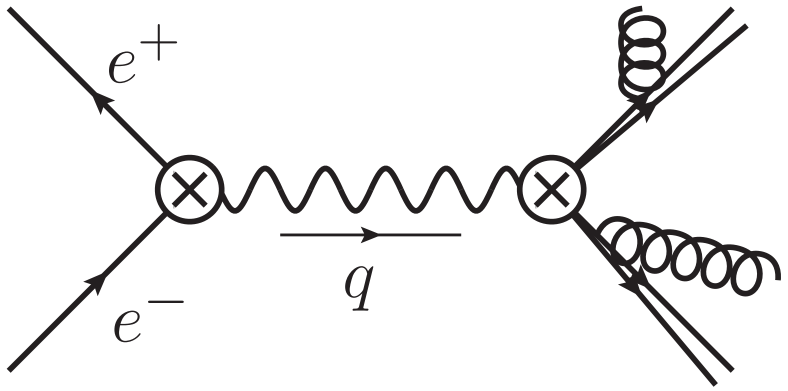


- Some smearing is necessary for perturbation theory to have any chance to describe data

- We also need to design the observable to be maximally sensitive to the heavy quark mass



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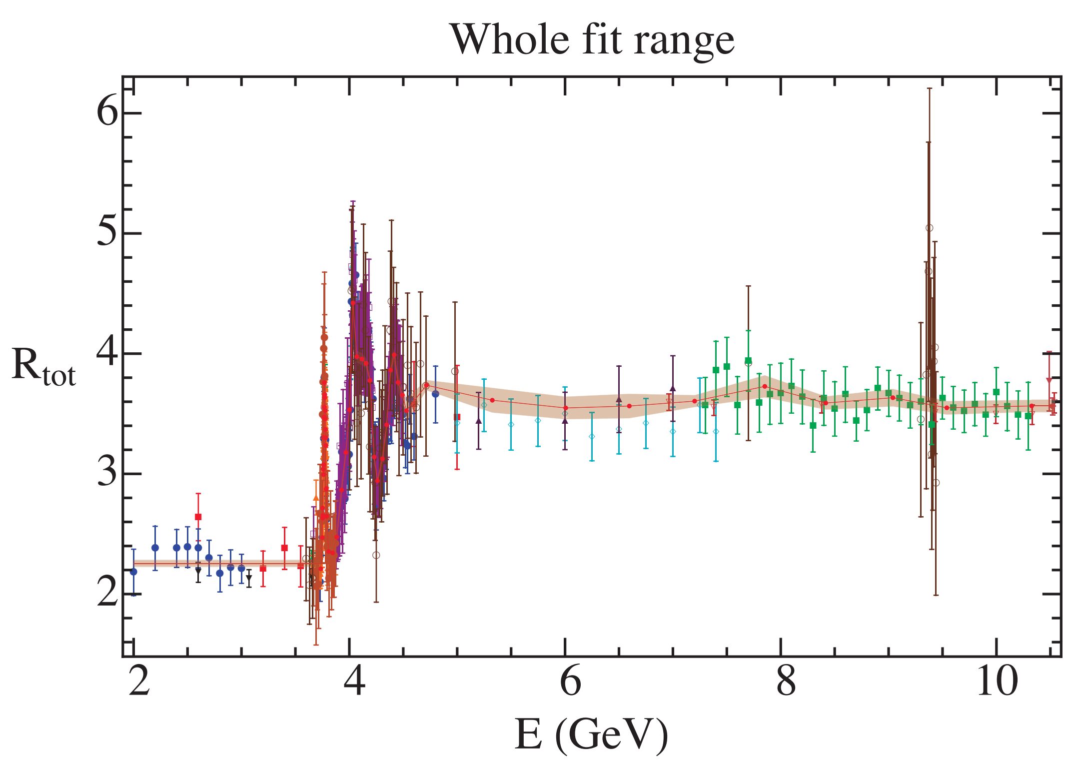

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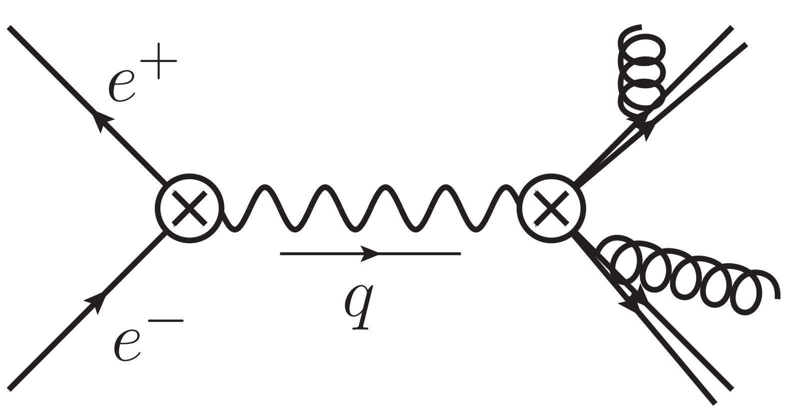

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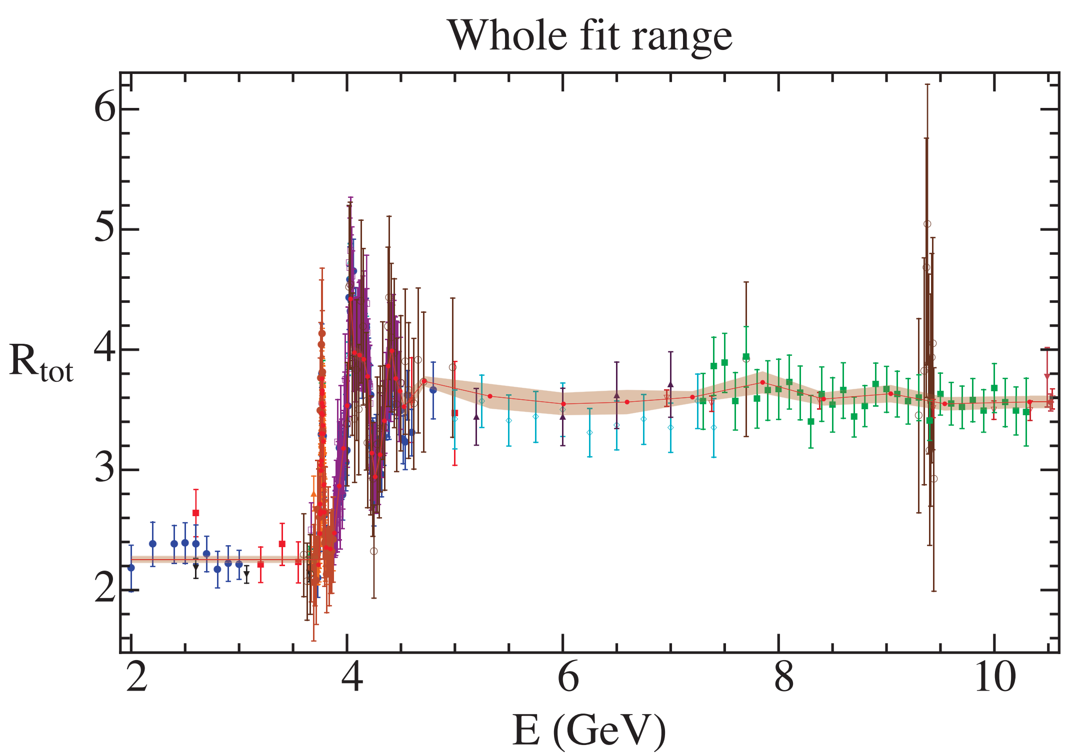

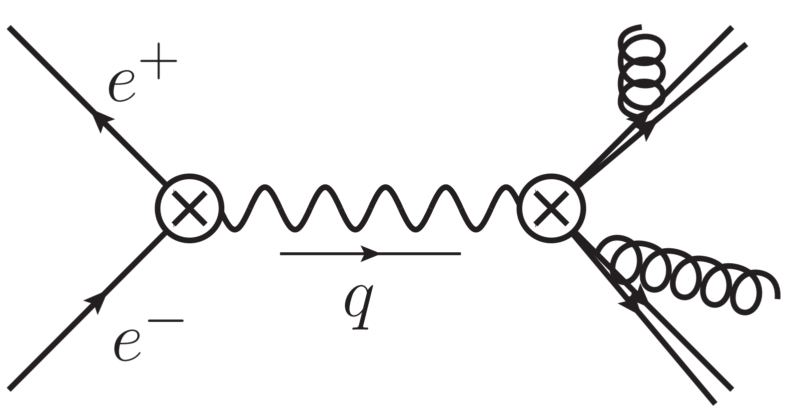

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change of variables

- Some smearing is necessary for perturbation theory to have any chance to describe data
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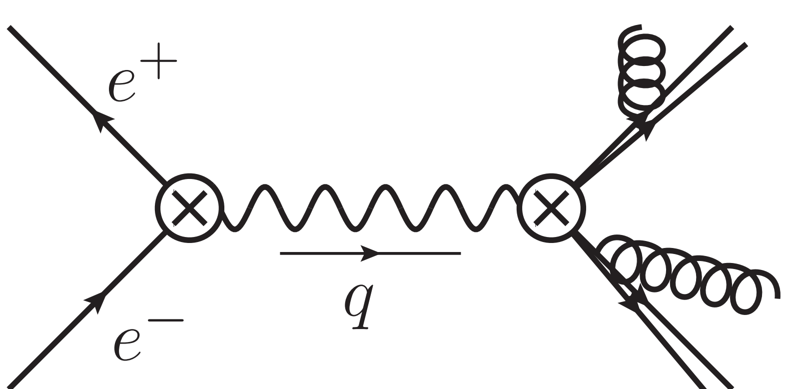

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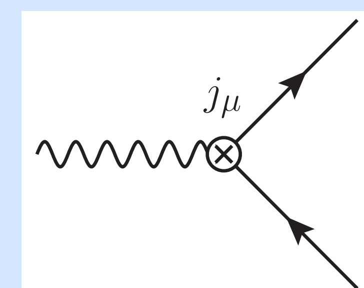
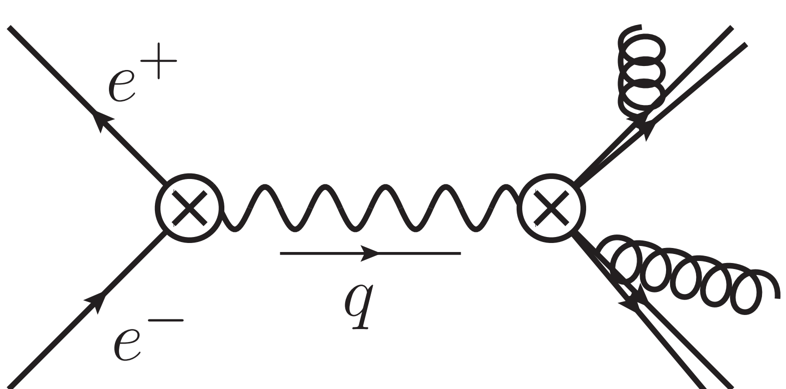
Vacuum polarization function

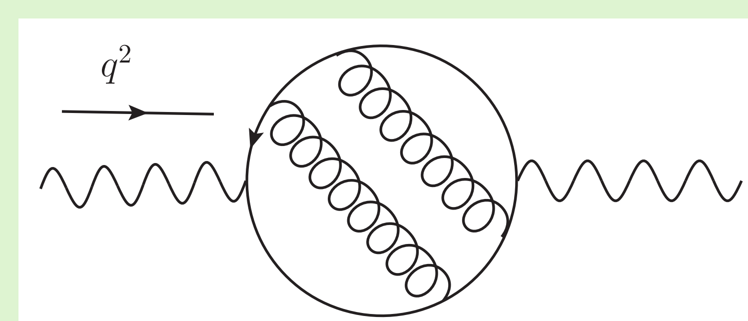
$$(q^2\, g_{\mu\nu} - q_\mu q_\nu)\Pi(q^2) = -\,i \int \mathrm{d}x\, e^{ix\cdot q} \langle 0 | \mathrm{T} j_\mu(x) j^\mu(0) | 0 \rangle$$

Vector current (electromagnetic)

$$J_\mu(x) = \bar{q}(x)\gamma_\mu q(x)$$







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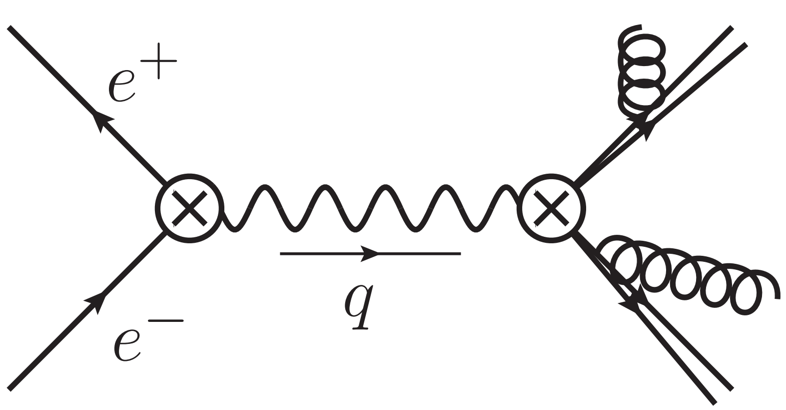
Optical theorem

electric charge

$$R(s) = 12\,\pi\, Q^2 \, \mathrm{Im}\, \Pi(s + i\,0^+)$$



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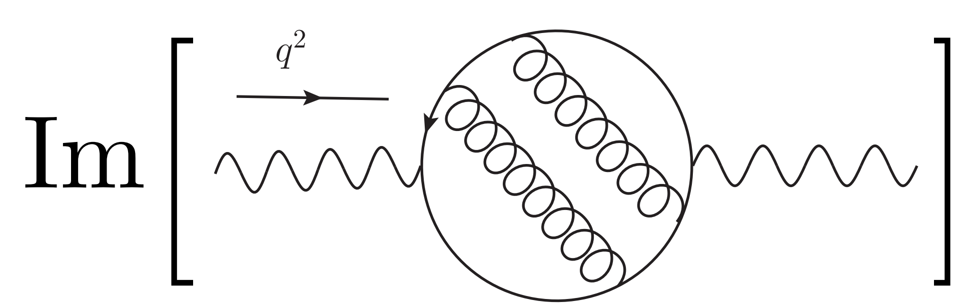

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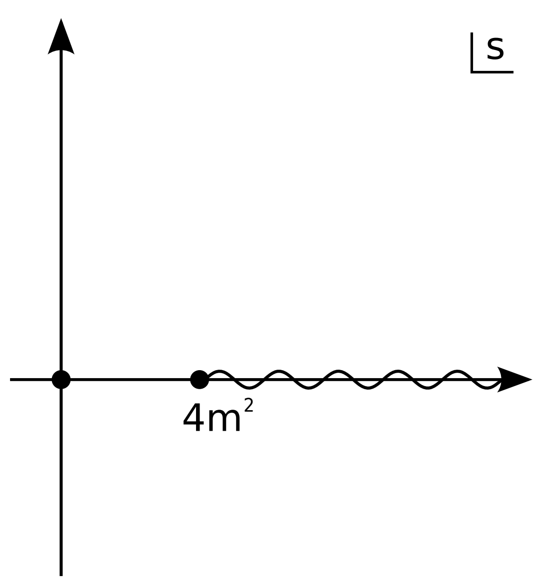

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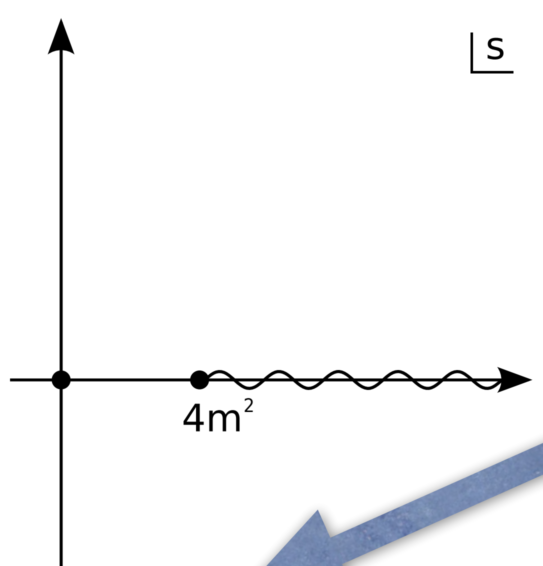

$$\Pi(q^2\sim 0)=\frac{1}{12\,\pi^2\,Q^2}\sum_{n=0}^{\infty}M_n q^{2n}\quad\Longleftrightarrow\quad M_n^{\mathrm{th}}=\frac{12\pi^2Q^2}{n!}\,\frac{\mathrm{d}^n}{\mathrm{d}q^{2n}}\,\Pi(q^2)\Big|_{q^2=0}$$

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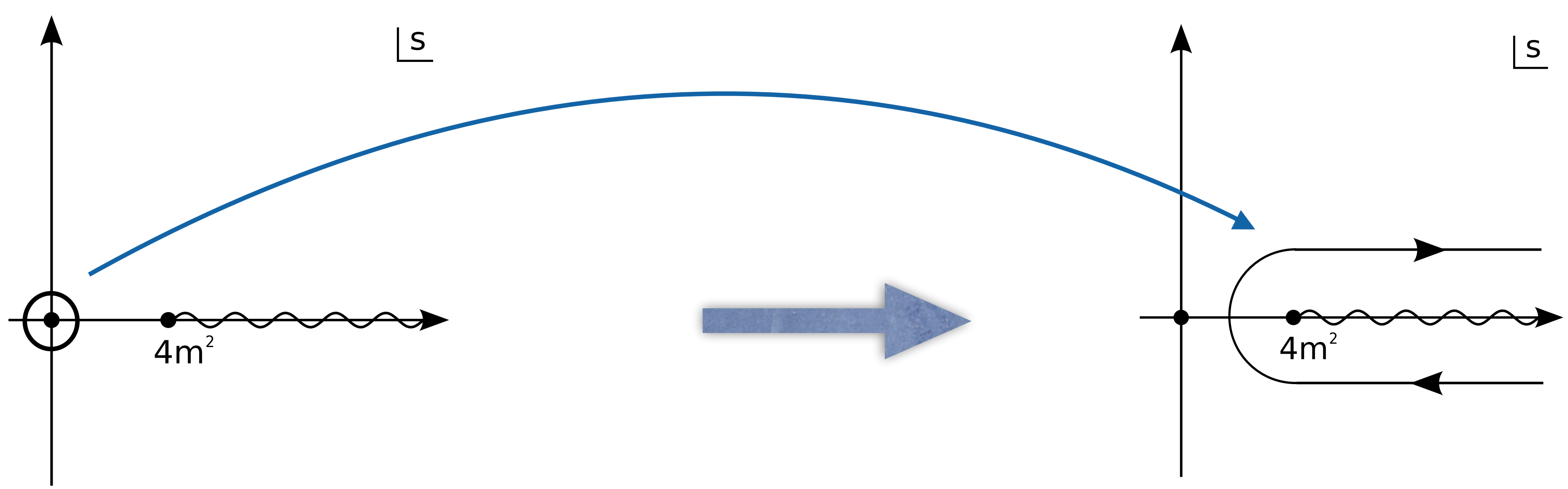

$$\Pi(q^2 \sim 0) = \frac{1}{12\,\pi^2\, Q^2} \sum_{n=0}^{\infty} M_n q^{2n} \quad \Longleftrightarrow \quad M_n^{\mathrm{th}} = \frac{12\pi^2 Q^2}{n!} \frac{\mathrm{d}^n}{\mathrm{d}q^{2n}} \Pi(q^2)\Big|_{q^2=0} \quad \Longleftrightarrow \quad M_n = 6\,\pi\, i\, Q^2 \oint \mathrm{d}s \, \frac{\Pi(s)}{s^{n+1}}$$

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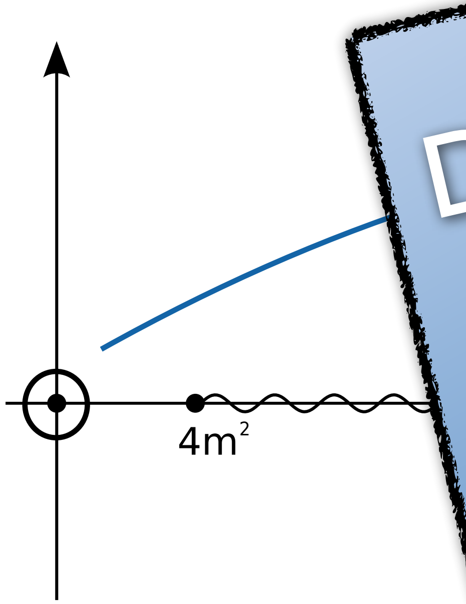
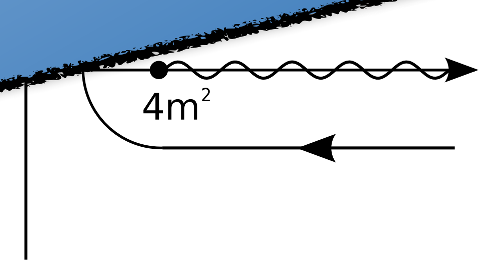
Di… on

$$\ldots \int_{4m^2}^{\infty} \mathrm{d}s\, \frac{R(s)}{s(s-q^2)}$$





Duality/non-relativistic bound:

n < m_c/Λ_{QCD} ~ 2

n < m_b/Λ_{QCD} ~ 4

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Alternative perturbative expansions

Methods in perturbation theory

One can use four different expansion methods, equivalent in perturbation theory, to test the convergence of the series expansion

All perturbative methods should give similar results when determining the charm and bottom mass (within theoretical uncertainties)

We use different renormalization scales for α_s (denoted by μ_α) and $\overline{m}_q$ (denoted by μ_m)

Methods in perturbation theory

Fixed order expansion

$$M_n^{\mathrm{pert}} = \frac{1}{(4\overline{m}_c^2(\mu_m))^n} \sum_{i,a,b} \left(\frac{\alpha_s(\mu_\alpha)}{\pi}\right)^i C_{n,i}^{a,b} \ln^a\left(\frac{\overline{m}_c^2(\mu_m)}{\mu_m^2}\right) \ln^b\left(\frac{\overline{m}_c^2(\mu_m)}{\mu_\alpha^2}\right)$$

Methods in perturbation theory

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Linearized expansion

$$\left(M_n^{\rm th,pert}\right)^{1/2n} = \frac{1}{2\overline{m}_c(\mu_m)}\sum_{i,a,b}\left(\frac{\alpha_s(\mu_\alpha)}{\pi}\right)^i \tilde{C}_{n,i}^{a,b}\ln^a\left(\frac{\overline{m}_c^2(\mu_m)}{\mu_m^2}\right)\ln^b\left(\frac{\overline{m}_c^2(\mu_m)}{\mu_\alpha^2}\right)$$

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Iterative linearized expansion

$$\overline{m}_c^{(0)} = \frac{1}{2\left(M_n^{\text{th,pert}}\right)^{1/2n}} \tilde{C}_{n,0}^{0,0}$$

$$\overline{m}_c(\mu_m) = \overline{m}_c^{(0)} \sum_{i,a,b} \left(\frac{\alpha_s(\mu_\alpha)}{\pi}\right)^i \hat{C}_{n,i}^{a,b} \ln^a\left(\frac{\overline{m}_c^{(0)\,2}}{\mu_m^2}\right) \ln^b\left(\frac{\overline{m}_c^{(0)\,2}}{\mu_\alpha^2}\right)$$

Solve analytically for mass, always has a solution

Methods in perturbation theory

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Iterative linearized expansion

$$\overline{m}_c^{(0)} = \frac{1}{2\left(M_n^{\mathrm{th,pert}}\right)^{1/2n}} \tilde{C}_{n,0}^{0,0}$$

$$\overline{m}_c(\mu_m) = \overline{m}_c^{(0)} \sum_{i,a,b} \left(\frac{\alpha_s(\mu_\alpha)}{\pi}\right)^i \hat{C}_{n,i}^{a,b} \ln^a\left(\frac{\overline{m}_c^{(0)\,2}}{\mu_m^2}\right) \ln^b\left(\frac{\overline{m}_c^{(0)\,2}}{\mu_\alpha^2}\right)$$

Solve analytically for mass, always has a solution

Contour improved expansion

$$M_n^{\mathrm{c,pert}} = \frac{6\pi Q_q^2}{i} \oint_{\mathcal{C}} \frac{\mathrm{d}s}{s^{n+1}} \Pi[\, s, \alpha_s(\mu_\alpha^c(s,\overline{m}_c^2)), \overline{m}_c(\mu_m), \mu_\alpha^c(s,\overline{m}_c^2), \mu_m \,]$$

$$(\mu_\alpha^c)^2(s,\overline{m}_c^2) = \mu_\alpha^2 \left(1 - \frac{s}{4\overline{m}_c^2(\mu_m)}\right)$$

Methods in perturbation theory

Fixed order expansion

$$M_n^{\text{pert}} = \frac{1}{(4\overline{m}_c^2(\mu_m))^n} \sum_{i,a,b} \left(\frac{\alpha_s(\mu_\alpha)}{\pi}\right)^i C_{n,i}^{a,b} \ln^a\left(\frac{\overline{m}_c^2(\mu_m)}{\mu_m^2}\right) \ln^b\left(\frac{\overline{m}_c^2(\mu_m)}{\mu_\alpha^2}\right)$$

Linearized expansion

$$\left(M_n^{\text{th,pert}}\right)^{1/2n} = \frac{1}{2\overline{m}_c(\mu_m)} \sum_{i,a,b} \left(\frac{\alpha_s(\mu_\alpha)}{\pi}\right)^i \tilde{C}_{n,i}^{a,b} \ln^a\left(\frac{\overline{m}_c^2(\mu_m)}{\mu_m^2}\right) \ln^b\left(\frac{\overline{m}_c^2(\mu_m)}{\mu_\alpha^2}\right)$$

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μ_α- and μ_m-independent

Contour improved expansion

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Methods in perturbation theory

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residual dependence on μ_α and μ_m due to truncation of series in α_s

Contour improved expansion

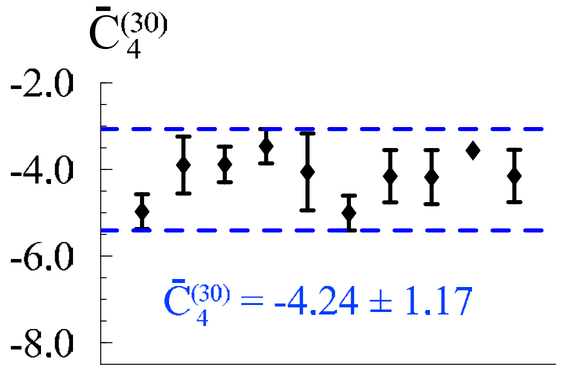
$$M_n^{\rm c,pert} = \frac{6\pi Q_q^2}{i} \oint_{\mathcal{C}} \frac{{\rm d}s}{s^{n+1}} \Pi[\, s, \alpha_s(\mu_\alpha^c(s,\overline{m}_c^2)), \overline{m}_c(\mu_m), \mu_\alpha^c(s,\overline{m}_c^2), \mu_m \,]$$

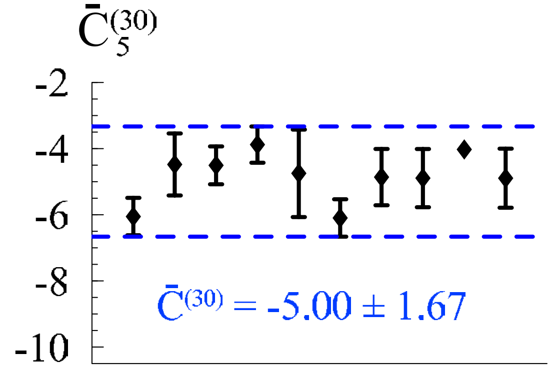
$$(\mu_\alpha^c)^2(s,\overline{m}_c^2) = \mu_\alpha^2\left(1 - \frac{s}{4\overline{m}_c^2(\mu_m)}\right)$$

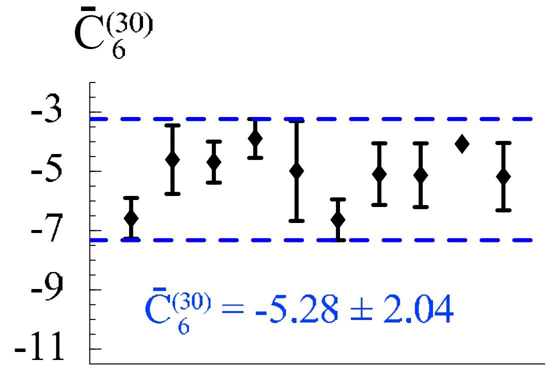
Status of computations

Moments

- For n = 1, 2, 3 the $C_n^{0,0}$ coefficients are known at $\mathcal{O}(\alpha_s^3)$
- For n ≥ 4, $C_n^{0,0}$ are known in a semi-analytic approach (Padé)
- The rest of $C_n^{a,b}$ can be deduced by RGE evolution







[Kühn et al]
[Boughezal et al]
[Sturm]
[Maier et al]
[Hoang, VM, Zebarjad]
[Greynat et al]

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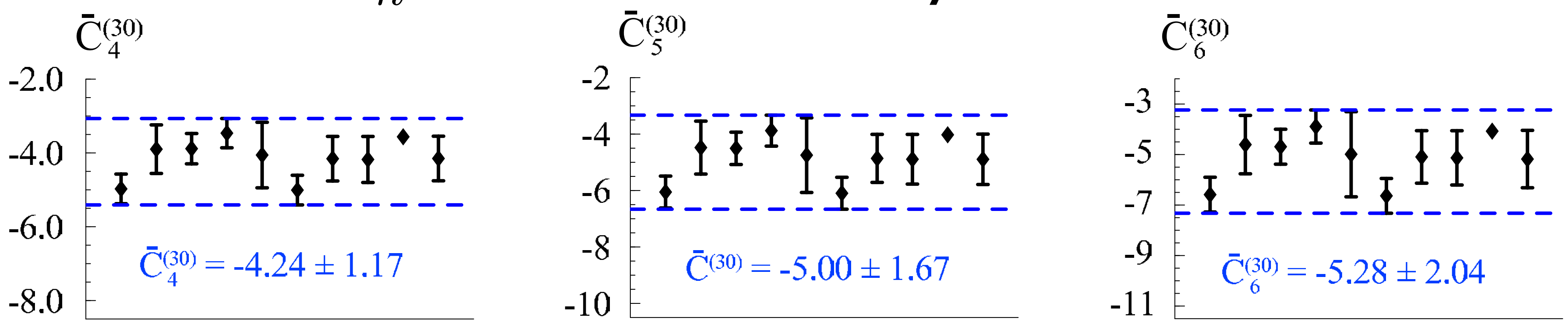
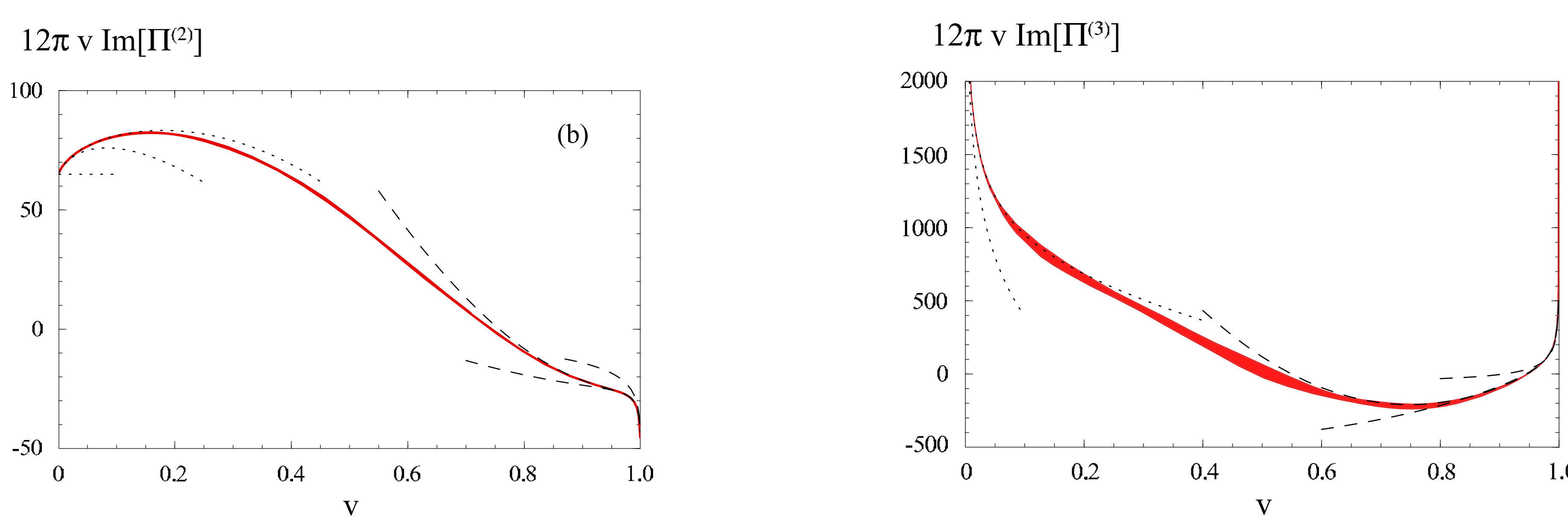


R-ratio for a massive pair of quarks

- Analytically known up to $\mathcal{O}(\alpha_s)$
- Known high-energy and threshold limits up to $\mathcal{O}(\alpha_s^3)$
- Semi-analytic approach (Padé) up to $\mathcal{O}(\alpha_s^3)$

