Top mass theory and connection with experiment

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I.: **Basic facts about the top quark**

II: **Basic facts about the mass in general**

III: **Implications on the precision for the top mass**

Why do we care about precision on the top mass?

- Obviously, the value of the top mass affects the measured top cross sections.
- Affects searches for new physics with top background, BSM decays into tops, etc.
- Top mass close to the electro-weak breaking scale, impact on precision physics of the Higgs sector.
  If there is new physics associated with electro-weak symmetry breaking top physics is a place to look for.
- If the Standard Model is assumed to be valid to very high scales, the stability of the electro-weak vacuum depends crucially on the precise numerical value of m_t.

Basic facts about top

The essential numbers:

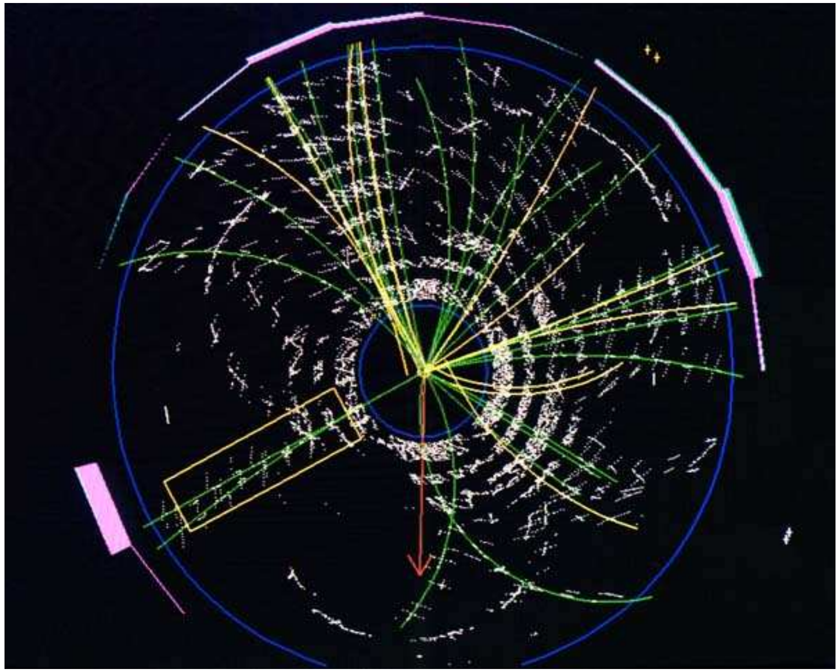
Mass:

$$m_t = 173.21 \pm 0.51 \pm 0.71 \text{ GeV}$$

Width:

$$\Gamma = 2.0 \pm 0.5 \text{ GeV}$$

Discovered at the Tevatron in 1995



A $t\bar{t}$ event from CDF.

Basic facts about top

The top quark is special:

+ The large top mass sets a hard scale.

+ Lifetime shorter than characteristic hadronization time scale.

$\Rightarrow$ Top physics is (mainly) described by perturbative QCD.

But, of course as any quark of the 2^{nd} or 3^{rd} generation:

- The top quark is a colour-charged particle.

- The top quark is not a stable particle.

$\Rightarrow$ There is no asymptotic free top state,
non-perturbative effects (might) enter here through the back door.

Why is there a theory talk on the top mass?

Up to now there is no “theory-free” experimental determination of the top mass.

Experimental measurements rely on theoretical input through template method / matrix element method.

The error on the top mass is approaching $O(\Lambda_{\mathrm{QCD}})$.

Can we in principle improve the error below $O(\Lambda_{\mathrm{QCD}})$?

We would like to be able to reduce the theory error systematically by calculating higher-order corrections.

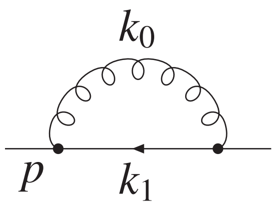
Basic facts about a fermion mass

Theorists like Lagrangians:

$$\mathcal{L}_{\text{fermion}} \quad = \quad \bar{\Psi}_{\text{bare}} \left(i\not{D} - m_{\text{bare}} \right) \Psi_{\text{bare}}$$

Beyond leading-order in perturbation theory: The (one-loop) self-energy:

$$-i\Sigma \quad =$$

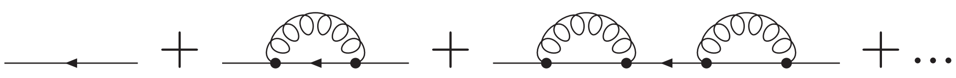


$$= \frac{g^2 C_F}{\mu^{D-4}} \int \frac{d^D k}{(2\pi)^D}\, i\gamma_\rho\, \frac{i}{\not{k}_1 - m_{\text{bare}}}\, i\gamma^\rho\, \frac{(-i)}{k_0^2} \quad = \quad -i\left(A\not{p} + B m_{\text{bare}}\right)$$

- In four space-time dimensions this integral is divergent.
- $D = 4 - 2\varepsilon$ is a regulator, divergences show up as poles $1/\varepsilon$.
- A and B depend on p^2, m^2_{bare} and μ^2.

