The role of top in the Standard Model

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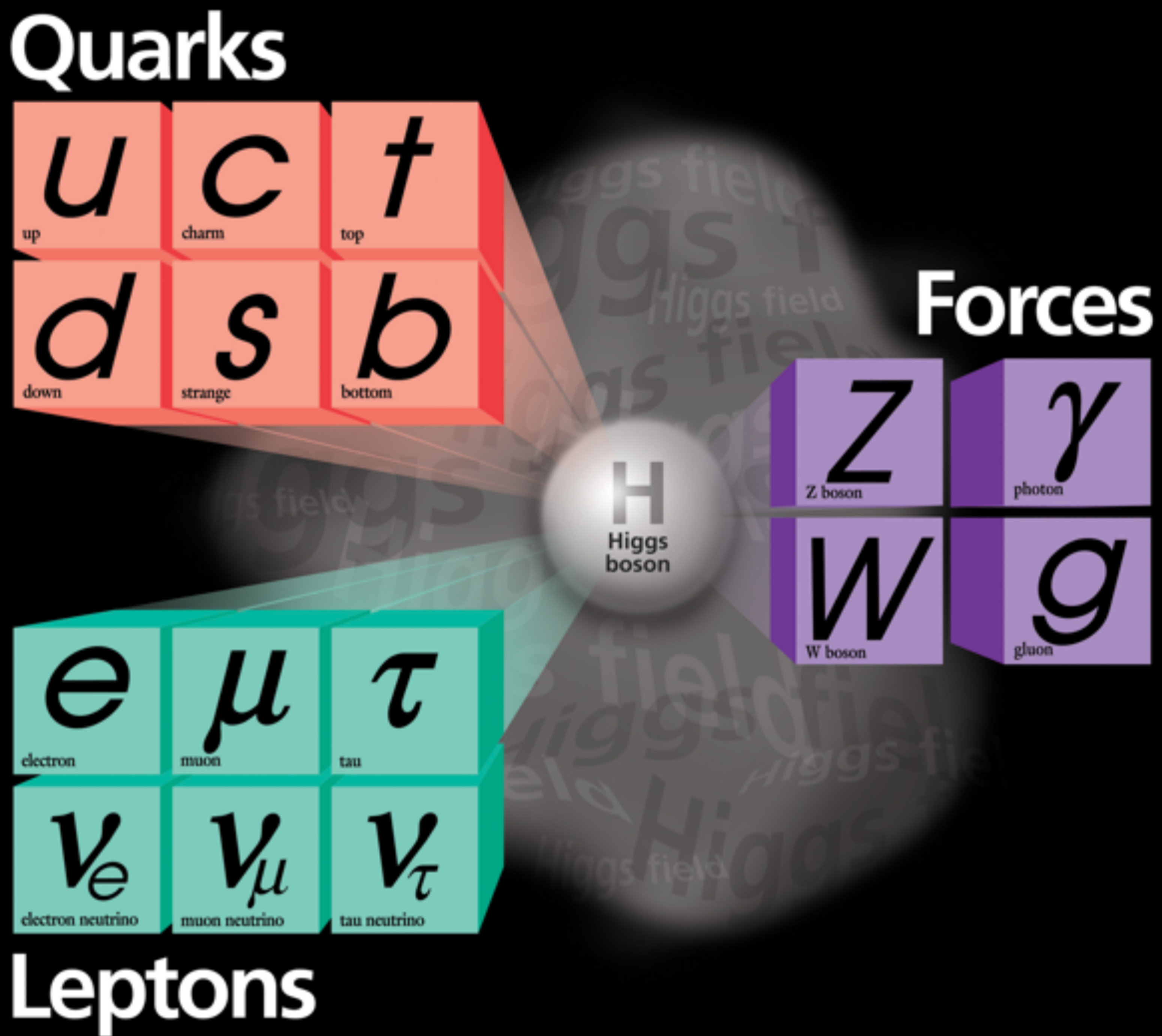
Top at Twenty, 9-10 April 2015, FNAL

Standard Model



Global flavor symmetry

[see e.g. Nir, hep-ph/0109090]

In absence of Yukawa couplings

$$\frac{vY_u}{\sqrt{2}} = L_u^\dagger \begin{pmatrix} m_u & & \\ & m_c & \\ & & m_t \end{pmatrix} R_u \qquad \frac{vY_d}{\sqrt{2}} = L_d^\dagger \begin{pmatrix} m_d & & \\ & m_s & \\ & & m_b \end{pmatrix} R_d$$

Standard Model (SM) globally symmetric under

$$G_{\text{flavor}} = SU(3)_{Q_L} \times SU(3)_{u_R} \times SU(3)_{d_R}$$

Charged vs. neutral currents

[see e.g. Nir, hep-ph/0109090]

Charge currents measure only left-handed misalignment

$$\bar{u}_L \not{W} d_L \;\rightarrow\; L_u^\dagger L_d \, \bar{u}_L \not{W} d_L = V_{ud} \, \bar{u}_L \not{W} d_L$$

parameterized by Cabibbo-Kobayashi-Maskawa (CKM) matrix V. Instead, neutral currents

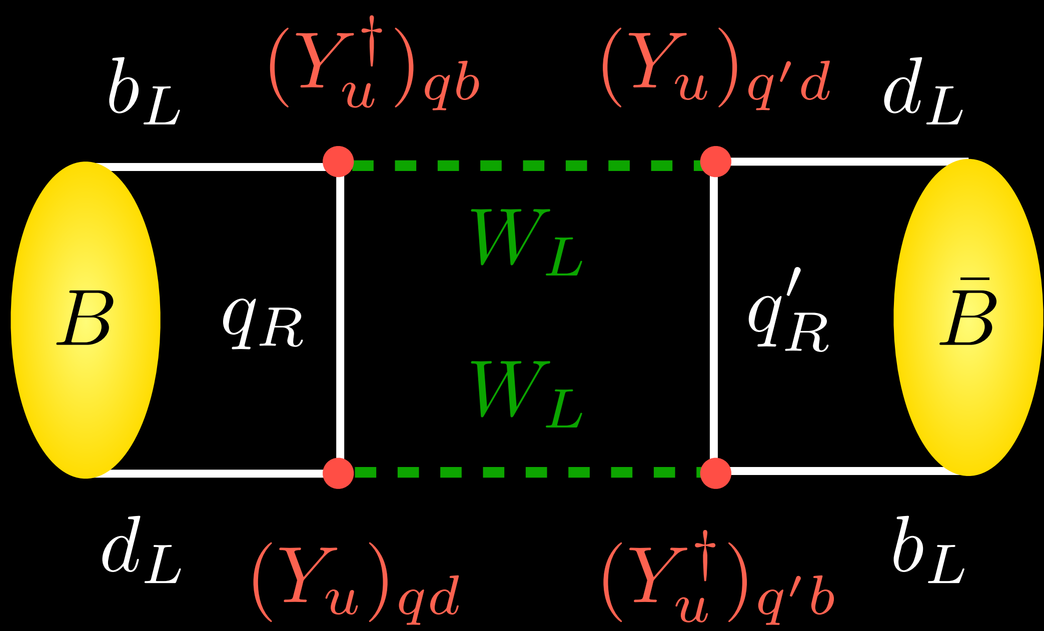
$$\bar{u}_L \not{Z} u_L \;\rightarrow\; L_u^\dagger L_u \, \bar{u}_L \not{Z} u_L = 1 \, \bar{u}_L \not{Z} u_L$$

remain flavor diagonal at tree level, due to enhanced $SU(3)_{u_L} \times SU(3)_{d_L}$ flavor symmetry

Flavor changing neutral currents

[see e.g. D'Ambrosio et al., hep-ph/0207036]