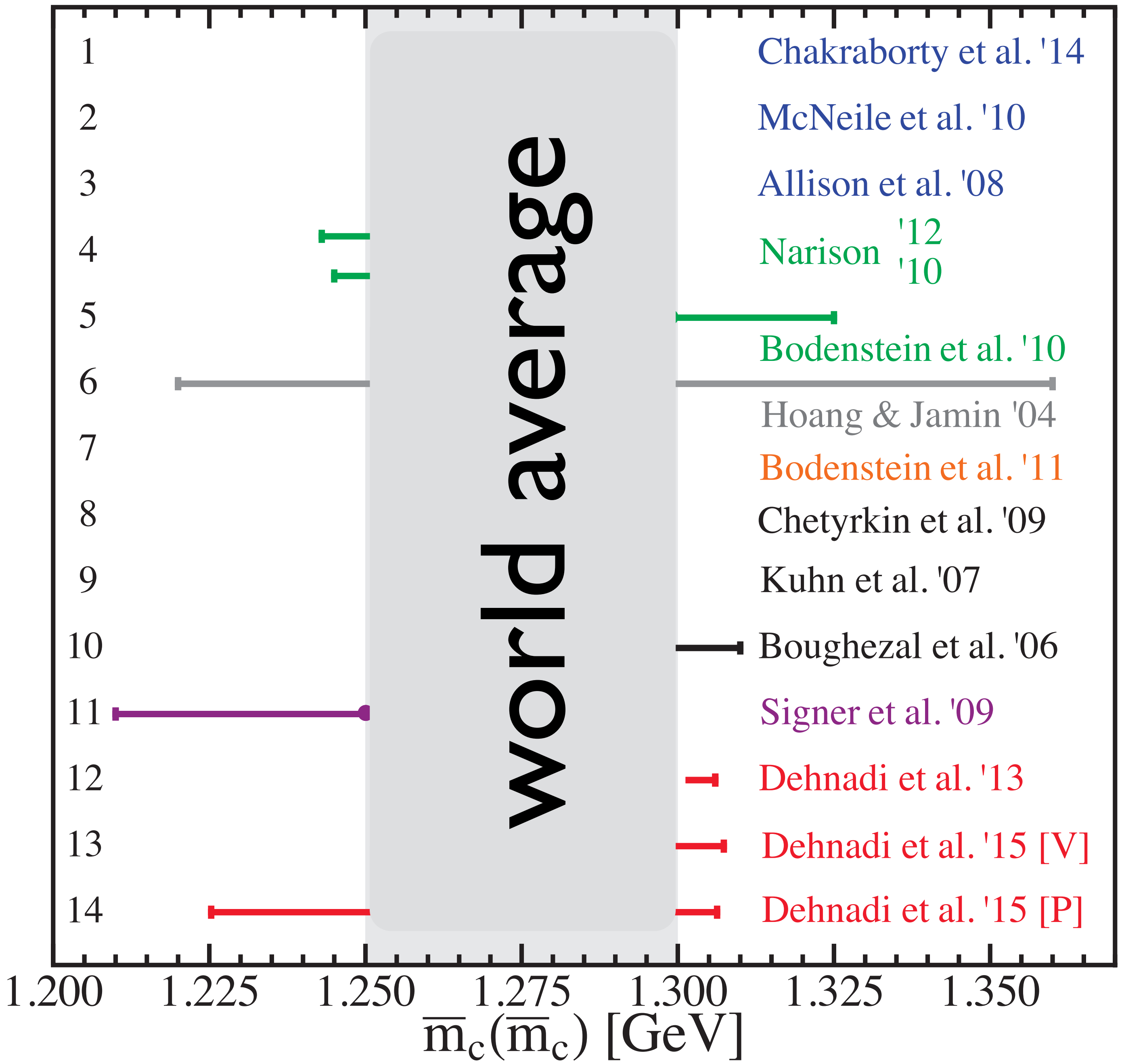
[Hoang,VM, Zebarjad]
[Greynat et al]



Charm mass from sum rules

Charm mass determinations

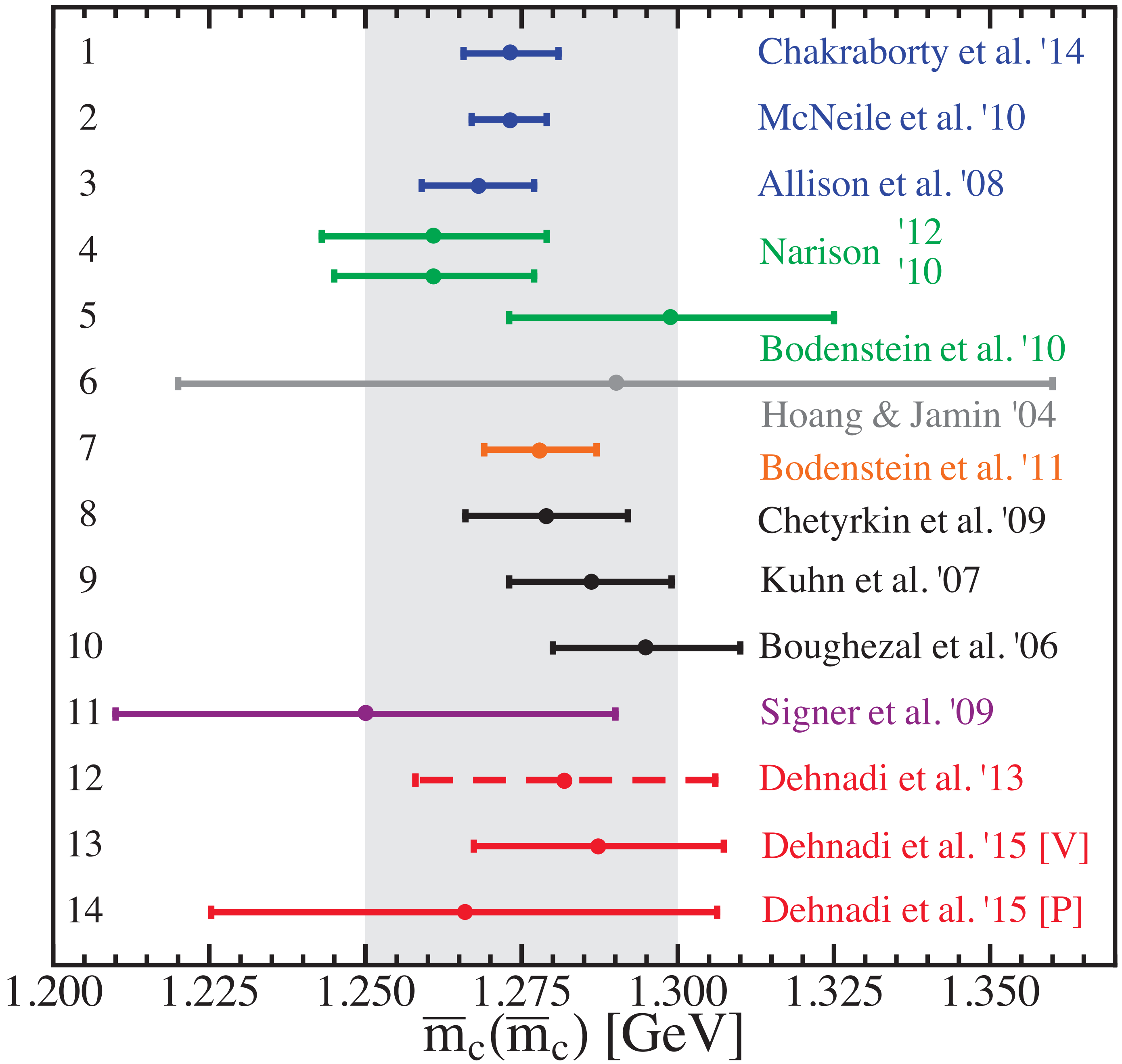
From QCD sum rules



[Dehnadi, Hoang, & VM '15]

Charm mass determinations

From QCD sum rules

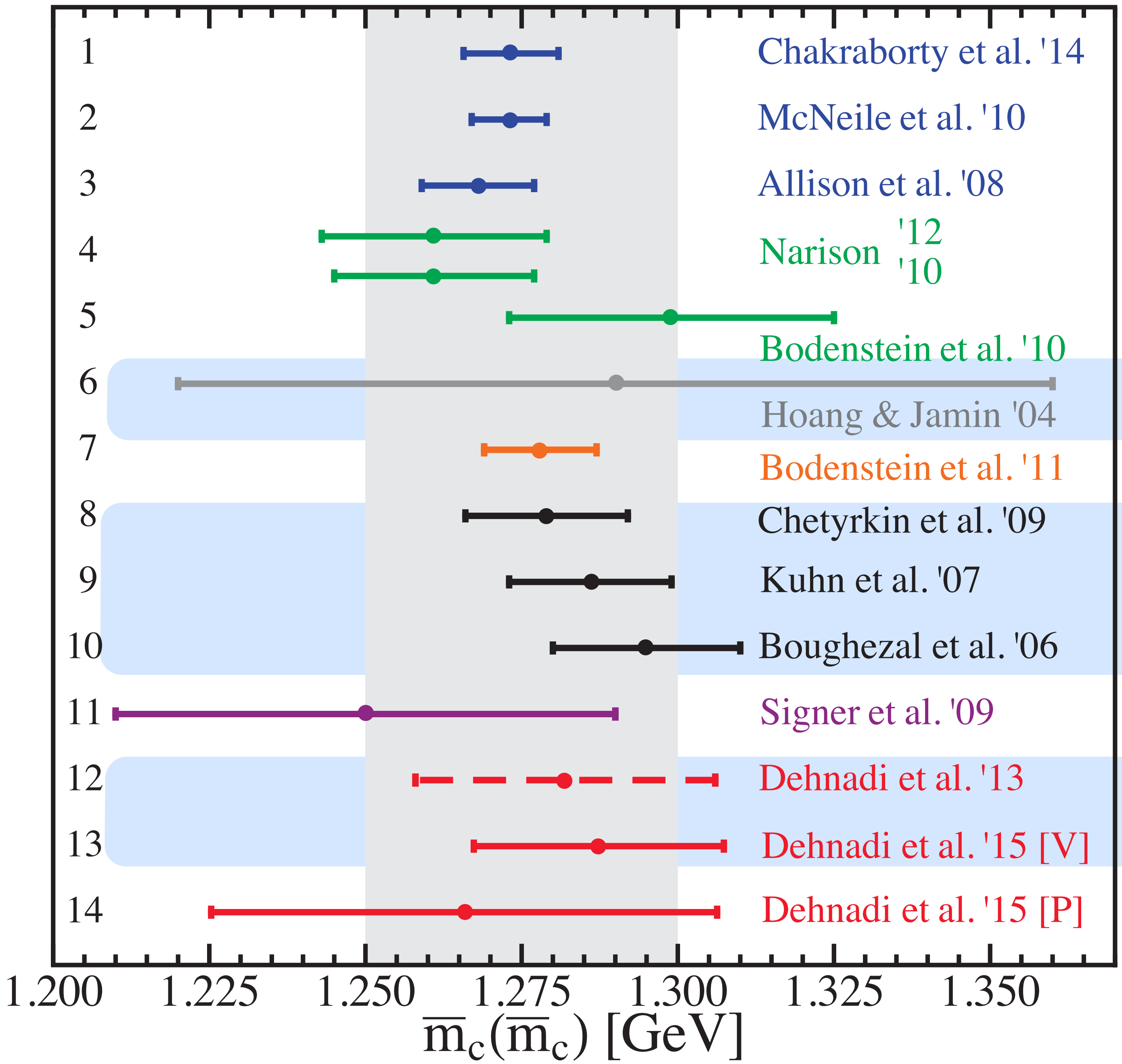


[Dehnadi, Hoang, & VM '15]

