Resolving m_c and m_b in precision Higgs boson analyses

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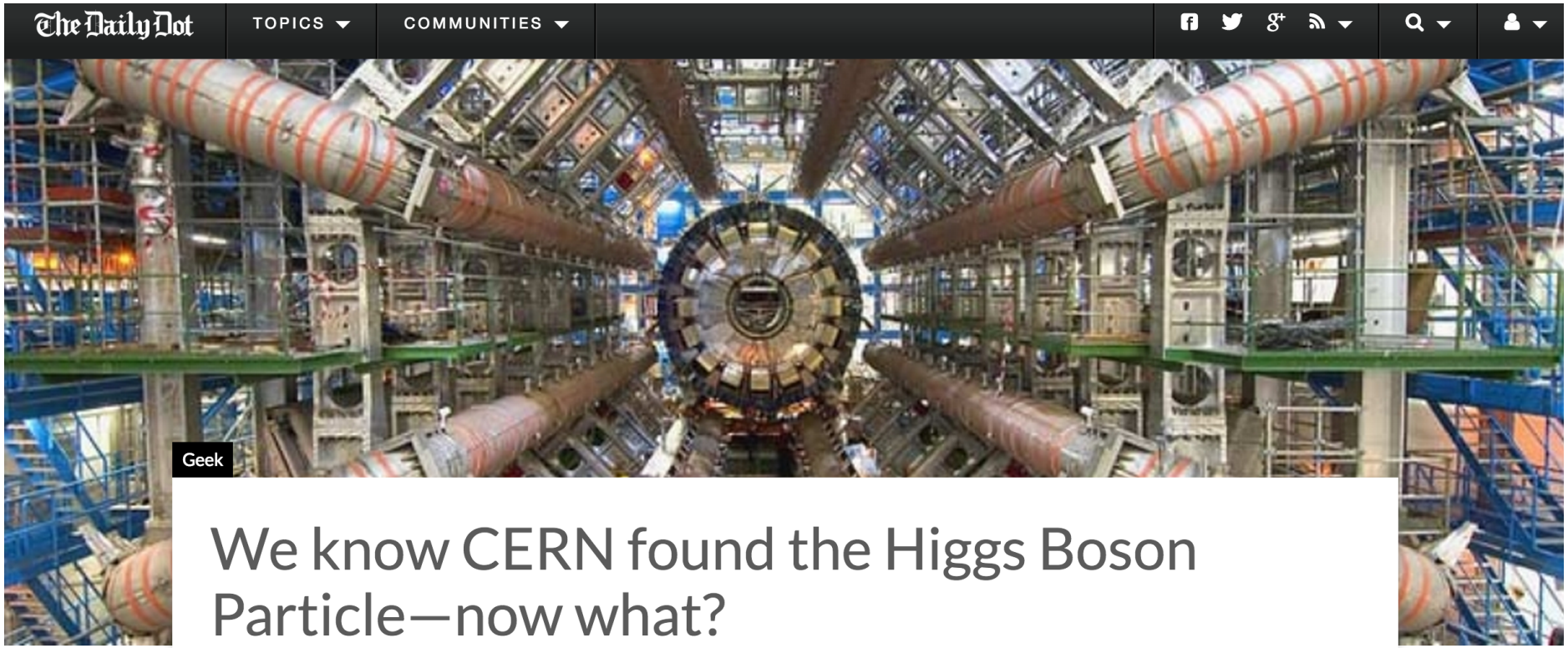
Based on A. A. Petrov, S. Pokorski, J. D. Wells, ZZ, Phys. Rev. D **91**, 073001 (2015)

[arXiv:1501.02803 [hep-ph]]



We know CERN found the Higgs Boson Particle—now what?

By Aja Romano

Jun 24, 2014, 3:35pm CT

Introduction: the precision frontier

Measure its properties very precisely! (BSM hints?)

- Theory expectation: $(\frac{v}{\text{TeV}})^2 \sim \mathcal{O}(1\%)$.
- Experiment expectation: (sub)percent-level measurements of $\Gamma_{H\to c\bar{c}}$, $\Gamma_{H\to b\bar{b}}$ at HL-LHC, ILC, FCC-ee, CEPC. [Asner *et al*, 1310.0763] [Peskin, 1312.4974] [Fan, Reece, Wang, 1411.1054] [Ruan, 1411.5606]

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Will future experiments be sensitive to %-level new physics effects?

