Lattice perspective on leptonic and semileptonic decays

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1. **Introduction**

**Goals in the study of leptonic and semileptonic $D$ decays**

* Precise determination of CKM matrix elements ($|V_{cd,cs}|$)

$$\text{Experiment} = (\text{known factors}) \times (V_{CKM}) \times \text{(hadronic matrix elements)}$$

** Check Standard Model

** Consistency of different determinations of CKM matrix elements

** Test unitarity of CKM matrix.

** Comparison of shape of form factors with experimental data.
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\text{Experiment} = (\text{known factors}) \times (V_{CKM}) \times (\text{hadronic matrix elements}) \text{ (lattice QCD)}
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* Validate lattice QCD techniques to use in $B$ physics

* Constraining possible NP models

Fajfer, Nisandzic and Rojec, 1502.07748, Barranco et al., 1303.3896, 1404.0454

** Correlated signals of NP in leptonic and semileptonic decays.
1. **Introduction: Lattice QCD**

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**Lattice QCD:** Numerical evaluation of QCD path integral (rely only on first principles).

**Goal:** Precise calculations ($\leq 5\%$ error)

* Control over systematic errors:

  ** Unquenched calculations: $N_f = 2$, $N_f = 2 + 1$ or $N_f = 2 + 1 + 1$.

  ** Discretization: improved actions $+$ simulations at several $a'$s $\rightarrow$ continuum limit.

  ** Chiral extrapolation: simulate at several $m_\pi$ and extrapolate to $m_\pi^{\text{phys}}$ using ChPT.

  ** Renormalization: non-perturbative, perturbative.

  ** Tuning lattice scale and masses

  ** Finite volume, *isospin effects, electromagnetic effects, ...*

**Systematically improvable**
1. Introduction: Overview of simulations parameters

Several $N_f = 2 + 1$ and even $N_f = 2 + 1 + 1$, and physical quark masses.

$N_f = 2$ : open
$N_f = 2 + 1$ : shaded
$N_f = 2 + 1 + 1$ : filled

plot by A. El-Khadra,

First results with simulations with physical light quark masses (*BMW*, PACS-CS, MILC, RBC/UKQCD, ETMC)
1. Introduction: Averaging lattice QCD results

# Flavor Lattice Averaging Group (FLAG-2): 28 people representing all big lattice collaborations.

- Advisory Board: S. Aoki, C. Bernard, C. Sachrajda
- Working Groups:
  - u, d and s quark masses: L. Lellouch, T. Blum, V. Lubicz
  - \(|V_{us}|, |V_{ud}|\): A. Jüttner, T. Kaneko, S. Simula
  - LEC’s: S. Dürr, H. Fukaya, S. Necco
  - \(B_K\): H. Wittig, J. Laiho, S. Sharpe
  - \(\alpha_s\): R. Sommer, R. Horsley, T. Onogi
  - \(f_{B(s)}, f_{D(s)}, \hat{B}_B\): A. El Khadra, Y. Aoki, M. Della Morte
  - \(B, D\) semileptonic and radiative decays: R. Van de Water, E. Lunghi, C. Pena

http://itpwiki.unibe.ch/flag/

2. Leptonic $D$ decays

\[ \mathcal{B}(D_q \rightarrow l\nu) \sim 4.5 - 3.5\% \]
\[ \tau_{D_q} < 1.5\% \]
\[ f_{D_q}^2 \sim 1\% \]
\[ \text{others} < 0.4\% \]

\[ \mathcal{B}(D_q \rightarrow l\nu) = \frac{G_F^2 \tau_{D_q}}{8\pi} m_l^2 m_{D_q} \left( 1 - \frac{m_l^2}{m_{D_q}^2} \right)^2 |V_{cq}|^2 f_{D_q}^2 \]

(with $q = d, s$ and $D_q = D^+, D_s$)

Simple matrix element \[ \langle 0 | \bar{c} \gamma_\mu \gamma_5 q | D_q (p) \rangle = i f_{D_q} p_\mu \rightarrow \text{precise calculations} \]

or, if using the same action for light and charm valence quarks,

\[ (m_c + m_q) \langle 0 | \bar{c} \gamma_5 q | D_{(s)} (p = 0) \rangle = f_{D_q} M_{D_q}^2 \]