Charm mass determinations

Type of sum rule

From QCD sum rules

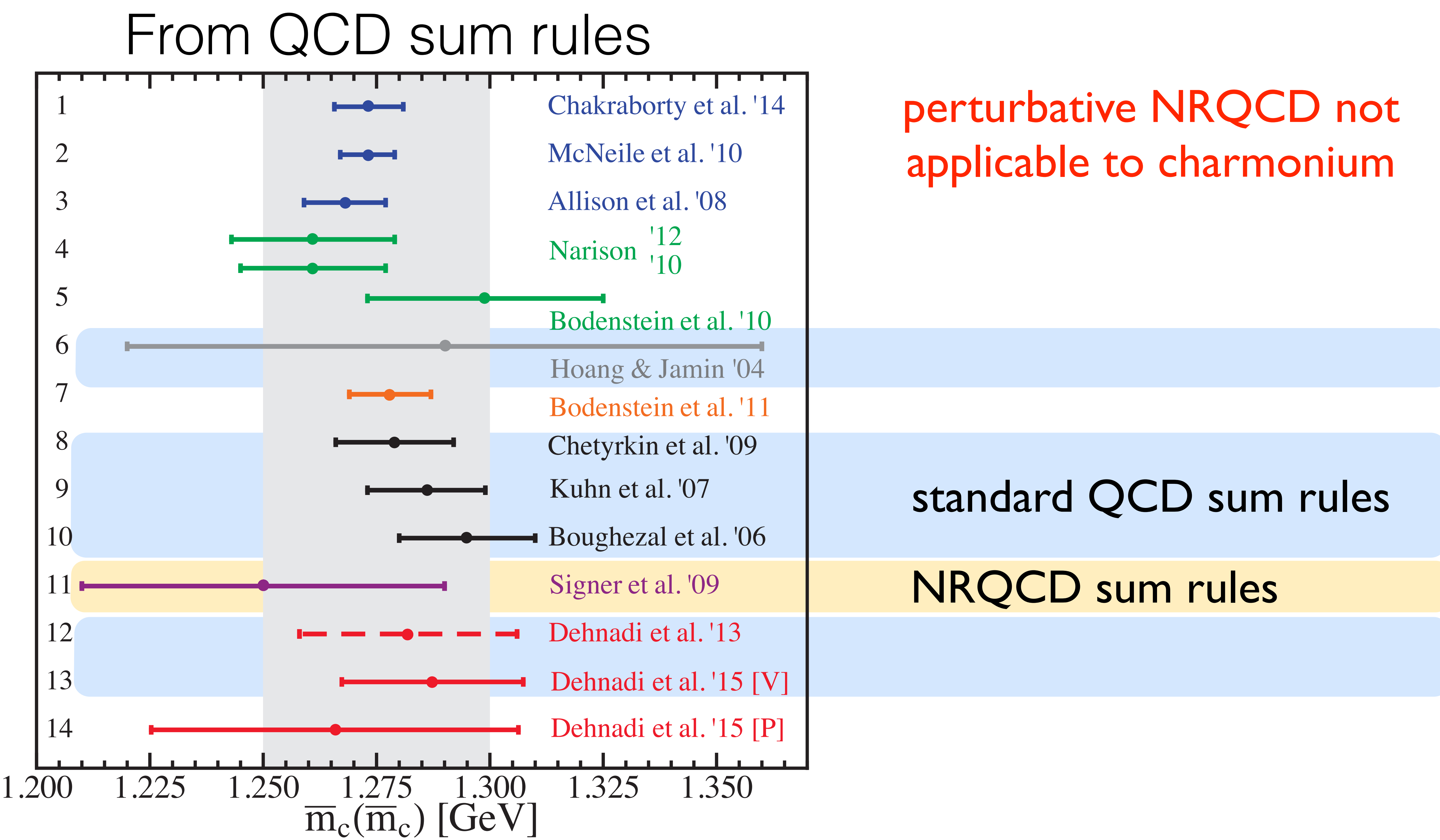


relativistic sum rules give the most precise determinations

standard QCD sum rules

Charm mass determinations

Type of sum rule

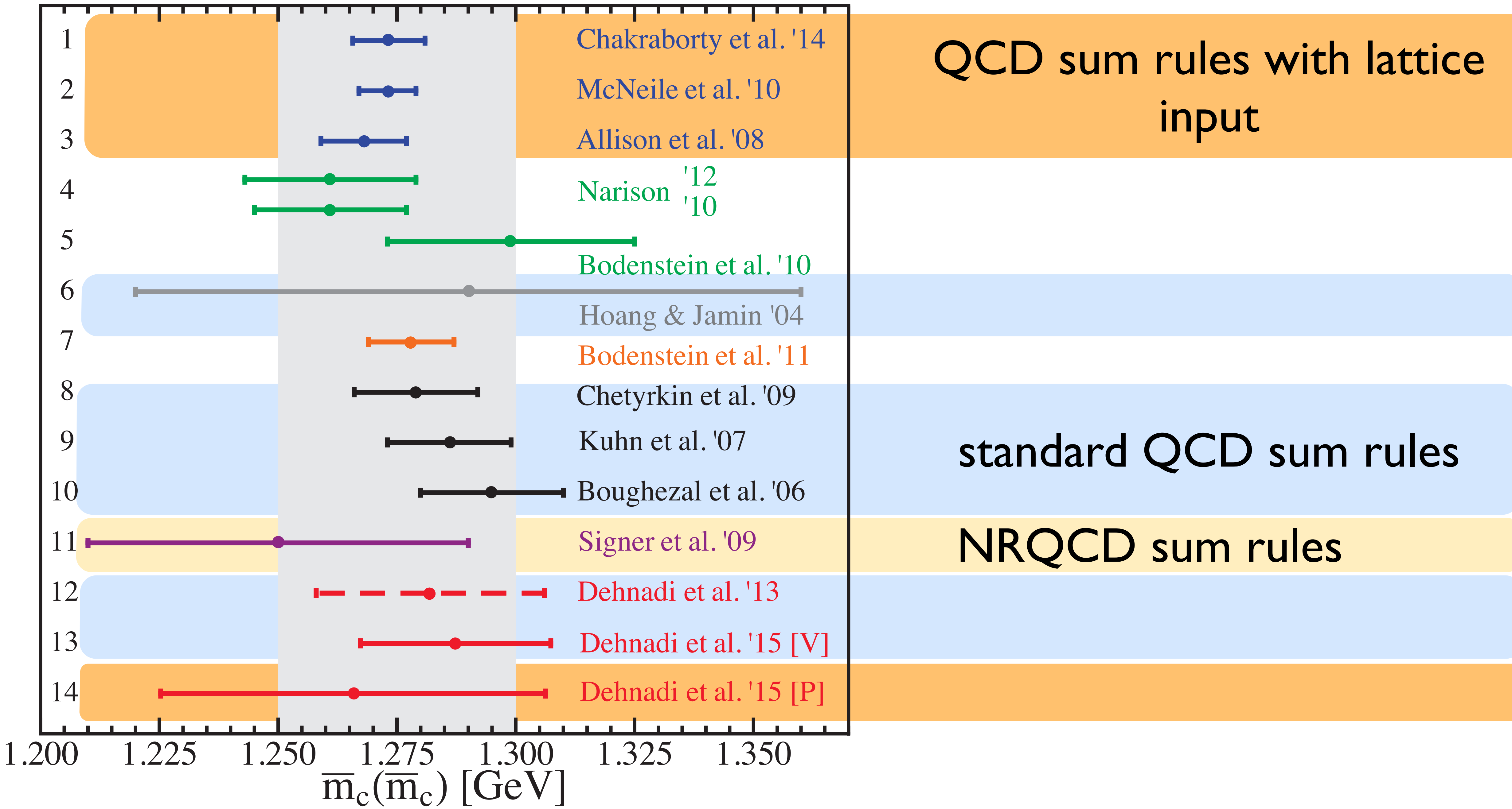


Charm mass determinations

Type of sum rule

only HPQCD has attempted
this kind of analysis

From QCD sum rules

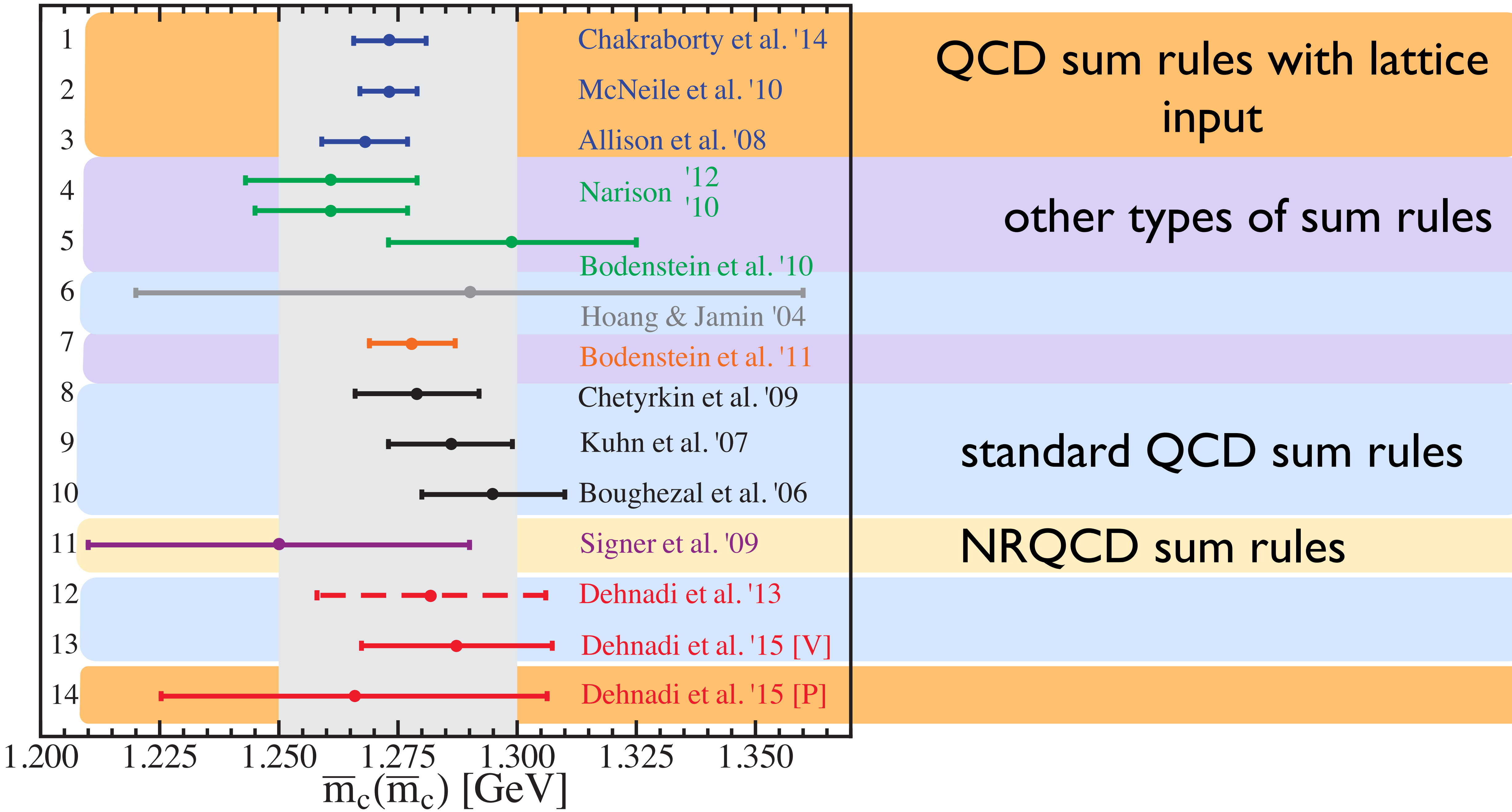


Charm mass determinations

Type of sum rule

From QCD sum rules

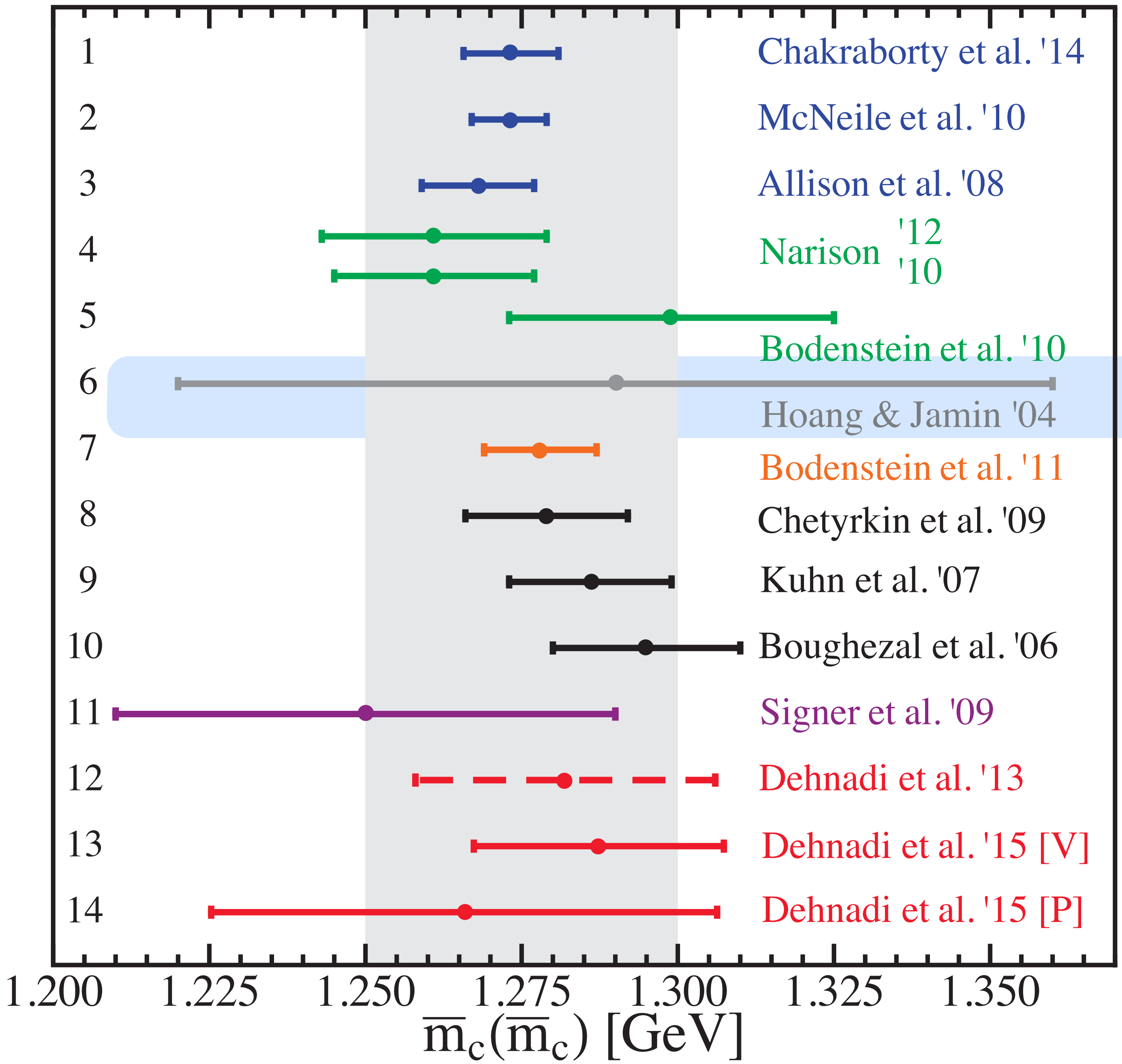
theoretically less sound



Charm mass determinations

Perturbative input

From QCD sum rules



expected large uncertainties

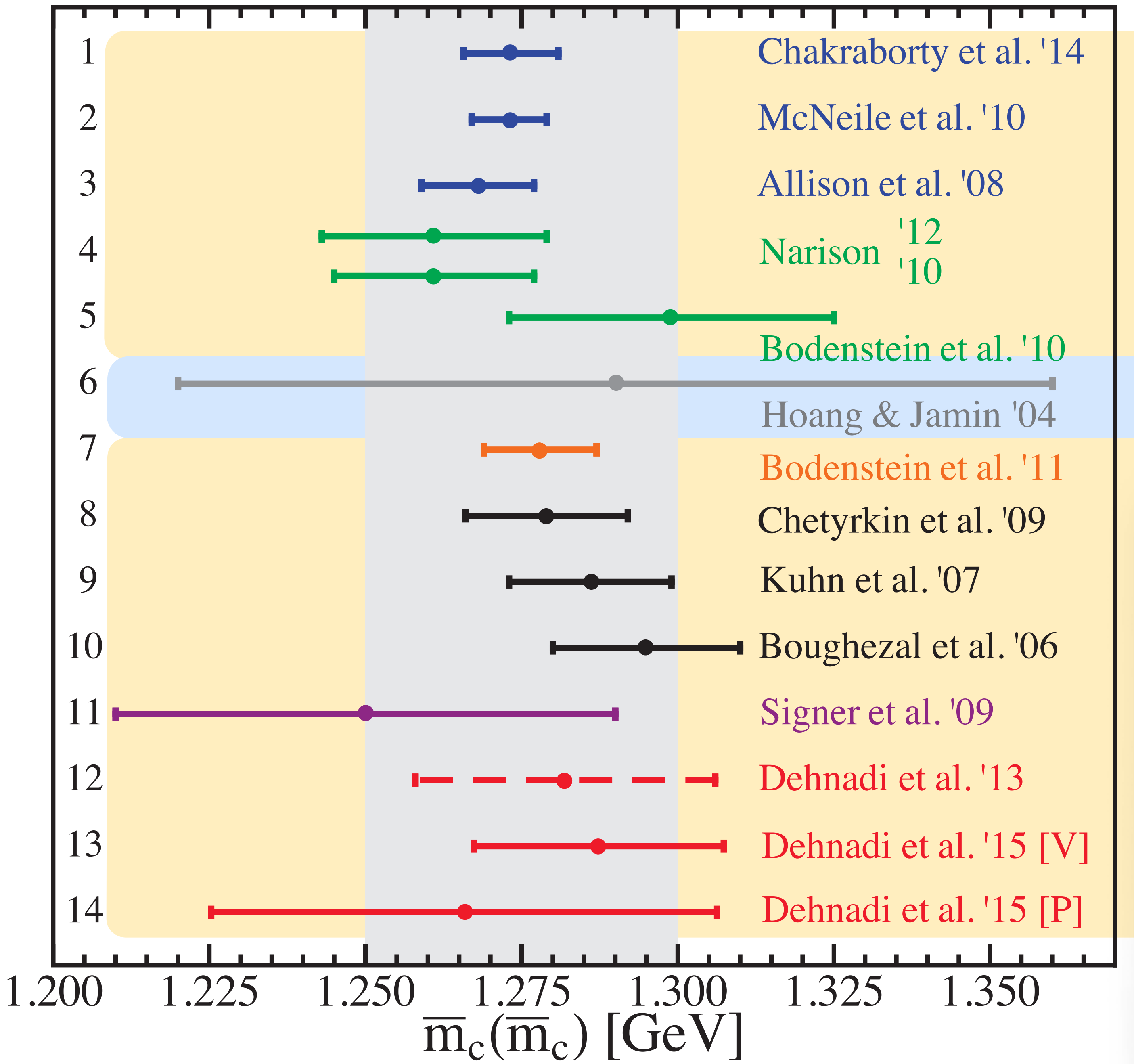
$\mathcal{O}(\alpha_s^2)$ input

Charm mass determinations

Perturbative input

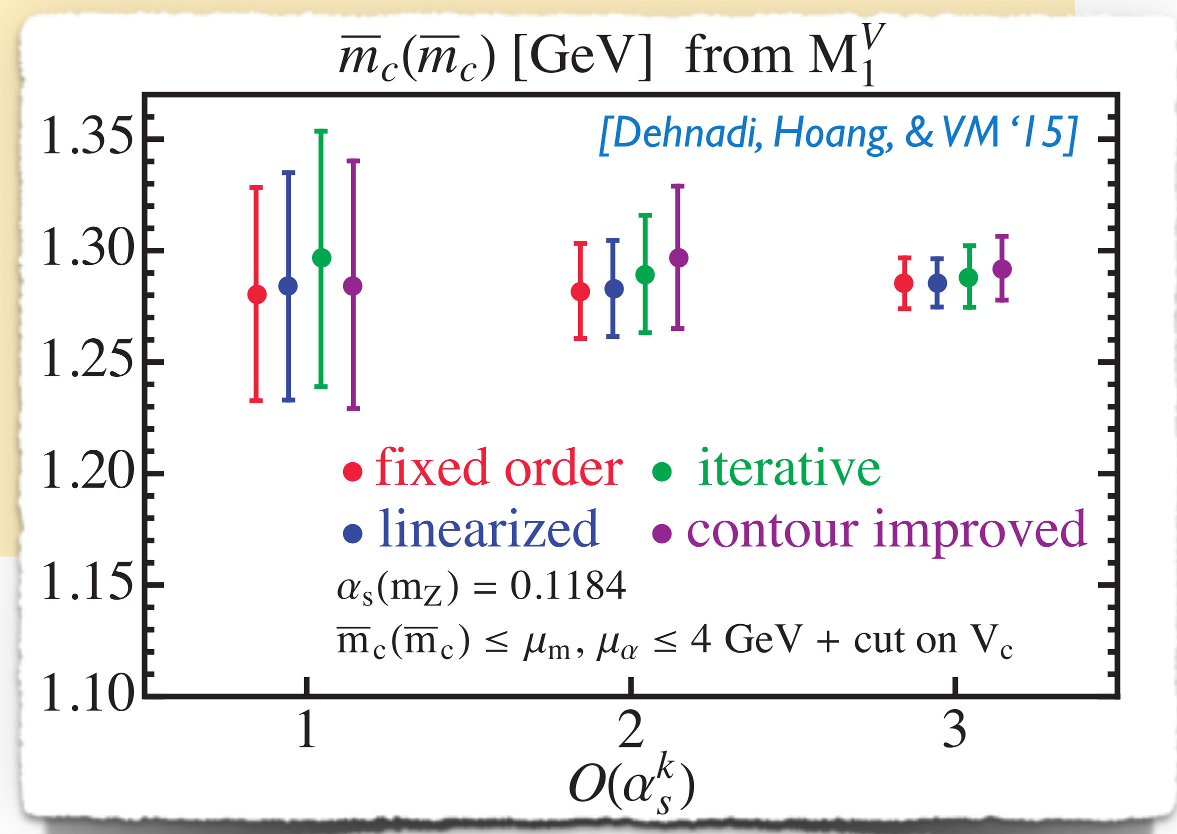
From QCD sum rules

much smaller uncertainties



$\mathcal{O}(\alpha_s^3)$ input

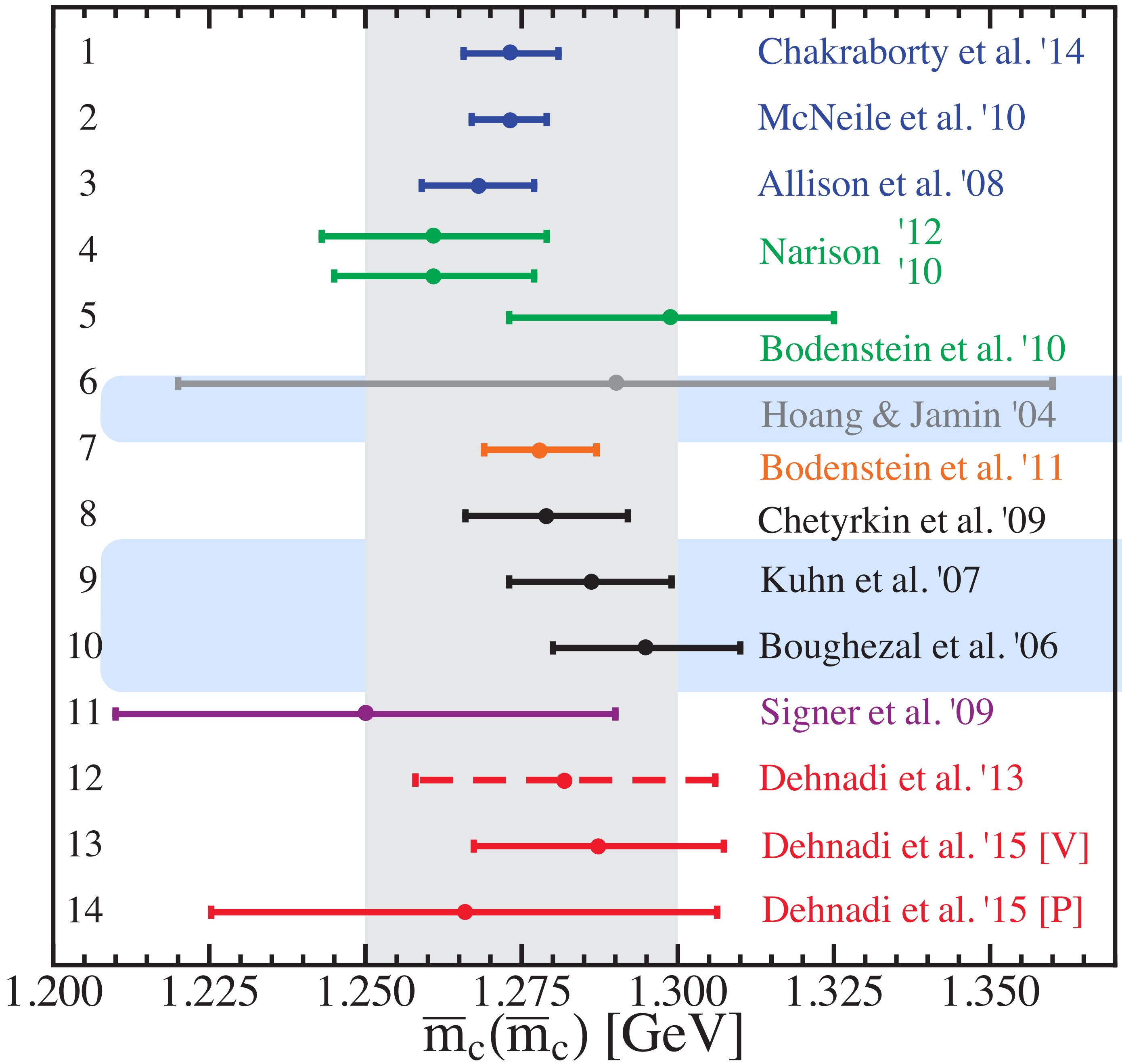
$\mathcal{O}(\alpha_s^2)$ input



Charm mass determinations

Experimental data used

From QCD sum rules



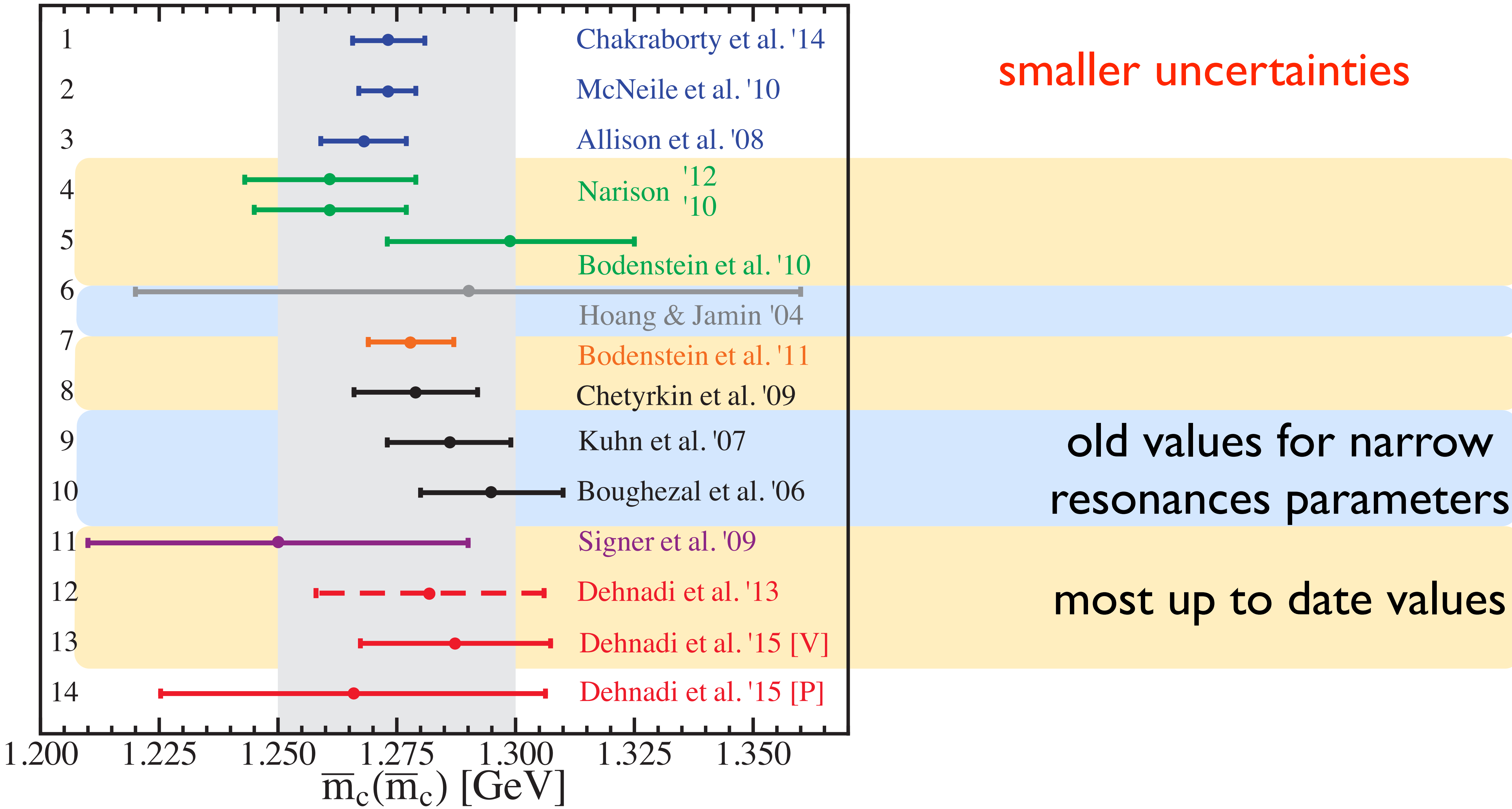
expected large uncertainties, since narrow resonances are the most important piece

old values for narrow resonances parameters

Charm mass determinations

Experimental data used

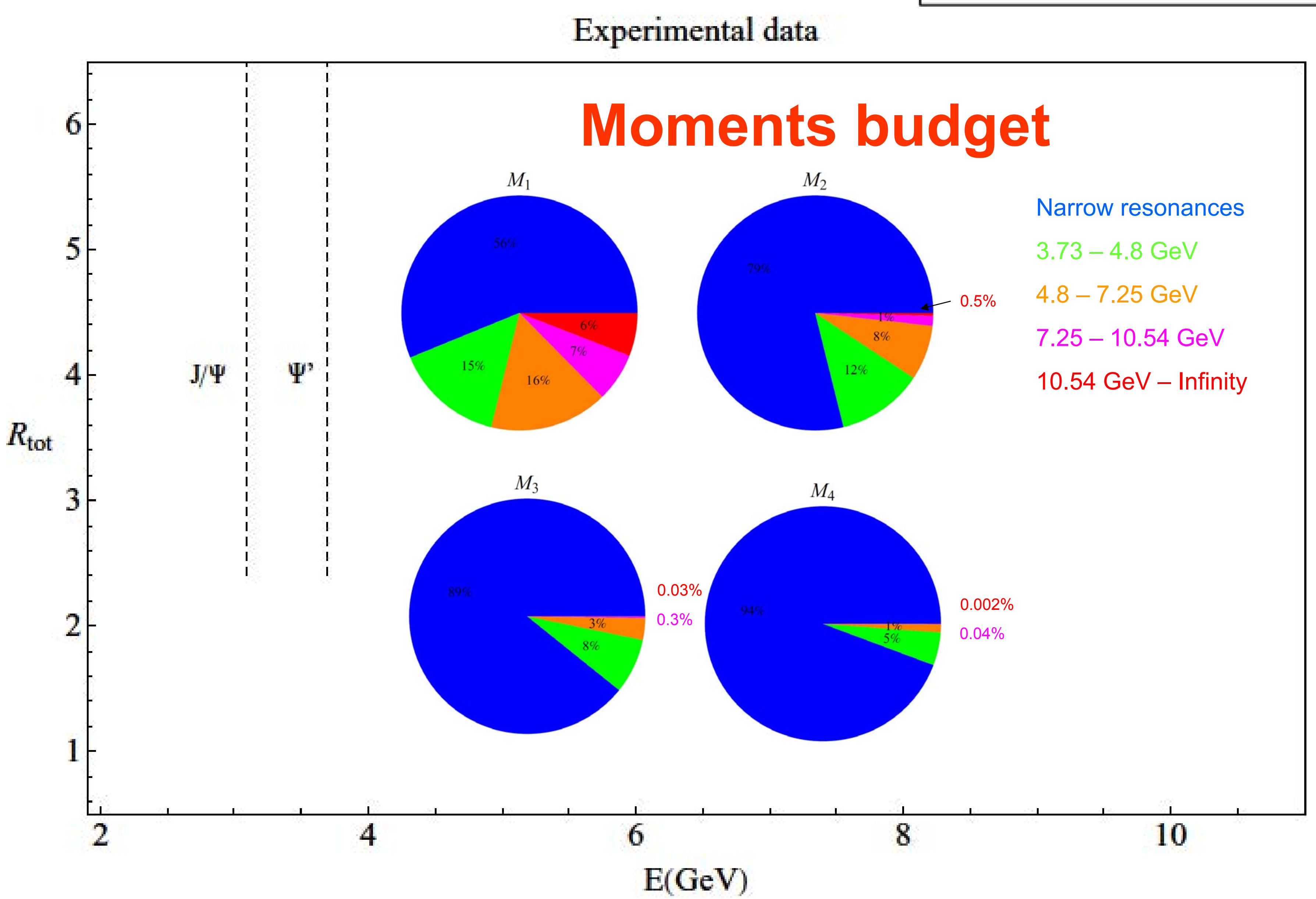
From QCD sum rules



Experimental data: charm

Narrow resonances

	J/Ψ	$\psi(2S)$
M (GeV)	3.096916(11)	3.686093(34)
Γ_{ee} (keV)	5.55(14)	2.48(6)
$(\alpha/\alpha(M))^2$	0.957785	0.95554



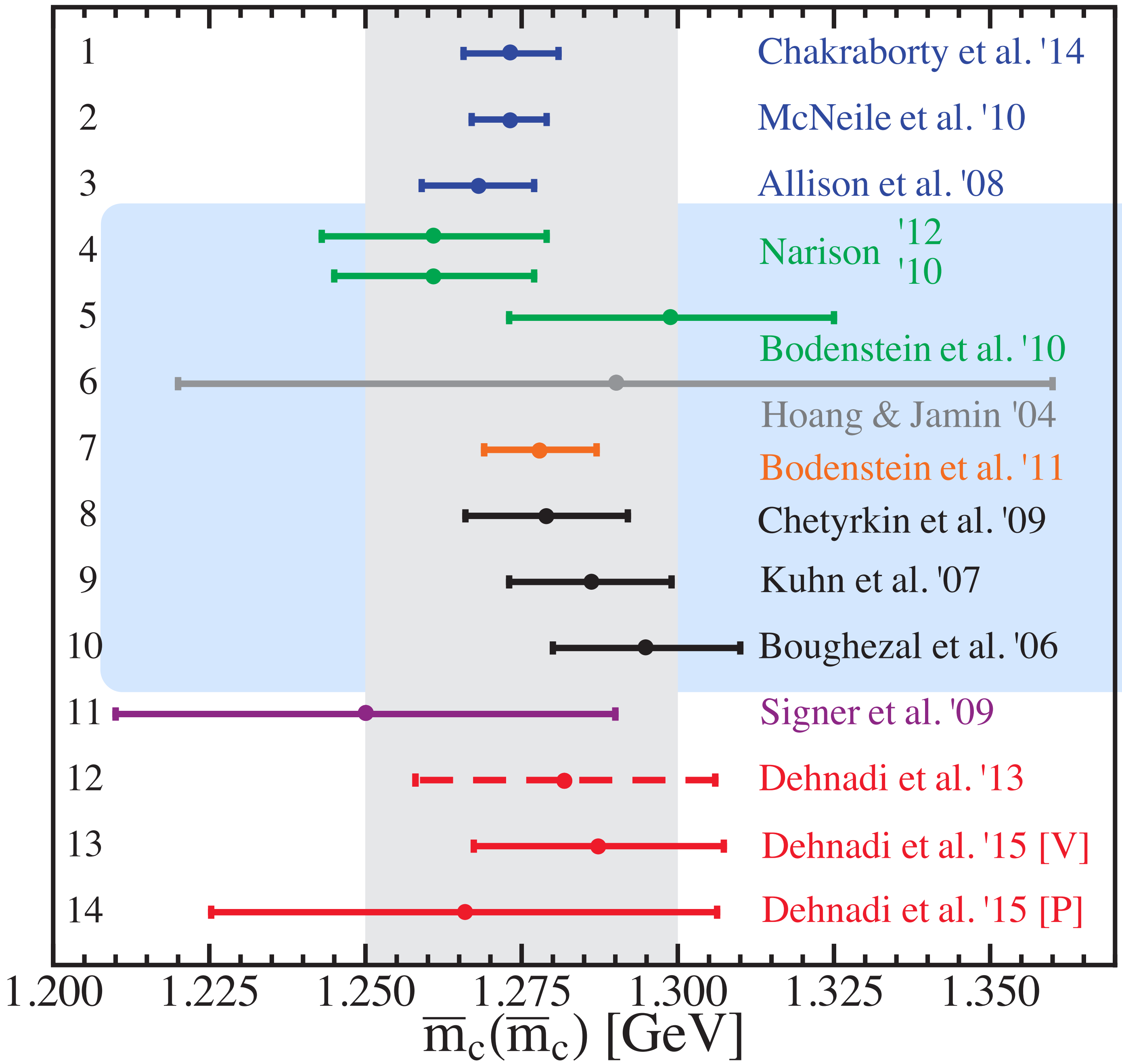
$$M_n^{\rm res} = \frac{9\,\pi\,\Gamma_{ee}}{\alpha(M)^2 M^{2n+1}}$$

Narrow-width approximation

Charm mass determinations

Experimental data used

From QCD sum rules

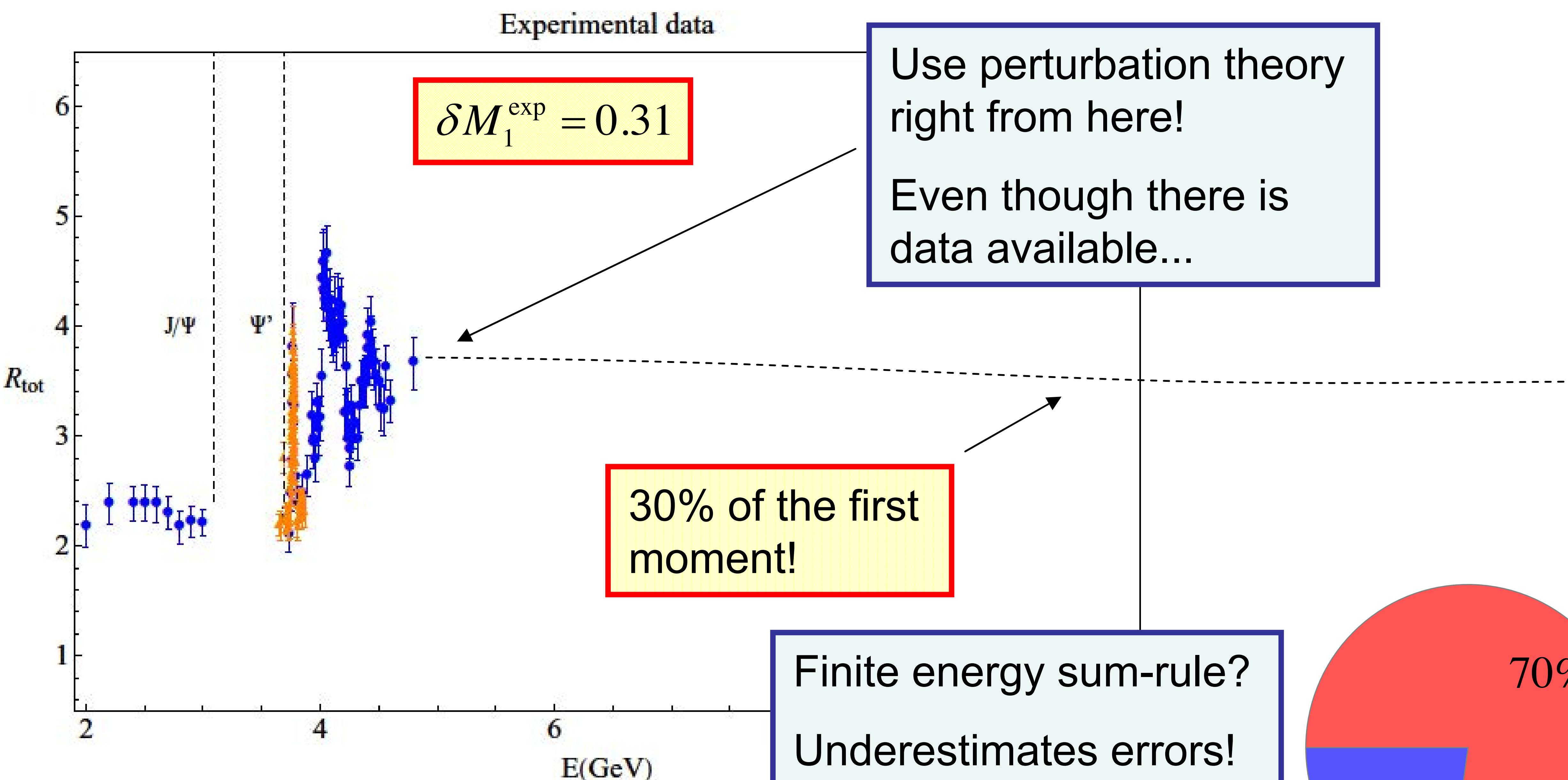


possible bias + underestimate of experimental uncertainties

Only BES data + pQCD instead of experimental info for the rest of the spectrum

Experimental data: charm

Data used in Kuhn et al (2004, 05) and Bodenstein et al



$\delta M_1^{exp} = 0.31$

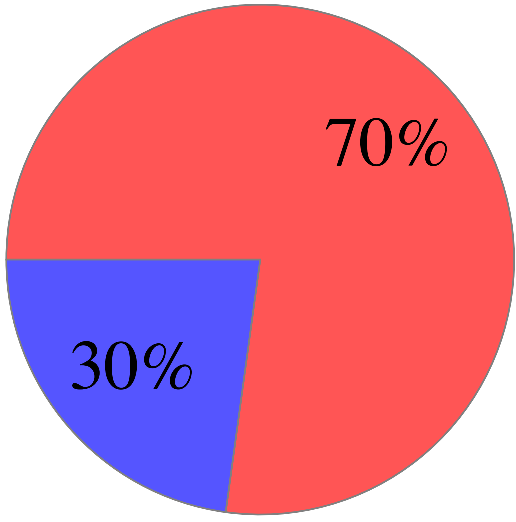
Use perturbation theory right from here!

Even though there is data available...

30% of the first moment!

Finite energy sum-rule?

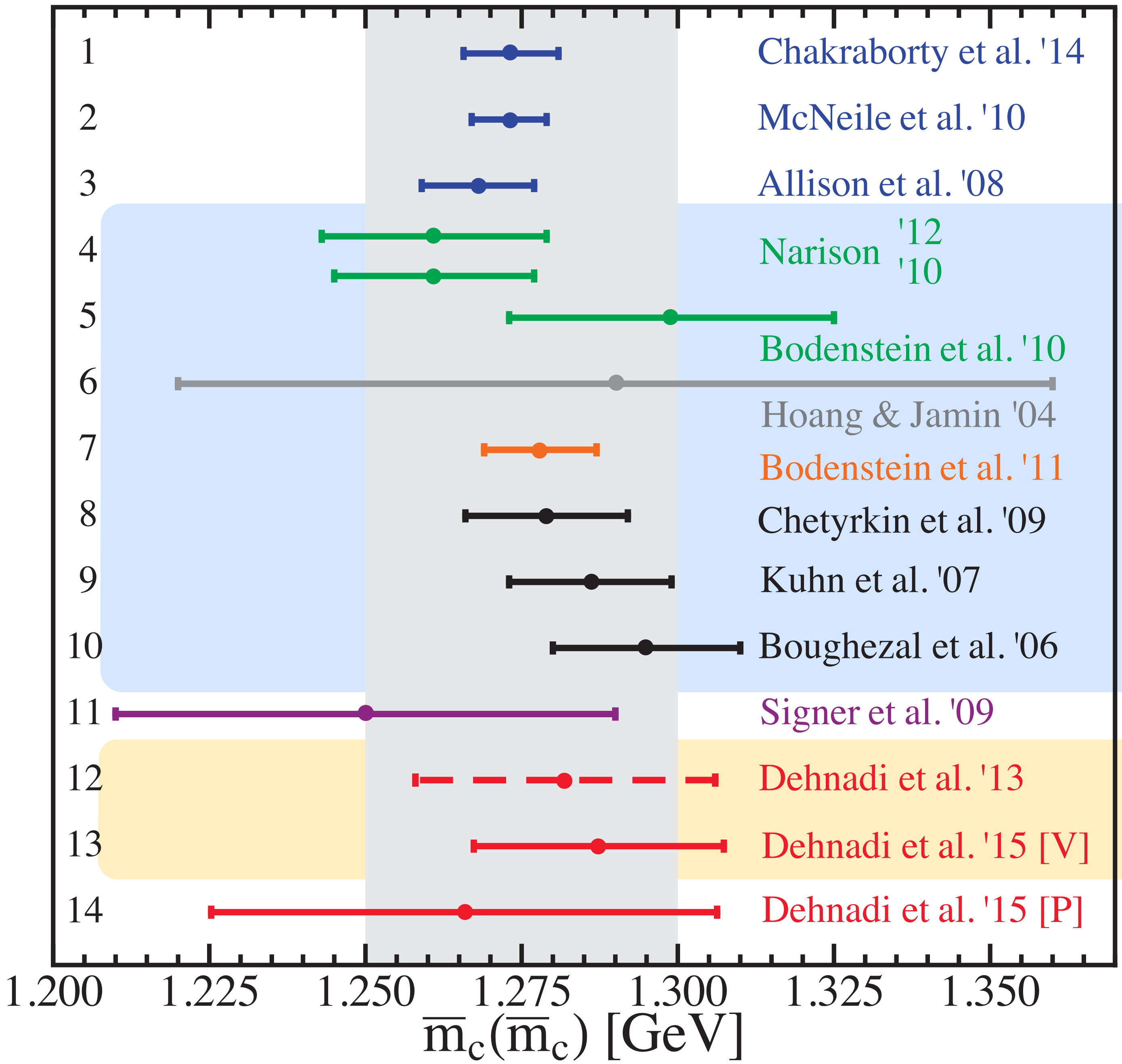
Underestimates errors! (they assing only naive theory error)



Charm mass determinations

Experimental data used

From QCD sum rules



minimal dependence on assumptions

Only BES data + pQCD instead of experimental info for the rest of the spectrum

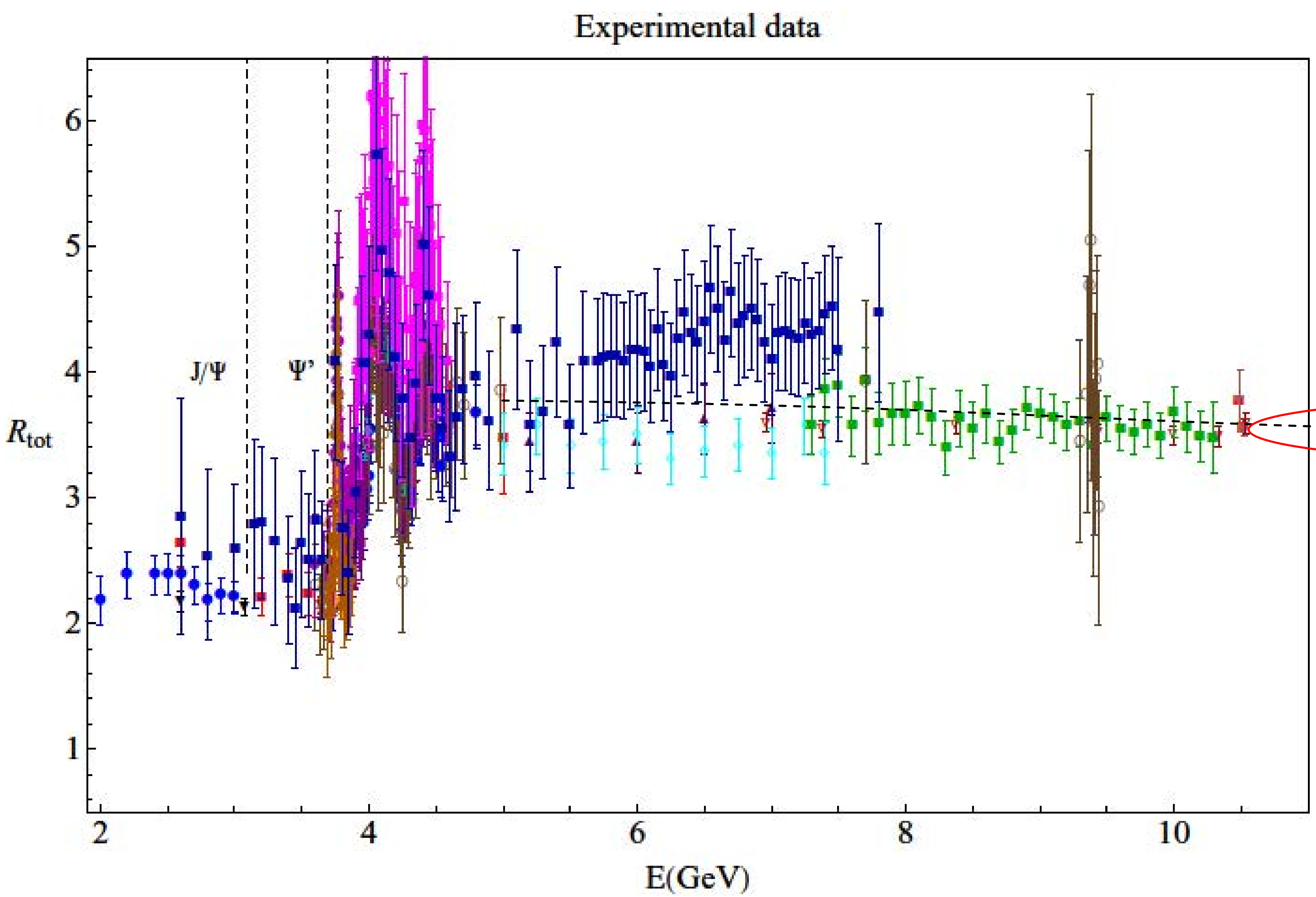
use all available data

Experimental data: charm

Perturbation theory

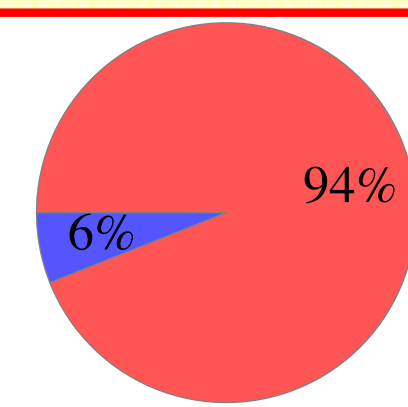
• Only where there is no data

• Assign a conservative 10% error to reduce model dependence



$M_1 \rightarrow 6\%$

$M_{n>1} \rightarrow < 1\%$



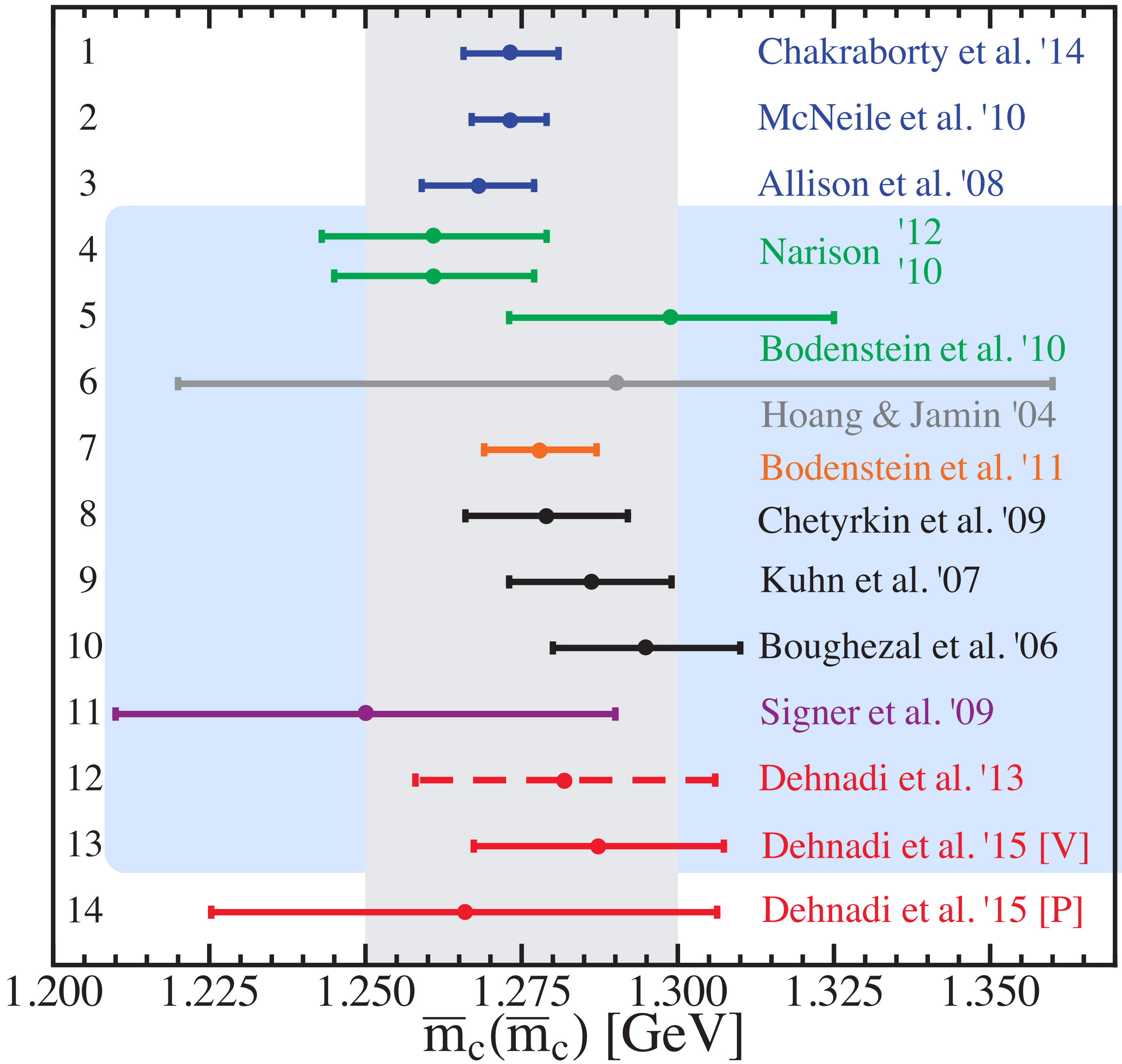
[Dehnadi, Hoang, VM & Zebarjad '15]

[Dehnadi, Hoang, & VM '15]