In fact, neutral meson mixing & other flavor changing processes test structure of Yukawa interactions beyond tree level



$$Y_u = V^\dagger \, \mathrm{diag}\,(y_u, y_c, y_t)$$

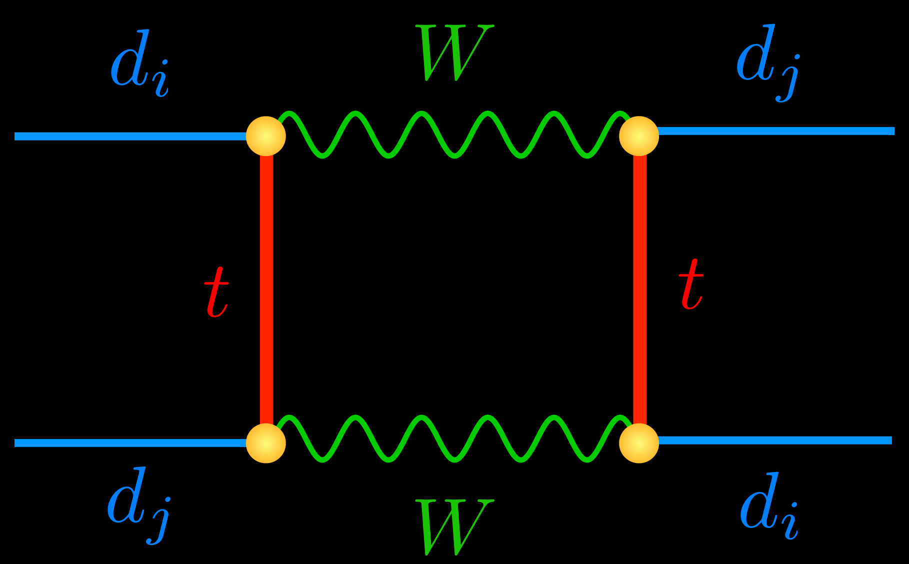
$$\approx V^\dagger \, \mathrm{diag}\,(0, 0, y_t)$$

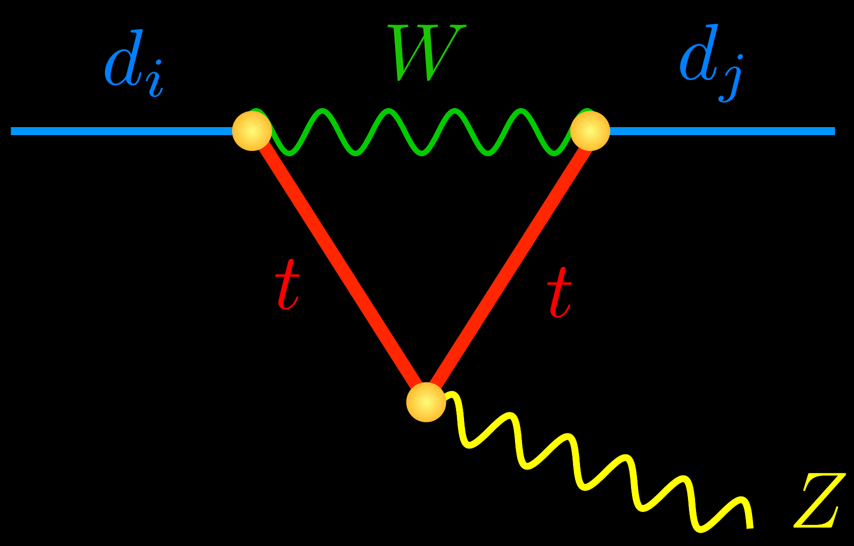
$$\Longrightarrow \quad \frac{m_t^2}{16\pi^2 m_W^4 m_t^4} \, y_t^4 \left(V_{tb}^* V_{td}\right)^2 \;\propto\; \frac{g_2^2}{16\pi^2 m_W^4} \, m_t^2 \left(V_{tb}^* V_{td}\right)^2$$

Boxes & Z penguins

[see e.g. Buras, hep-ph/9806471]

Within SM, only two 1-loop topologies lead to a quadratic dependence on top mass





$\Downarrow$

$\Delta M_K, \Delta M_{B_d}, \Delta M_{B_S}, \epsilon_K$

$\Downarrow$

$B_{d,s} \to \mu^+\mu^-, B \to K^{(*)}, X_s\mu^+\mu^-$

$K \to \pi\nu\bar{\nu}, K \to \pi\mu^+\mu^-, \epsilon'/\epsilon, Z \to b\bar{b}$

SM Higgs sector

[see e.g. Sikivie et al., Nucl. Phys. B173, 189 (1980)]

$$SU(2)_L \times SU(2)_R$$

$$\mathcal{L}_{\text{Higgs}} = \text{Tr}\left[(D_\mu \Phi)^\dagger (D^\mu \Phi)\right] + \mu^2\, \text{Tr}\left(\Phi^\dagger \Phi\right) - \lambda \left[\text{Tr}\left(\Phi^\dagger \Phi\right)\right]^2$$

$$\Phi = \frac{1}{\sqrt{2}}\left(\epsilon\phi^*, \phi\right) = \frac{1}{\sqrt{2}}\begin{pmatrix} \phi^{0*} & \phi^+ \\ -\phi^- & \phi^0 \end{pmatrix} \to L\,\Phi R^\dagger$$

In limit of vanishing hypercharge coupling ($g_1 \to 0$), SM Higgs sector has global $SU(2)_L$ symmetry & accidental global $SU(2)_R$ symmetry

SM Higgs sector

[see e.g. Sikivie et al., Nucl. Phys. B173, 189 (1980)]

$$SU(2)_L \times SU(2)_R \xrightarrow[\langle\Phi\rangle]{} SU(2)_{L=R}$$

$$\mathcal{L}_{\text{Higgs}} = \text{Tr}\left[(D_\mu \Phi)^\dagger (D^\mu \Phi)\right] + \mu^2\, \text{Tr}\left(\Phi^\dagger \Phi\right) - \lambda \left[\text{Tr}\left(\Phi^\dagger \Phi\right)\right]^2$$

$$\langle\Phi\rangle = \frac{1}{2}\begin{pmatrix} v & 0 \\ 0 & v \end{pmatrix} \to L\,\langle\Phi\rangle\, L^\dagger = \langle\Phi\rangle$$

Higgs vacuum expectation value v breaks global $SU(2)_L \times SU(2)_R$ down to diagonal subgroup $SU(2)_{L=R}$ aka custodial symmetry

Electroweak gauge sector

[see e.g. Sikivie et al., Nucl. Phys. B173, 189 (1980)]

$$M_W^2 = \frac{g_2^2 v^2}{4} \qquad M_Z^2 = \frac{\left(g_1^2 + g_2^2\right) v^2}{4} \qquad \cos^2\theta_w = \frac{g_2^2}{g_1^2 + g_2^2}$$

$$\Longrightarrow \quad \boxed{\rho = \frac{M_W^2}{M_Z^2 \cos^2\theta_w} = 1}$$

Custodial symmetry guarantees that in SM, ρ parameter equal 1 at tree level. What happens at loop level?

Yukawa sector in SM

[see e.g. Sikivie et al., Nucl. Phys. B173, 189 (1980)]

$$\mathcal{L}_{\text{Yukawa}} = -y_t \bar{Q}_L \epsilon \phi^* t_R - y_b \bar{Q}_L \phi b_R + \text{h.c.}$$