(no need of renormalization)

Reduction of error: use relativistic (improved) formulations for $c$. 
2. **Leptonic $D$ decays: New results ($> 2013$)**

$N_f = 2$:

* **TWQCD, 1404.3648**: $a \sim 0.06\,fm$ and $m_\pi \leq 260\,\text{MeV}$

\[
f_D = 202.3(3.4)\,\text{MeV} \quad f_{D_s} = 258.7(3.1)\,\text{MeV} \quad f_{D_s}/f_D = 1.279(26)
\]
2. Leptonic $D$ decays: New results (> 2013)

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$$ f_D = 202.3(3.4) \, MeV \quad f_{D_s} = 258.7(3.1) \, MeV \quad f_{D_s}/f_D = 1.279(26) $$

# $N_f = 2 + 1$:

* $\chi$QCD, 1410.3343: Different set of configurations (RBC/UKQCD) and valence quark formulation (overlap) than previous calculations: two lattice spacings.

$$ f_{D_s} = 254(2)(4) \, MeV $$
2. Leptonic $D$ decays: New results (> 2013)

# $N_f = 2 + 1 + 1$:

* FNAL/MILC, 1407.3772: highly improved action, MILC configurations with phys. quark masses and small lattice spacing (4 $a'$s, smallest $a \approx 0.06$ fm)
2. Leptonic $D$ decays: New results (> 2013)

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* FNAL/MILC, 1407.3772: highly improved action, MILC configurations with
  phys. quark masses and small lattice spacing ($4 a', $smallest $a \approx 0.06$ fm)

\[
\begin{align*}
  f_{D^+} &= 212.6^{+1.1}_{-1.2} \text{ MeV} \\
  f_{D_s} &= 249.0^{+1.3}_{-1.5} \text{ MeV} \\
  f_{D_s}/f_{D^+} &= 1.1712^{+0.031}_{-0.034}
\end{align*}
\]

** ~ 0.5% error dominated by continuum extrapolation error

** They calculate the difference between $f_{D^+}$ and the isospin limit value, $f_D$:

\[
f_{D^+} - f_D = 0.47^{+11}_{-5} \text{ MeV}
\]
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* **FNAL/MILC, 1407.3772:** highly improved action, MILC configurations with phys. quark masses and small lattice spacing ($4 \ a'$s, smallest $a \approx 0.06 \ fm$)

$$f_{D^+} = 212.6^{+1.1}_{-1.2} \ MeV \quad f_{D_s} = 249.0^{+1.3}_{-1.5} \ MeV \quad f_{D_s}/f_{D^+} = 1.172^{+31}_{-34}$$

** $\sim 0.5\%$ error dominated by continuum extrapolation error

** They calculate the difference between $f_{D^+}$ and the isospin limit value, $f_D$:

$$f_{D^+} - f_D = 0.47^{+11}_{-5} \ MeV$$

* **ETMC, 1411.7908:** ETMC configurations with 3 $a'$s (smallest $a \approx 0.06 \ fm$), $m_\pi \geq 210 \ MeV$

$$f_D = 207.2(3.8) \ MeV \quad f_{D_s} = 247.2(4.1) \ MeV \quad f_{D_s}/f_D = 1.192(22)$$

** Error dominated by stat.+ chiral extrapolation error
2. Leptonic $D$ decays

<table>
<thead>
<tr>
<th>$N_f$</th>
<th>$f_D$ (MeV)</th>
<th>$f_{D_s}$ (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>(208 ± 7)</td>
<td>(250 ± 7)</td>
</tr>
<tr>
<td>2+1</td>
<td>(209.2 ± 3.3)</td>
<td>(248.6 ± 2.7)</td>
</tr>
<tr>
<td>2+1+1</td>
<td></td>
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</table>

**FLAG – 2, $N_f = 2$**

$f_D = (208 \pm 7) \text{ MeV}$

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FNAL/MILC $N_f = 2 + 1 + 1$

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2. **Leptonic \( D \) decays**

\[
\frac{f_{D_s}}{f_D}
\]