Charm mass determinations

Type of QCD current

From QCD sum rules



good convergence

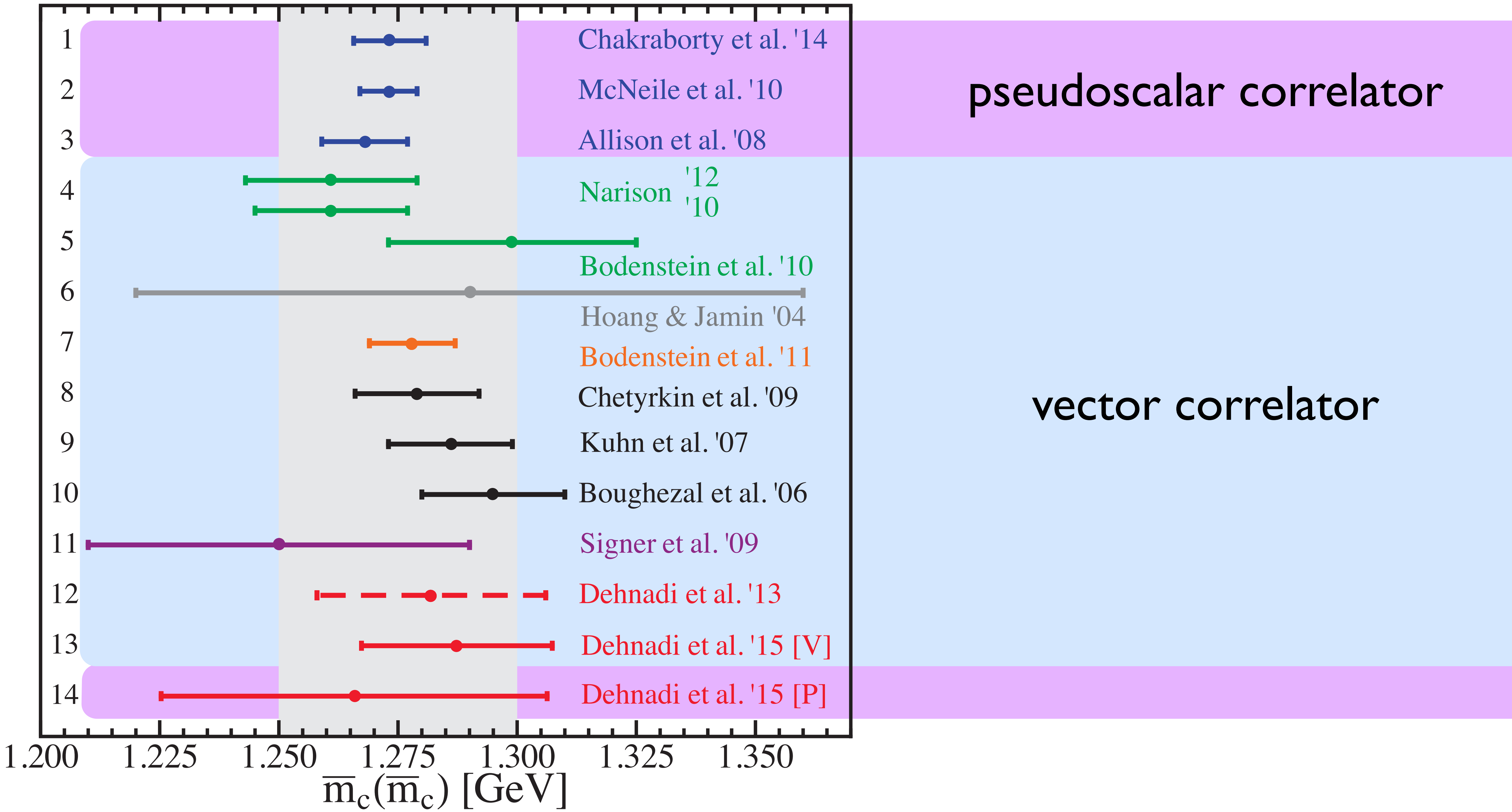
vector correlator

Charm mass determinations

Type of QCD current

From QCD sum rules

not so good convergence



Convergence test

Cauchy root convergence test: $S[a] = \sum_n a_n$ $\qquad V_\infty \equiv \limsup_{n\to\infty} (a_n)^{1/n}$

$$V_\infty = \begin{cases} > 1 & \text{divergent} \\ = 1^+ & \text{inconclusive} \\ \leq 1 & \text{convergent} \end{cases}$$

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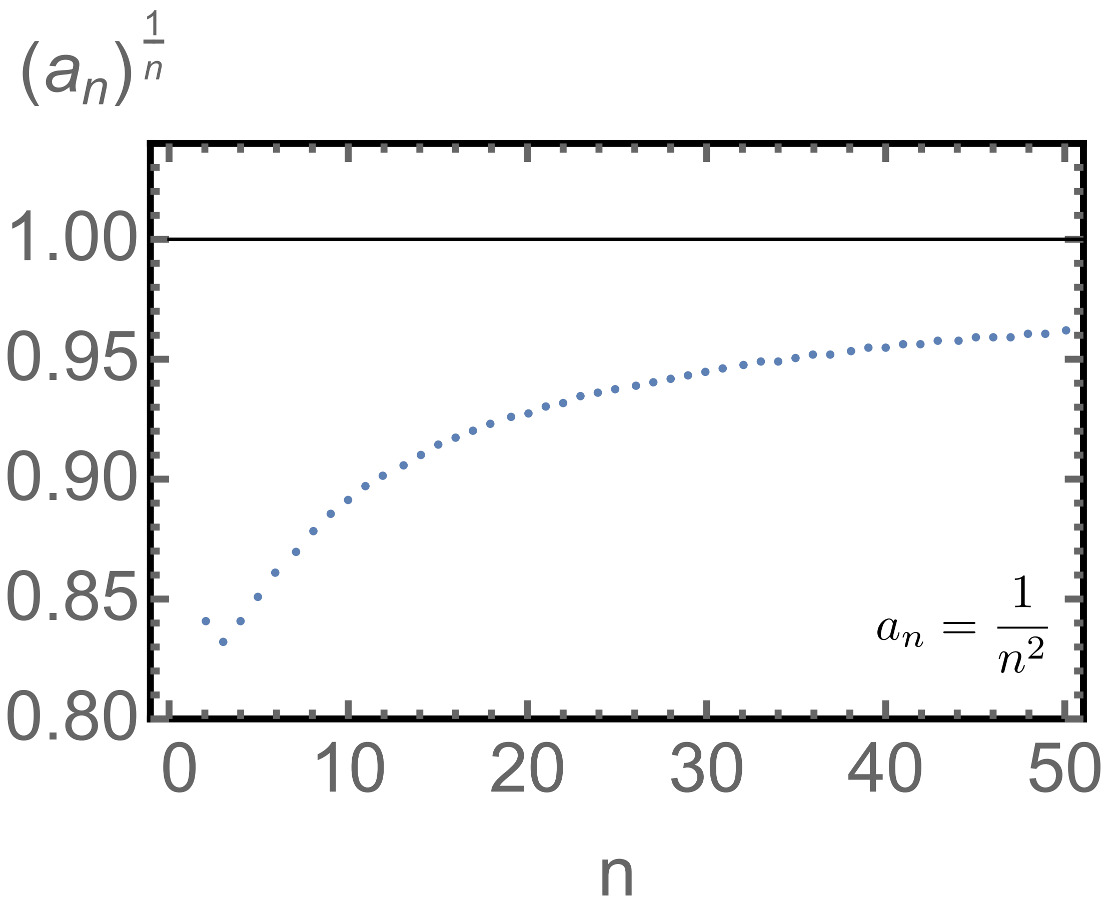

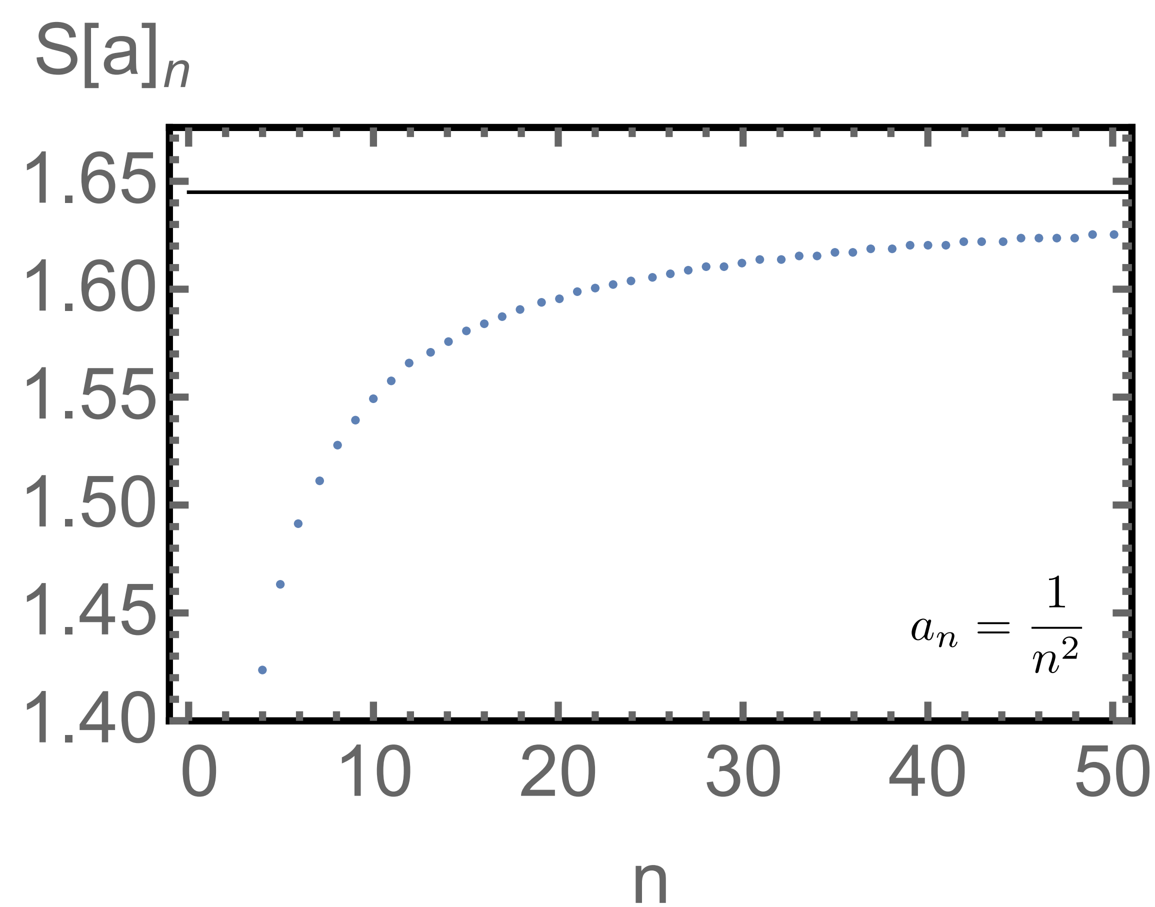

We do not known so many terms in QCD… need to adapt the test !

Convergence test

[Dehnadi, Hoang, & VM '15]

For each pear (μ_m, μ_α) we define

$$\overline{m}_c(\overline{m}_c) = m^{(0)} + \delta m^{(1)} + \delta m^{(2)} + \delta m^{(3)}$$

from the mass extractions at $\mathcal{O}(\alpha_s^{0,1,2,3})$ and define the convergence parameter

$$V_c = \max\left[\frac{\delta m^{(1)}}{m^{(0)}}, \Big(\frac{\delta m^{(2)}}{m^{(0)}}\Big)^{1/2}, \Big(\frac{\delta m^{(3)}}{m^{(0)}}\Big)^{1/3}\right]$$

Convergence test

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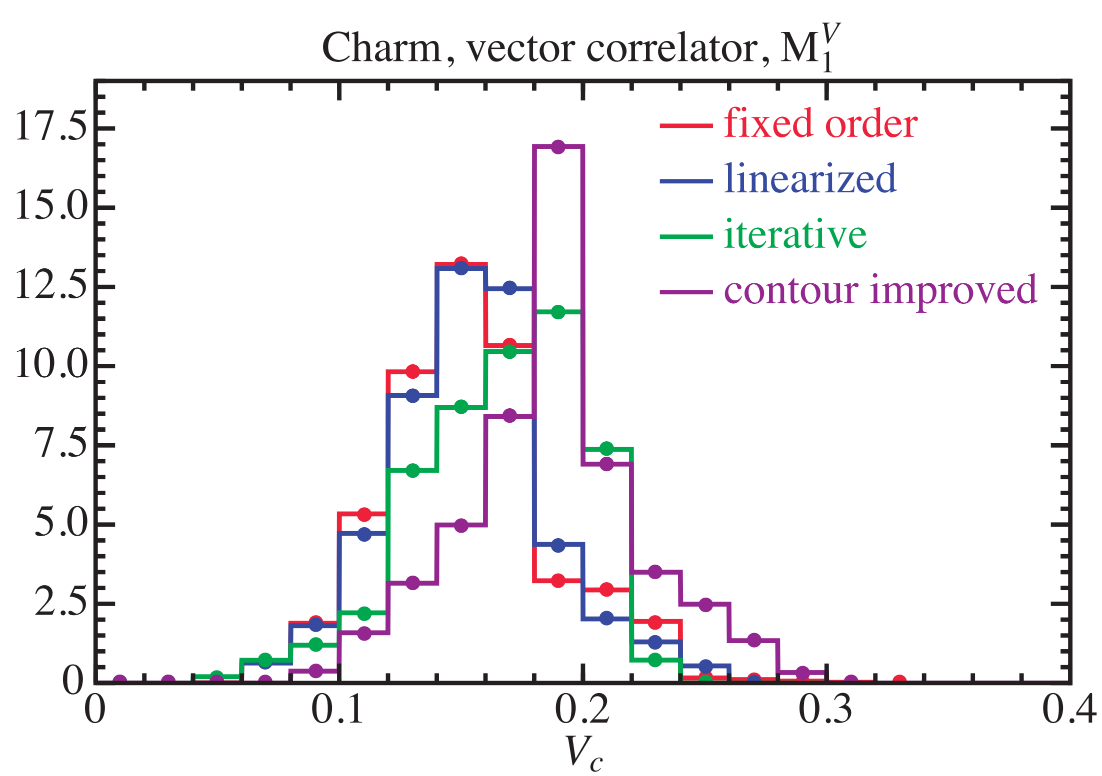
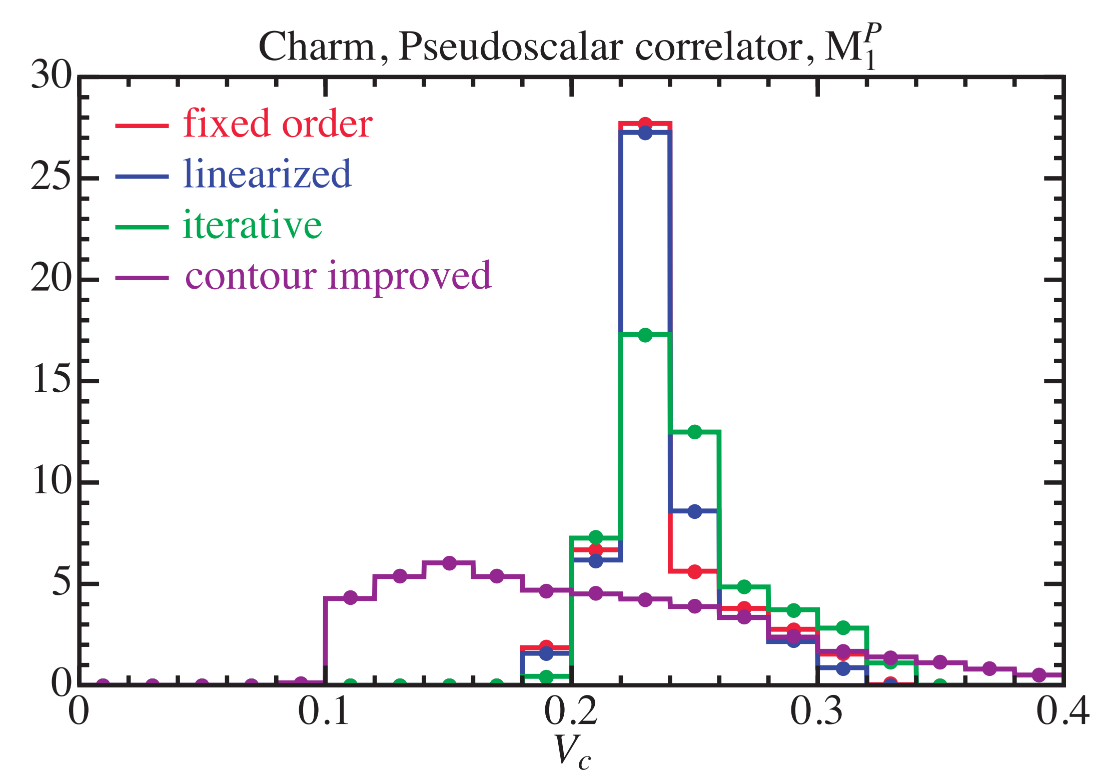
It is convenient to plot histograms, and see if there is a peaked structure





Smaller value of V_c means better convergence.

Convergence test

[Dehnadi, Hoang, & VM '15]

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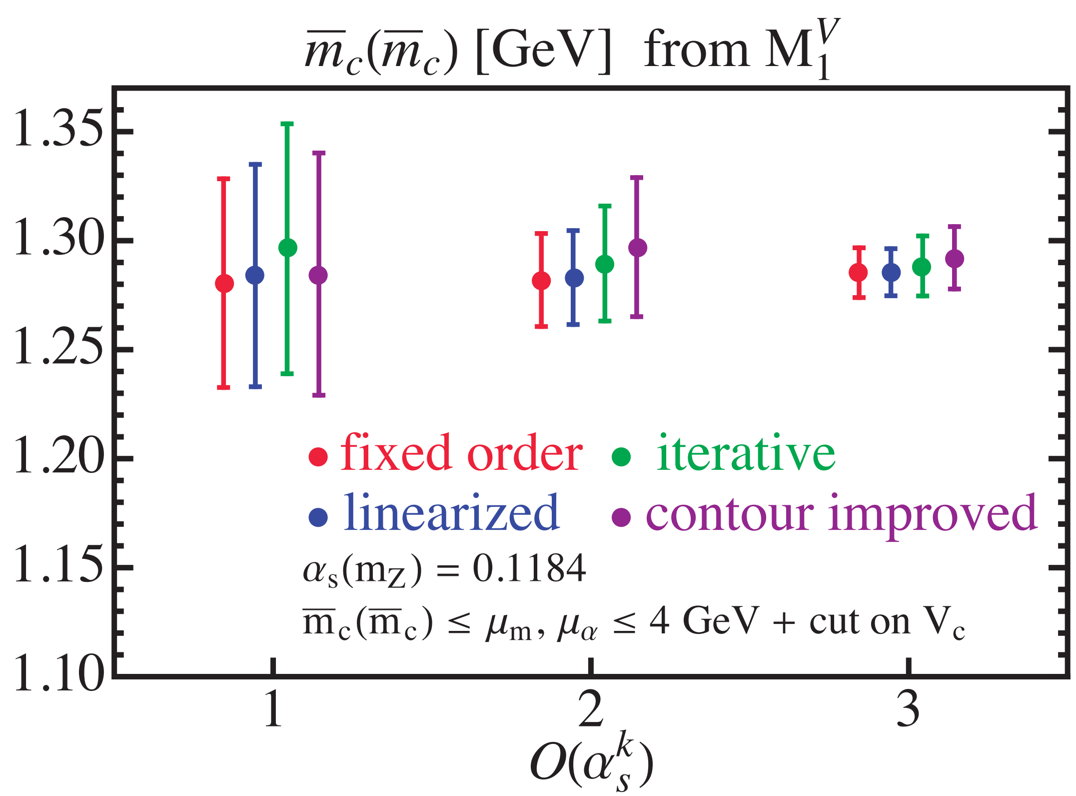
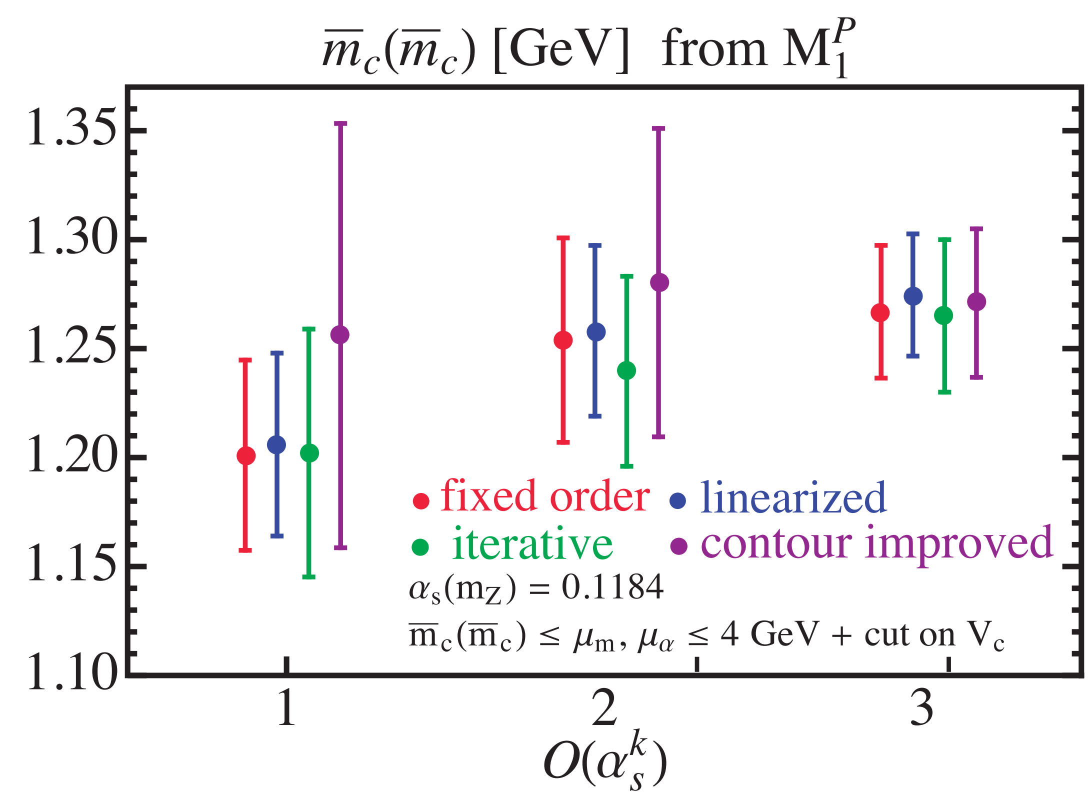


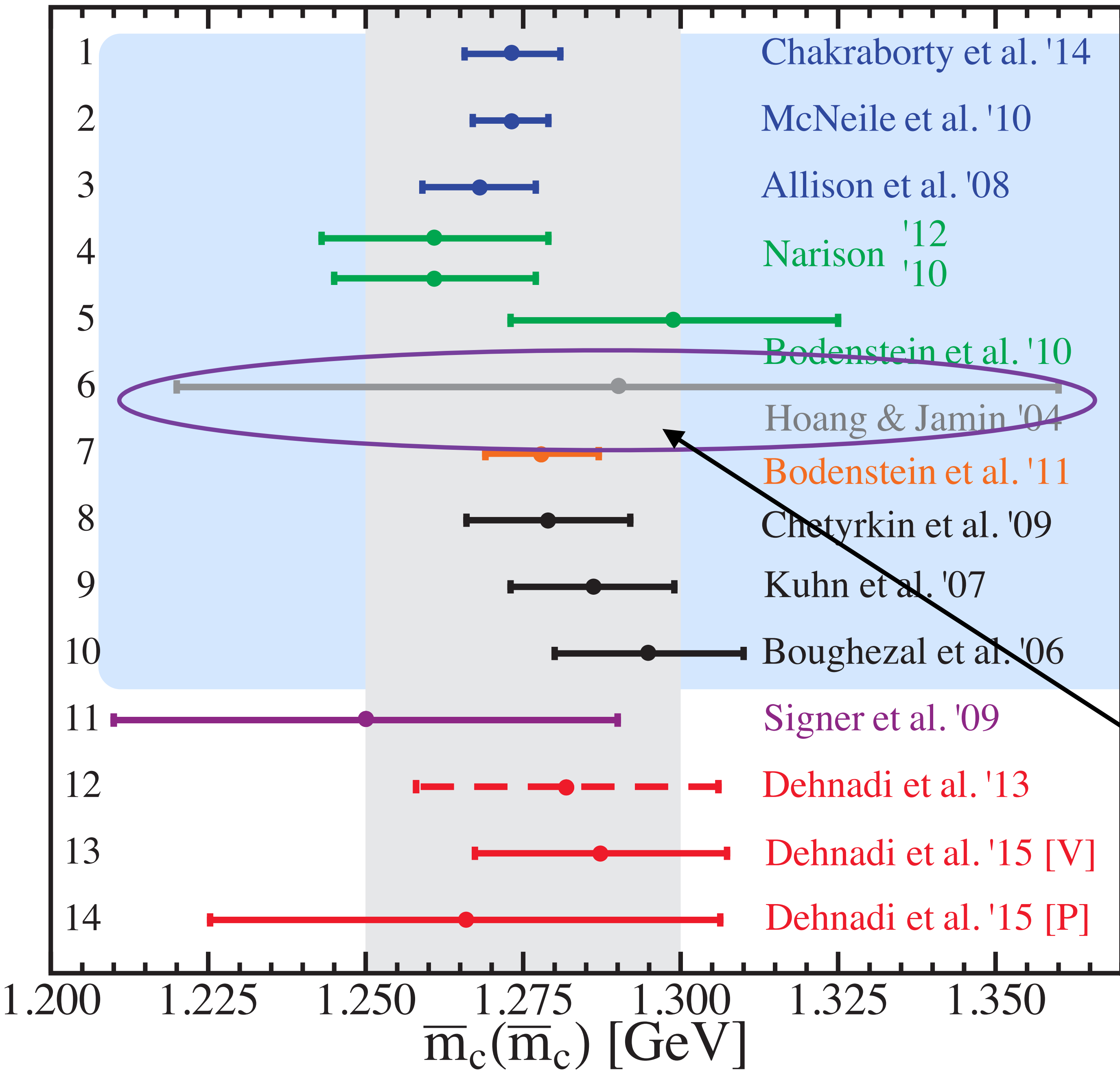


For our final analysis we discard series with $V_c \gg \langle V_c \rangle$ (3% of series only)

Charm mass determinations

Estimate of perturbative uncertainties

From QCD sum rules



inconsistent results for different methods and orders

correlated scale variation

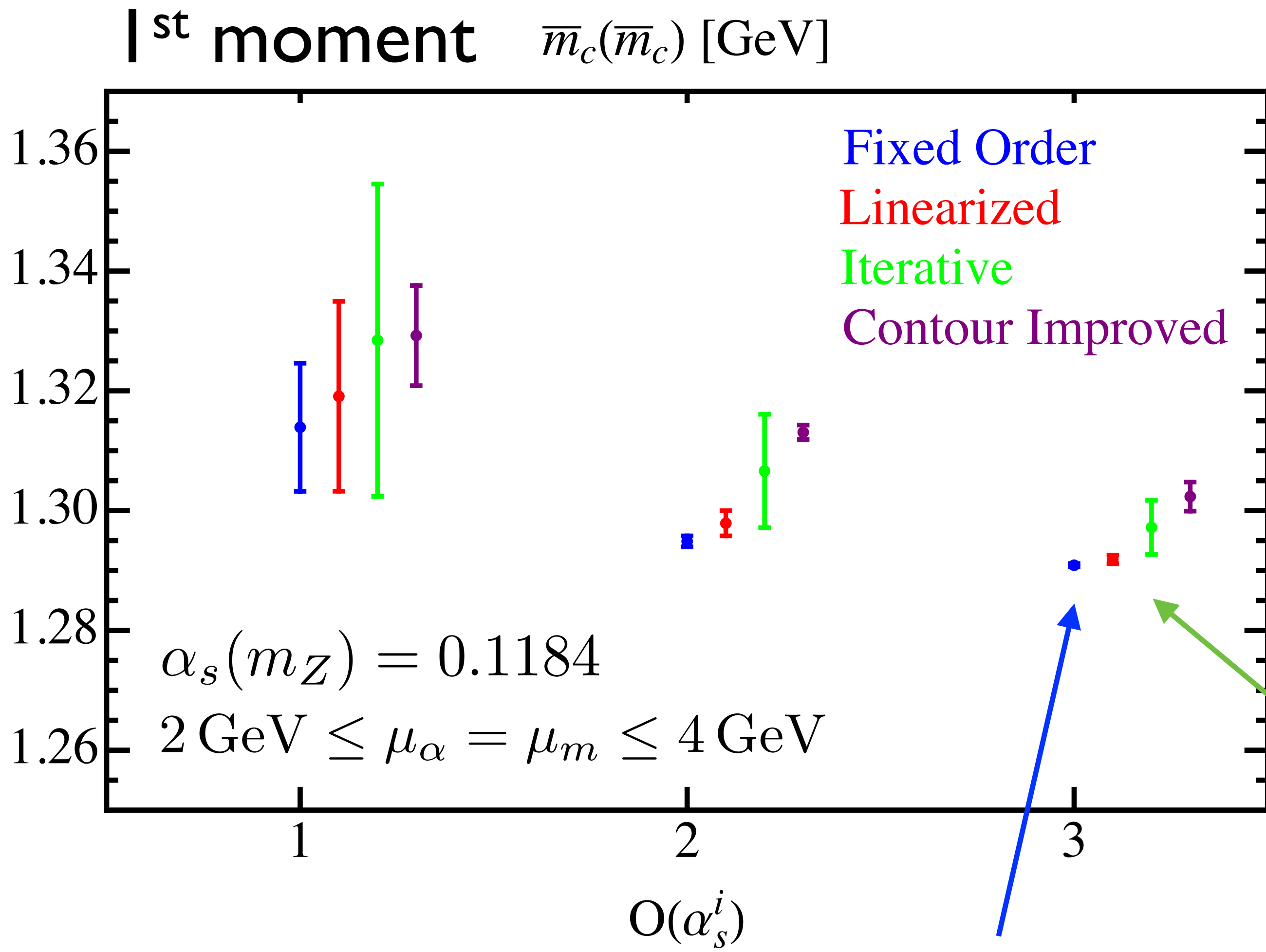
$$2\,\mathrm{GeV} \leq \mu_\alpha = \mu_m \leq 4\,\mathrm{GeV}$$

$$2\,\mathrm{GeV} \leq \mu_\alpha \leq 4\,\mathrm{GeV}$$
$$\mu_m = \overline{m}_c(\overline{m}_c)$$

Exploration of scale variation

[Dehnadi, Hoang, & VM '15]

1st moment

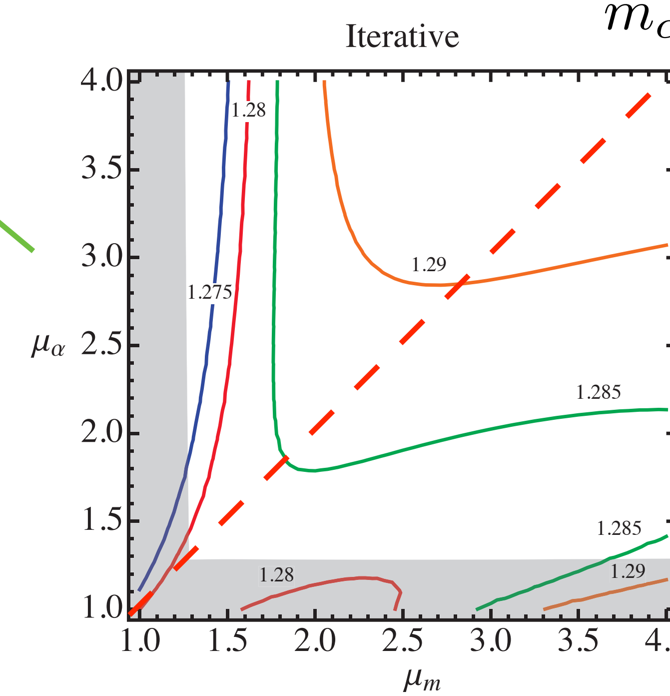


correlated variation

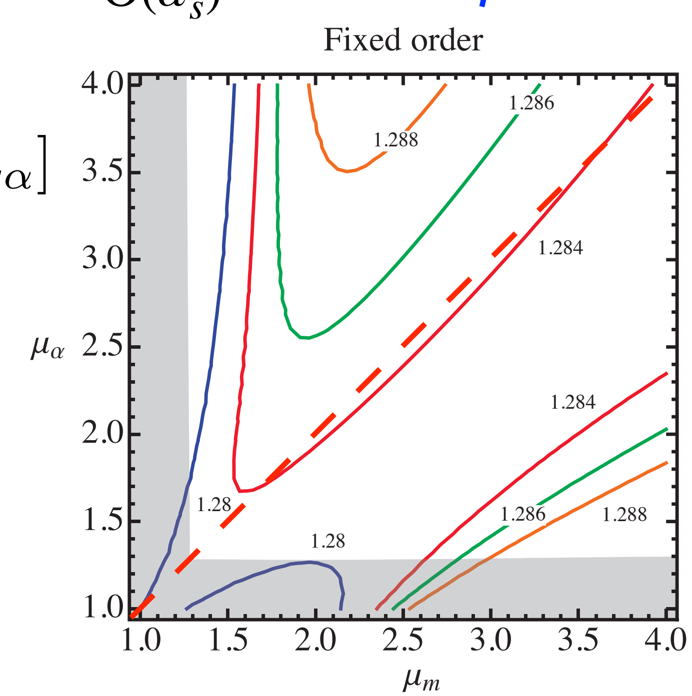
$$2\,\text{GeV} \leq \mu_\alpha = \mu_m \leq 4\,\text{GeV}$$