No, *unless* theory uncertainties can be reduced to below $\mathcal{O}(1\%)$!

Motivation: theory uncertainties in $\Gamma_{H\to c\bar{c}}$, $\Gamma_{H\to b\bar{b}}$

Where are the theory uncertainties from?

- Perturbative uncertainty well below 1%, thanks to N^{4}LO calculations [Baikov, Chetyrkin, Kuhn, hep-ph/0511063].

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$$\frac{\Delta\Gamma_{H\to c\bar{c}}}{\Gamma_{H\to c\bar{c}}} \simeq \frac{\Delta m_c(m_c)}{10\text{ MeV}} \times 2.1\%, \quad \frac{\Delta\Gamma_{H\to b\bar{b}}}{\Gamma_{H\to b\bar{b}}} \simeq \frac{\Delta m_b(m_b)}{10\text{ MeV}} \times 0.56\%.$$

where $m_Q(m_Q) \equiv m_Q^{\overline{\text{MS}}}(\mu = m_Q)$.

[Denner, Heinemeyer, Puljak, Rebuzzi, Spira, 1107.5909]
[Almeida, Lee, Pokorski, Wells, 1311.6721]
[Lepage, Mackenzie, Peskin, 1404.0319]
etc.

cf. PDG: $m_c(m_c) = 1.275(25)$ GeV, $m_b(m_b) = 4.18(3)$ GeV.

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Goal: understand this uncertainty propagation in more detail.

Precision Higgs analyses: conventional approach

Use PDG quark masses or other averaged quark masses as inputs.

c-QUARK MASS

The c-quark mass corresponds to the "running" mass m_c ($\mu = m_c$) in the $\overline{\text{MS}}$ scheme. We have converted masses in other schemes to the $\overline{\text{MS}}$ scheme using two-loop QCD perturbation theory with $\alpha_s(\mu{=}m_c) = 0.38 \pm 0.03$. The value 1.275 ± 0.025 GeV for the $\overline{\text{MS}}$ mass corresponds to 1.67 ± 0.07 GeV for the pole mass (see the "Note on Quark Masses").

VALUE (GeV)	*DOCUMENT ID*		*TECN*	*COMMENT*
1.275±0.025 OUR EVALUATION	See the ideogram below.			
$1.26 \pm 0.05 \pm 0.04$	[1] ABRAMOWICZ	13C	COMB	$\overline{\text{MS}}$ scheme
$1.24 \pm 0.03 {}^{+0.03}_{-0.07}$	[2] ALEKHIN	13	THEO	$\overline{\text{MS}}$ scheme
$1.282 \pm 0.011 \pm 0.022$	[3] DEHNADI	13	THEO	$\overline{\text{MS}}$ scheme
1.286 ± 0.066	[4] NARISON	13	THEO	$\overline{\text{MS}}$ scheme
1.159 ± 0.075	[5] SAMOYLOV	13	NOMD	$\overline{\text{MS}}$ scheme
$1.36 \pm 0.04 \pm 0.10$	[6] ALEKHIN	12	THEO	$\overline{\text{MS}}$ scheme
1.261 ± 0.016	[7] NARISON	12A	THEO	$\overline{\text{MS}}$ scheme
1.278 ± 0.009	[8] BODENSTEIN	11	THEO	$\overline{\text{MS}}$ scheme
$1.28 {}^{+0.07}_{-0.06}$	[9] LASCHKA	11	THEO	$\overline{\text{MS}}$ scheme
$1.196 \pm 0.059 \pm 0.050$	[10] AUBERT	10A	BABR	$\overline{\text{MS}}$ scheme
1.28 ± 0.04	[11] BLOSSIER	10	LATT	$\overline{\text{MS}}$ scheme
1.273 ± 0.006	[12] MCNEILE	10	LATT	$\overline{\text{MS}}$ scheme
1.279 ± 0.013	[13] CHETYRKIN	09	THEO	$\overline{\text{MS}}$ scheme
1.25 ± 0.04	[14] SIGNER	09	THEO	$\overline{\text{MS}}$ scheme
1.295 ± 0.015	[15] BOUGHEZAL	06	THEO	$\overline{\text{MS}}$ scheme
1.24 ± 0.09	[16] BUCHMULLER	06	THEO	$\overline{\text{MS}}$ scheme
$1.224 \pm 0.017 \pm 0.054$	[17] HOANG	06	THEO	$\overline{\text{MS}}$ scheme

Unsatisfactory:

- Correlations among the entries neglected
- Correlation with α_s neglected
- Uncertainties underestimated and inflated [Dehnadi, Hoang, Mateu, Zebarjad, 1102.2264]

Precision Higgs analyses: proposed approach

PDG averaged quark masses are dominated by m_c, m_b determinations from low-energy observables†, e.g.