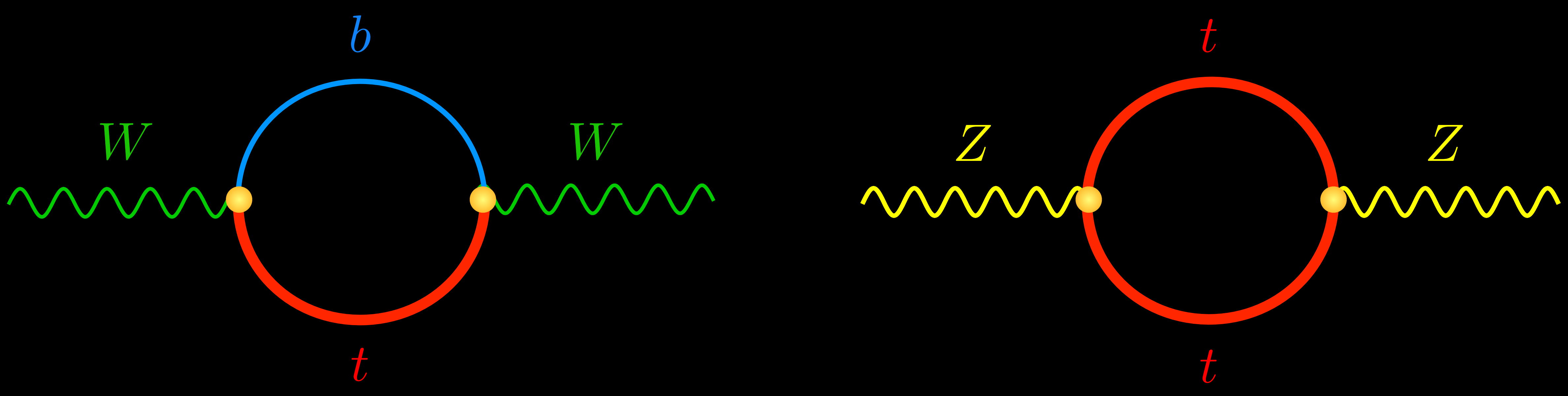
$$\overset{y=y_t=y_b}{\Longrightarrow} \quad \boxed{-y \bar{Q}_L \Phi Q_R + \text{h.c.}}$$

$$Q_L = \begin{pmatrix} t_L \\ b_L \end{pmatrix} \to L Q_L \qquad Q_R = \begin{pmatrix} t_R \\ b_R \end{pmatrix} \to R Q_R$$

Yukawas would be $SU(2)_L \times SU(2)_R$ invariant if $y_t = y_b$. Symmetry breaking proportional to squared mass difference $(m_t - m_b)^2$ of top & bottom

1-loop corrections to ρ

[cf. Veltman, Nucl. Phys. B123, 89 (1977)]



$$\Delta\rho = \alpha\Delta T = \frac{3G_F}{8\sqrt{2}\pi^2} m_t^2 \left\{ 1 + \frac{m_b^2}{m_t^2}\left[1 + \frac{2\ln\left(\frac{m_b^2}{m_t^2}\right)}{1-\frac{m_b^2}{m_t^2}}\right]\right\}$$

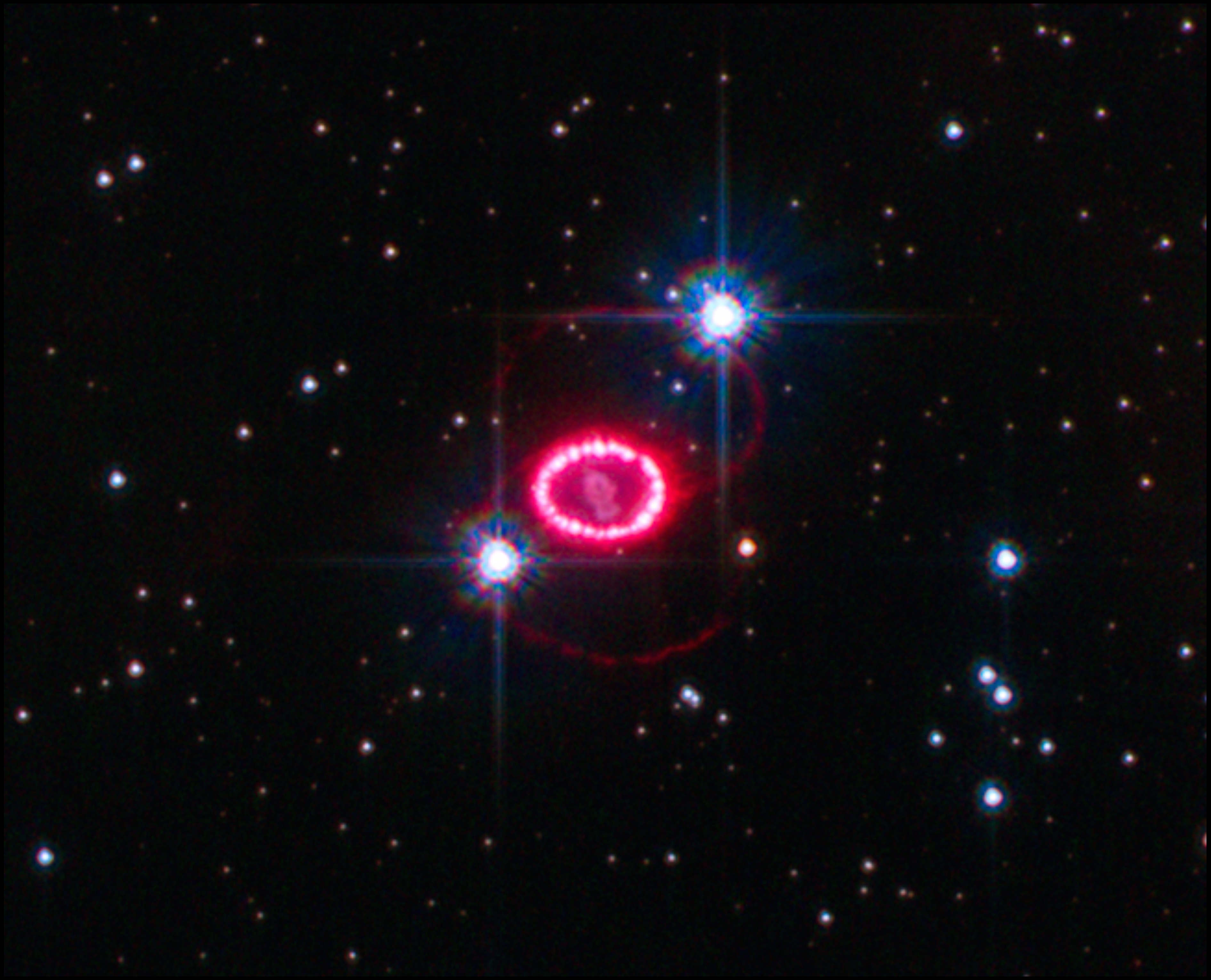
Dominant 1-loop corrections due to top exchange & proportional to y_t^2. In contrast, Higgs contribution scales as $g_1^2 \ln(m_h^2/m_Z^2)$

1987

[http://en.wikipedia.org/wiki/1987]

Events:

- …
- February 23 – SN 1987A, the first "naked-eye" supernova since 1604, is observed



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- June 12 – During a visit to Berlin, Germany, U.S. President Ronald Reagan challenges Soviet Premier Mikhail Gorbachev to tear down the Berlin Wall


