$c$ and $b$ quark masses from lattice QCD.

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Intro & Motivation

- Quark masses – fundamental parameters of the Standard Model.

- Many applications to phenomenology and BSM physics. Example: Higgs partial widths.
  - Couplings proportional to quark masses.
  - Main source of uncertainty in partial [1404.0319] widths from $m_b, m_c, \alpha_s$.

- Focus on recent lattice results for charm and bottom masses.
Outline

• Background
  ▶ Theory background.
  ▶ Lattice determinations.

• Charm mass
  ▶ Time moments of $\langle JJ \rangle$ correlators.
  ▶ Comparison with perturbation theory.

• Bottom mass
  ▶ Different approaches.
  ▶ $\langle JJ \rangle$ moments in NRQCD.

• Mass ratios
  ▶ $m_c/m_b$
  ▶ $m_s/m_c$

• Future Work & Conclusions
Quark mass – definitions

- Quarks are not asymptotic (physical) states.
- Quark masses are scheme and scale dependent, $m_{q}^{\text{scheme}}(\mu)$.
- Generally will quote results $m_{q}^{\overline{\text{MS}}}(\mu_{\text{ref}})$. 
Lattice determination of quark mass

Bare quark masses are input parameters to lattice simulations. These parameters are tuned to reproduce physical quantities, e.g.

- $m_{ud0} \rightarrow m_{\pi}^2$
- $m_{s0} \rightarrow m_{K}^2$
- $m_{c0} \rightarrow m_{\eta_c}$

Tuning performed at multiple lattice spacings, defining a continuum trajectory for which $a^2 \rightarrow 0$ limit can be taken.

- Rest of physics is then prediction of QCD.
- Parameters can be varied away from physical values to understand effect of quark mass, quantify systematics, etc.
c-quark mass
Simulating charm

Heavy quarks are challenging to simulate.

- Requires $am_0 < 1$ to keep discretization effects under control.

- Need large enough box to minimize finite-volume effects $\rightarrow N_{\text{site}}$ large.

These conditions can be satisfied by using a highly improved action (e.g. HISQ).
HISQ action

- No $\mathcal{O}(a^2)$ discretization errors (begin at $\mathcal{O}(\alpha_s a^2)$).
- Significant $\mathcal{O}(\alpha_s a^2)$ effects are in turn suppressed.

$n_f = 4$ simulations

- Charm quarks in the sea.
- Avoid applying perturbation theory at $m_c$ (matching $n_f = 4 \rightarrow 3$).

It is increasingly feasible to simulate the $b$ quark relativistically.
Calculate time-moments of $J_5 \equiv \bar{\psi} h \gamma_5 \psi_h$ correlators:

$$G(t) = a^6 \sum_x (am_{0h})^2 \langle J_5(t, x) J_5(0, 0) \rangle$$

- Currents are absolutely normalized (no $Z$s required).
- $G(t)$ is UV finite $\rightarrow G(t)_{\text{cont}} = G(t)_{\text{latt}} + \mathcal{O}(a^2)$. 

Lattice QCD is best method to determine quark masses $m_{q,latt}$ determined very accurately by fixing a meson mass to be correct. e.g. for $m_c$ fix $M(\pi_c)$.

Issue is conversion to the $\overline{\text{MS}}$ scheme.

- Direct method
  $$m_{\overline{\text{MS}}} (\mu a) = Z(\mu a) m_{\text{latt}}$$

  Calculate $Z$ perturbatively or partly nonperturbatively.

- Indirect methods: (after tuning $\ldots$) match a quantity from lattice QCD to continuum pert. th. in terms of $\overline{\text{MS}}$ mass

Chetyrkin et al, 0907.2110

- $q^2$-derivative moments of current-current correlators (vac. pol.function) for heavy quarks known through $\ldots$.

Calc. on lattice as time-moments of 'local' meson correlation function.