- $e^+e^- \to Q\bar{Q}$ cross sections;
- Kinematic distributions of semileptonic B decay.

†For the prospect of lattice calculations see [Lepage, Mackenzie, Peskin, 1404.0319].

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A global analysis!

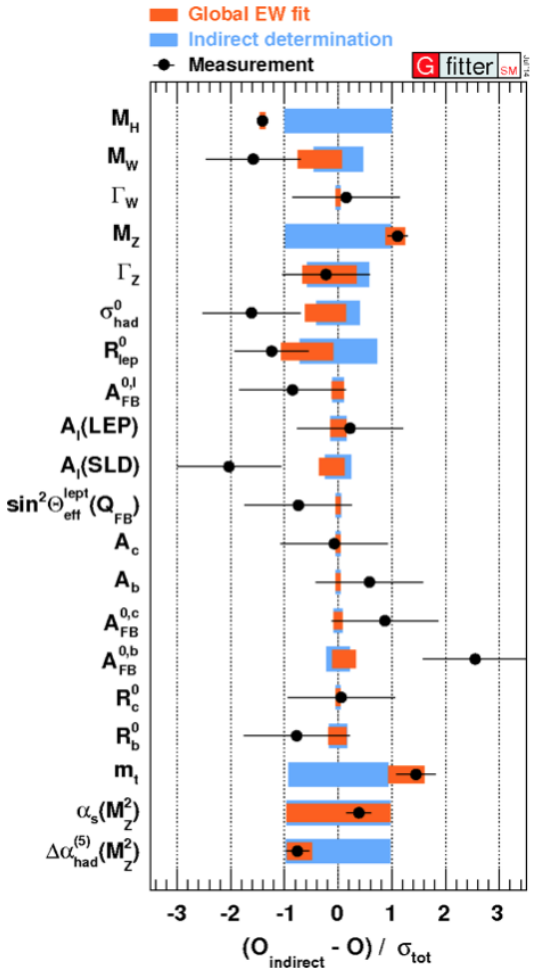
$$\left\{\begin{array}{c} \widehat{O}_1^{\text{low}}(m_c, m_b, \alpha_s, \dots) \\ \widehat{O}_2^{\text{low}}(m_c, m_b, \alpha_s, \dots) \\ \widehat{O}_3^{\text{low}}(m_c, m_b, \alpha_s, \dots) \\ \vdots \end{array}\right\} \Leftarrow \left\{\begin{array}{c} \underline{\text{Inputs}} \\ m_c \\ m_b \\ \alpha_s \\ \vdots \end{array}\right\} \Rightarrow \left\{\begin{array}{c} \widehat{O}_1^{\text{Higgs}}(m_c, m_b, \alpha_s, \dots) \\ \widehat{O}_2^{\text{Higgs}}(m_c, m_b, \alpha_s, \dots) \\ \widehat{O}_3^{\text{Higgs}}(m_c, m_b, \alpha_s, \dots) \\ \vdots \end{array}\right\}$$

† For the prospect of lattice calculations see [Lepage, Mackenzie, Peskin, 1404.0319].

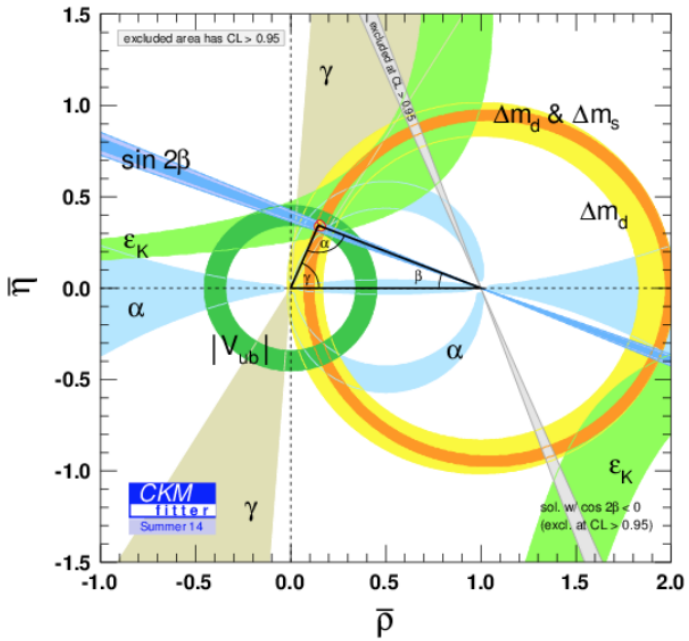
Precision Higgs analyses: proposed approach

... just like what we have done before!