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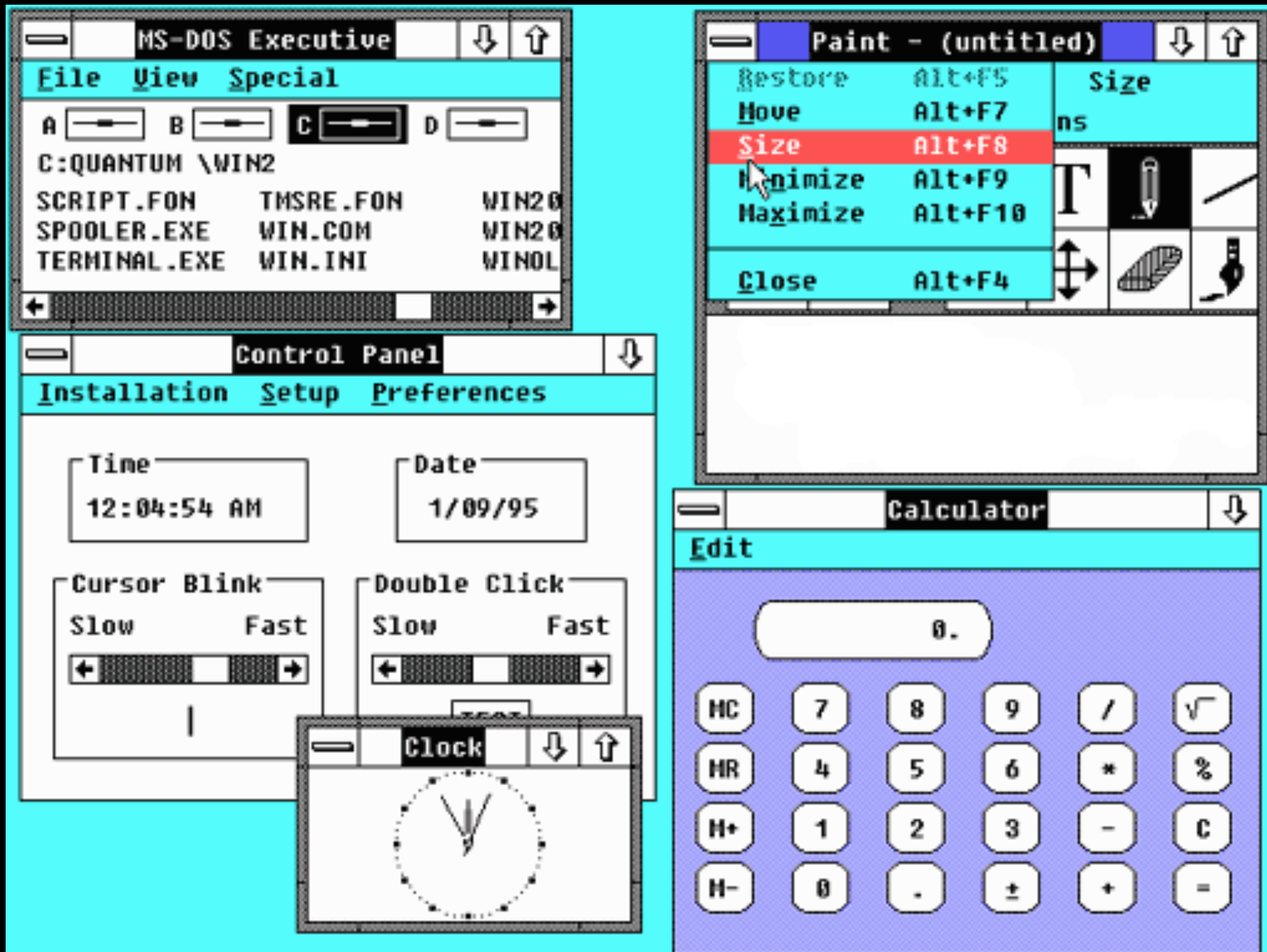
- …
- December 9 – Microsoft releases Windows 2.0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Maximize Alt+F10
Close Alt+F4
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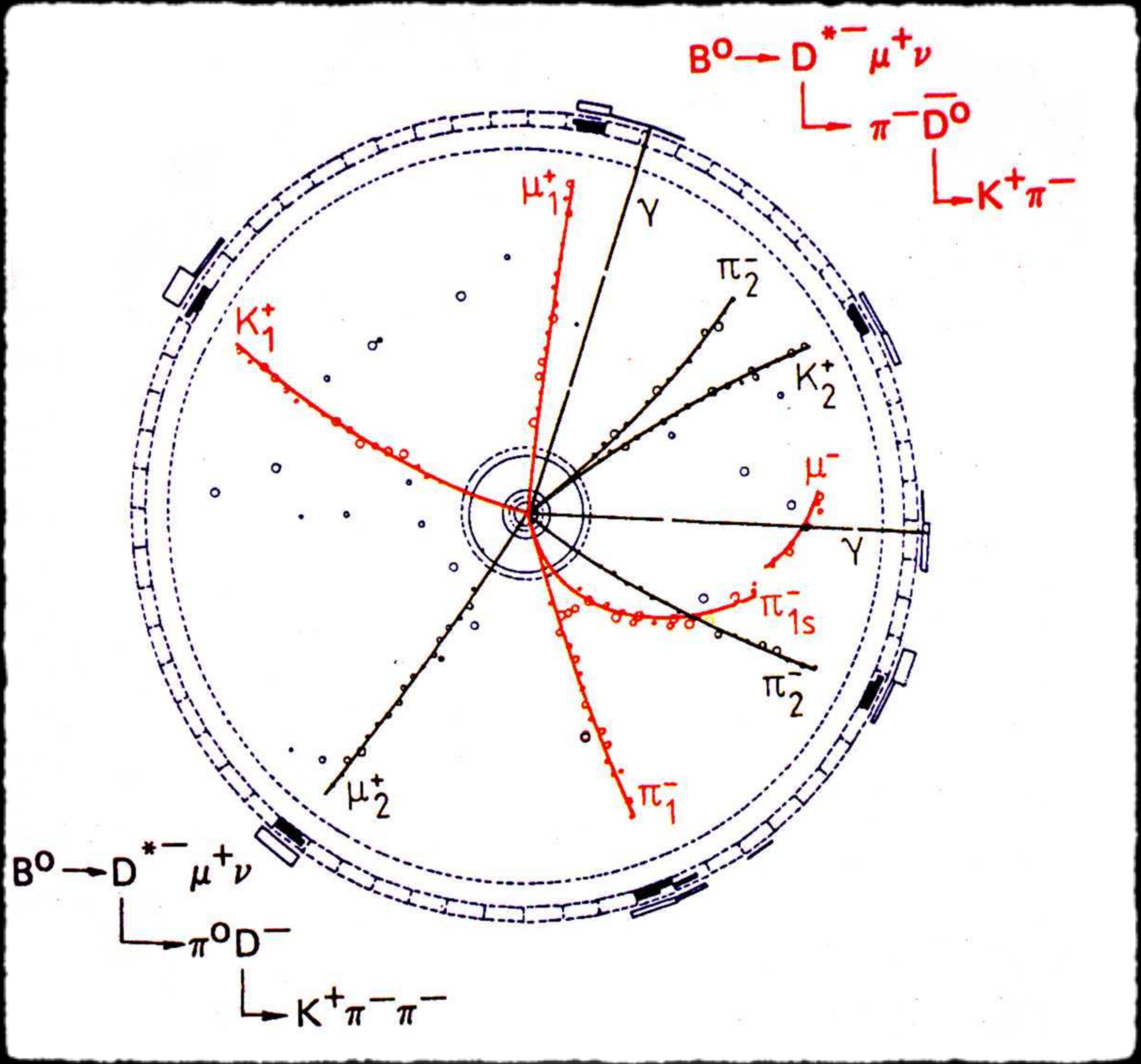
Not on list: $\Upsilon(4S) \to B^0\bar{B}^0 \to B^0B^0$

[ARGUS, Phys. Lett. B192, 245 (1987)]