Chetyrkin et al, 0805.2999, C. Mcneile et al, HPQCD, 1004.4285

*masses important for Higgs cross-sections*
Moments

The time-moments $G_n = \sum_t (t/a)^n G(t)$ can be computed in perturbation theory. For $n \geq 4$,

$$G_n = \frac{g_n(\alpha_{\overline{\text{MS}}}, \mu)}{a m_h(\mu)^{n-4}}.$$ 

Basic strategy:

1. Calculate $G_{n,\text{latt}}$ for a variety of lattice spacings and $m_{h0}$.
2. Compare continuum limit $G_{n,\text{cont}}$ with $G_{n,\text{pert}}$ (at reference scale $\mu = m_h$, say).
3. Determine best-fit values for $\alpha_{\overline{\text{MS}}}(m_h)$, $m_h(m_h)$. 
In practice comparison carried out using reduced moments.

\[ R_4 = \frac{G_4}{G_4^{(0)}} \]

\[ R_n = \frac{1}{m_{0c}} \left( \frac{G_n}{G_n^{(0)}} \right)^{1/(n-4)} \quad (n \geq 6). \]

On the perturbative side,

\[ R_4 = r_4(\alpha_{\overline{\text{MS}}}, \mu) \]

\[ R_n = \frac{1}{m_{c}(\mu)} r_n(\alpha_{\overline{\text{MS}}}, \mu) \quad (n \geq 6). \]

Reference scale is taken as \( \mu = 3m_h (= m_c \frac{m_{h0}}{m_{c0}}) \).
Some details

- Calculate moments for $n = 4, 6, 8, 10$.
- Three lattice spacings: $a \approx 0.12, 0.09, 0.06$ fm. (MILC)
- Seven input masses from $m_h = m_c - 0.7m_b$.

All data points fit simultaneously with perturbative $R_n$ expressions $\rightarrow m_c^{\overline{\text{MS}}} (\mu)$, $\alpha_{\overline{\text{MS}}} (\mu)$ for $\mu \approx 3 - 9$ GeV.
Uncertainties

- Non-perturbative terms/condensates.
  - $r_n(\alpha_{\overline{\text{MS}}}) \rightarrow r_n(\alpha_{\overline{\text{MS}}}) \left[ 1 + d_n(\alpha) \left( \frac{\alpha G^2}{\pi} \right) + \cdots \right]$
  - Effects suppressed by $(\frac{\Lambda}{2m_h})^4$.

- Truncation of perturbation theory.
  - $r_n = 1 + \sum_j \alpha^j(m_h) r_{nj}$.
  - $j = 1, 2, 3$ known for $n \leq 10$

- Lattice artifacts.
  - Grow like $\alpha_s \times (am_h)^2$.
  - Decrease with increasing $n$. 
Results for $n_f = 4$ [1408.4169]

$$m_c(3m_h) = \frac{r_n(\alpha_{\overline{MS}}, \mu = 3m_h)}{R_n}$$

- Discretization effects grow with $am_h$ and decrease with $n$.
- Grey band shows best-fit $m_c(3m_c)$ evolved perturbatively.

$$m_c(3 GeV) = 0.9851(63) \text{ GeV}$$
$m_c$ comparison plot

- **HPQCD HISQ** $n_f=3$ [1004.4285]  
- **HPQCD HISQ** $n_f=4$ [1408.4169]  
- **χQCD** $n_f=3$ [1410.3343]  
- **ETMC** $n_f=4$ [1403.4504]
\[ \alpha_s^{\overline{\text{MS}}} (m_Z) \]

**HPQCD \langle JJ \rangle result:**

- \[ \alpha_s^{\overline{\text{MS}}} (m_Z) = 0.1182(7) \]
- Agrees with \( n_f = 3 \) result.
- Agrees well with world average.
$b$-quark mass
Approaches to calculating.

It is challenging to treat $b$-quark in LQCD calculations.

- Fully relativistic treatment.
  - Requires $am_b \ll 1$.
  - Now becoming possible using highly-improved actions.

- Effect field theories.
  - NRQCD
  - HQET

Focus on NRQCD approach.
NRQCD approach

- Expansion in $v^2$ where $v^2 \sim 0.1$ for $\Upsilon$.
- Want $am_{b_0} > 1$.
- $H = H_0 + \delta H$
- Unlike relativistic case, current needs normalized: $J_{\mu}^{\text{NRQCD}} = Z_V J_{\mu}^{\text{cont}}$.
- Effective theory $\rightarrow$ no continuum limit.

$$G_n^{\text{NRQCD}} = Z_V^2 \frac{g_n(\alpha_{\overline{\text{MS}}}, \mu)}{am_b(\mu)^{n-2}}.$$
NRQCD moments  [1408.5768]

- Study ratios of successive moments to cancel factors of $Z_V$.
- Look for a “plateau” in the moment number $n$.