**FLAG – 2, \( N_f = 2 \)**

\[
\frac{f_{D_s}}{f_D} = 1.20 \pm 0.020
\]

**FLAG – 2, \( N_f = 2 + 1 \)**

\[
\frac{f_{D_s}}{f_D} = 1.187 \pm 0.012
\]

\( N_f = 2 + 1 + 1: \)

\[
\left| \frac{f_{D_s}}{f_D} \right|_{\text{FNAL/MILC}} = 1.1712^{+0.031}_{-0.034}
\]

\[
\left| \frac{f_{D_s}}{f_D} \right|_{\text{ETMC}} = 1.192 \pm 0.022
\]
2. Leptonic $D$ decays

**Experiment:** Average from G. Rong, CKM2014, 1411.3868 and unitarity values

$|V_{cs}| = 0.97343 \pm 0.00015$, $|V_{cd}| = 0.22522 \pm 0.00061$ from PDG2014:

$$f_{D_s}/f_D|_{\text{exp.}} = 1.270 \pm 0.036$$

2.7σ larger than $N_f = 2 + 1 + 1$ FNAL/MILC result
and 2.3σ larger than $N_f = 2 + 1$ FLAG-2 average

**FLAG – 2, $N_f = 2$**

$$f_{D_s}/f_D = 1.20 \pm 0.020$$

**FLAG – 2, $N_f = 2 + 1$**

$$f_{D_s}/f_D = 1.187 \pm 0.012$$

$N_f = 2 + 1 + 1$:

$$f_{D_s}/f_D|_{\text{FNAL/MILC}} = 1.1712^{+0.031}_{-0.034}$$
$$f_{D_s}/f_D|_{\text{ETMC}} = 1.192 \pm 0.022$$
2. **Leptonic $D$ decays:** $N_f = 2 + 1$ calculations in progress

* RBC/UKCD, 1502.00845: plans to calculate several charm physics observables (and extrapolate to $a m_b$).
2. **Leptonic $D$ decays:** $N_f = 2 + 1$ calculations in progress

* **RBC/UKCD, 1502.00845:** plans to calculate several charm physics observables (and extrapolate to $am_b$).

* **FNAL/MILC, 1501.01991:** Fermilab $c +$ asqtad (relativistic) $s, u = d$ calculation with 5 $a'$s and high statistics

** Estimated total error: $f_D \sim 2.1\%, f_{Ds} \sim 1.8\%, f_{Ds}/f_D \sim 1\%$

(larger error is heavy-quark mass tuning for $f_{D(s)}$)

** Errors reduced by a factor of $\sim 2.5$, now comparable to HPQCD

** Use same action for $b$ and $c \rightarrow$ precise calculations of ratios $f_B/f_D$ and $f_{Bs}/f_{Ds}$ (many systematics cancel)
2. Leptonic decays: Charm-light and charm-charm vector mesons

\[ \langle 0 | V_i | M^* (\vec{0}, \lambda) \rangle = f_{M^*} m_{M^*} e_\lambda^i \]

where \( e_\lambda^i \) is the polarization vector of the meson \( M^* \).
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where \( e^\lambda_i \) is the polarization vector of the meson \( M^* \).

# Predictions for \( f_{D^*} \) and \( f_{D^*_s} \) (using relativistic action for \( c \))

* \( N_f = 2 \) on ETM configurations

\[
\frac{f_{D^*_s}}{f_{D_s}} = 1.26(3) \quad \frac{f_{D^*}}{f_D} = 1.208(27)
\]

Becirevic et al, 1201.4039 \hspace{1cm} Becirevic et al, 1407.1019

* \( N_f = 2 + 1 \) calculation by HPQCD, 1312.5264

\[
\frac{f_{D^*_s}}{f_{D_s}} = 1.10(2) \rightarrow f_{D^*_s} = 274(6) \text{ MeV}
\]
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\[ f_{D_s^*}/f_{D_s} = 1.10(2) \rightarrow f_{D_s^*} = 274(6) \text{ MeV} \]