Charm mass scale is excluded from the scale variation

$\overline{m}_c(\overline{m}_c)[\mu_m, \mu_\alpha]$



$\overline{m}_c(\overline{m}_c)[\mu_m, \mu_\alpha]$

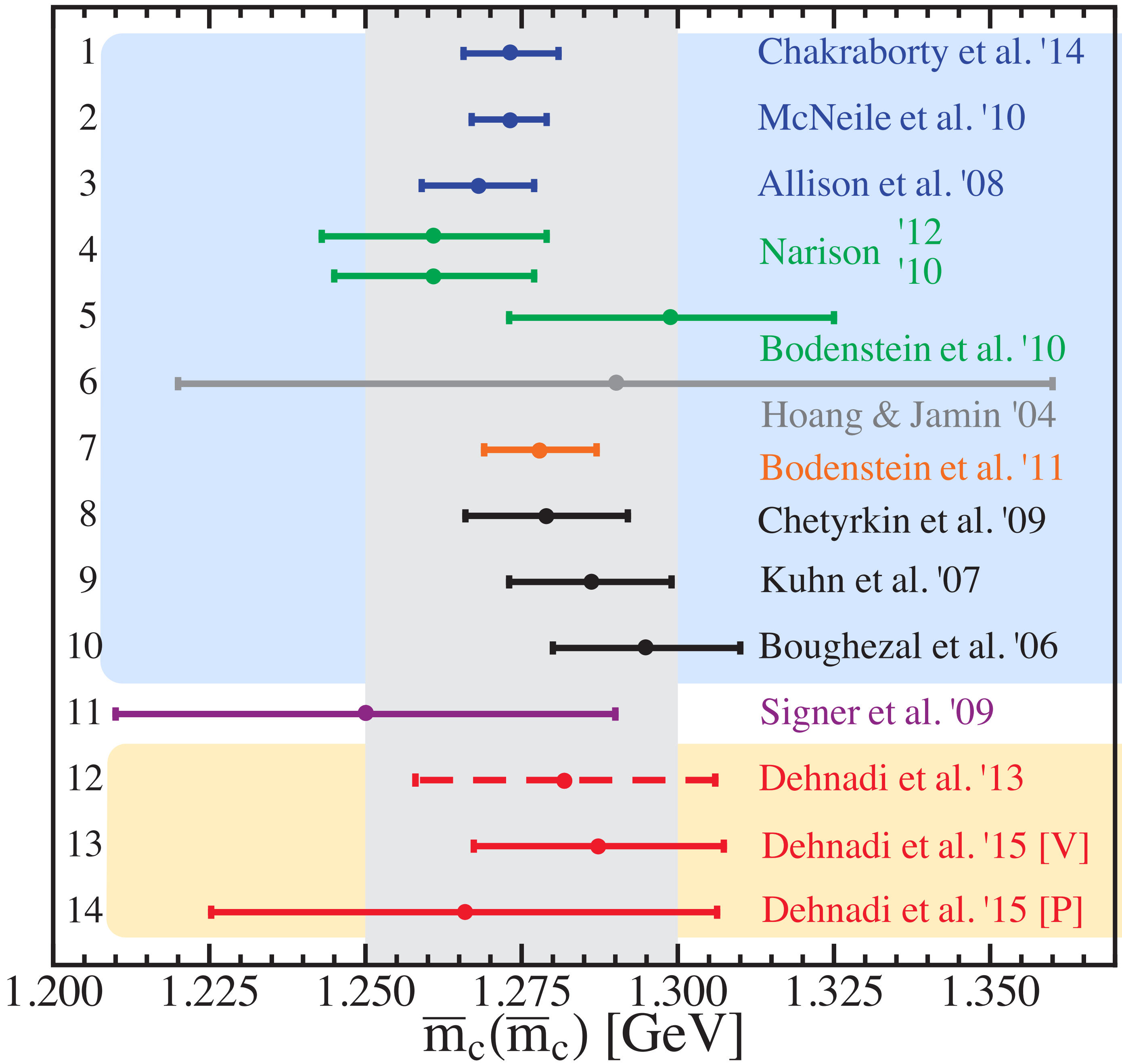


Charm mass determinations

Estimate of perturbative uncertainties

From QCD sum rules

provides consistent results, reflects actual series convergence



correlated scale variation

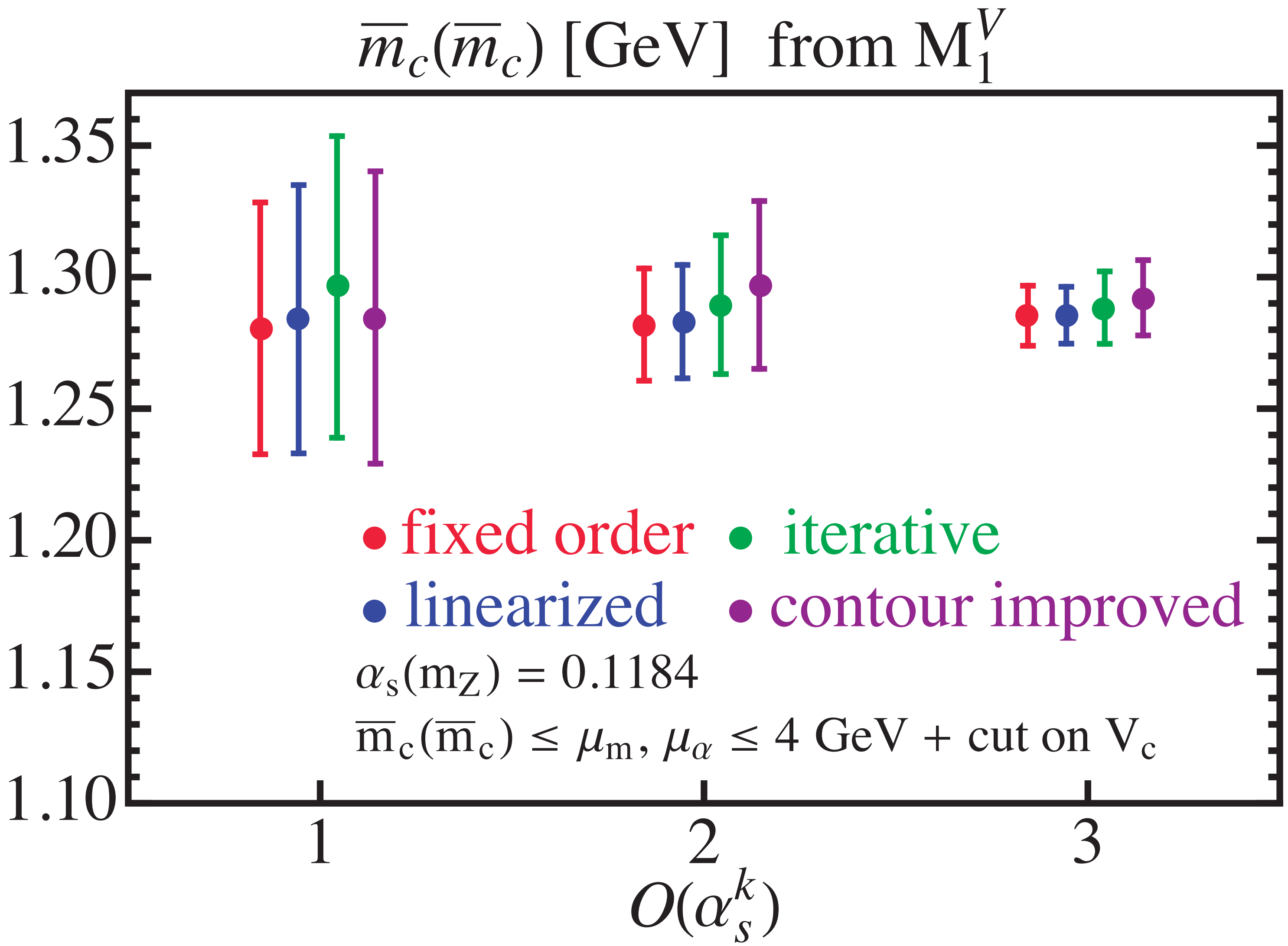
$$2\,\mathrm{GeV} \leq \mu_\alpha = \mu_m \leq 4\,\mathrm{GeV}$$

uncorrelated scale variation

$$\overline{m}_c(\overline{m}_c) \leq \mu_\alpha, \mu_m \leq 4\,\mathrm{GeV}$$

Exploration of scale variation

[Dehnadi, Hoang, & VM '15]



our approach

$$\overline{m}_c(\overline{m}_c) \leq \mu_\alpha, \mu_m \leq 4\,\mathrm{GeV}$$

Charm mass scale should not be excluded in the perturbative extraction of the charm mass

Our default is iterative method

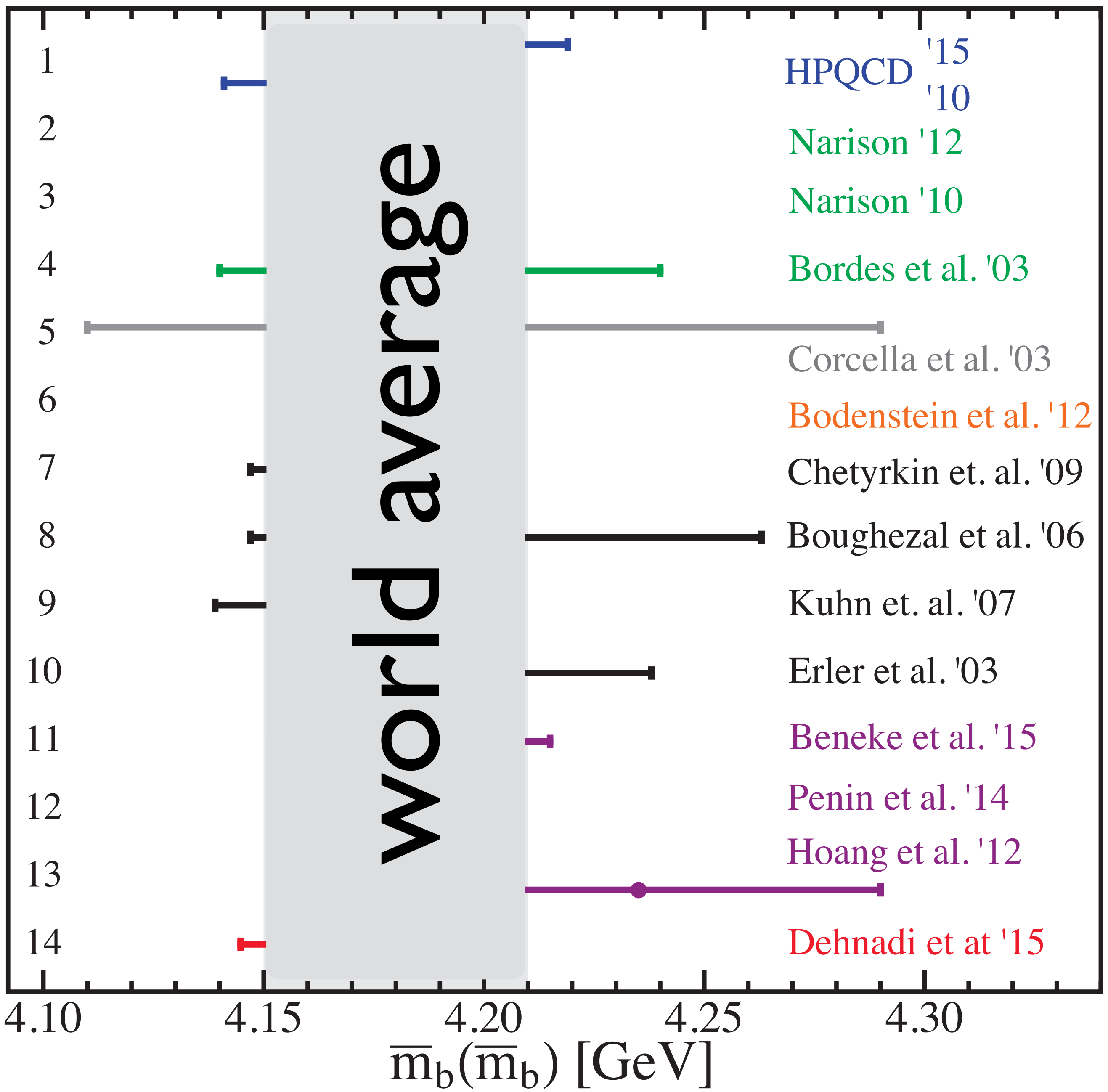
We implement a cut on badly convergent series (mild effect on error)

conclusions: independent variation of scales down to $\overline{m}_c(\overline{m}_c)$ so that using different expansions does not matter

Bottom mass from sum rules

Bottom mass determinations

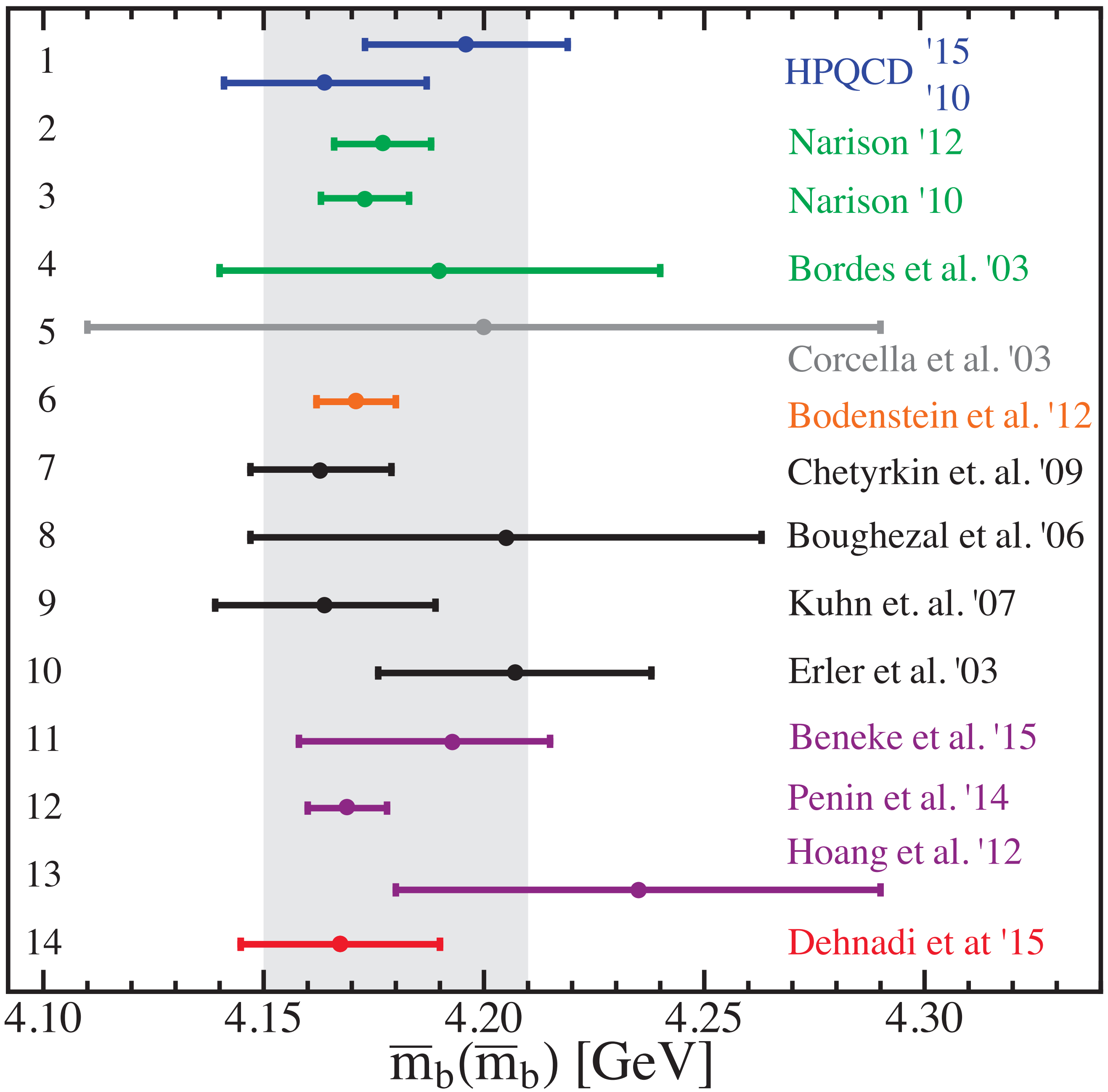
From QCD sum rules



[Dehnadi, Hoang, & VM '15]

Bottom mass determinations

From QCD sum rules

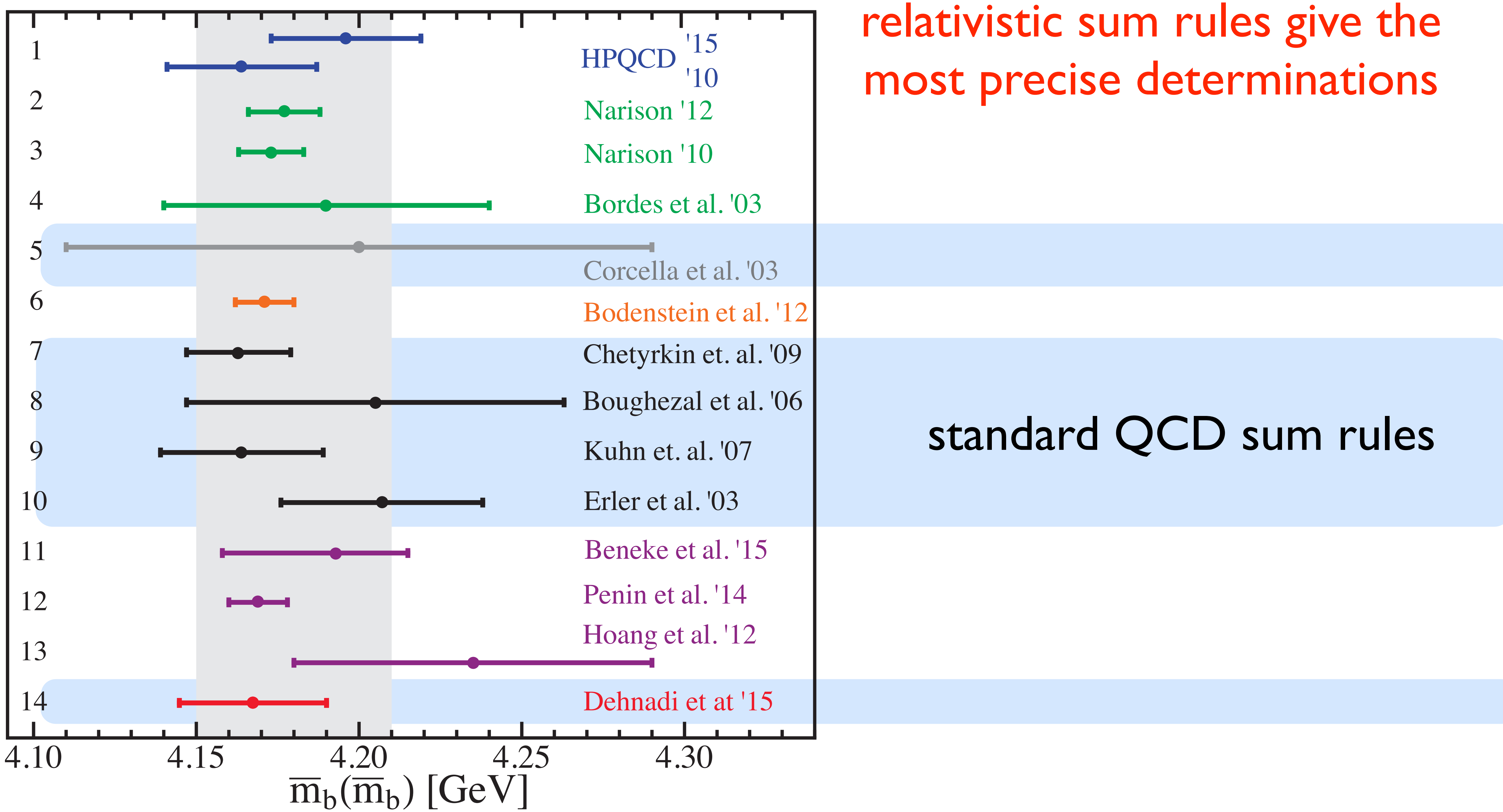


[Dehnadi, Hoang, & VM '15]