Implications for top mass

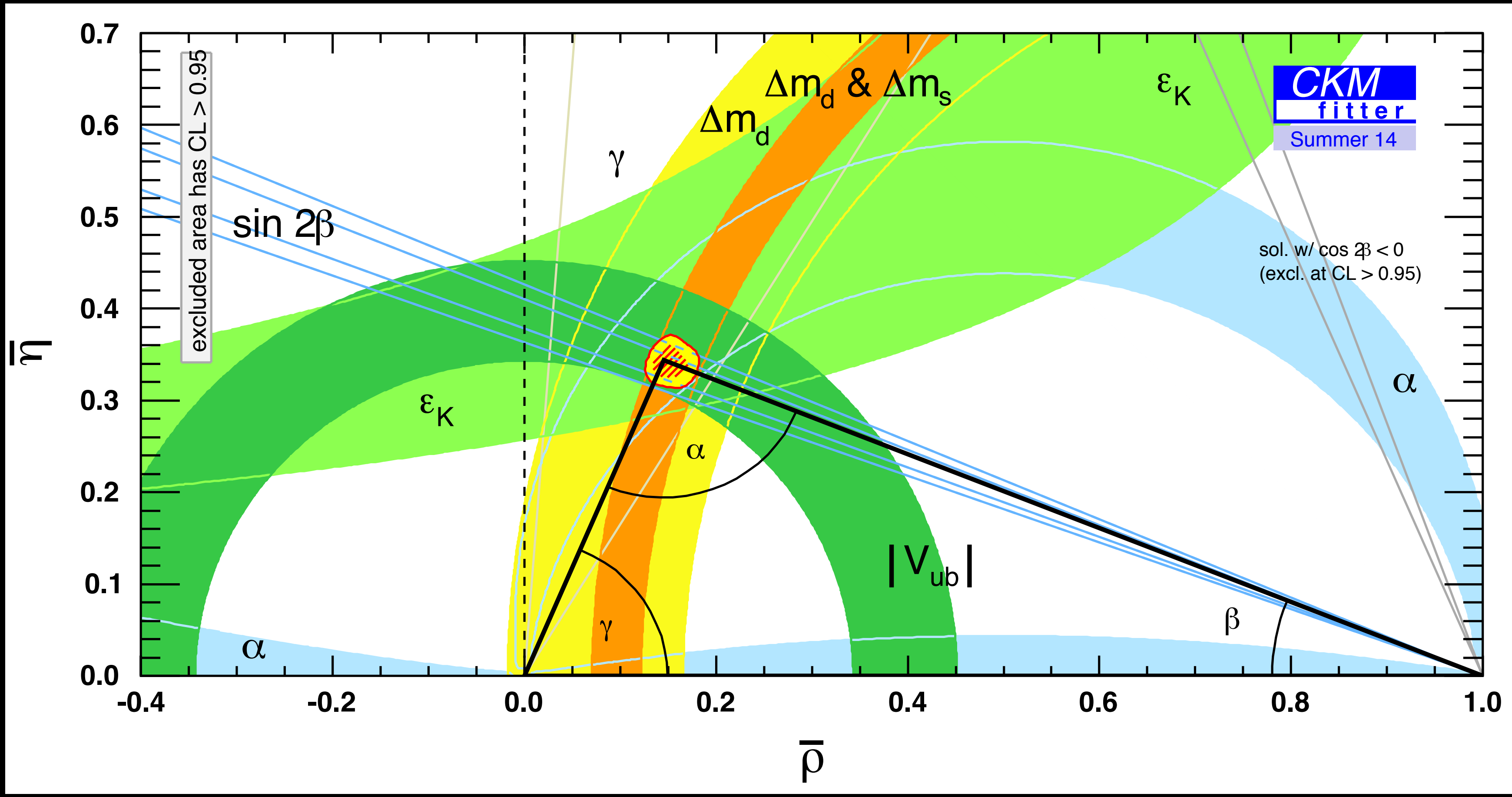
[ARGUS, Phys. Lett. B192, 245 (1987)]

$r > 0.09$ (90%CL)	this experiment
$x > 0.44$	this experiment
$B^{1/2} f_B \approx f_\pi < 160$ MeV	B meson ($\approx$ pion) decay constant
$m_b < 5$ GeV$/c^2$	b-quark mass
$\tau < 1.4 \times 10^{-12}$s	B meson lifetime
$\lvert V_{td} \rvert < 0.018$	Kobayashi–Maskawa matrix element
$\eta_{QCD} < 0.86$	QCD correction factor
$m_t > 50$ GeV$/c^2$	t quark mass

By 1987 it was general belief that top mass was much smaller than 50 GeV, but ARGUS found that it is (probably significantly) larger

Top mass from unitarity triangle

[CKMfitter, CKM14 results]

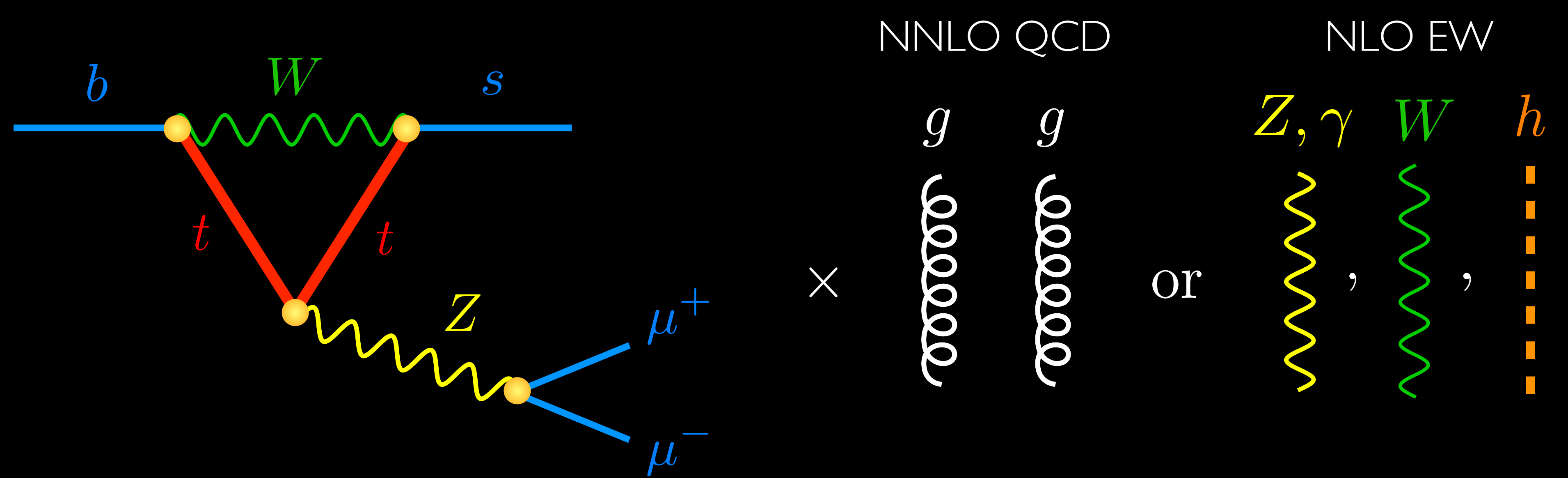


$\Downarrow$

$$m_t^{\text{pole}} = (169 \pm 5)\ \text{GeV}$$

Top mass from $B_s \to \mu^+\mu^-$: Present

[Bobeth et al., 1311.0903]



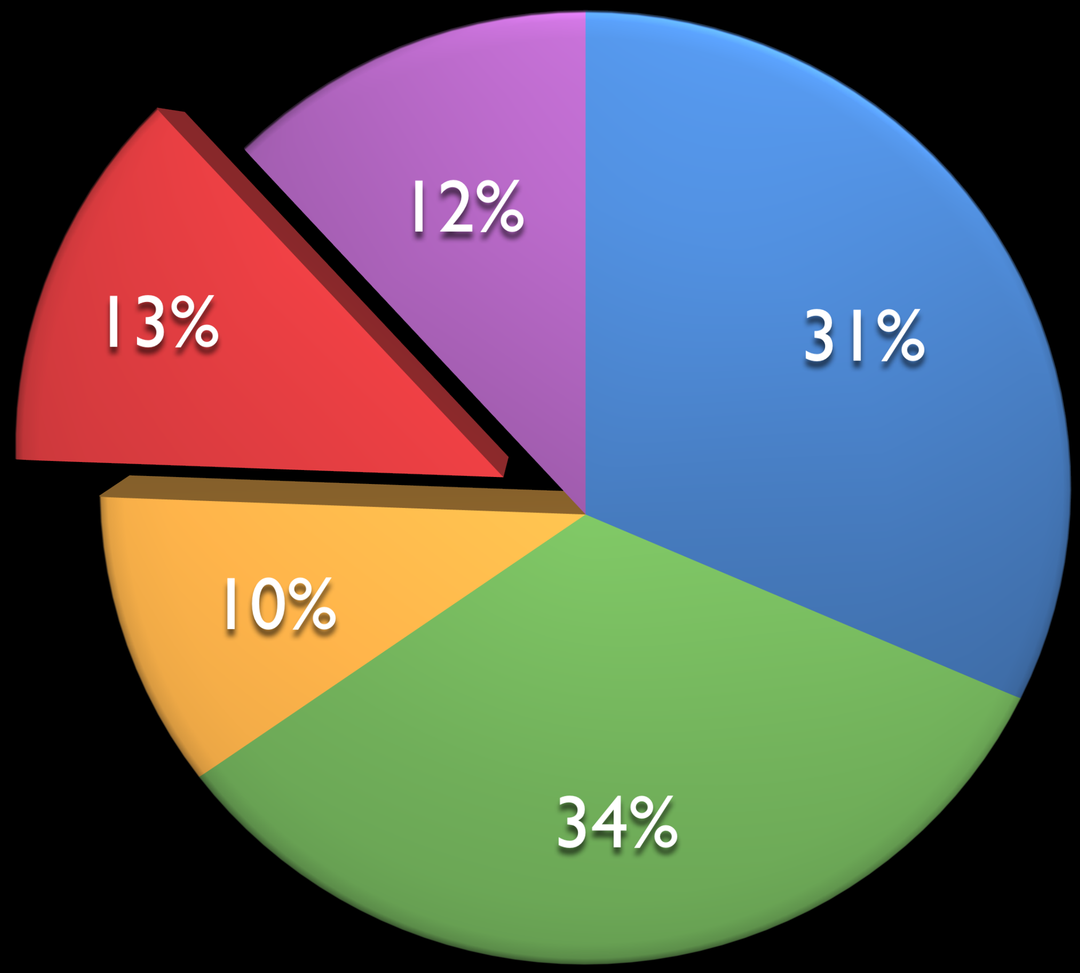
$$\mathrm{Br}\left(B_s \to \mu^+\mu^-\right)_{\mathrm{SM}} = 3.65 \left(\frac{m_t^{\mathrm{pole}}}{173.1\,\mathrm{GeV}}\right)^{3.06} (1 \pm 6.4\%) \cdot 10^{-9}$$