Basic facts about a fermion mass

Resummed self-energy insertions:



$$= \frac{i}{\not{p} - m_{\text{bare}} - \Sigma}$$

$$= \frac{i(1+A)}{\not{p} - (1+A+B)m_{\text{bare}}} + O\left(\alpha_s^2\right)$$

Renormalisation:

$$\psi_{\text{bare}} = \sqrt{Z_2}\,\psi_{\text{renorm}}$$

$$m_{\text{bare}} = Z_m\, m_{\text{renorm}}$$

All renormalisation schemes entail:

- Wave function renormalisation: Absorb UV-divergences of $(1+A)$ in the numerator.
- Mass renormalisation: Absorb UV-divergences of $(1+A+B)$.

The $\overline{\rm MS}$-scheme

Absorb only the parts proportional to $\frac{1}{\varepsilon} - \gamma_E + \ln(4\pi)$ and nothing else into Z_m:

$$Z_m \quad = \quad 1 - (A+B)_{\rm div}$$

The propagator is then

$$\frac{i}{\not p - m_{\overline{\rm MS}} - (A+B)_{\rm fin} m_{\overline{\rm MS}}}$$

- $m_{\overline{\rm MS}}$ depends on the scale μ: Running mass.
- Presence of $(A+B)_{\rm fin} m_{\overline{\rm MS}}$: The propagator does not have a pole at $m_{\overline{\rm MS}}$, matrix elements do not factor at $p^2 = m_{\overline{\rm MS}}^2$.
- $(A+B)_{\rm fin}$ depends on p^2: Propagator does not yield Breit-Wigner shape.

The $\overline{\mathrm{MS}}$-scheme

$m_{\overline{\mathrm{MS}}}$ is an example of a short-distance mass.

Can extract $m_{\overline{\mathrm{MS}}}$ from an infrared safe observable for a process like $pp \to l\bar{\nu} j j b\bar{b}$ at high energies by comparing

$$\sigma_{\mathrm{exp}} \quad \text{with} \quad \sigma_{\mathrm{theo}}\left(m_{\overline{\mathrm{MS}}}\right)$$

Moch, Langenfeld, Uwer, '09;

Czakon, Fiedler, Mitov, '13;

Dowling, Moch, '13

Can also use $pp \to t\bar{t} + \text{jet}$.

Dittmaier, Uwer, S.W., '07,

Melnikov, Schulze, '10

The on-shell-scheme

Define Z_m such that the propagator has a pole at $m_{\rm pole}$.

The propagator is then by definition

$$\frac{i}{\not{p} - m_{\rm pole}}$$

+ $m_{\rm pole}$ is complex, includes the width.

+ Matrix elements factor at $p^2 = m^2_{\rm pole}$.

+ Propagator corresponds to a Breit-Wigner shape.

- The pole mass is not a short distance mass.

Non-perturbative sensitivity related to the pole mass

The pole mass is ambigous by an amount $O(\Lambda_{\mathrm{QCD}})$:

- In the on-shell scheme, the renormalisation constant Z_m contains contributions from all momentum scales, not just the ultraviolet region.

- In higher orders, subsets of diagrams are dominated by the IR-region.

- Therefore, the full perturbative series can only be summed up to an (infrared) renormalon ambiguity.

- The renormalon ambiguity is of $O(\Lambda_{\mathrm{QCD}})$.

Bigi, Shifman, Uraltsev, Vainshtein, '94, Beneke, '94, Smith, Willenbrock, '96

Conversion between the pole mass and the $\overline{\mathrm{MS}}$-mass

In perturbation theory one has with $\bar{m} = m_{\overline{\mathrm{MS}}}(\mu = m_{\overline{\mathrm{MS}}})$

$$m_{\text{pole}} \quad = \quad \bar{m} \times \left[1 + c_1 \frac{\alpha_s(\bar{m})}{\pi} + c_2 \left(\frac{\alpha_s(\bar{m})}{\pi} \right)^2 + c_3 \left(\frac{\alpha_s(\bar{m})}{\pi} \right)^3 + c_4 \left(\frac{\alpha_s(\bar{m})}{\pi} \right)^4 + \ldots \right]$$

Melnikov, van Ritbergen, ’99; Chetyrkin, Steinhauser, ’99; Marquard, A. Smirnov, V. Smirnov, Steinhauser, ’15

Numerically for the top quark:

$$m_{\text{pole}} \quad = \quad \bar{m} \times [1 + 0.046 + 0.010 + 0.003 + 0.001 + \ldots]$$

The conversion formula is again only an asymptotic series and has an renormalon ambiguity as well.