Bottom mass determinations

Type of sum rule

From QCD sum rules

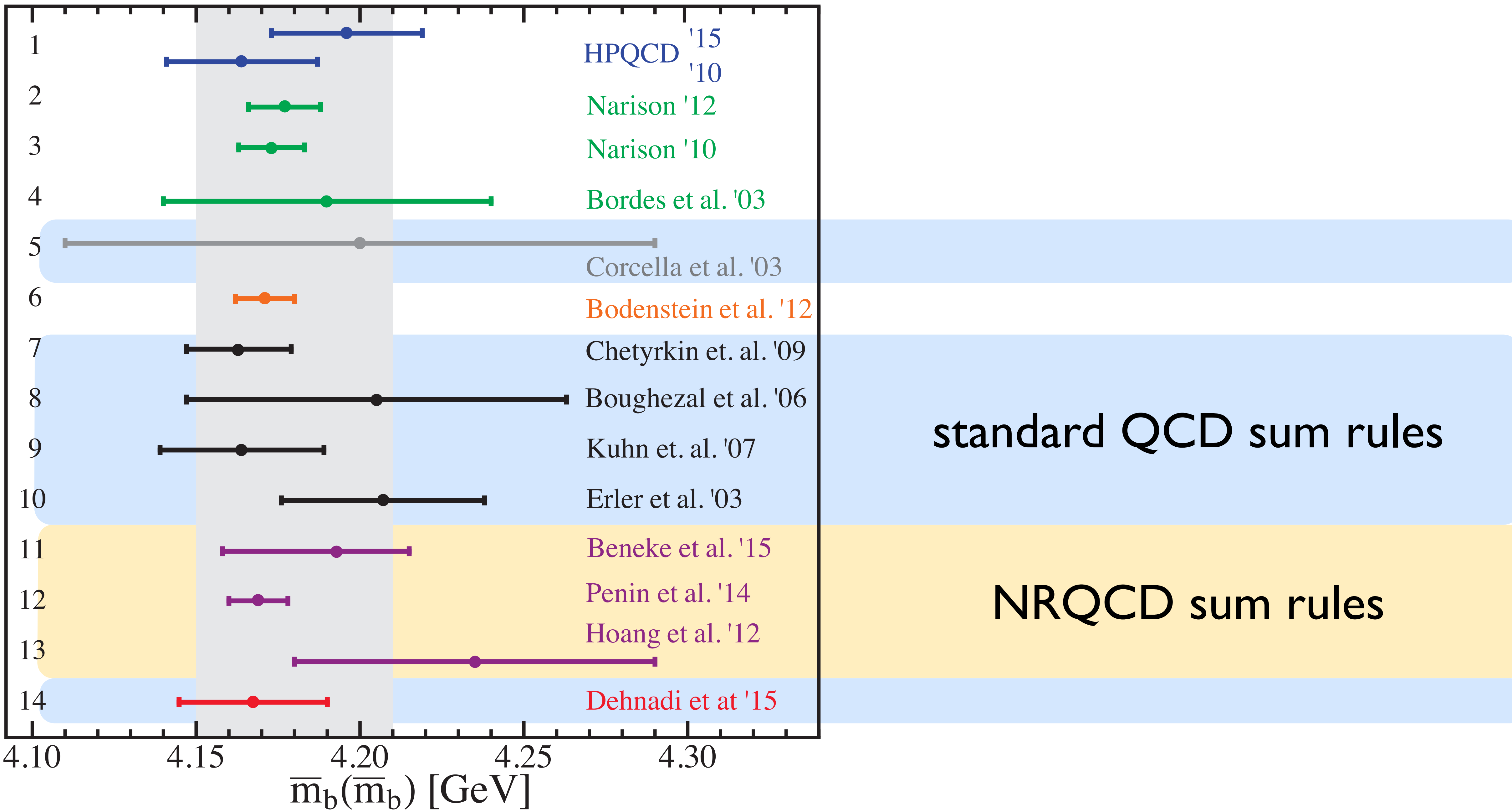


relativistic sum rules give the most precise determinations

Bottom mass determinations

Type of sum rule

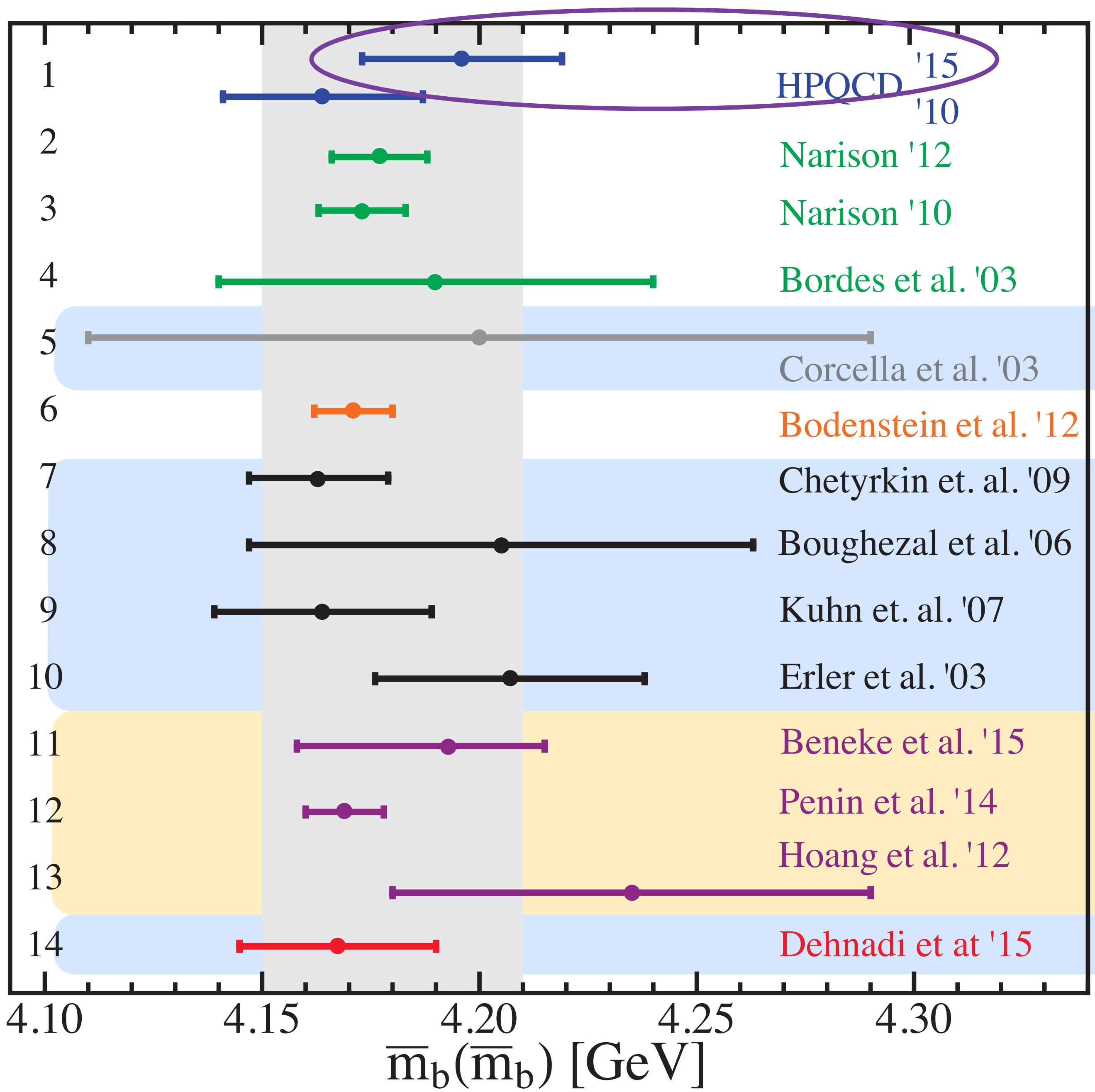
From QCD sum rules



Bottom mass determinations

Type of sum rule

From QCD sum rules



uses NRQCD lattice action, but relativistic pQCD for large-n moments

standard QCD sum rules

NRQCD sum rules

Bottom mass determinations

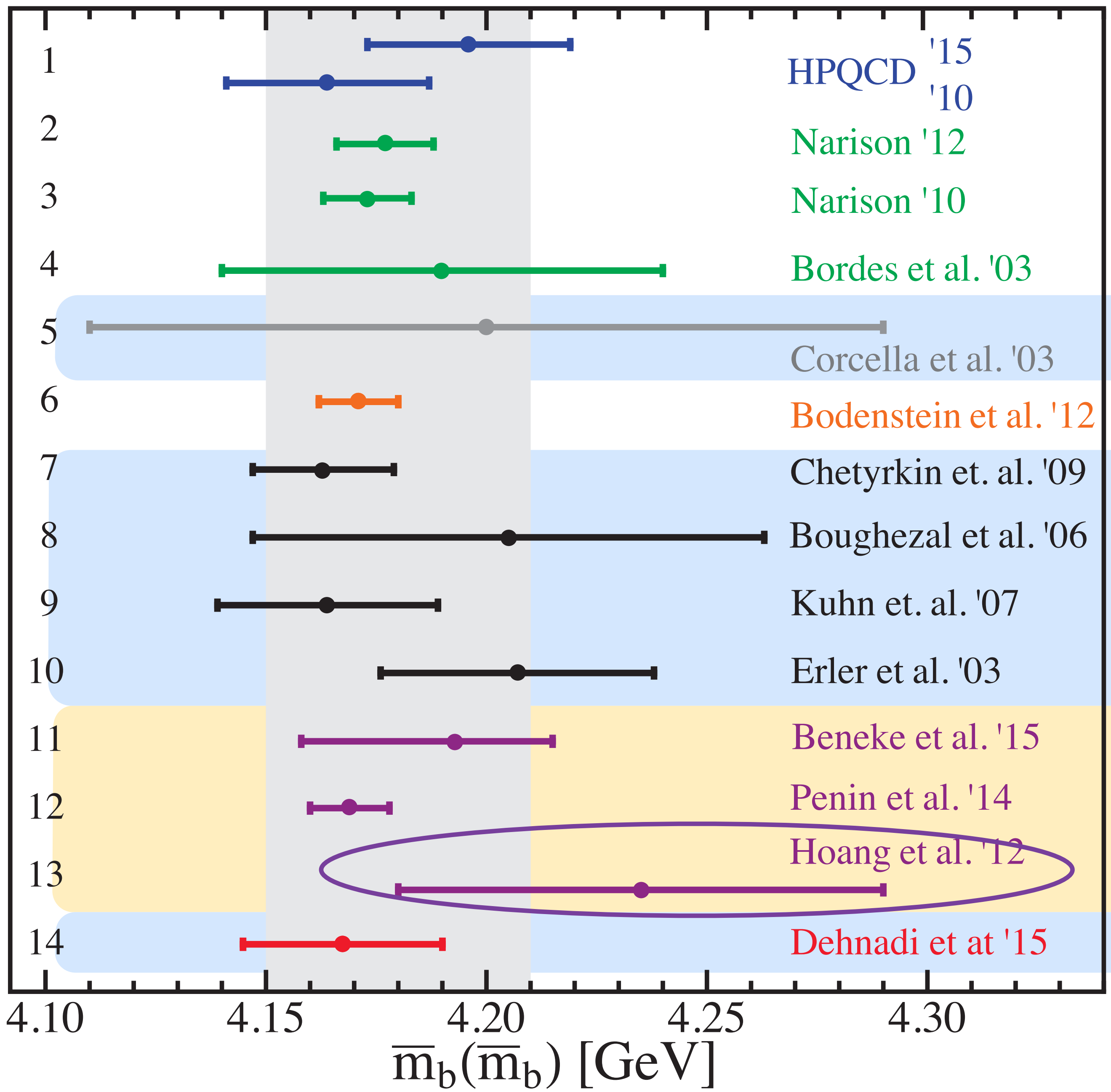
Type of sum rule

uses vNRQCD to sum up Coulomb and log singularities

standard QCD sum rules

NRQCD sum rules

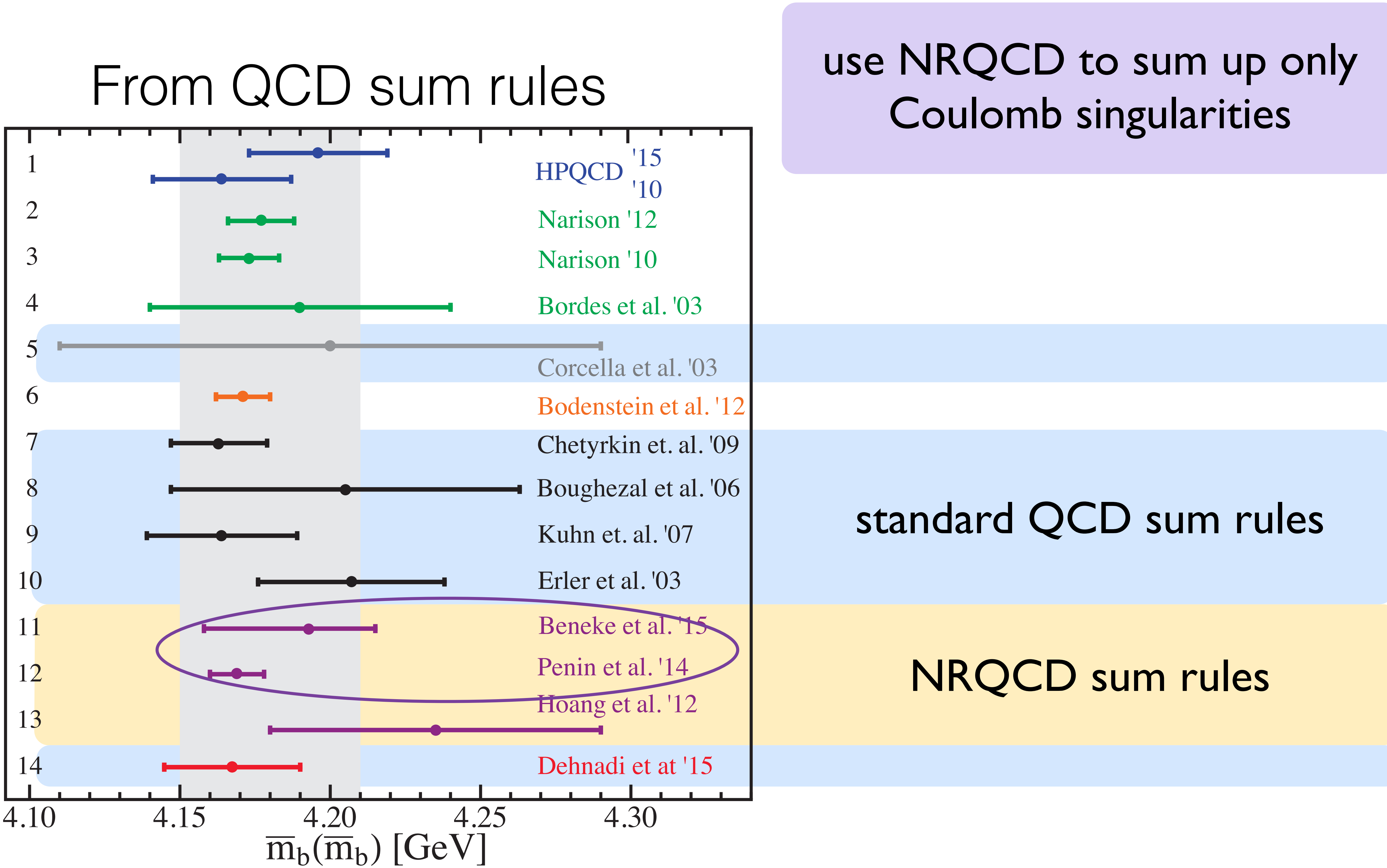
From QCD sum rules



Bottom mass determinations

Type of sum rule

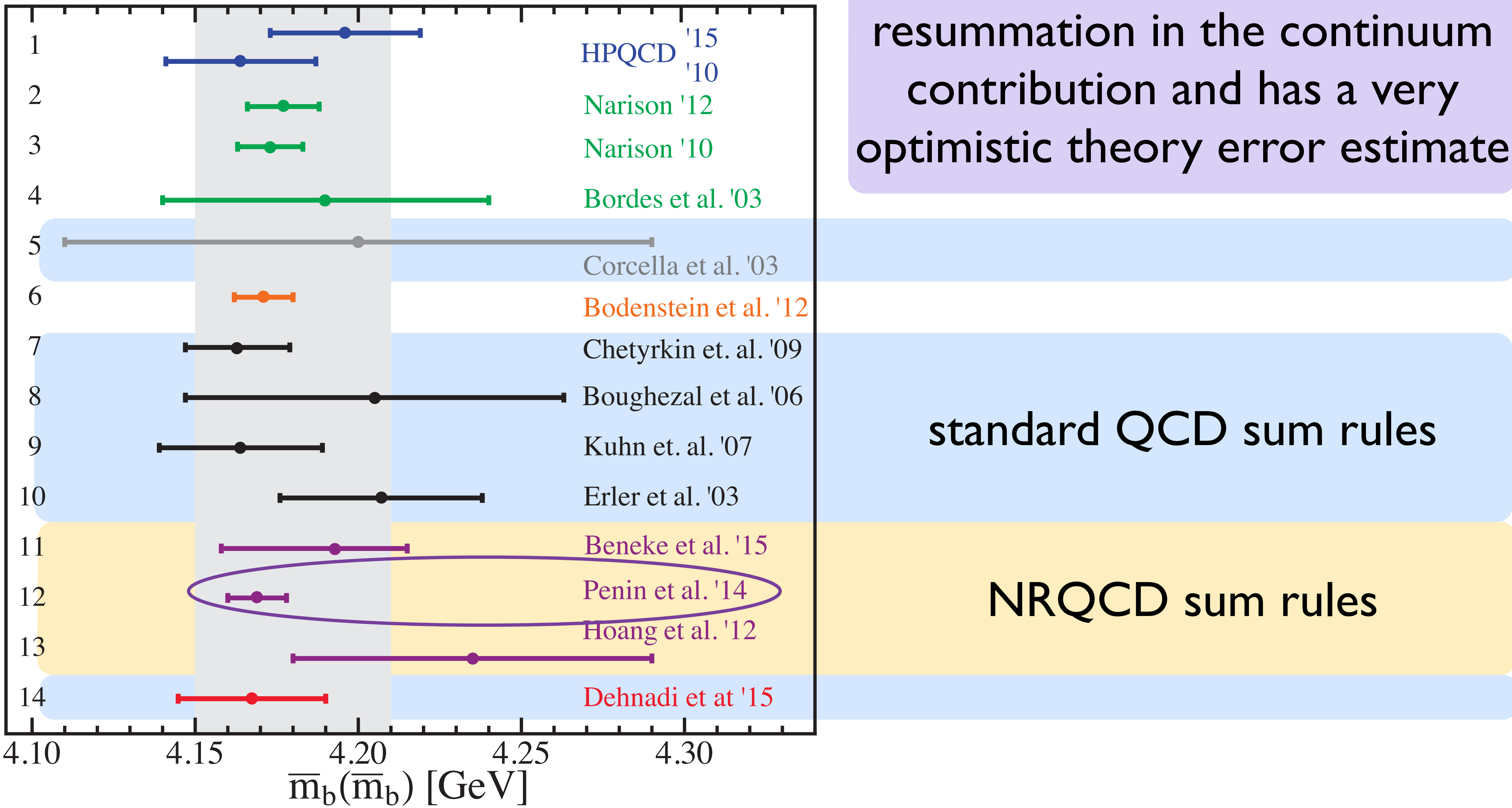
From QCD sum rules



Bottom mass determinations

Type of sum rule

From QCD sum rules



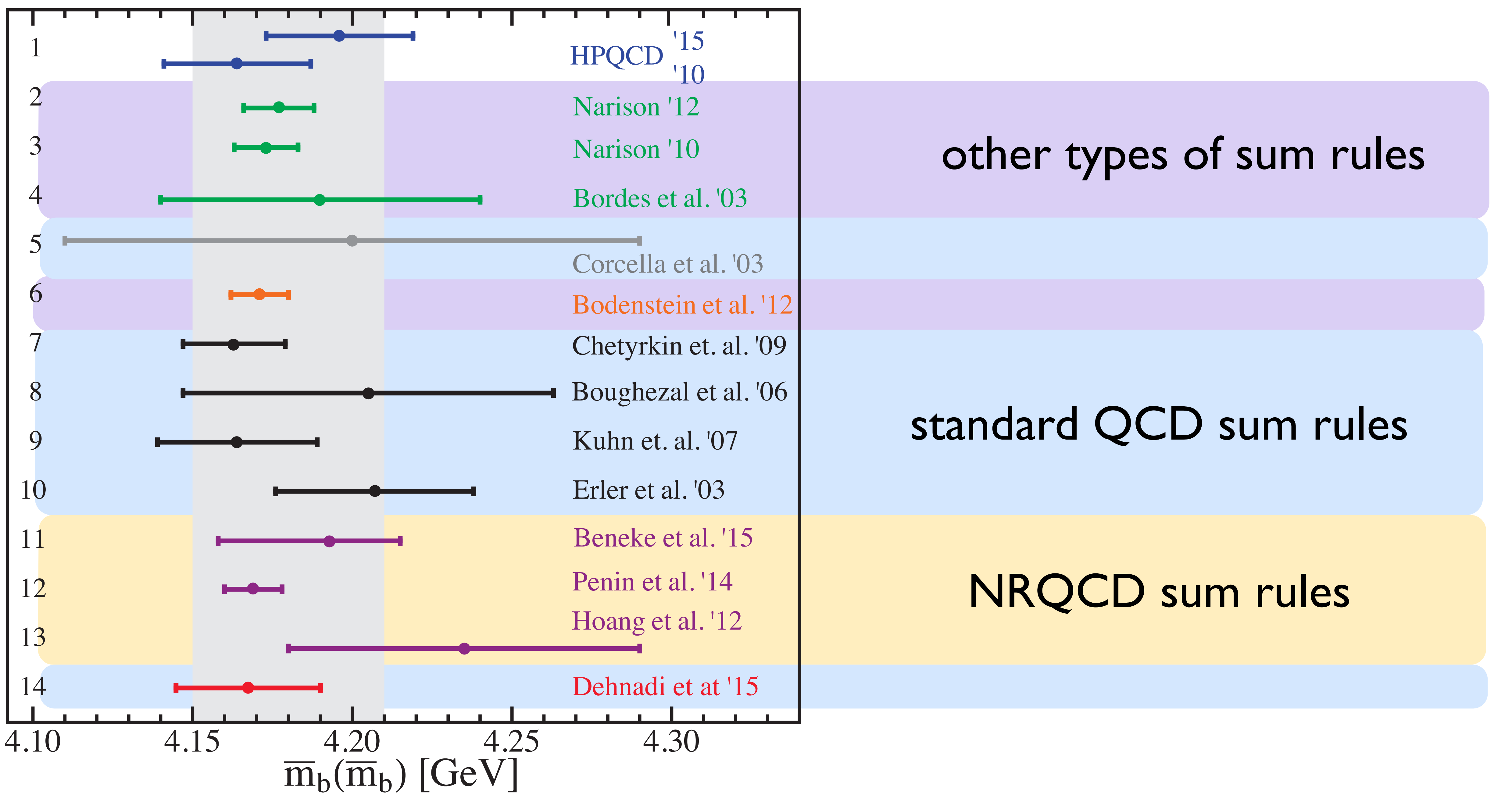
incomplete perturbative information, misses resummation in the continuum contribution and has a very optimistic theory error estimate

Bottom mass determinations

Type of sum rule

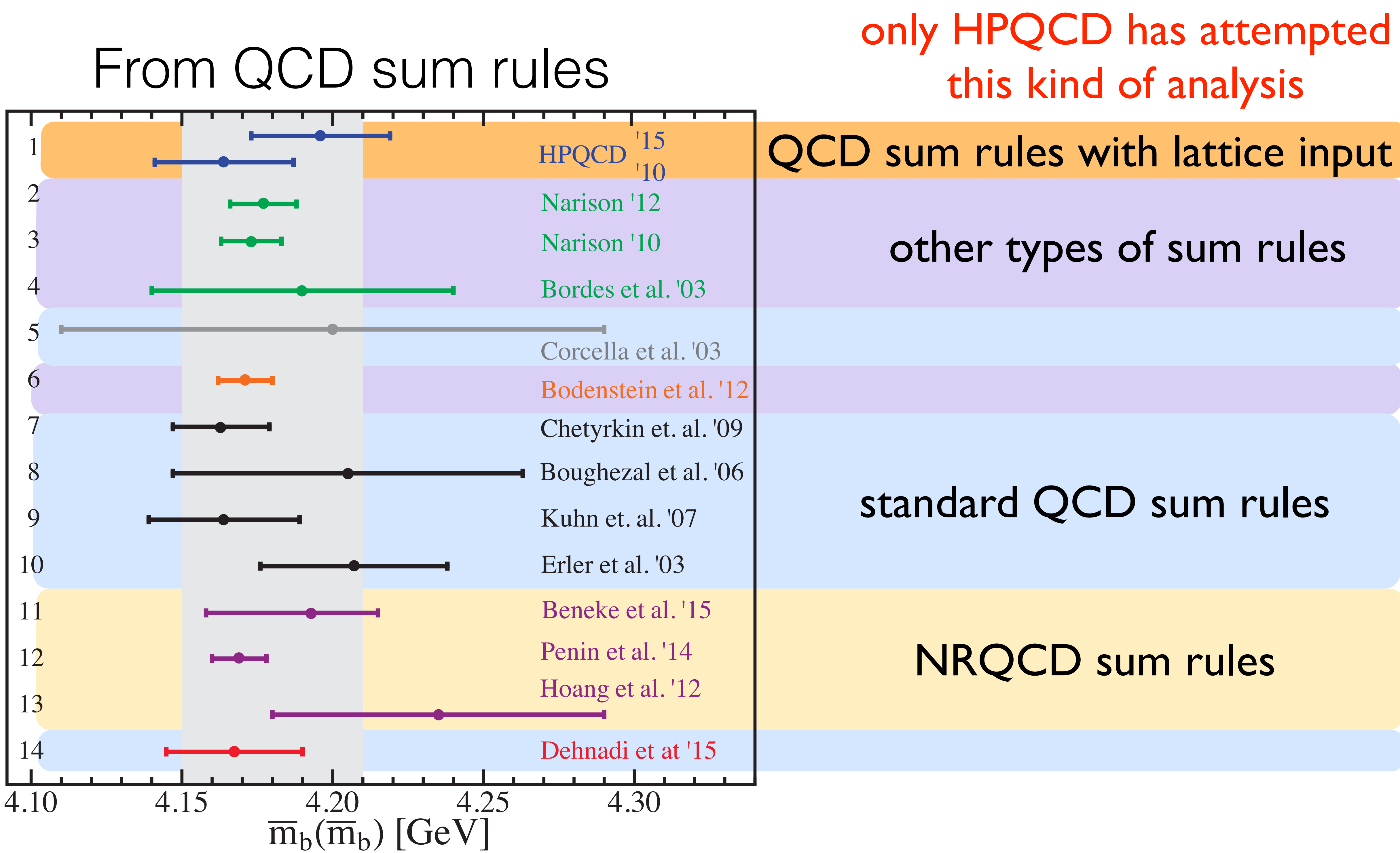
theoretically less sound

From QCD sum rules



Bottom mass determinations

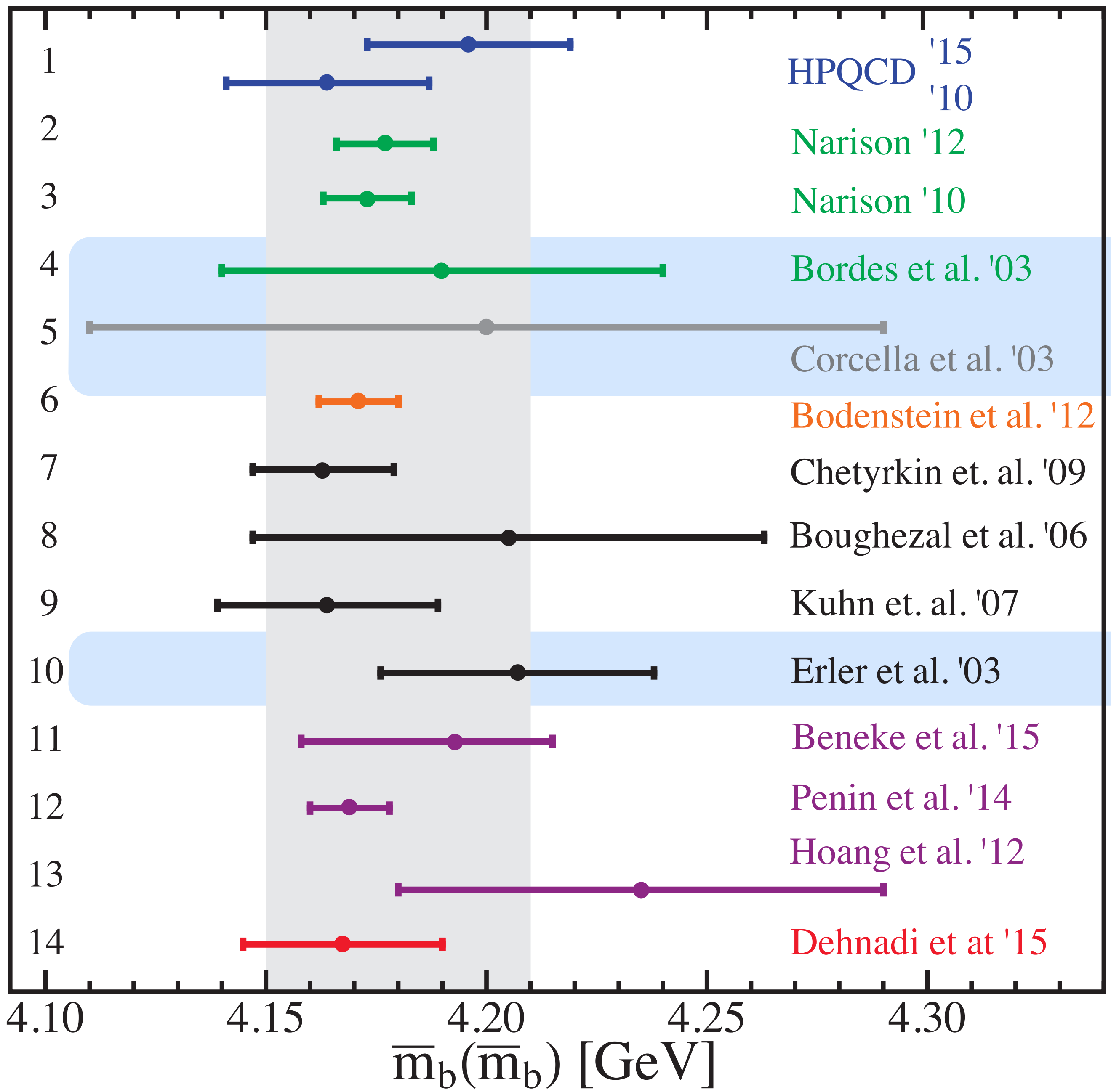
Type of sum rule



Bottom mass determinations

Perturbative input

From QCD sum rules



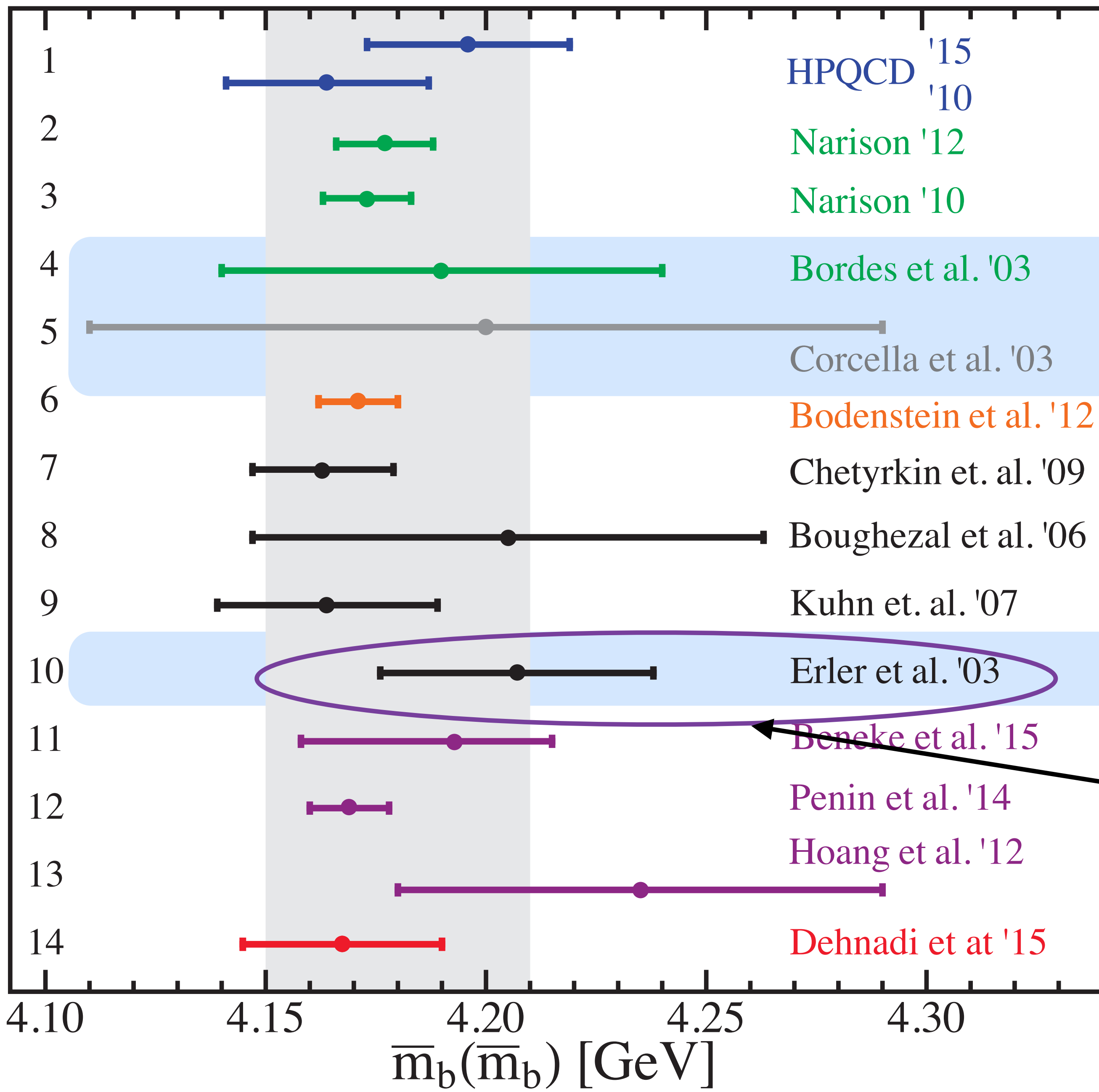
expected large uncertainties

$\mathcal{O}(\alpha_s^2)$ input

Bottom mass determinations

Perturbative input

From QCD sum rules



expected large uncertainties

$\mathcal{O}(\alpha_s^2)$ input

too precise !?!?

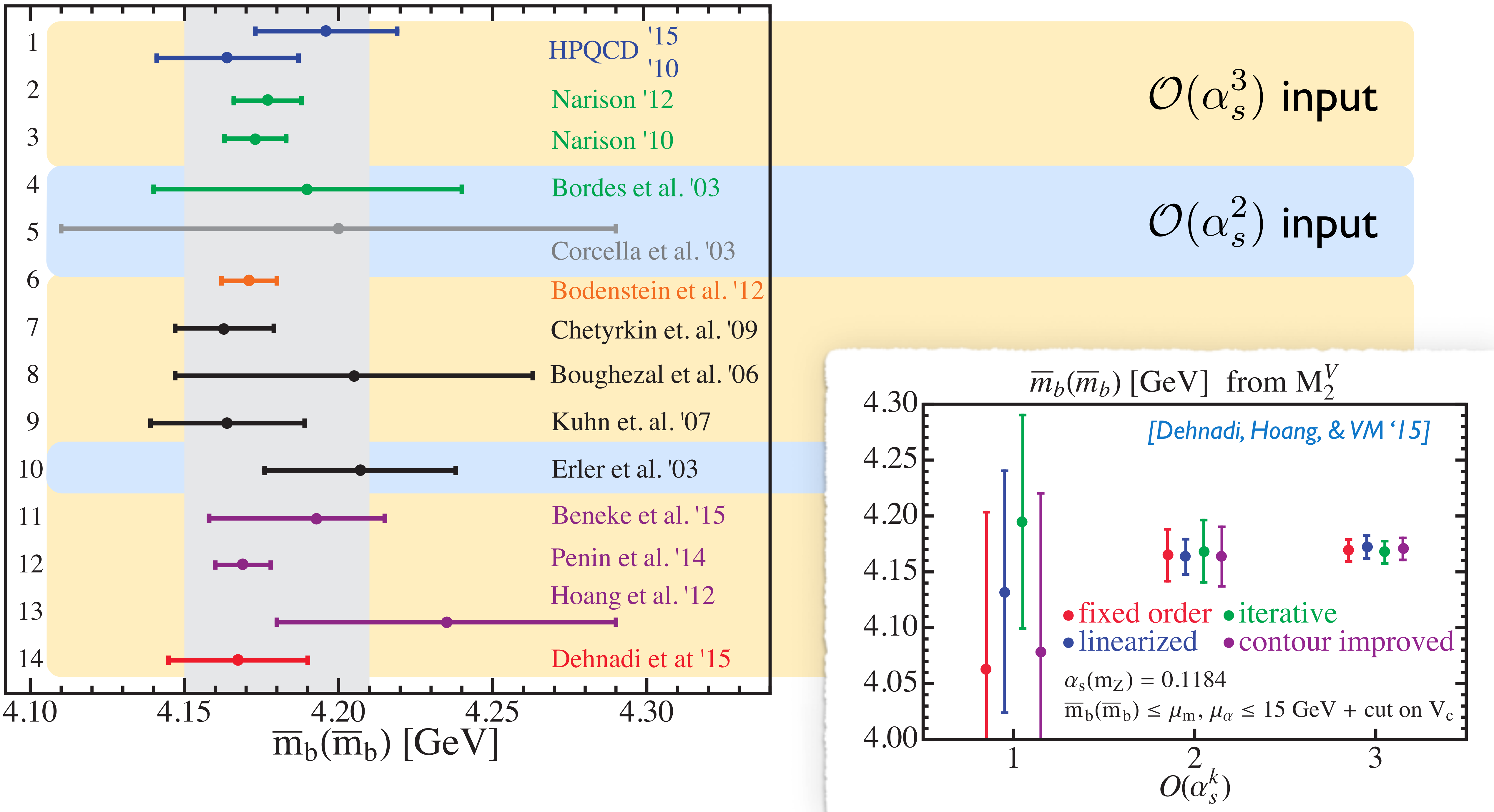
no scale variation, only narrow resonances included + theory model for the rest

Bottom mass determinations

Experimental data used

much smaller uncertainties

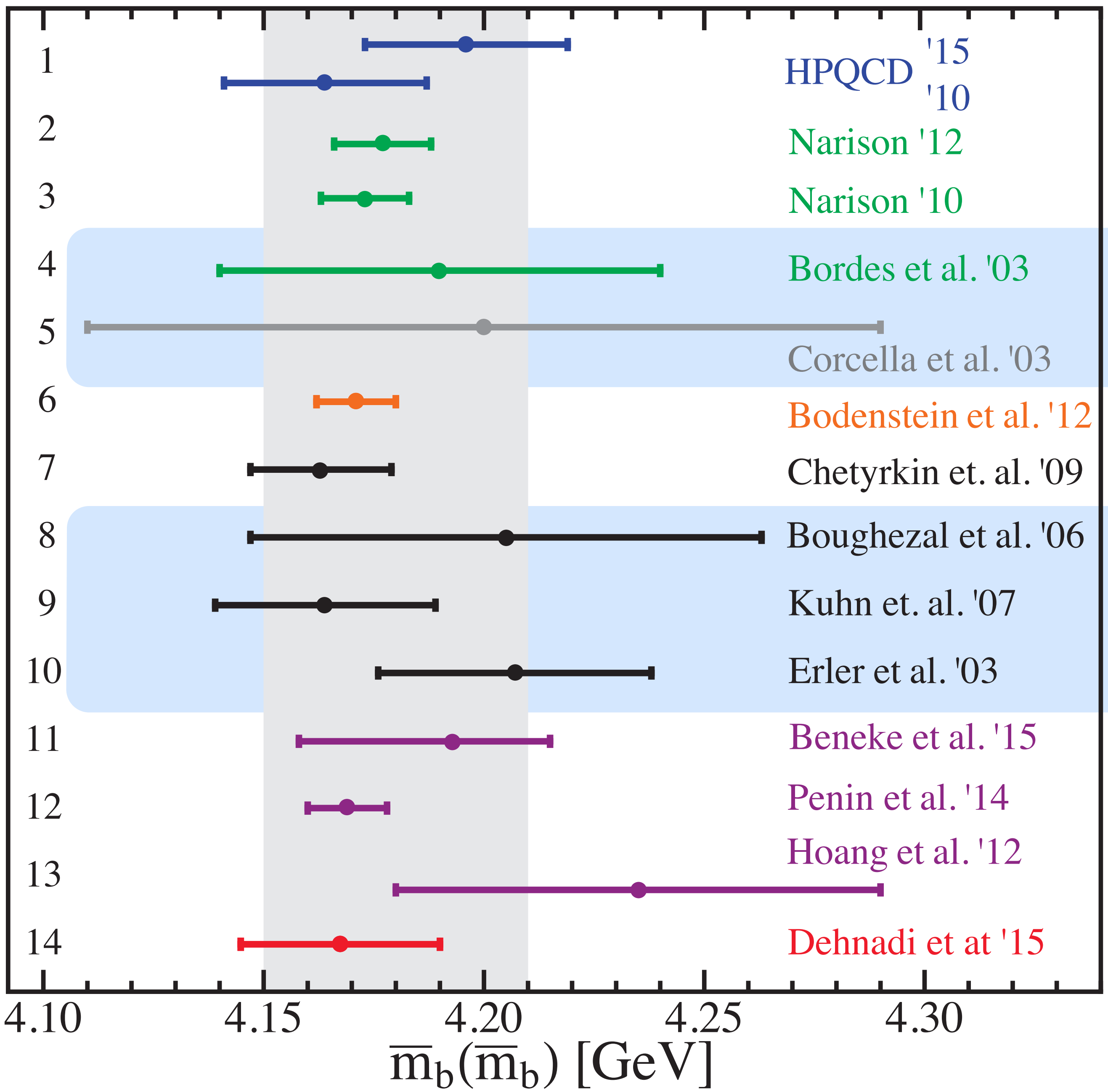
From QCD sum rules



Bottom mass determinations

Experimental data used

From QCD sum rules



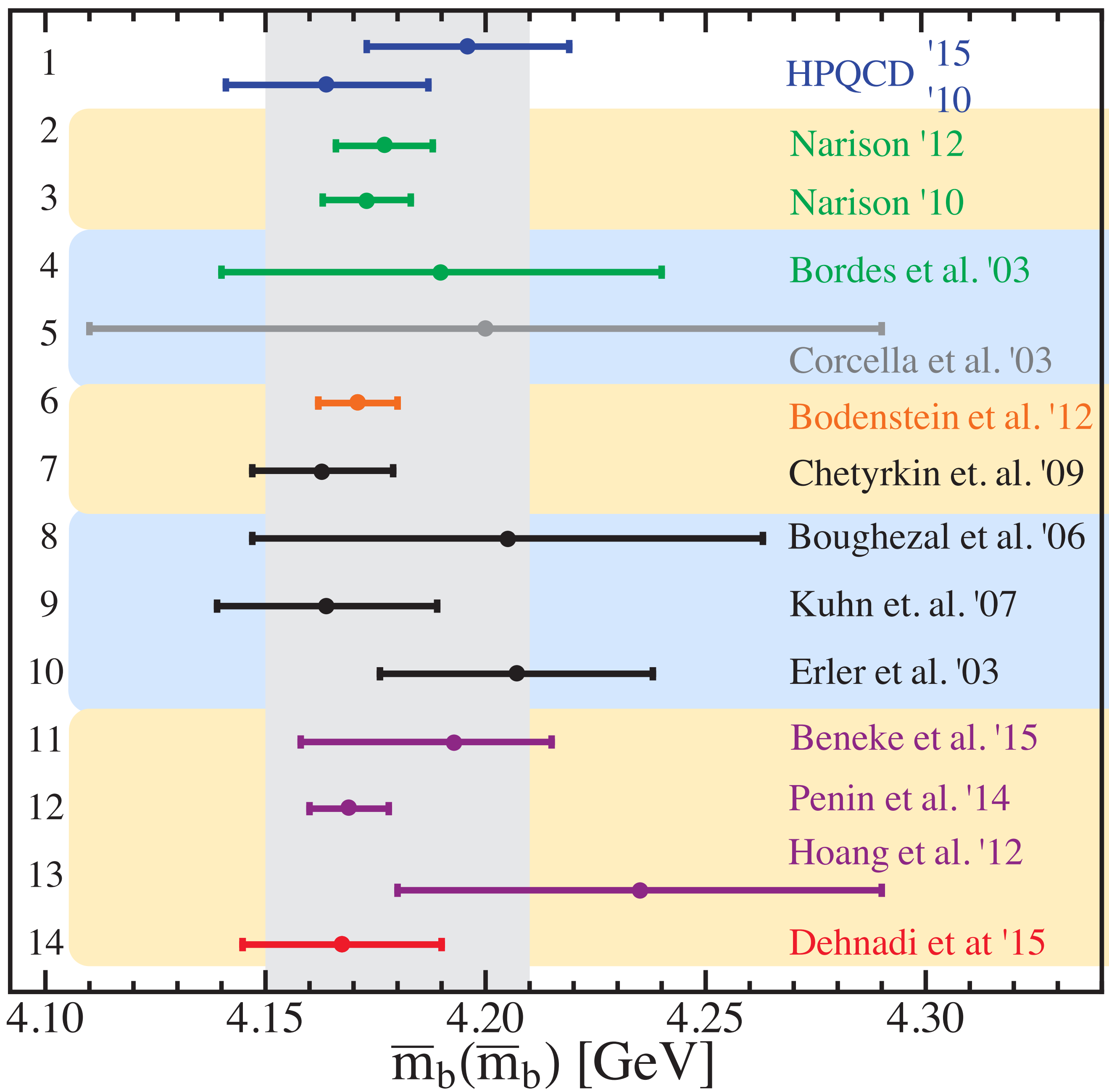
expected large uncertainties

old values for narrow resonances parameters and threshold region

Bottom mass determinations

Strong impact on experimental uncertainties

From QCD sum rules



smaller uncertainties

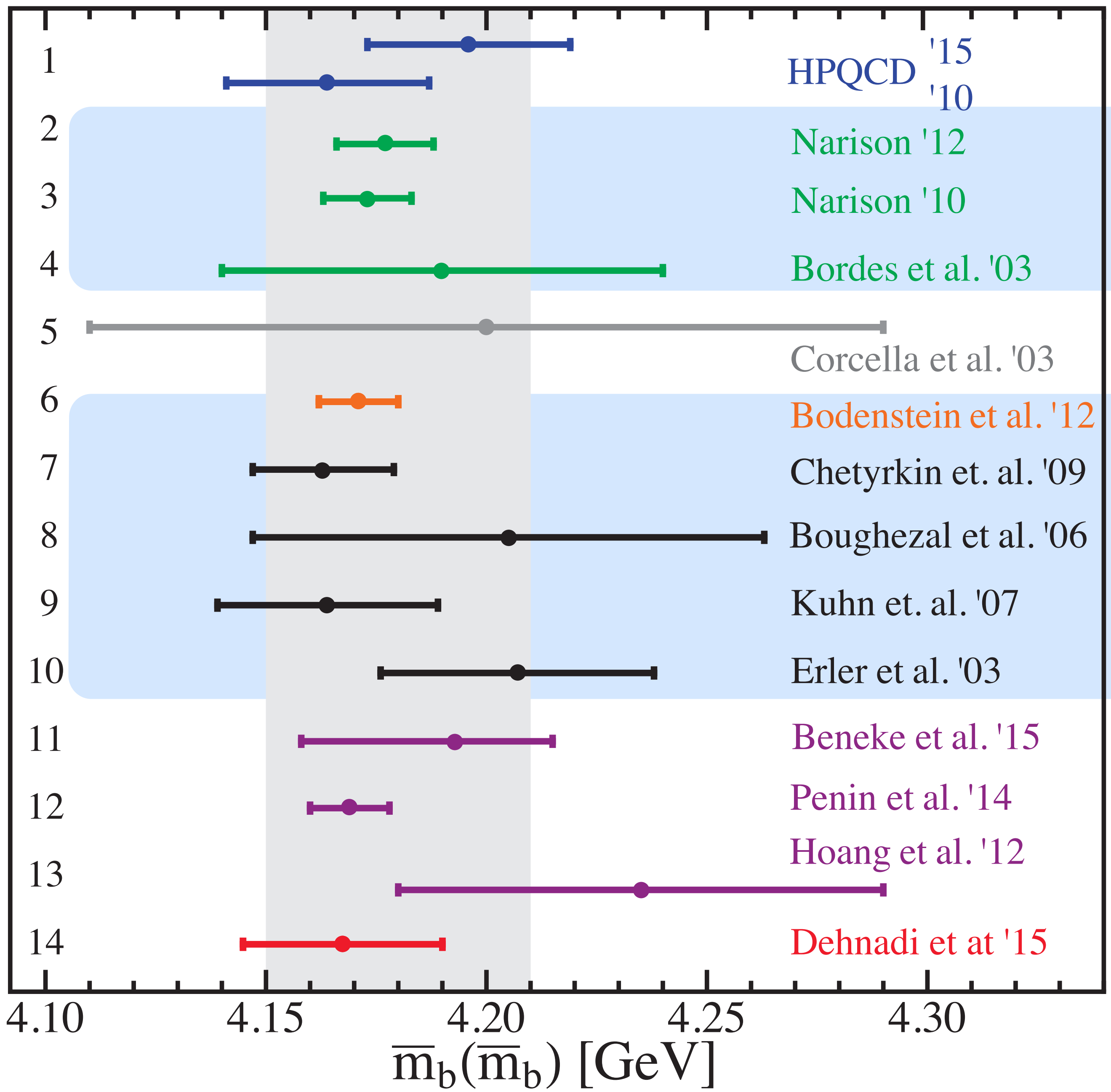
old values for narrow resonances parameters and threshold region