## Calculations of \( f_{J/\Psi} \) (experimental value \( f_{J/\Psi}^{exp} = 407(5) \text{ MeV} \))

* Calculation on \( N_f = 2 \) ETM configurations by Becirevic and Sanfilippo, 1206.1445:

\[ f_{J/\Psi} = 414 \pm 8^{+9}_{-0} \text{ MeV} \]

* \( N_f = 2 + 1 \) calculation by HPQCD, 1208.2855:

\[ f_{J/\Psi} = 405(6)(2) \text{ MeV} \]
3. **Semileptonic $D$ decays**

$$P = \pi, K$$

$$x = d, s \text{ daughter light quark}$$

$$q = (p_D - p_P) \text{ (momentum of lepton pair)}$$

$$\frac{d\Gamma(D \rightarrow Pl\nu)}{dq^2} = \frac{G_F^2}{24\pi^3} \frac{(q^2 - m_l^2)^2 \sqrt{E_P^2 - m_P^2}}{q^4 m_D^2} |V_{cx}|^2$$

$$\left[ \left(1 + \frac{m_l^2}{2q^2} \right) m_D^2 (E_P^2 - m_P^2) |f_+(q^2)|^2 + \frac{3m_l^2}{8q^2} (m_D^2 - m_P^2)^2 |f_0(q^2)|^2 \right]$$

With vector and scalar form factors $f_+(q^2)$ and $f_0(q^2)$ defined by

$$\langle P(p_P)|V_\mu|D(p_D)\rangle = \left( p_{P\mu} + p_{D\mu} - \frac{m_D^2 - m_P^2}{q^2} q_\mu \right) f_+(q^2) + \frac{m_D^2 - m_P^2}{q^2} q_\mu f_0(q^2)$$
3. Semileptonic $D$ decays

For $l = e, \mu$ the contribution from $f_0(q^2)$ can be neglected and

$$\frac{d\Gamma(D \to Pl\nu)}{dq^2} = \frac{G_F^2}{24\pi^3} |\vec{p}_P|^3 |V_{cx}|^2 |f_{+}^{DP}(q^2)|^2$$

- experimental
- lattice QCD
3. Semileptonic $D$ decays

For $l = e, \mu$ the contribution from $f_0(q^2)$ can be neglected and

\[
\frac{d\Gamma(D \to Pl\nu)}{dq^2} \propto \frac{G_F^2}{24\pi^3} \left| \vec{p}_P \right|^3 |V_{cx}|^2 \underbrace{\left| f_+^{DP}(q^2) \right|^2}_{\text{lattice QCD}}
\]

The errors on those studies are still dominated by errors in the calculation of the relevant form factors.

\[
\frac{d}{dq^2} \Gamma(D \to K(\pi)l\nu) \propto |V_{cs(cd)}|^2 \left| f_+^{D \to K(\pi)}(q^2) \right|^2
\]

$1.1(2.8)\%$ error  $\quad 5(8.7)\%$ error
3. **Semileptonic** $D$ **decays:** $q^2 = 0$

Two main strategies to eliminate the need of renormalize the lattice currents

- **Double ratios of 3-point correlators**  
  Becirevic, Haas, Mescia 0710.1741  
  (get the form factors from linear combinations of the double ratios)
3. Semileptonic $D$ decays: $q^2 = 0$

Two main strategies to eliminate the need of renormalize the lattice currents

- Double ratios of 3-point correlators \cite{Becirevic2007}
  (get the form factors from linear combinations of the double ratios)

- Use the Ward identity ($S = \bar{x}c$) \cite{HPQCD2010}

\[
q^\mu \langle P|V^\text{cont.}_\mu |D\rangle = (m_c - m_x)\langle P|S^\text{cont.}|D\rangle
\]

that relates matrix elements of vector and scalar currents. In the lattice

\[
q^\mu \langle P|V^\text{lat.}_\mu |D\rangle Z = (m_c - m_x)\langle P|S^\text{lat.}|D\rangle
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3. Semileptonic $D$ decays: $q^2 = 0$

Two main strategies to eliminate the need of renormalize the lattice currents

- Double ratios of 3-point correlators \textbf{Becirevic, Haas, Mescia} 0710.1741 (get the form factors from linear combinations of the double ratios)