\[ \bar{m}_b(\mu = 4.18, n_f = 4) (\text{GeV}) \]

\[ m_b^{\overline{\text{MS}}}(m_b, n_f = 5) = 4.196(23) \text{ GeV} \]
$m_b$ comparison plot

\begin{itemize}
\item HPQCD NRQCD JJ [1408.5768]
\item HPQCD HISQ JJ $n_f=3$ [1004.4285]
\item HPQCD HISQ ratio $n_f=4$ [1408.4169]
\item HPQCD NRQCD $E_0$ [1302.3739]
\item ETMC ratio [1411.0484]
\end{itemize}
Heavy-charm HISQ moments

- ultrafine hh
- superfine hh
- ultrafine hc
- superfine hc

![Graph showing the relationship between $m_h(m_h,\eta_f=3)$ [GeV] and $M_{\eta_h}$ [GeV].](image)
Mass ratios
Mass ratios

\[ \frac{m_{10}}{m_{20}} = \frac{m_{1 \overline{\text{MS}}} (\mu)}{m_{2 \overline{\text{MS}}} (\mu)} + \mathcal{O}(a^2) \]

- Tuning of simulation \(\rightarrow\) accurate determination of bare ratios.

- Precise determination of one renormalized mass can be translated to other masses.
\[ m_c/m_s \ (n_f = 4) \]

\[ m_c/m_s = 11.652(65) \rightarrow m_s^{\overline{\text{MS}}}(2\text{GeV}) = 93.6(8)\text{MeV}. \]
\[ \frac{m_b}{m_c} = 4.49(4) \quad \text{HPQCD [1004.4285]} \]

\[ \frac{m_b}{m_c} = 4.40(8) \quad \text{ETMC [1411.0484]} \]
Conclusions & Future Work

- Accurate determinations of quark masses are of fundamental importance for (B)SM physics.

- LQCD simulations provide an effective and controlled way to determine quark masses.
  - Systematically improveable.
  - Multiple complementary approaches → assess systematics, check consistency.
  - Control of input parameters.

- In the future we can expect:
  - Fully relativistic $b$.
  - Additional approaches:
    - Heavy-light $\langle JJ \rangle$, RI/MOM - type determinations.
    - More independent calculations by different groups.
Thank you!
Additional Slides
## Error budget – ⟨JJ⟩ HISQ

<table>
<thead>
<tr>
<th>Source of Uncertainty</th>
<th>m_c(3)</th>
<th>α_{MS}(M_Z)</th>
<th>m_c/m_s</th>
<th>m_b/m_c</th>
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<tbody>
<tr>
<td>Perturbation theory</td>
<td>0.3</td>
<td>0.5</td>
<td>0.0</td>
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<tr>
<td>Statistical errors</td>
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<td>0.2</td>
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<tr>
<td>a^2 → 0</td>
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<td>0.3</td>
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<td>1.0</td>
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<tr>
<td>δm_{sea}^{uds} → 0</td>
<td>0.2</td>
<td>0.1</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>δm_{sea}^{c} → 0</td>
<td>0.3</td>
<td>0.1</td>
<td>0.0</td>
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<tr>
<td>m_h ≠ m_c</td>
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<tr>
<td>Uncertainty in w_0, w_0/a</td>
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<td>0.0</td>
<td>0.1</td>
<td>0.4</td>
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<tr>
<td>α_0 prior</td>
<td>0.0</td>
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<tr>
<td>Uncertainty in m_{η_s}</td>
<td>0.0</td>
<td>0.0</td>
<td>0.4</td>
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<tr>
<td>m_h/m_c → m_b/m_c</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.4</td>
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<tr>
<td>δm_{η_c}: electromag., annih.</td>
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<td>0.1</td>
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<tr>
<td>δm_{η_b}: electromag., annih.</td>
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<td>0.0</td>
<td>0.1</td>
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<tr>
<td><strong>Total:</strong></td>
<td><strong>0.64%</strong></td>
<td><strong>0.63%</strong></td>
<td><strong>0.55%</strong></td>
<td><strong>1.20%</strong></td>
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Error budget – $\langle JJ \rangle$ NRQCD

<table>
<thead>
<tr>
<th>Error</th>
<th>$f_\gamma \sqrt{M_\gamma}$</th>
<th>$\bar{m}_b(10\text{GeV})$</th>
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<td>Statistics</td>
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<td>$Z_V/k_1$</td>
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<td>perturbation theory/$\alpha_s$</td>
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<td>uncertainty in $a$</td>
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<td>lattice spacing dependence</td>
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<td>NRQCD systematics</td>
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<tr>
<td>total</td>
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<td>0.7</td>
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