most up to date values + BABAR data

Bottom mass determinations

Treatment of continuum

From QCD sum rules



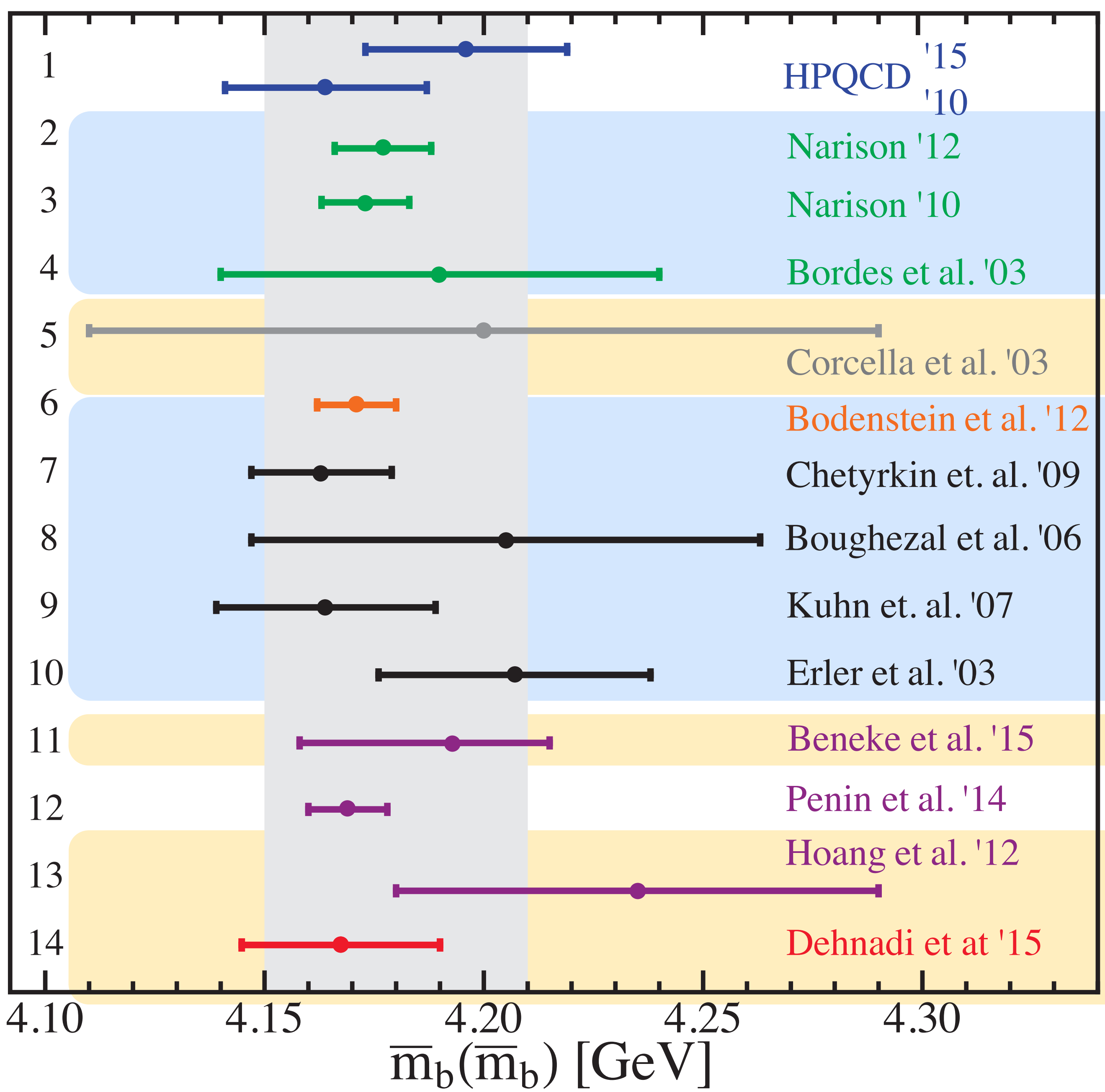
underestimate the error due to modeling

use pQCD with perturbative uncertainties to model region with no data

Bottom mass determinations

Treatment of continuum

From QCD sum rules



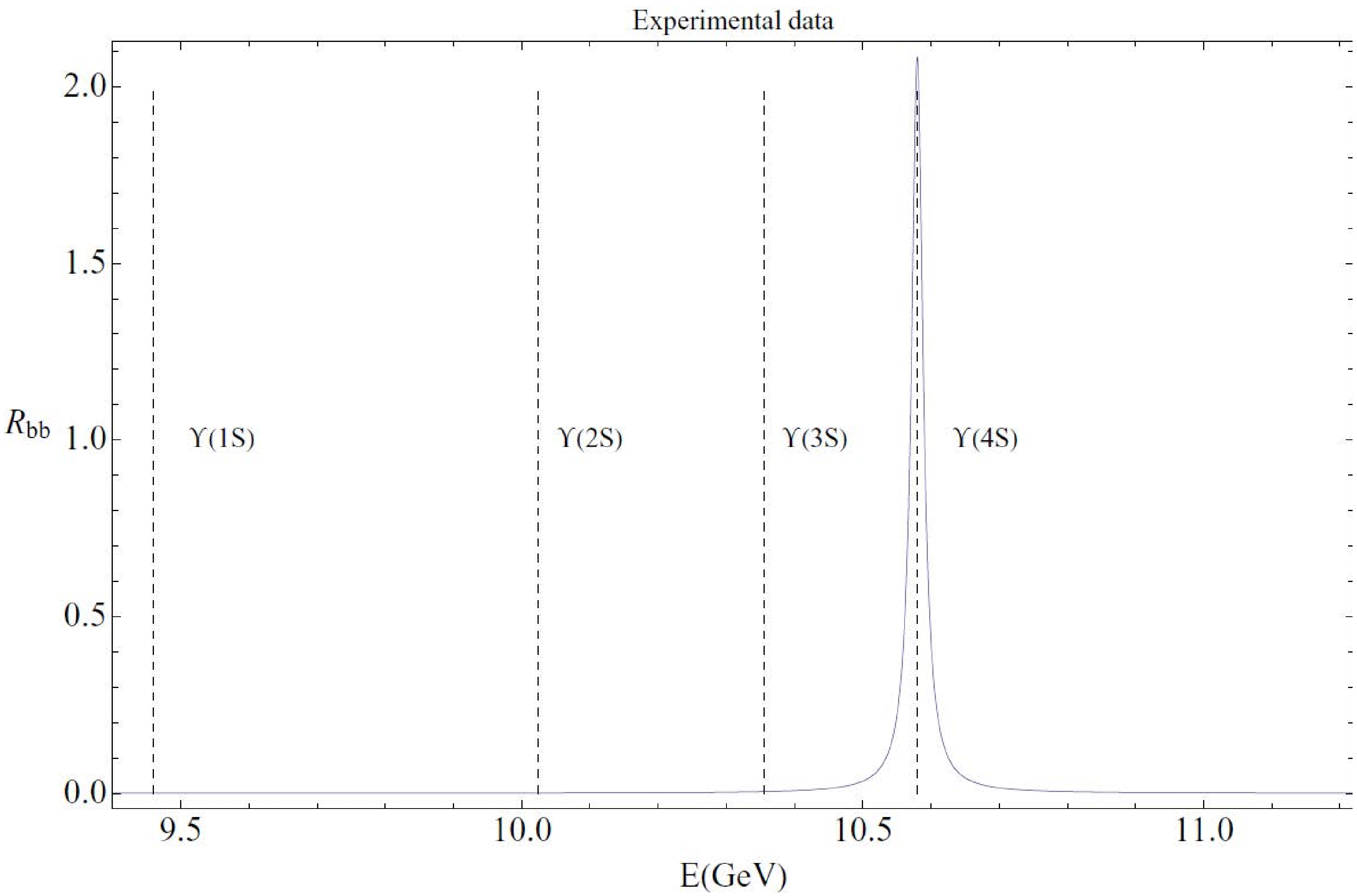
more realistic uncertainties

use pQCD with perturbative uncertainties to model region with no data

use pQCD with 4% systematic uncertainty

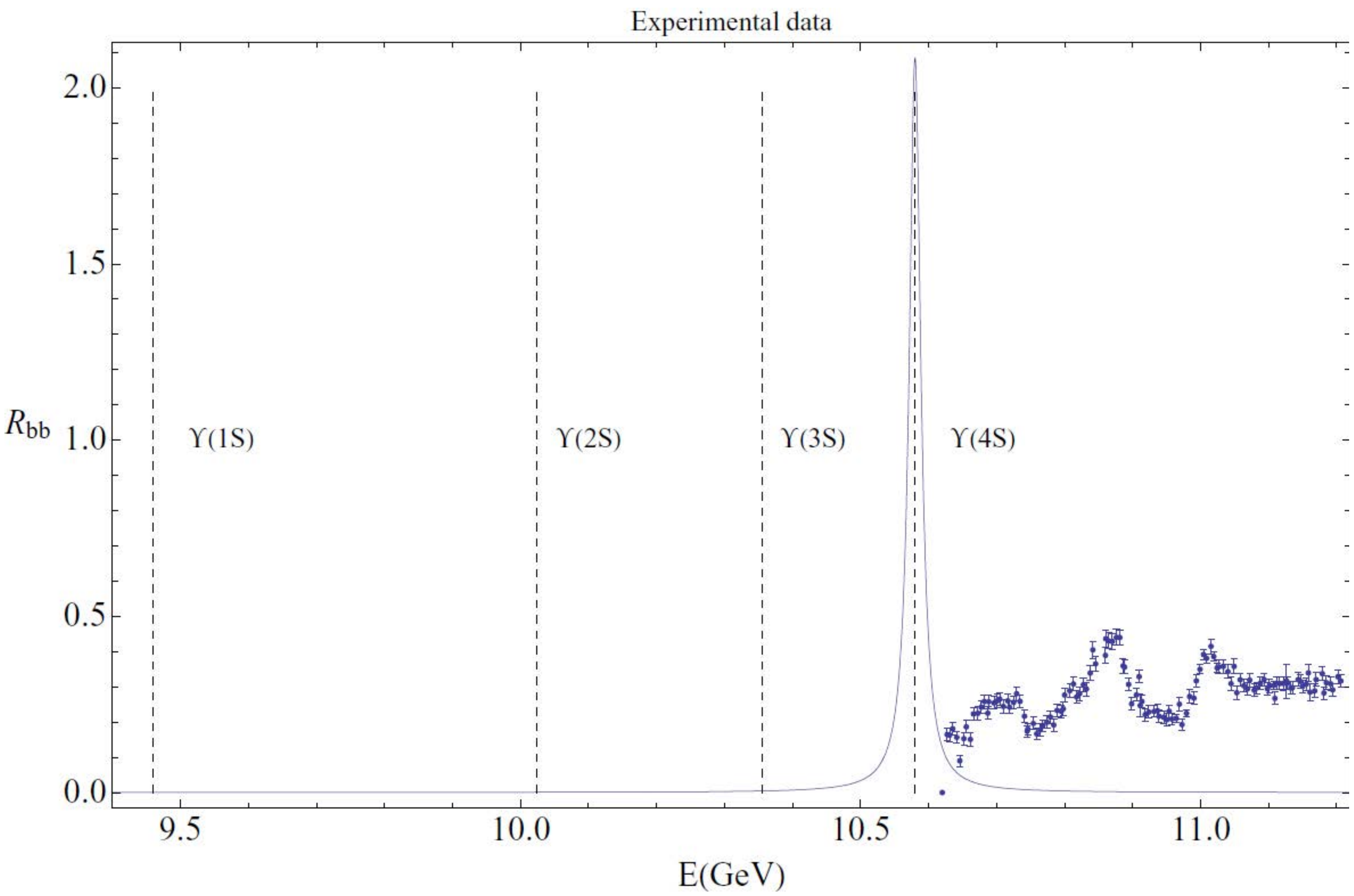
Experimental data: bottom

Narrow resonances



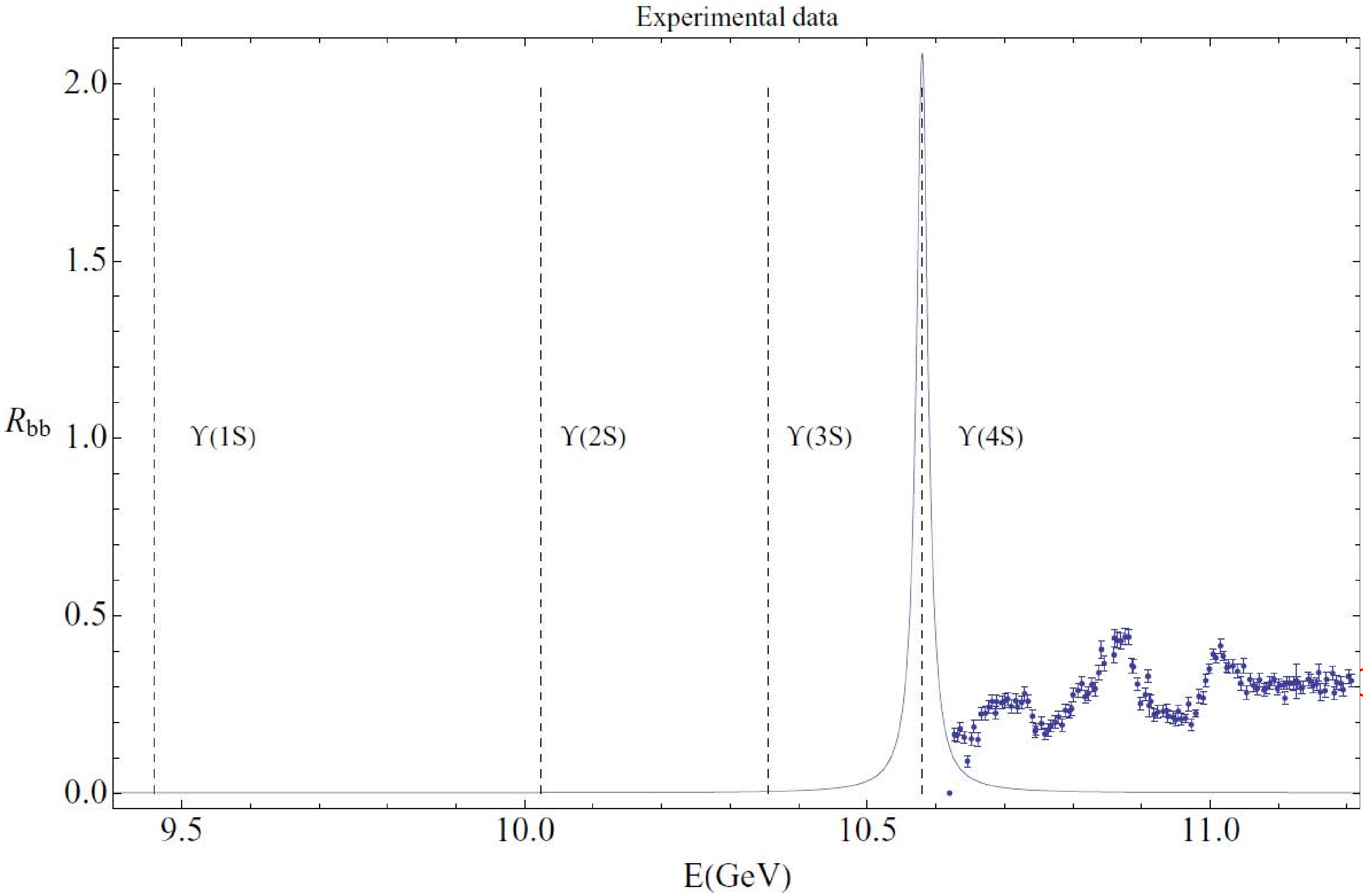
Experimental data: bottom

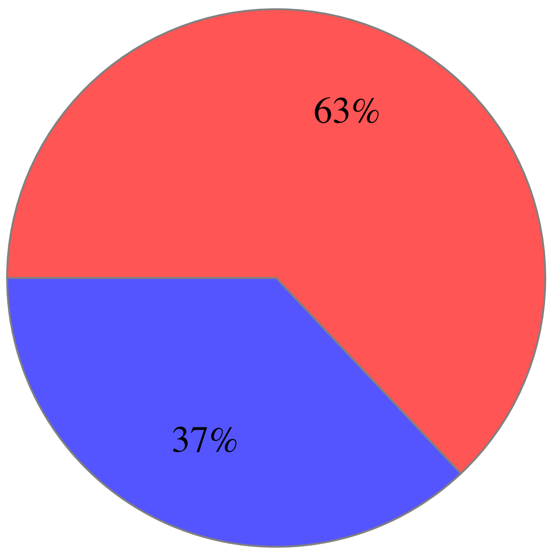
Babar data



Experimental data: bottom

Perturbation theory





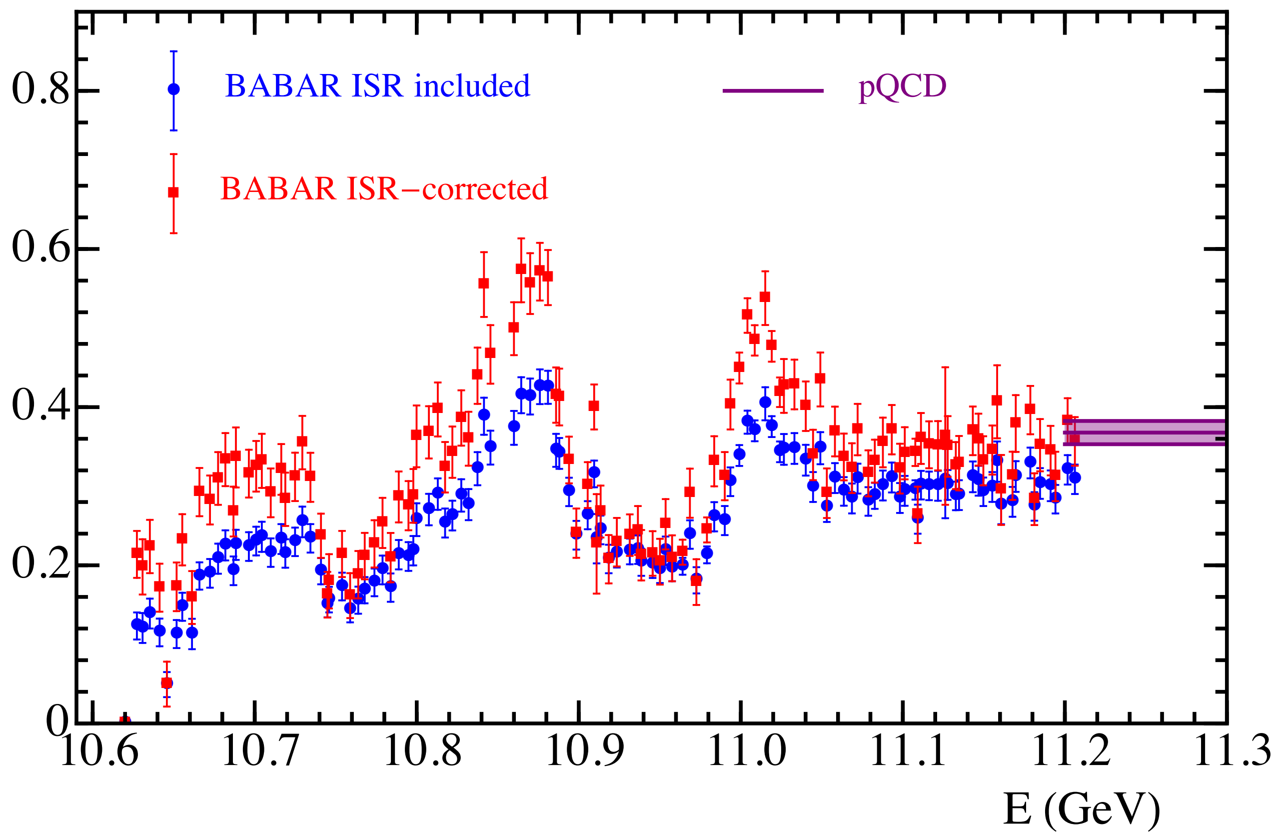
Perturbative QCD

Aren't we comparing theory to theory?

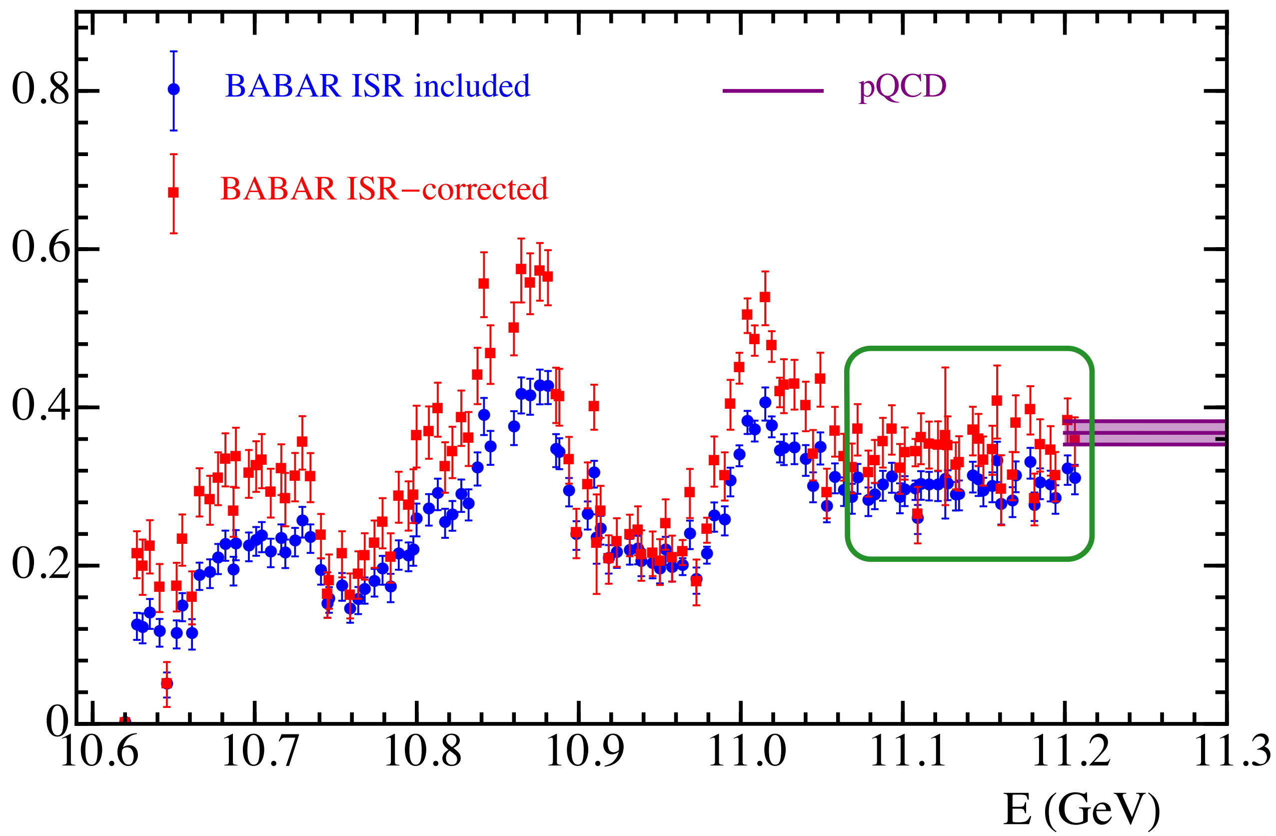
4% error gives a huge uncertainty to the first moment !!

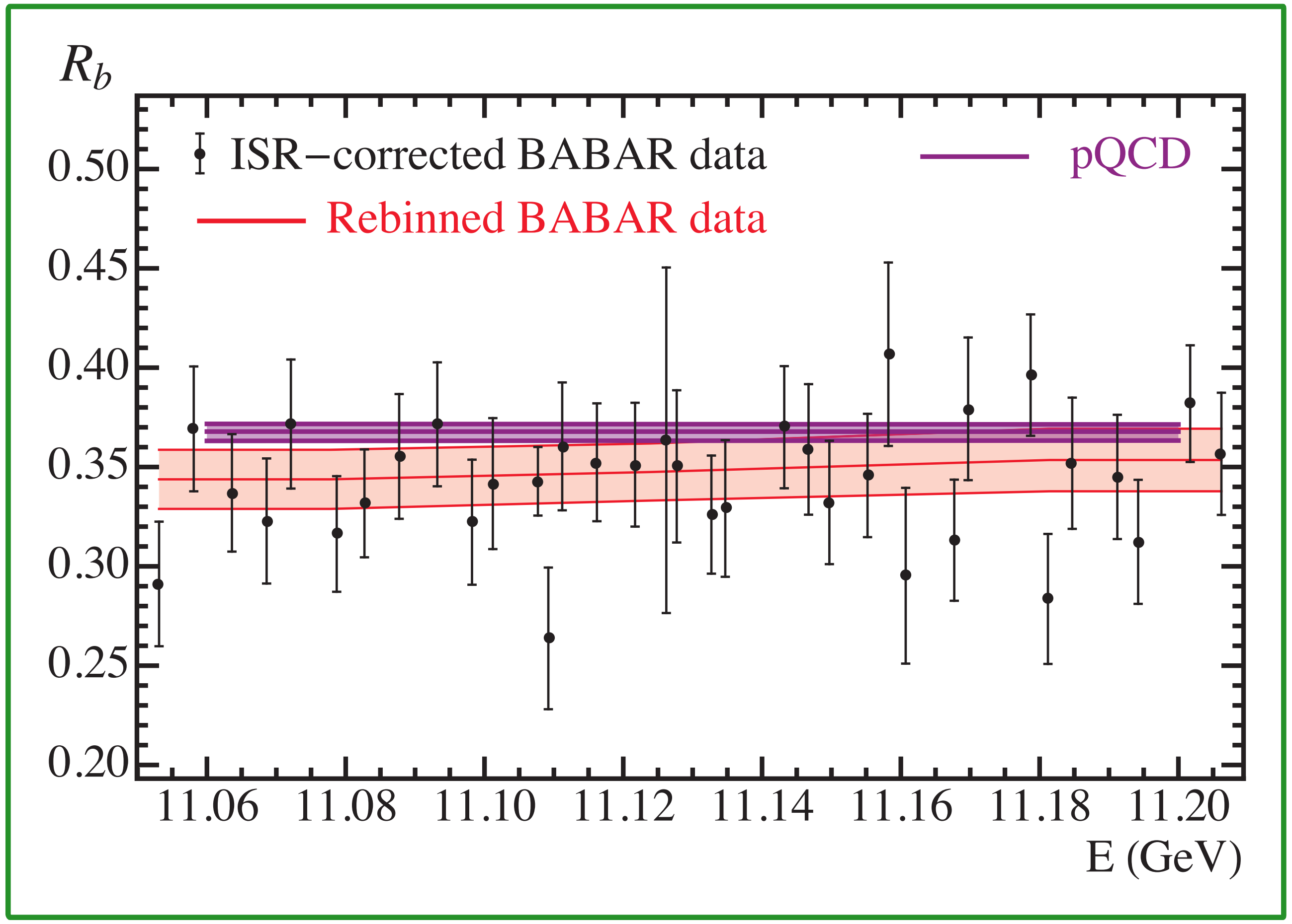
63% of the first moment from region without data !