Precision electroweak



Precision flavor



Precision Higgs?

?

Higgs observables

+

low-energy observables

A first calculation: $\Gamma_{H\to c\bar{c}}$, $\Gamma_{H\to b\bar{b}}$ in terms of $\mathcal{M}_1^c$, $\mathcal{M}_2^b$

To see the role of low-energy observables in this precision Higgs boson analyses, we will

- focus on $\Gamma_{H\to c\bar{c}}$, $\Gamma_{H\to b\bar{b}}$, and
- eliminate m_c, m_b from the input in favor of $\mathcal{M}_1^c$, $\mathcal{M}_2^b$.

"nth moment of R_Q":

$$\mathcal{M}_n^Q \equiv \int \frac{\mathrm{d}s}{s^{n+1}} R_Q(s), \quad \text{where } R_Q \equiv \frac{\sigma(e^+e^- \to Q\bar{Q}X)}{\sigma(e^+e^- \to \mu^+\mu^-)}.$$

A first calculation: $\Gamma_{H\to c\bar{c}}$, $\Gamma_{H\to b\bar{b}}$ in terms of $\mathcal{M}_1^c$, $\mathcal{M}_2^b$

Moments of R_Q are calculated by relativistic quarkonium sum rules

[Novikov, Okun, Shifman, Vainshtein, Voloshin, Zakharov, Phys. Rept. 41, 1 (1978)]

$$\mathcal{M}_n^Q = \int \frac{\mathrm{d}s}{s^{n+1}} R_Q(s) = \frac{12\pi^2}{n!}\left(\frac{\mathrm{d}}{\mathrm{d}q^2}\right)^n \Pi_Q(q^2)\bigg|_{q^2=0}, \quad \text{where}$$

$$(q^2 g_{\mu\nu} - q_\mu q_\nu)\Pi_Q(q^2) = -i\int \mathrm{d}^4x\, e^{iq\cdot x}\langle 0|T j_\mu(x) j_\nu^\dagger(0)|0\rangle,$$

via an operator product expansion (OPE)

$$\mathcal{M}_n^Q = \frac{\left(Q_Q/(2/3)\right)^2}{\left(2m_Q(\mu_m)\right)^{2n}} \sum_{i,a,b} C_{n,i}^{(a,b)}(n_f)\left(\frac{\alpha_s(\mu_\alpha)}{\pi}\right)^i \ln^a \frac{m_Q(\mu_m)^2}{{\mu_m}^2} \ln^b \frac{m_Q(\mu_m)^2}{{\mu_\alpha}^2} + \mathcal{M}_n^{Q,\mathrm{np}}.$$

Low moments (small n) are preferred to suppress $\mathcal{M}_n^{Q,\mathrm{np}}$.

A first calculation: $\Gamma_{H\to c\bar{c}}$, $\Gamma_{H\to b\bar{b}}$ in terms of $\mathcal{M}_1^c$, $\mathcal{M}_2^b$

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Best calculations available:

- $C_{n,i}^{(a,b)}(n_f)$: up to $i = 3$ [Maier, Maierhofer, Marquard, Smirnov, 0907.2117].
- $\mathcal{M}_n^{Q,\mathrm{np}}$: up to NLO [Broadhurst, Baikov, Ilyin, Fleischer, Tarasov, Smirnov, hep-ph/9403274], kept only for charm.

Renormalization scales: μ_m for m_Q, μ_α for α_s.

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$$\Rightarrow \begin{cases} m_c(m_c) = m_c(m_c)\left[\alpha_s, \mathcal{M}_1^c, \mu_m^c, \mu_\alpha^c, \mathcal{M}_1^{c,\mathrm{np}}\right], \\ m_b(m_b) = m_b(m_b)\left[\alpha_s, \mathcal{M}_2^b, \mu_m^b, \mu_\alpha^b\right]. \end{cases}$$

[Kuhn, Steinhauser, hep-ph/0109084]

[Kuhn, Steinhauser, Sturm, hep-ph/0702103]

[Chetyrkin, Kuhn, Maier, Maierhofer, Marquard, Steinhauser, Sturm, 0907.2110]

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[Chetyrkin, Kuhn, Maier, Maierhofer, Marquard, Steinhauser, Sturm, 0907.2110]

Should keep $\mu_m \neq \mu_\alpha$, otherwise perturbative uncertainty will be underestimated (common in the literature).