- Use the Ward identity ($S = \bar{c}c$) \textbf{HPQCD}, Phys.Rev.D82:114506(2010)

$$q^\mu \langle P|V^{\text{cont.}}_\mu |D\rangle = (m_c - m_x) \langle P|S^{\text{cont}}|D\rangle$$

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$$q^\mu \langle P|V^{\text{lat.}}_\mu |D\rangle Z = (m_c - m_x) \langle P|S^{\text{lat.}}|D\rangle$$

→ replace the $V_\mu$ with an $S$ current in the 3-point function

$$f_0^{DP}(q^2) = \frac{m_c - m_x}{m_D^2 - m_P^2} \langle P|S|D\rangle_{q^2} \implies f_0^{PD}(0) = f_0^{PD}(0) = \frac{m_c - m_x}{m_D^2 - m_P^2} \langle S \rangle_{q^2=0}$$
3. **Semileptonic $D$ decays: $q^2 = 0$**

Important reduction of errors in the lattice determination of the form factors $f_+^{D\pi(K)}(0)$ by the **HPQCD Collaboration**, Phys.Rev.D82:114506(2010), due mainly to

* Use a relativistic action, HISQ, to describe light and charm quarks.

* Absolutely normalized current

**HPQCD**, 1008.4562, 1109.1501

$$f_+^{D\pi}(0) = 0.666(29)$$

$$f_+^{DK}(0) = 0.747(19)$$

# Work in progress: $N_f = 2 + 1 + 1$ **FNAL/MILC**, 1411.1651 with physical quark masses.
3. **Semileptonic $D$ decays: $q^2 \neq 0$**

Determination of $|V_{cs}|$ from $D \to Kl\nu$ at non-zero momentum transfer

*HPQCD*, 1305.1462

Calculation of $f_{DK}^0(q^2)$ (using Ward identity method) and $f_{DK}^+(q^2)$ (using its definition)

* Global fit to available experimental data (using $z-$expansion) $\rightarrow$ extraction of $|V_{cs}|$ using all experimental $q^2$ bins.
3. **Semileptonic** $D$ **decays:** $q^2 \neq 0$

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* Global fit to available experimental data (using $z$—expansion) $\rightarrow$ extraction of $|V_{cs}|$ using all experimental $q^2$ bins.

![Graph showing the ratio of experimental to lattice values for $V_{cs}$ with $q^2$ bins in GeV$^2$.]

$|V_{cs}| = 0.963(5)_{\text{exp}}(14)_{\text{lat}}$

**Unitarity value PDG2014:** $|V_{cs}| = 0.97343 \pm 0.00015$
3. Semileptonic $D$ decays: $q^2 \neq 0$

# Work in progress $(N_f = 2)$:

* **ETM**: Preliminary results in PoSLattice 2013, 391 (2013)

# Work in progress $(N_f = 2 + 1)$:

* **FNAL/MILC**: Preliminary results in 1211.4964. Fermilab charm and staggered light, four lattice spacings, $m_\pi \geq 180$ MeV.

** Same actions used for $B \to \pi l \nu$ form factors calculation → can calculate accurately $\left| \frac{f_{B\pi}}{f_{D\pi}} \right|$ → alternative calculation of $|V_{ub}|$ (see A. Oyanguren talk)

# Work in progress $(N_f = 2 + 1 + 1)$:

* **ETM, Lattice2014**: Twisted mass, three lattice spacings, $m_\pi \geq 210$ MeV

* **FNAL/MILC**: relativistic action for $c$, physical quark masses, four lattice spacings.
3. **Semileptonic $D$ decays:** beyond gold-platted quantities

- Alternative determination of $|V_{cs}|$: $D_s \to \phi l\nu$ [HPQCD, 1311.6669]

- More challenging: five form factors (vector meson), unstable meson ...

- Treat $\phi$ as stable and estimate the error.
3. Semileptonic $D$ decays: beyond gold-platted quantities

# Alternative determination of $|V_{cs}|$: $D_s \to \phi l \nu$ HPQCD, 1311.6669

More challenging: five form factors (vector meson), unstable meson ...

* Treat $\phi$ as stable and estimate the error.