High energy region



High energy region



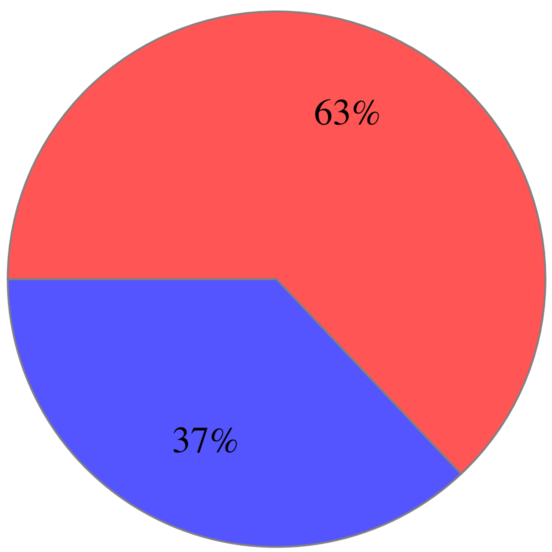


Discrepancy: (rebinned) data vs theory: 4%

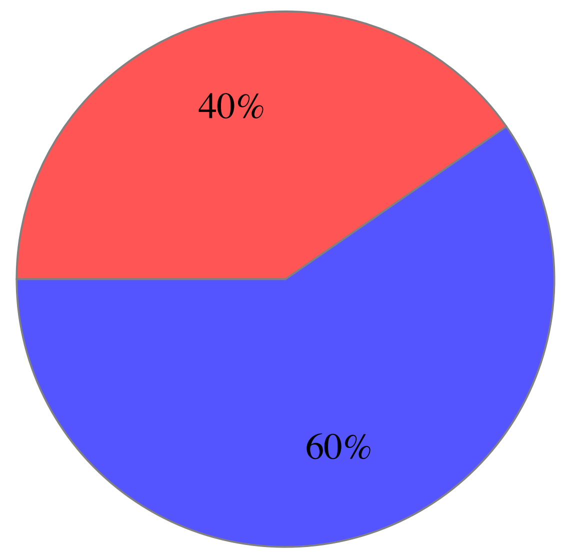
- Conservative continuum model: $R_b^{\rm model} = R_b^{\rm theory} \pm 4\%$
- Size of systematic error in rebinned data

High energy region contribution

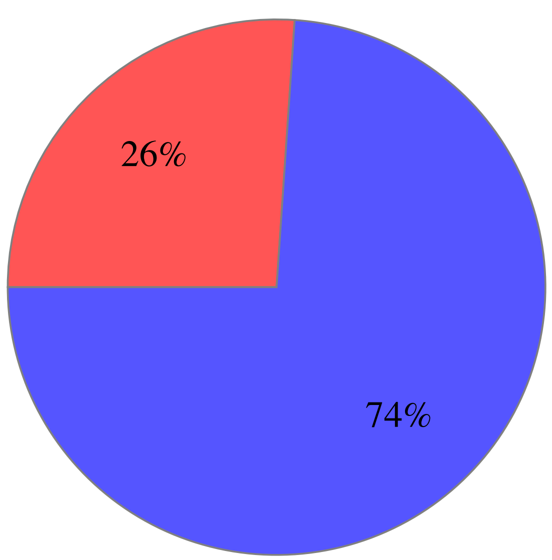
n = 1



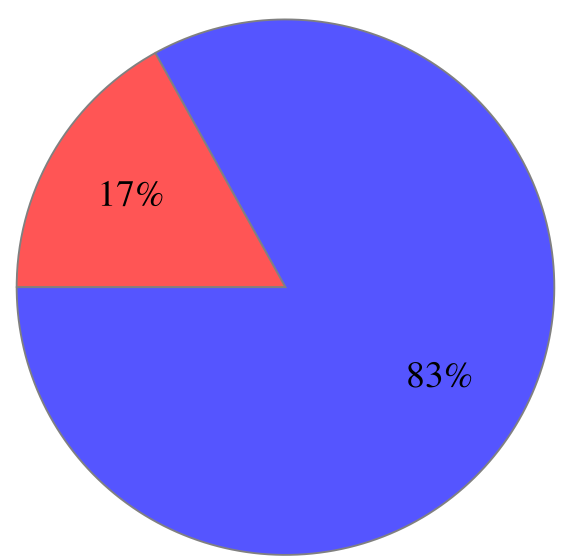
n = 2



n = 3



n = 4



Situation is less dramatic for higher moments

For n > 2 we find issues with perturbation theory

Therefore we use the 2nd moments as our default

High-energy region contributes “only” 39% of total error if 4% error assigned to theory

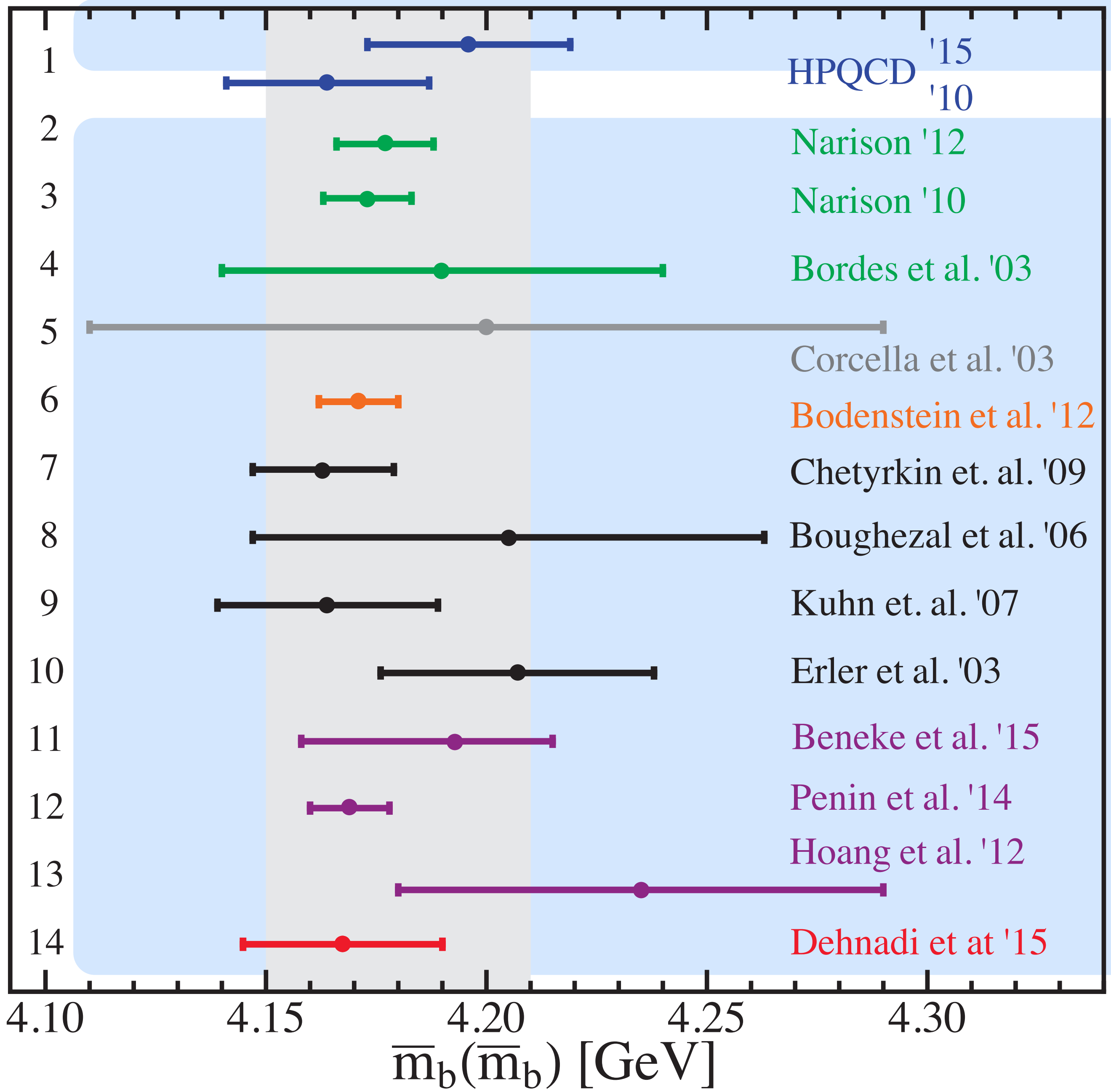
New experimental data in high-energy region: dramatic impact to precision!

Bottom mass determinations

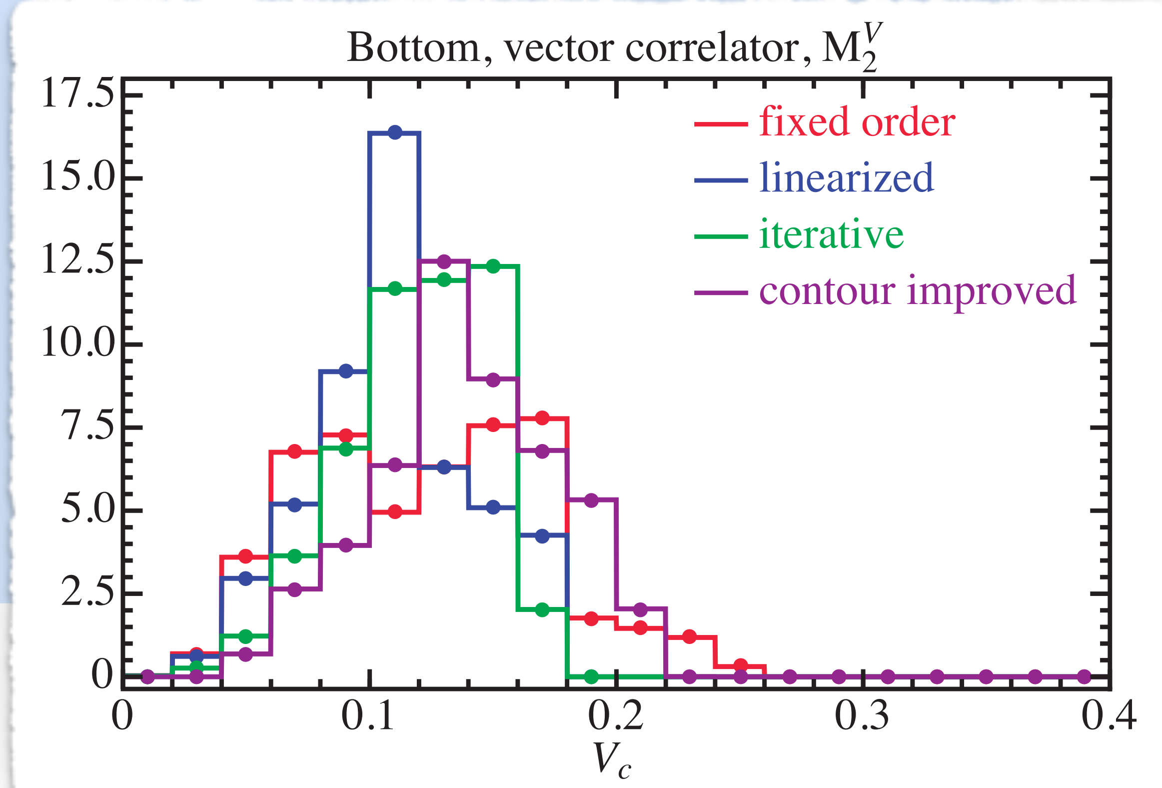
Type of QCD current

good convergence

From QCD sum rules



vector correlator

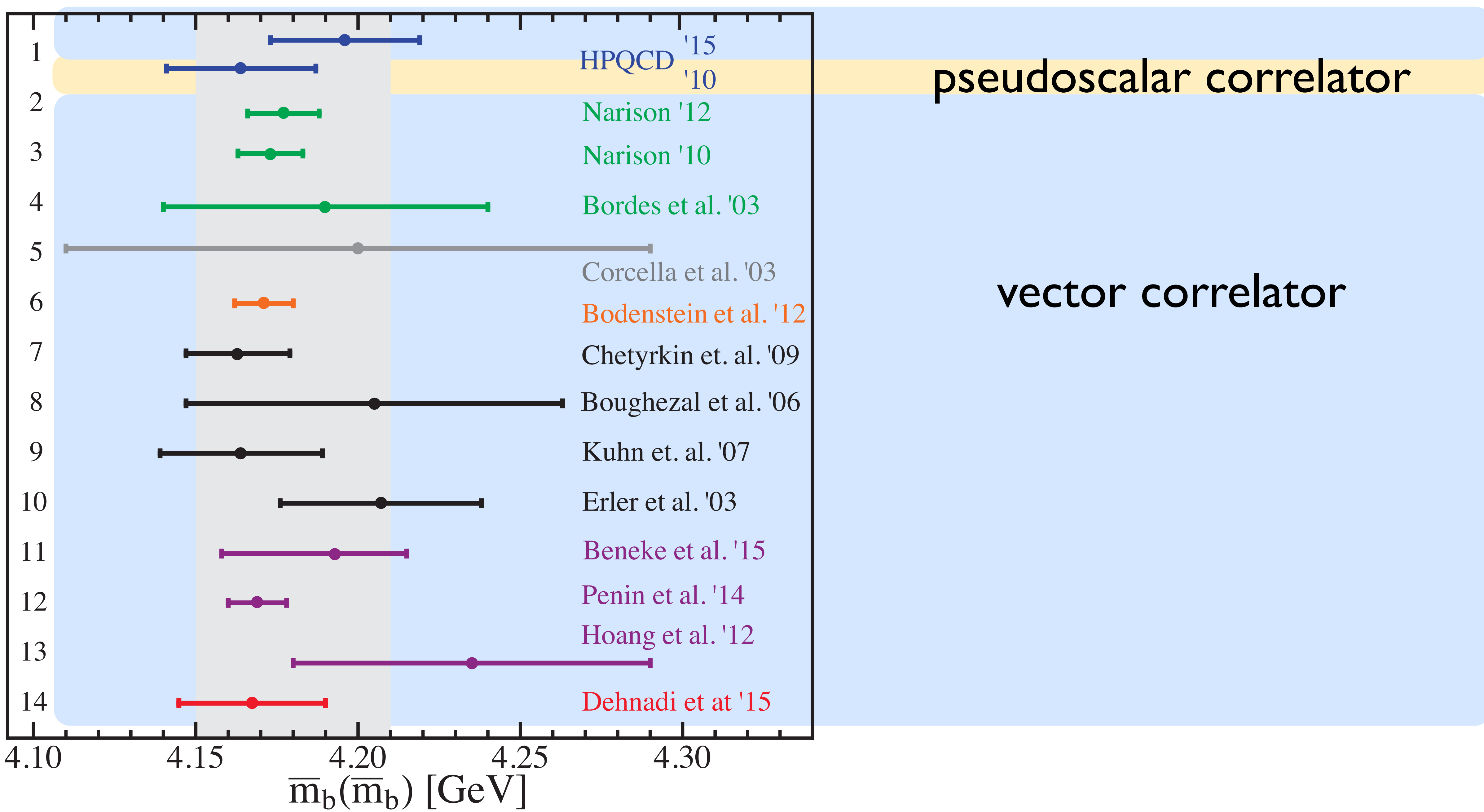


Bottom mass determinations

Type of QCD current

From QCD sum rules

not so good convergence

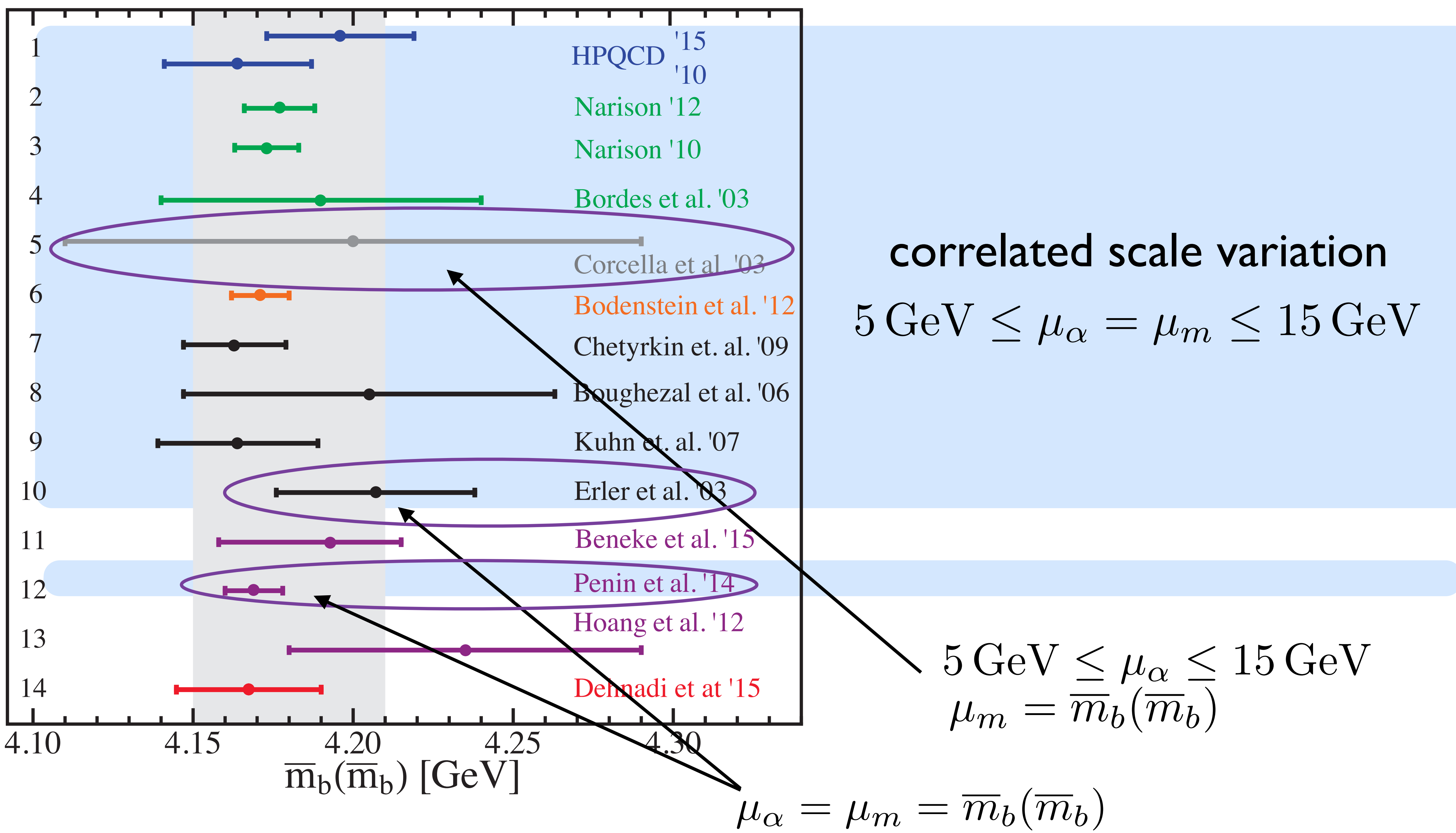


Bottom mass determinations

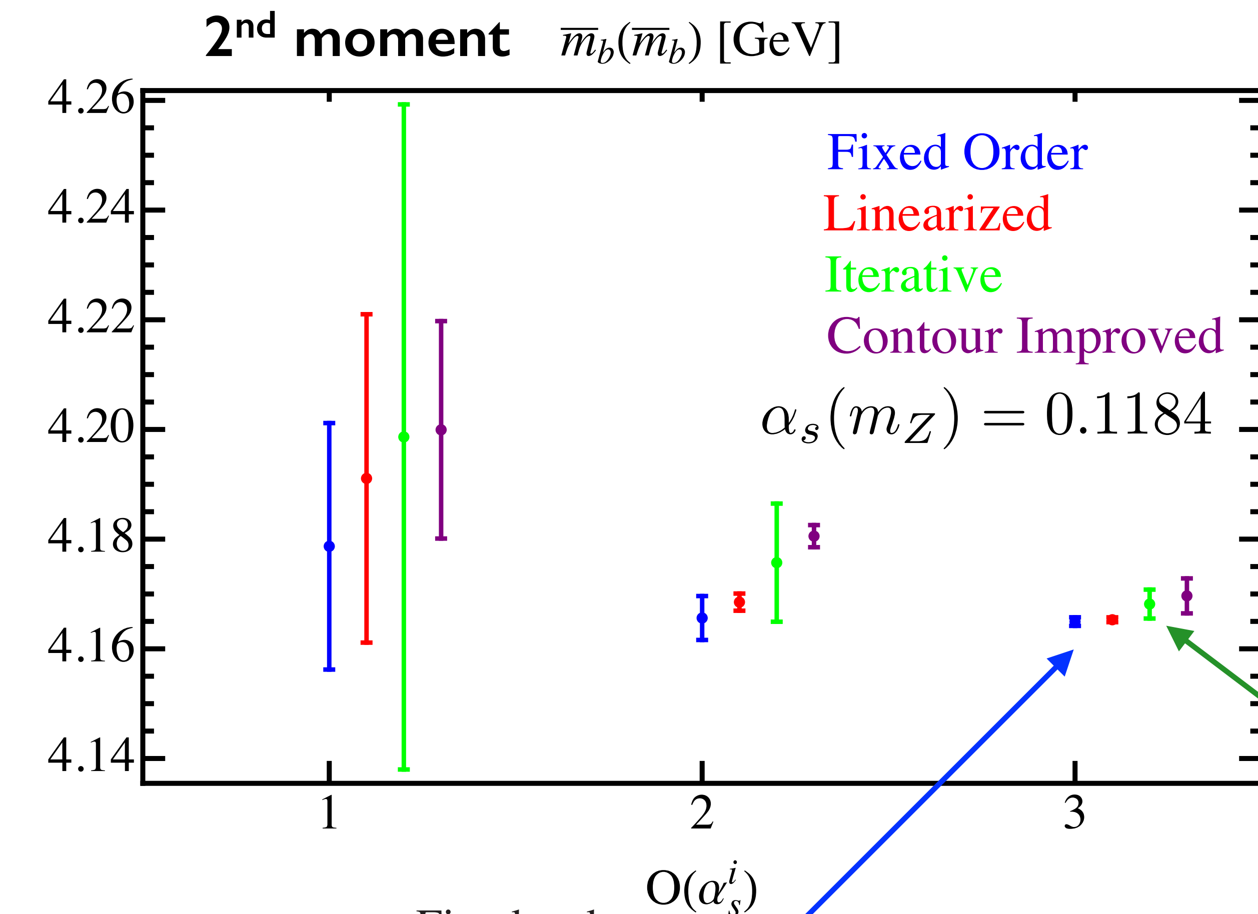
Estimate of perturbative uncertainties

inconsistent results for different methods and orders

From QCD sum rules

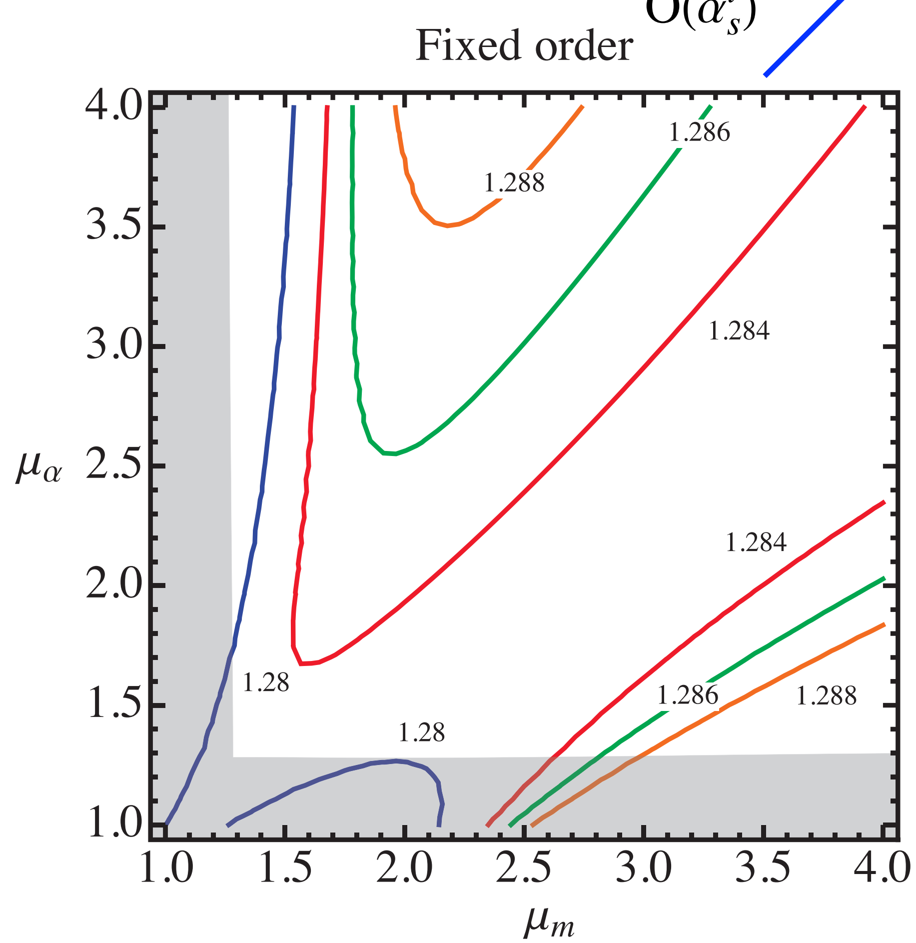


Exploration of scale variation

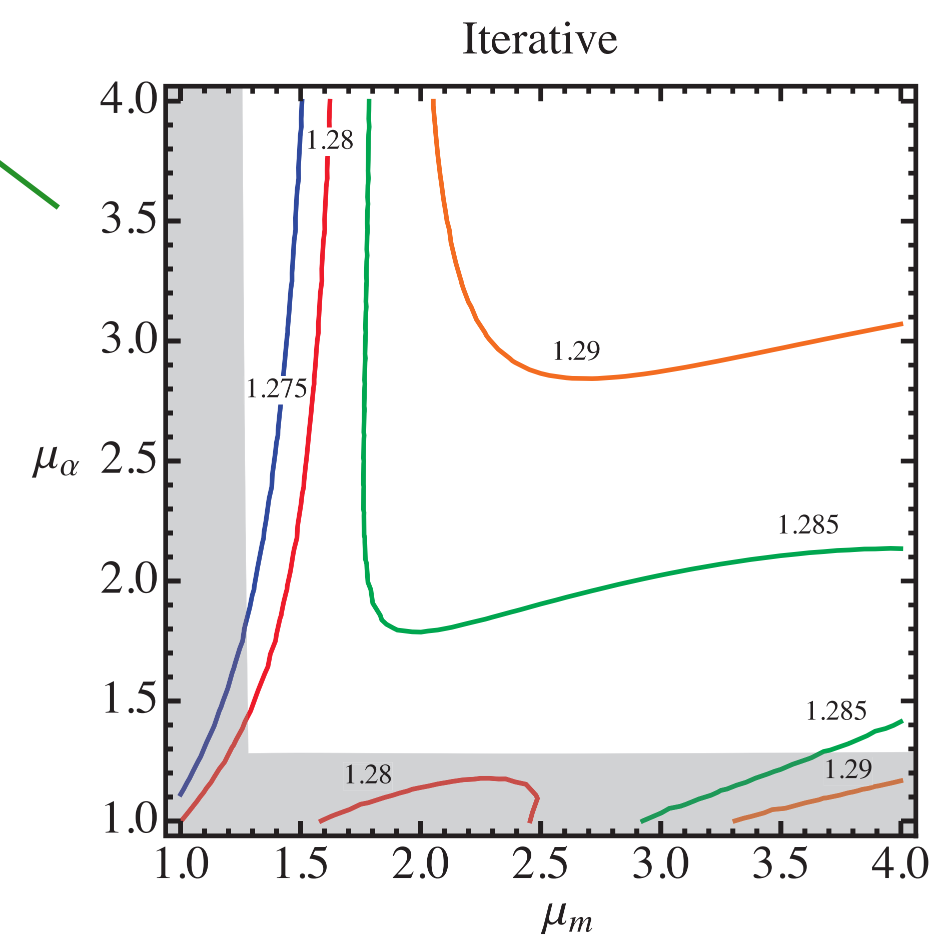


correlated

$$5\,\mathrm{GeV} \leq \mu_\alpha = \mu_m \leq 15\,\mathrm{GeV}$$



$$\overline{m}_b(\overline{m}_b)[\mu_m, \mu_\alpha]$$

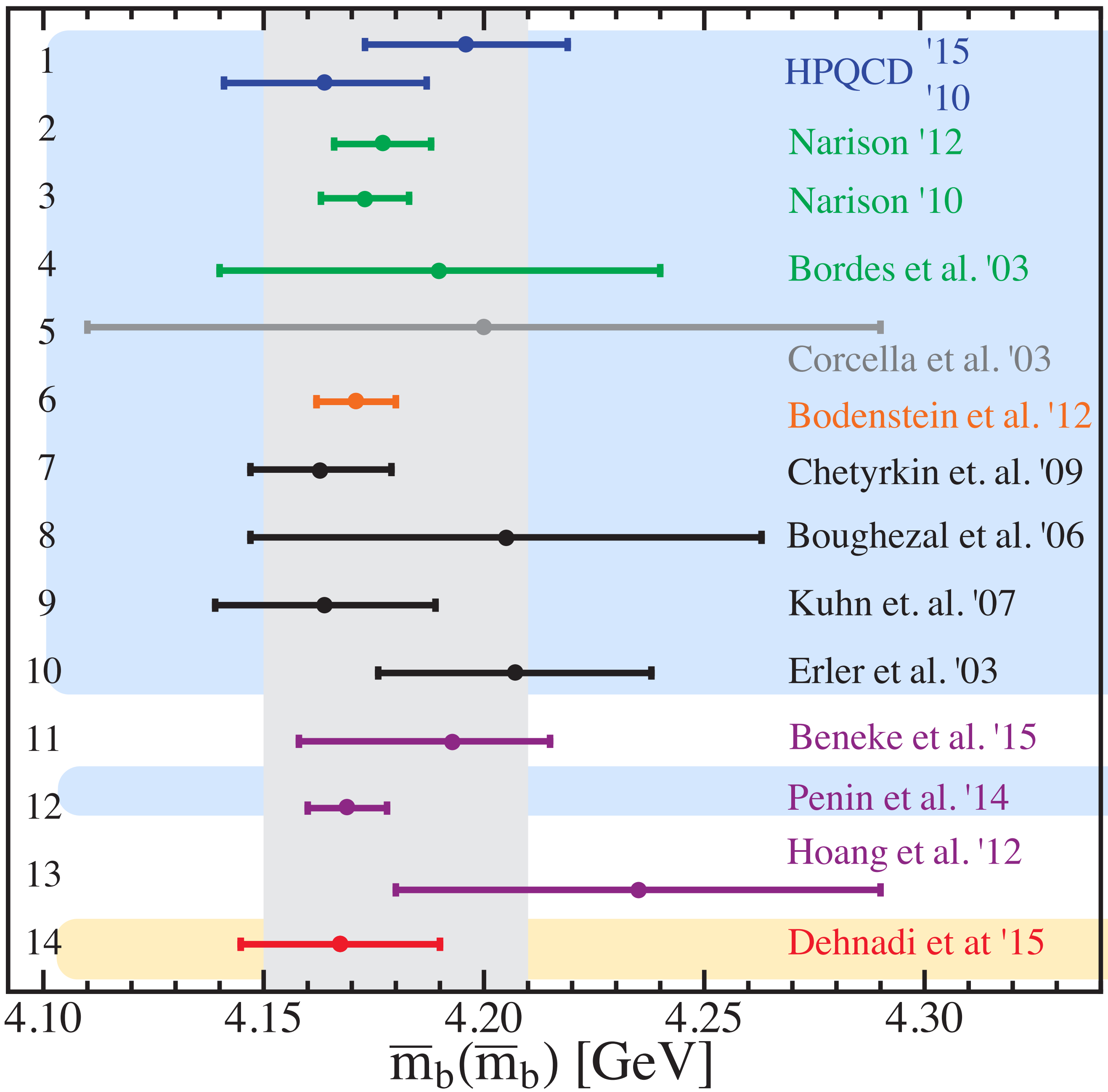


Bottom mass determinations

Estimate of perturbative uncertainties

provides consistent results, reflects actual series convergence

From QCD sum rules



correlated scale variation

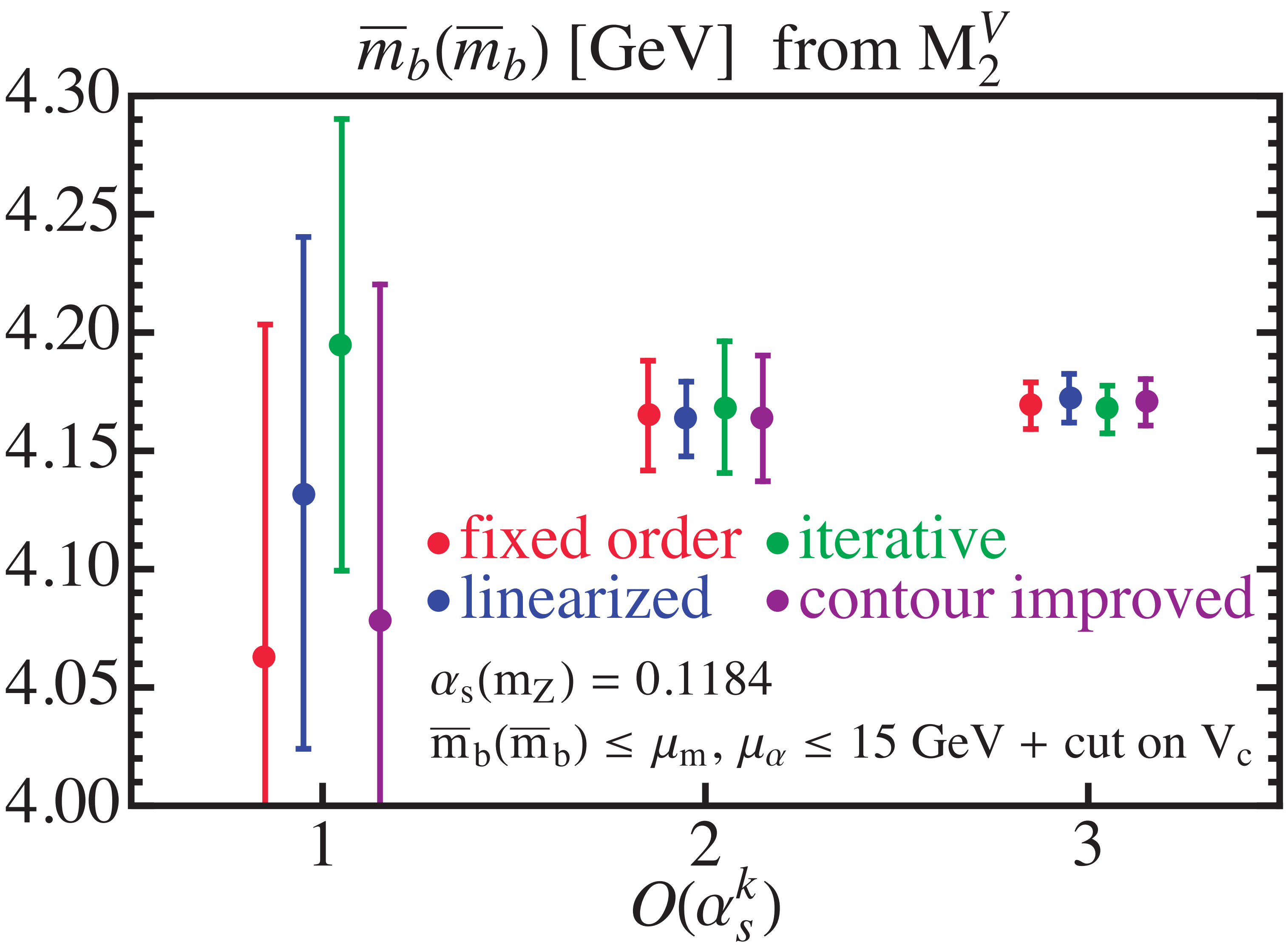
$$5\,\mathrm{GeV} \leq \mu_\alpha = \mu_m \leq 15\,\mathrm{GeV}$$

uncorrelated scale variation

$$\overline{m}_b(\overline{m}_b) \leq \mu_\alpha, \mu_m \leq 15\,\mathrm{GeV}$$

Exploration of scale variation

[Dehnadi, Hoang, & VM '15]



our approach

$\overline{m}_b(\overline{m}_b) \leq \mu_\alpha, \mu_m \leq 15\,\mathrm{GeV}$

bottom mass scale should not be excluded in the perturbative extraction of the charm mass

Our default is iterative method

We implement a cut on badly convergent series (mild effect on error)

conclusions: independent variation of scales down to $\overline{m}_b(\overline{m}_b)$ so that using different expansions does not matter

Conclusions

Conclusions & Outlook

- Sum rules provide the most accurate extractions of the charm and bottom masses
- Double scale variation appears to provide best uncertainty estimate (charm, bottom, pseudo)
- Pseudo-scalar correlator has worse convergence
- Comparisons with lattice, important cross check
- Bottom: 2nd moment smaller experimental error