$$\mathrm{Br}\left(B_s \to \mu^+\mu^-\right)_{\mathrm{exp}} = 2.8\left(1^{+25\%}_{-21\%}\right) \cdot 10^{-9}$$ [CMS & LHCb, 1411.4413]

$$\Longrightarrow \quad m_t^{\mathrm{pole}} = (158 \pm 13)\ \mathrm{GeV}$$

$B_s \to \mu^+\mu^-$ relative error budget

[Bobeth et al., 1311.0903]



6.4% total uncertainty

Improvements in lattice QCD calculations may reduce errors due to decay constant f_{Bs} & V_{cb}. Might result in future total uncertainty of 3%

Top mass from $B_s \to \mu^+\mu^-$: Reach

[Bobeth et al., 1311.0903]

$$\mathrm{Br}\big(B_s \to \mu^+\mu^-\big)_{\mathrm{SM}} = 3.65 \left(\frac{m_t^{\mathrm{pole}}}{173.1\,\mathrm{GeV}}\right)^{3.06} (1 \pm 3\%) \cdot 10^{-9}$$

$$\mathrm{Br}\big(B_s \to \mu^+\mu^-\big)_{\mathrm{exp}} = 3.65\,(1 \pm 4\%) \cdot 10^{-9}$$

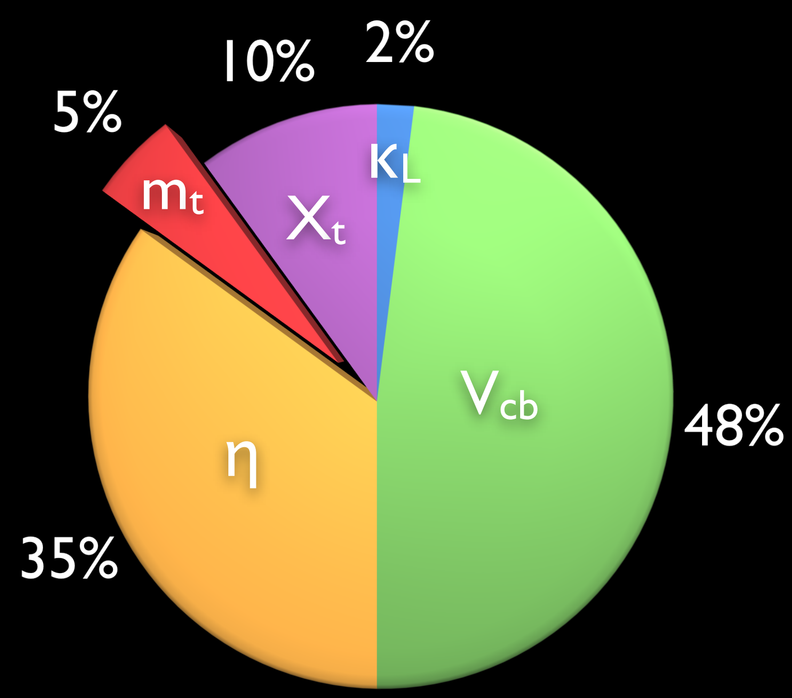
[LHCb, 1208.3355]

$$\Downarrow$$

$$m_t^{\mathrm{pole}} = (173.0 \pm 2.8)\ \mathrm{GeV}$$

Top mass from $K_L \to \pi^0 \nu\bar{\nu}$

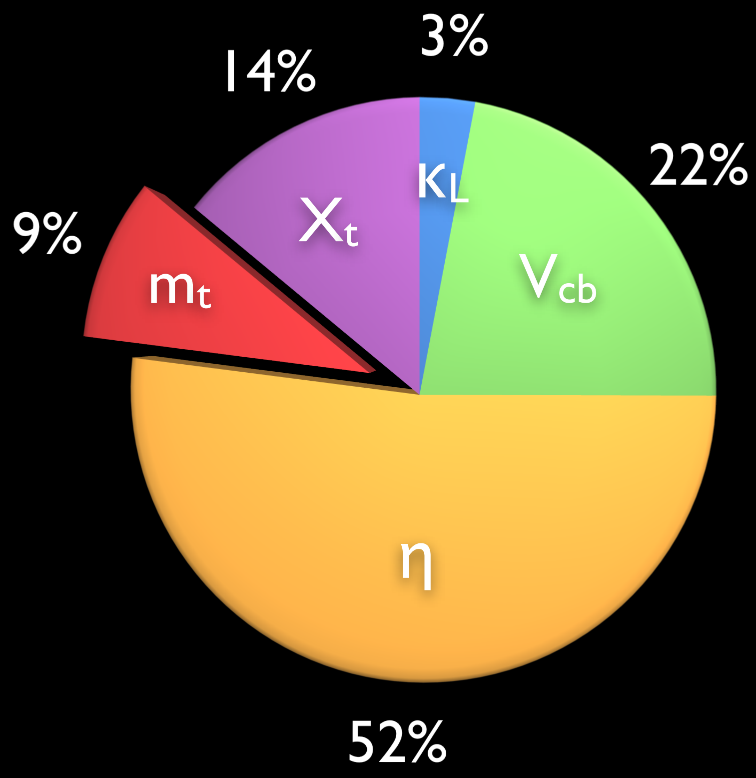
[Brod et al., 1009.0947]



$$\mathrm{Br}\big(K_L \to \pi^0 \nu\bar{\nu}\big)_{\mathrm{SM}} = 2.4\,(1 \pm 15\%) \cdot 10^{-11}$$

$$\underset{10\%\ \text{measurement}}{\Longrightarrow} \quad \boxed{\delta m_t^{\mathrm{pole}} = 14\ \mathrm{GeV}}$$