[Dehnadi, Hoang, Mateu, Zebarjad, 1102.2264]

[Dehnadi, Hoang, Mateu, 1504.07638]

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$$\Downarrow$$

$$\begin{cases} \Gamma_{H\to c\bar{c}} = \Gamma_{H\to c\bar{c}}\left[\{\widehat{O}_k^{\mathrm{in}}\}, m_c(m_c), \mu_H^c\right] = \Gamma_{H\to c\bar{c}}\left[\{\widehat{O}_k^{\mathrm{in}}\}, \mathcal{M}_1^c, \mu_m^c, \mu_\alpha^c, \mu_H^c, \mathcal{M}_1^{c,\mathrm{np}}\right], \\ \Gamma_{H\to b\bar{b}} = \Gamma_{H\to b\bar{b}}\left[\{\widehat{O}_k^{\mathrm{in}}\}, m_b(m_b), \mu_H^b\right] = \Gamma_{H\to b\bar{b}}\left[\{\widehat{O}_k^{\mathrm{in}}\}, \mathcal{M}_2^b, \mu_m^b, \mu_\alpha^b, \mu_H^b\right]. \end{cases}$$

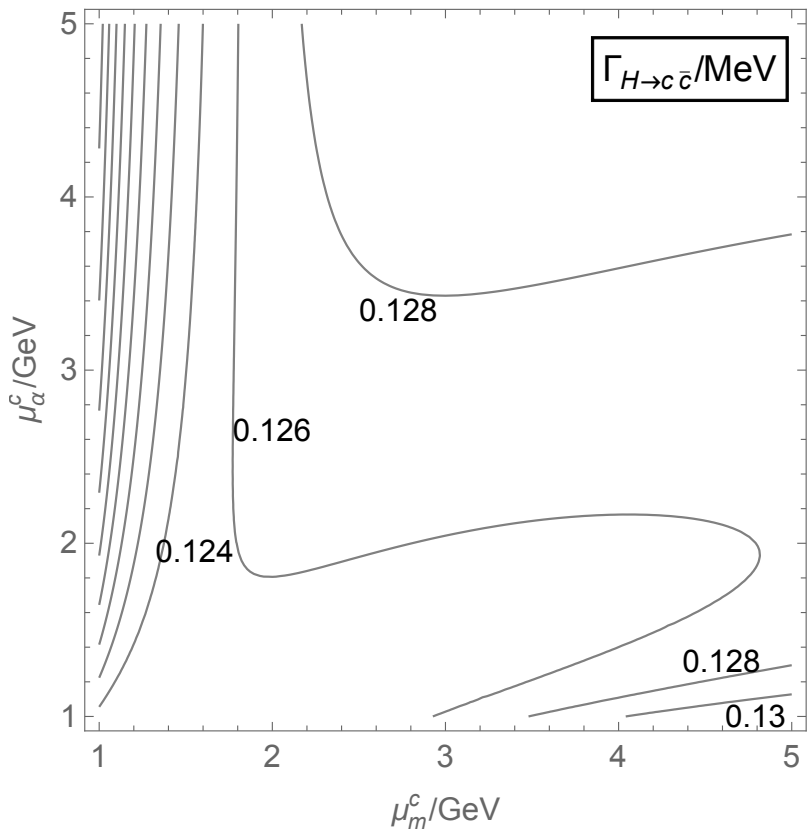
"Uncertainties from m_Q" are decomposed into concrete sources.