* $q^2$ and angular distributions agree with BaBar data.

$$|V_{cs}| = 1.017(44)_{lat}(35)_{exp}(30)_{K\bar{K}}$$

* Expected reduction of exper. errors at BESIII $\to$ need improvement of theor. calculation (lattice error dominated by statistical error)

* Are the heavy meson form factors at a given $q^2$ insensitive to the spectator $m_q$?
  (compare $D_s \to \phi$ and $D \to K^*$).
3. **Semileptonic $D$ decays:** beyond gold-platted quantities

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- Exploratory $N_f = 2 + 1$ calculation of $D \rightarrow \eta^{(')} l\nu$ **G. Bali et al**, 1406.5449

  * Calculate $\eta - \eta'$ mixing angles and disconnected contributions
3. **Semileptonic $D$ decays: correlations with leptonic decays**

Cancel CKM matrix elements building ratios of semileptonic and leptonic decay widths

* $N_f = 2 + 1$ **HPQCD** calculation

$$\left[ \frac{f_{D^+}^{D\pi}(0)}{f_{D^+}} \right]_{\text{lat}} = (3.20 \pm 0.15) \text{ GeV}^{-1}$$

* Using **HPQCD** $f_{D^+}^{D\pi}(0)$ and $N_f = 2 + 1 + 1$ **FNAL/MILC** $f_{D^+}$

$$\left[ \frac{f_{D^+}^{D\pi}(0)}{f_{D^+}} \right]_{\text{lat}} = (3.13 \pm 0.14) \text{ GeV}^{-1}$$
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\]

\[
\left[ \frac{f_{+D\pi}(0)}{f_{D^+}} \right]_{exp} = (3.11 \pm 0.08) \text{ GeV}^{-1}\]

**G. Rong et al, 1410.3232**

Good agreement experiment-theory
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\]

Good agreement experiment-theory

# Several $N_f = 2 + 1$ and $N_f = 2 + 1 + 1$ calculations in progress.
4. $|V_{cd}|$, $|V_{cs}|$: CKM unitarity in the second row

# Extracting CKM matrix elements $|V_{cd(cs)}|$ from leptonic decays

Experimental averages: BaBar, Belle, CLEO-c, BESIII

$$f_{D_s}|V_{cs}| = (252.0 \pm 3.7 \pm 1.8) \text{ MeV} \quad f_{D^+}|V_{cd}| = (45.92 \pm 1.04 \pm 0.15) \text{ MeV}$$

Y. Fang et al, 1409.8049 \quad G. Rong et al, 1410.3232

Decay constant errors $\sim 0.5\%$ $\to$ need EM effects when combining with experiment
4. $|V_{cd}|$, $|V_{cs}|$: CKM unitarity in the second row

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Following FNAL/MILC, 1407.3772

* Universal long-distance EM: $\downarrow \sim 2.5\%$ Kinoshita, PRL2, 1959

* Universal short-distance EM: $\uparrow \sim 1.8\%$ Sirlin, NPB196, 1982

* Hadronic structure dependent EM effects: rough estimate $\sim 0.6\%$.

(phenomelogical estimates available only in the $K$ sector)
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$$|V_{cd}| = 0.220 \pm 0.004_{lat} \pm 0.005_{exp} \pm 0.001_{EM}$$

$$|V_{cs}| = 1.017 \pm 0.011_{lat} \pm 0.017_{exp} \pm 0.006_{EM} \quad \text{FLAG-2 } N_f = 2 + 1$$
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\[
|V_{cd}| = 0.217 \pm 0.001_{lat} \pm 0.005_{exp} \pm 0.001_{EM}
\]

\[
|V_{cs}| = 1.016 \pm 0.005_{lat} \pm 0.017_{exp} \pm 0.006_{EM} \quad \text{FNAL/MILC, } N_f = 2 + 1 + 1
\]
4. \(|V_{cd}|, |V_{cs}|: \) CKM unitarity in the second row

# Extracting CKM matrix elements \(|V_{cd(cs)}|\) from semileptonic decays

Experimental averages:

\[
f_{+}^{DK}(0)|V_{cs}| = 0.717 \pm 0.004 \quad f_{+}^{D\pi}(0)|V_{cd}| = 0.143 \pm 0.002
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Y. Fang et al, 1409.8049 \quad G. Rong et al, 1410.3232