$$\Big\Downarrow \ \text{if } V_{cb} \text{ known to } 1\%$$

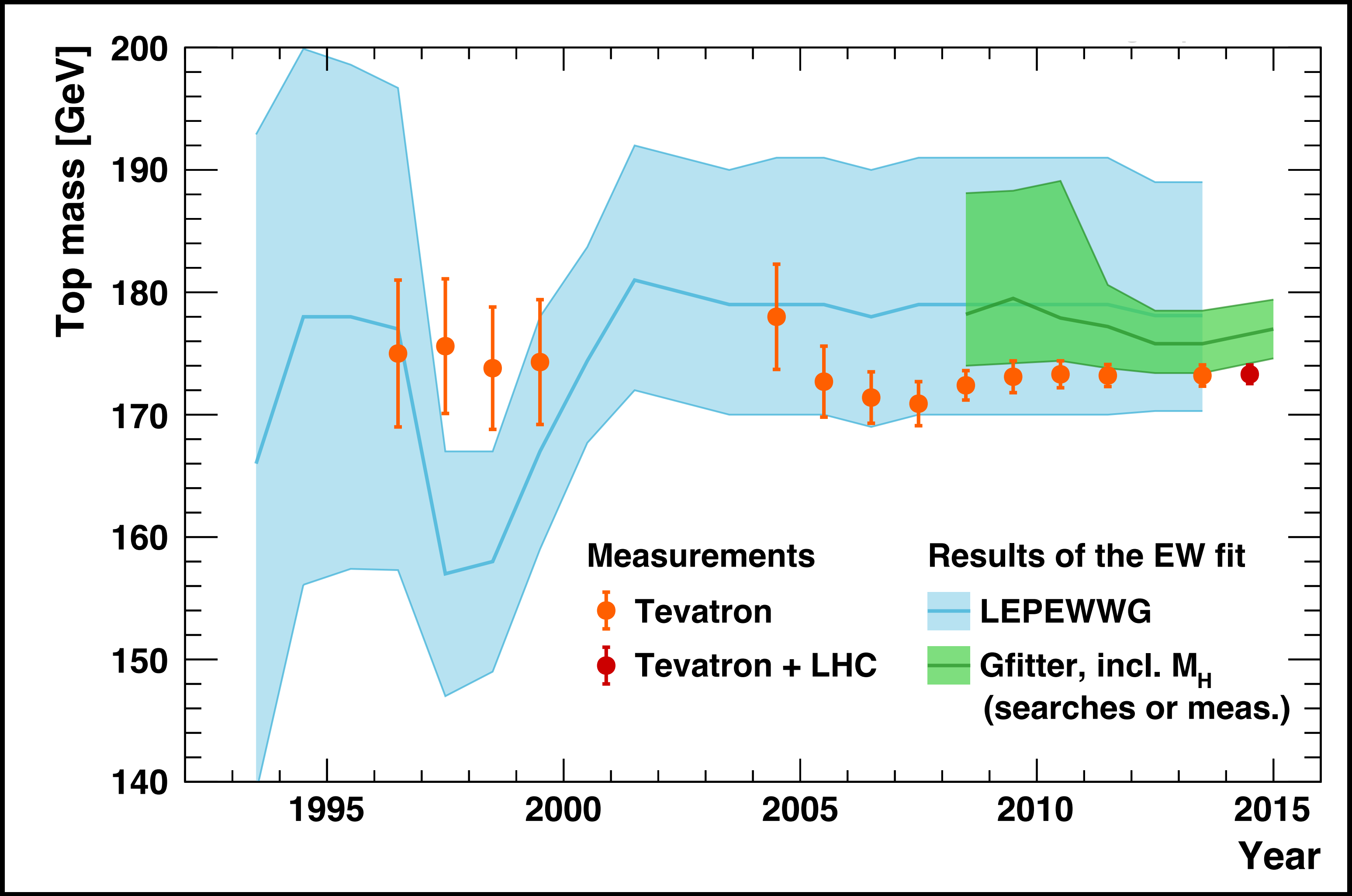


$$\mathrm{Br}\big(K_L \to \pi^0 \nu\bar{\nu}\big)_{\mathrm{SM}} = 2.4\,(1 \pm 10\%) \cdot 10^{-11}$$

$$\underset{10\%\ \text{measurement}}{\Longrightarrow} \quad \boxed{\delta m_t^{\mathrm{pole}} = 11\ \mathrm{GeV}}$$

History of m_t from electroweak fit

[Gfitter, November 2014]



Even before top discovery at Fermilab in 1995, global electroweak (EW) fits have always been able to predict mass correctly

Theory behind EW fits

- W-boson mass: full 2-loop EW corrections as well as higher-order contributions, including 4-loop QCD effects in ρ parameter

[Awramik et al., hep-ph/0311148; Chetyrkin et al., hep-ph/0605201]

- Weak mixing angle: complete 2-loop EW corrections & dominant higher-order effects

[Awramik et al., hep-ph/0608099, 0811.1364]

- Partial widths of Z & W boson: full EW corrections up to 2 loops & leading higher-order effects for Z, 1-loop for W

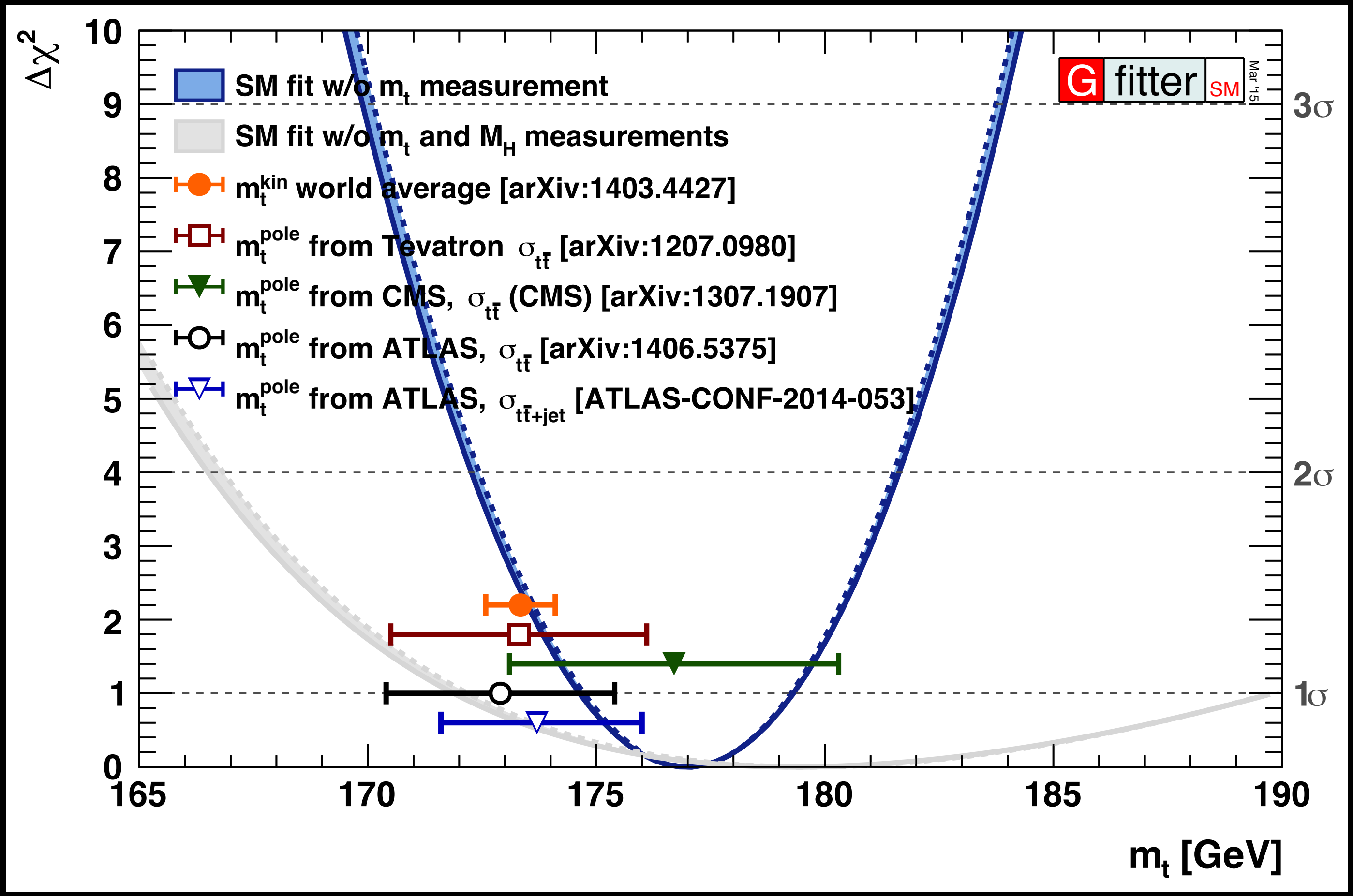
[Freitas, 1401.2447; Cho et al., 1104.1769]