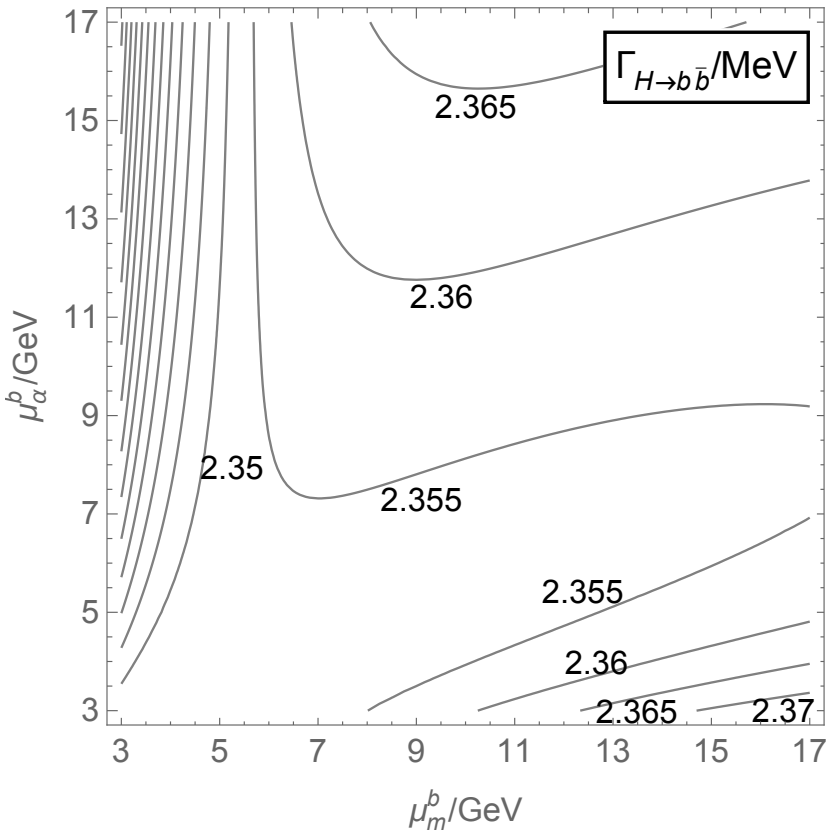
Uncertainty source	$\Delta\Gamma_{H\to c\bar{c}}/\Gamma_{H\to c\bar{c}}$	$\Delta\Gamma_{H\to b\bar{b}}/\Gamma_{H\to b\bar{b}}$
$\mathcal{M}_n^Q$ measurement[†]	2%	0.6%
$\mathcal{M}_n^Q$ calculation	see next 3 slides	
α_s (vs. no correlation)	1% (1.6%)	0.5% (0.6%)
$\mathcal{M}_n^{Q,\mathrm{np}}$	<0.8%	→0
m_H	<0.3%	<0.3%

[†] This also includes a sizable uncertainty from pQCD input for $\sqrt{s} > 11.2$ GeV where no data is available, but the situation will be improved by Belle-II.

Perturbative uncertainty from $\mathcal{M}_n^Q$ calculation

Renormalization scale dependence of finite-order calculation:





Vary μ_m, μ_α within $[\mu_{\rm min}, \mu_{\rm max}]$ $\Rightarrow$ estimated perturbative uncertainty is very sensitive to $\mu_{\rm min}$.

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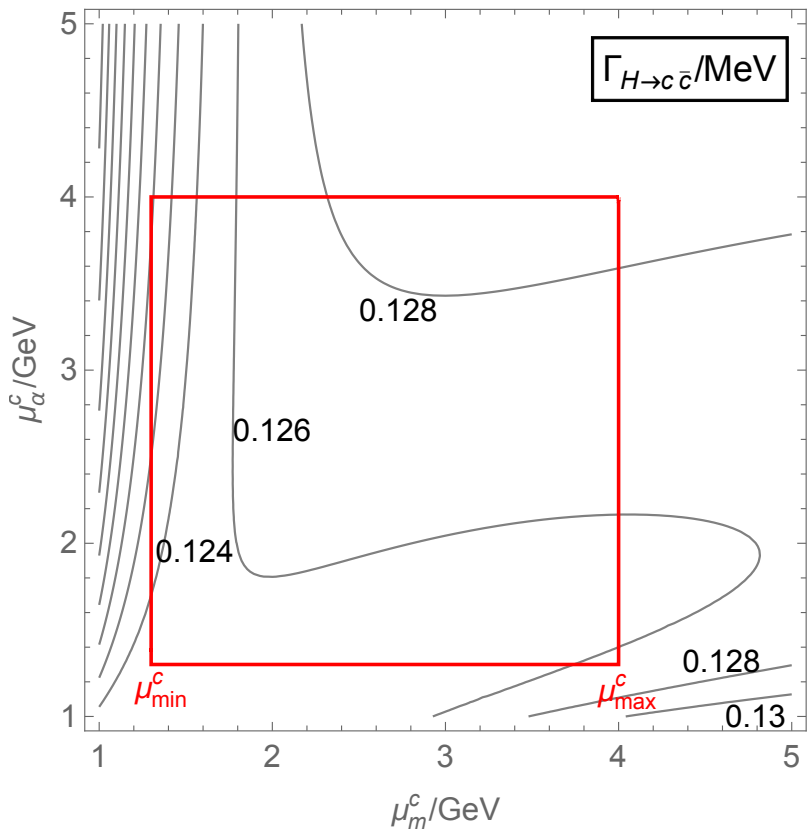