(not included Babar, Phys.Rev.D 91 052022 (2015)

\(|V_{cd}|f_{+}^{D\pi}(0) = 0.1374 \pm 0.0038 \pm 0.0022 \pm 0.0009, \) talk by A. Oyanguren)

* Experimental averages for neutral and charged \(D\) do not remove corrections from Coulomb attraction between charged FS particles in neutral mode \(\sim 1\%\)

→ Needed when lattice errors are reduced (forthcoming calculations from FNAL/MILC, ETM, RBC/UKQCD, ...
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\[
|V_{cd}| = 0.215 \pm 0.009_{lat} \pm 0.003_{exp} \quad |V_{cs}| = 0.960 \pm 0.024_{lat} \pm 0.005_{exp}
\]

(with HPQCD, \( N_f = 2 + 1 \) \( f_+^{D\pi}(0) = 0.666(29) \) and \( f_+^{DK}(0) = 0.747(19) \))
4. $|V_{cd}|, |V_{cs}|$: CKM unitarity in the second row

* $|V_{cd}|$: Pretty good agreement between different determinations, but some tension

$N_f = 2 + 1 + 1$ FNAL/MILC leptonic-unitarity.

* $|V_{cs}|$: Slight tensions leptonic-semileptonic ($D \to KL\nu$) and leptonic-unitarity.
4. $|V_{cd}|$, $|V_{cs}|$: CKM unitarity in the second row

Using the most precise leptonic numbers ($N_f = 2 + 1 + 1$ FNAL/MILC)

$$1 - |V_{cd}|^2 - |V_{cs}|^2 - |V_{cb}|^2 = -0.07(4)$$

Using $N_f = 2 + 1$ FLAG-2 averages for decay constants

$$1 - |V_{cd}|^2 - |V_{cs}|^2 - |V_{cb}|^2 = -0.08(4)$$
4. Conclusions and outlook

# Relativistic description of charm → important reduction of lattice QCD errors in decay constants and semileptonic form factors ...

\[
\text{Error } f_{D(s)} \sim 0.5\% \quad \text{Error } f_{DK(\pi)}^{D^+} \sim 2.5 - 4.3\%
\]

... still theory errors are dominant in \(|V_{cd(cs)}|\) extractions from semileptonic decays.

* Several on-going calculations of the shape of \(f_{+(0)}(q^2)\) will further reduce errors FNAL/MILC (with two different descriptions of the \(c\)), ETM, HPQCD ...

** Need experimental results reported in a model independent way, i.e., in \(q^2\) bins (including full covariance matrix).

* Physical quark masses also important in the reduction of errors, especially for \(D^+\) quantities.
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At current level of precision we need to include subdominant effects:

* **EM effects** → Eventually will do QCD+QED simulations.


* Include **charm** in the sea

* **Strong isospin breaking effects:** leading order corrections included via tuning light valence quarks (effects of degenerate sea are NNLO in CHPT).
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# Extend the same techniques to $B$ physics
2. Leptonic $D$ decays: New results ($> 2013$)

Reduction of errors in $f_D$ and $f_{D_s}$ mainly due to the use of relativistic actions, and using the same action for light and charm quarks.

# $N_f = 2 + 1 + 1$:

* FNAL/MILC, 1407.3772: highly improved action, MILC configurations with phys. quark masses and small lattice spacing (smallest $a \approx 0.06$ fm)

\[
\begin{align*}
  f_{D_s} &= 249.0 \pm 0.3_{\text{stat}} \pm 0.2_{\text{extr.}} \pm 0.4_{FV} \pm 0.1_{em} \pm 0.1_{FV} \pm 0.3_{em} \pm 0.4_{f_\pi} \text{ MeV} \\
  f_{D^+} &= 212.6 \pm 0.4_{\text{stat}} \pm 0.2_{\text{extr.}} \pm 0.3_{FV} \pm 0.1_{em} \pm 0.3_{f_\pi} \text{ MeV}
\end{align*}
\]