Crude estimates of the ambiguity

From the truncation of the conversion formula between $m_{\rm pole}$ and $\bar{m}$:

$$\delta m_{\rm pole} \quad \approx \quad \mathcal{O}(\, 200\,{\rm MeV}\,)$$

From the estimate of the renormalon:

$$\delta m_{\rm pole} \quad \approx \quad \mathcal{O}(\, 270\,{\rm MeV}\,)$$

What about determining the non-perturbative effects by comparing two different non-perturbative models?

Engineer A: $13^2 = 172$ (sic)

Engineer B: $13^2 = 174$ (sic)

This does not imply $13^2 = 173 \pm 1$ (sic)

Measuring the peak position

Can one translate a measurement of the peak position into a theoretical well defined short-distance top mass?

Remark: Experimentalists can measure many things to high precision (average number of pions in pp collisions, etc.), the question is if and how a quantity can be related to a quantity depending only on short-distance physics.

Let's split up this question:

- Which scales are involved?
- How to define a short-distance mass at a given scale?
- How to translate the measurement?

The involved scales

In order to avoid large logarithms:

- Describe physics at a particular scale μ by an appropriate effective theory.
- Evolution operators sum up large logarithms.

From a study of $e^+e^- \to t\bar{t}$:

Scale	Matrix elements	Effective theory	Affects	Remarks
$Q ... m_t$	hard function	QCD	norm of the distribution	depends on m_t
$m_t ... \Gamma_t$	jet function	SCET	shape and position	depends on m_t
$\Gamma_t ... \Lambda_{QCD}$	soft function	top-HQET	shape and position	independent of m_t

$\Rightarrow$ Need a short-distance mass definition for scales down to Γ_t.

Fleming, Hoang, Mantry, Stewart, '07

The MSR mass

Short-distance mass: any mass definition not affected by a renormalon ambiguity.

Idea for construction: Remove contributions giving rise to this ambiguity (known from bottomium, potential subtracted mass).

This will involve apart from the UV-renormalisation scale μ a second scale R.

The $\overline{\mathrm{MS}}$-mass is a short-distance mass, and $R = \bar{m}$ in this case.

The MSR-mass (read: $\bar{m}$ substituted by R) is the two-scale generalisation with a UV-scale μ and an IR-scale R, such that

$$m_{\mathrm{MSR}}(R=0) = m_{\mathrm{pole}}, \qquad m_{\mathrm{MSR}}(R=\bar{m}) = \bar{m}.$$

Hoang, Jain, Scimemi, Stewart, '08

An analogy

Jet cross section: Jets defined by

- an infrared-safe jet algorithm (SISCone, k_t-algorithm, anti-k_t-algorithm, etc.)
- parameters associated to this algorithm (R, f, n_{pass}, y_{cut}, etc.)

Top mass: Mass defined by

- a short-distance renormalisation scheme ($\overline{\mathrm{MS}}$-scheme, MSR-scheme, etc.)
- parameters associated to this scheme (μ, R, etc.)

Translating the measurement

Theory sneaks in though template method / matrix element method.

Analogy of factorisation:

Effective theory:	Hard function / jet function / soft function
Monte Carlo:	Hard matrix element / parton shower / hadronisation

Parton shower has a lower cut-off.

$\Rightarrow$ Monte Carlo mass is something like a short-distance mass.

Translation for Pythia:

$$m_{\rm Pythia} \quad = \quad m_{\rm MSR}\,(R = 1...9\ {\rm GeV})$$

This introduces an uncertainty of the order of $1\ {\rm GeV}$ on the translation from the Monte Carlo mass to a theoretically well defined short-distance mass.

Hoang, Stewart, ’08

Work to do

- Work out in detail factorisation and short-distance mass in pp-collisions.
  - Coloured initial states.
  - Jets instead of hemisphere masses.
- Compare in detail MC mass with a well defined short-distance mass.
  - Establish that shower cut-off effectively implements some short-distance mass.
  - Improve translation from MC mass to well-defined short-distance mass.
- Consider practical issues:
  - Can we write a dedicated event generator, based on a well-defined short distance mass?

    Proposals (not specific to top) to go from SCET to exclusive event generators

    Bauer and Schwartz, '06; Bauer, Tackmann and Thaler, '08

Summary

- The value of the top mass is essential for many precision measurements.
- Want to have a well defined short-distance mass.
  - At high scales the $\overline{\mathrm{MS}}$-mass can be used.
  - The pole mass is not a short-distance mass.
  - The MSR-mass can be used as a short-distance mass at lower scales.
- State of the art: Conceptionally understood,
  details have to be worked out.
- Outlook below $100\ \mathrm{MeV}$ uncertainty: Threshold scan at an e^+e^--machine with a potential subtracted mass or $1\mathrm{S}$ mass.