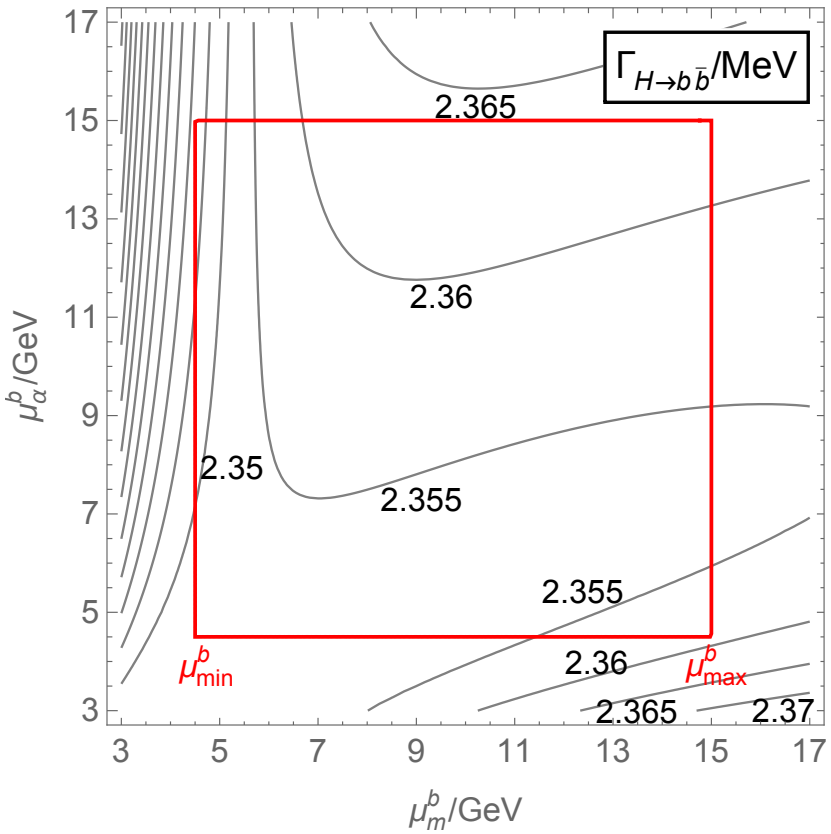

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Perturbative uncertainty from $\mathcal{M}_n^Q$ calculation

Plot estimated perturbative uncertainty vs. $\mu_{\rm min}$ and compare with uncertainties from $\mathcal{M}_n^Q$, α_s, $\mathcal{M}_n^{Q,\rm np}$, m_H.

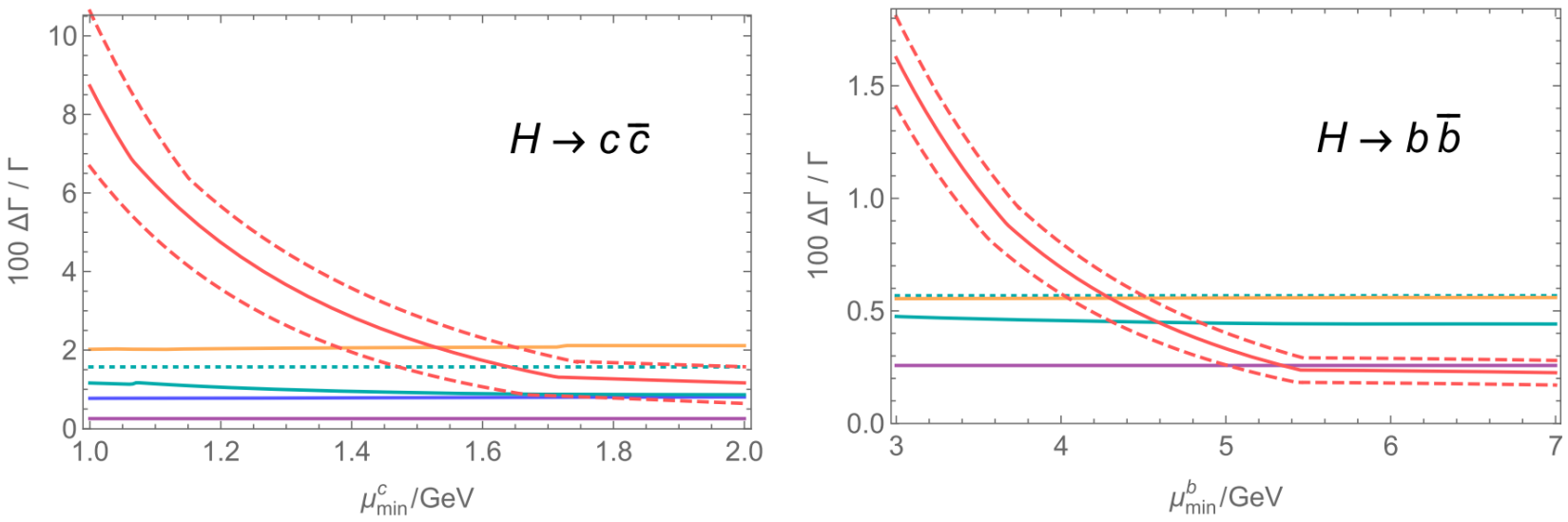

FIG. 2 (color online). Percent relative uncertainties in $\Gamma_{H\to c\bar{c}}$ (left) and $\Gamma_{H\to b\bar{b}}$ (right) as functions of $\mu_{\rm min}$ from various sources: perturbative uncertainty with $\mu^c_{\rm max} = 4$ GeV, $\mu^b_{\rm max} = 15$ GeV (red solid) or alternatively $\mu^c_{\rm max} = 3, 5$ GeV, $\mu^b_{\rm max} = 13, 17$ GeV (red dashed), parametric uncertainties from $\mathcal{M}_1^c$ or $\mathcal{M}_2^b$ (orange), $\alpha_s(m_Z)$ (cyan solid), $\mathcal{M}_1^{c,\rm np}$ (blue, for $\Gamma_{H\to c\bar{c}}$ only), and m_H (purple). The parametric uncertainty from $\alpha_s(m_Z)$ incorrectly calculated assuming no correlation with m_Q (cyan dotted) is also shown for comparison.

Big challenge for higher-precision $\Gamma_{H\to Q\bar{Q}}$ calculations!

Perturbative uncertainty from $\mathcal{M}_n^Q$ calculation

We need to get the perturbative uncertainty under control.

- $\mathcal{O}\left(\alpha_s^4\right)$ calculation of $\mathcal{M}_n^Q$, or equivalently, $\left(\frac{\mathrm{d}}{\mathrm{d}q^2}\right)^n \Pi_Q(q^2)\Big|_{q^2=0}$?
- Other algorithms to estimate perturbative uncertainty?
  - BLM [Brodsky, Lepage, Mackenzie, PRD28, 228 (1983)] (not directly applicable)
  - Convergence test [Dehnadi, Hoang, Mateu, 1504.07638] (still arbitrary)
- Other low-energy observables? (future work)
  - Variants of $\mathcal{M}_n^Q$
    [Bodenstein, Bordes, Dominguez, Penarrocha, Schilcher, 1102.3835, 1111.5742]
  - High moments of R_Q (nonrelativistic sum rules for $n \geq 10$)
    [Signer, 0810.1152] [Hoang, Ruiz-Femenia, Stahlhofen, 1209.0450] [Penin, Zerf, 1401.7035] [Beneke, Maier, Piclum, Rauh, 1411.3132]
  - Semileptonic B decay observables
    [Bauer, Ligeti, Luke, Manohar, Trott, hep-ph/0408002] [Buchmuller, Flacher, hep-ph/0507253] [Gambino, Schwanda, 1307.4551]

Conclusions

- m_c, m_b bring large theory uncertainties into $\Gamma_{H\to c\bar{c}}$, $\Gamma_{H\to b\bar{b}}$ calculations that should be understood better.
- The conventional approach to precision Higgs analyses using m_c and m_b as inputs hides various uncertainties and correlations.
- We propose a global analysis involving low-energy observables as well as Higgs observables.
- A first calculation in this direction shows how the uncertainties from m_c, m_b are resolved into concrete sources.

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There is much theoretical work to be done for the precision Higgs program to succeed in the future!

Thank you!