- Determination of α_s from hadronic Z width: all $O(\alpha_s^4)$ QCD effects

[Baikov et al., 1201.5804]

Top mass from EW fit: Present

[Kogler, Moriond EW 2015]

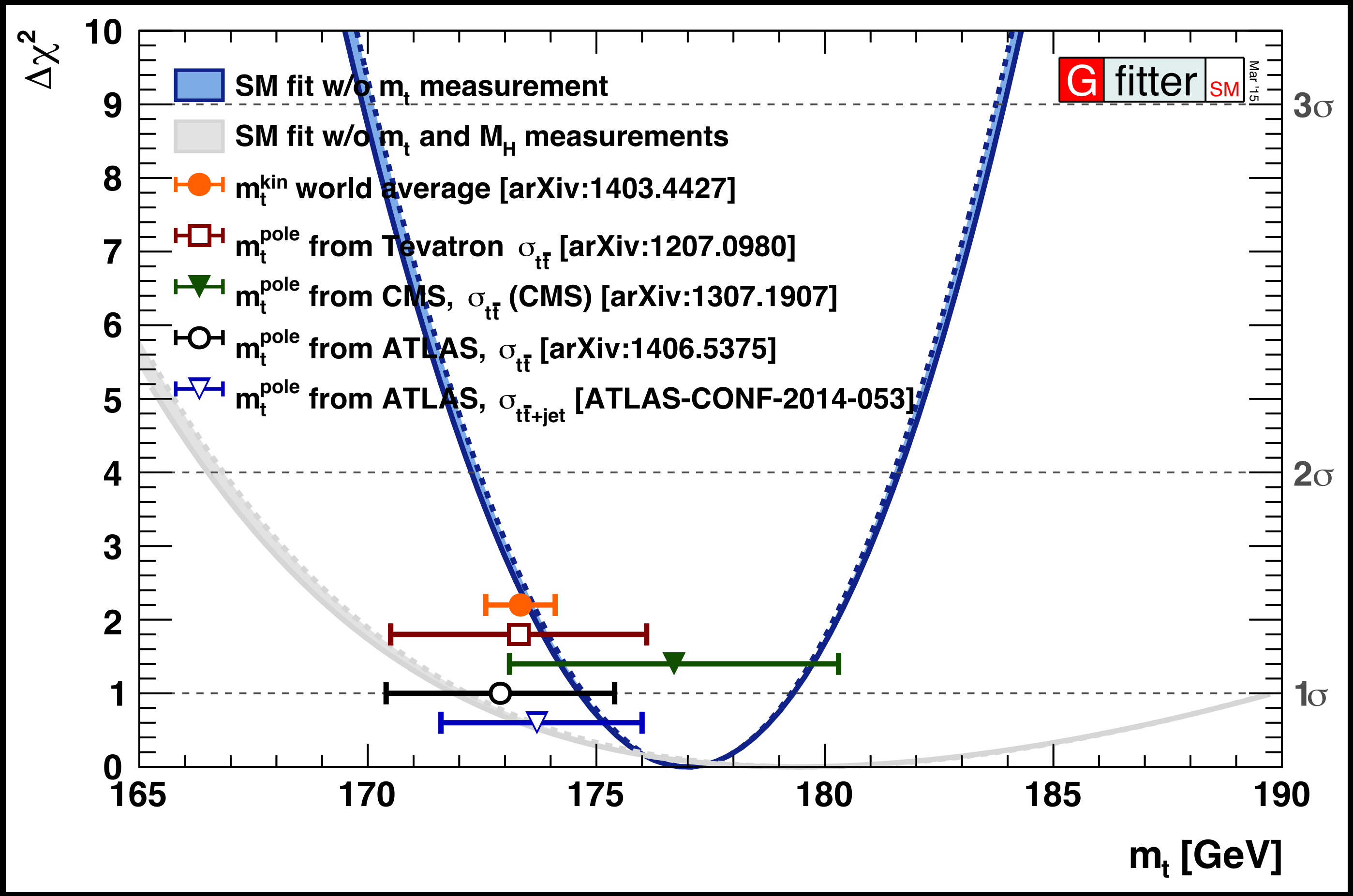


$$\Downarrow$$

$$m_t^{\rm pole} = \left(177.0 \pm 2.3_{M_W,\sin^2\theta^f_{\rm eff}} \pm 0.6_{\alpha_s} \pm 0.5_{\Delta\alpha_{\rm had}} + 0.4_{M_Z}\right)\ {\rm GeV}$$

Top mass from EW fit: Present

[Kogler, Moriond EW 2015]

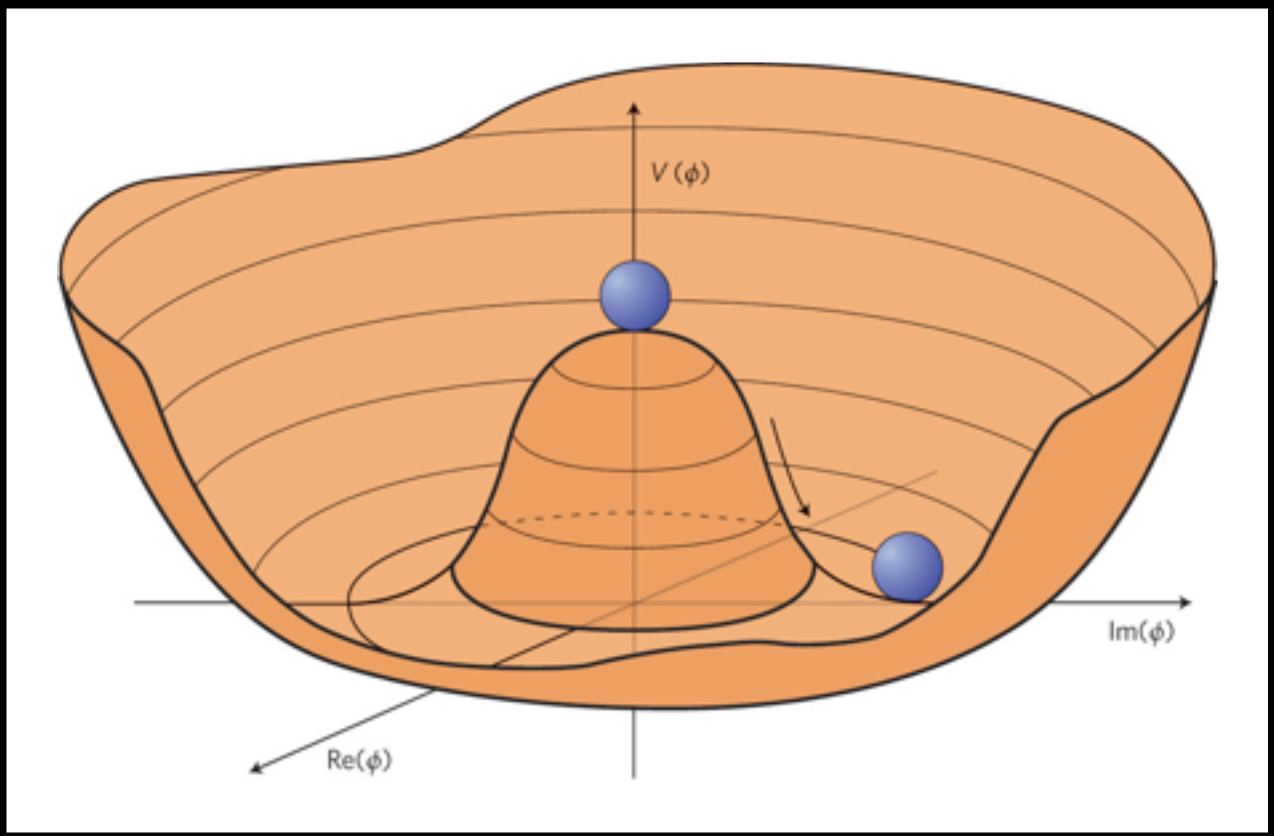


$$\Downarrow$$

$$m_t^{\text{pole}} = (177.0 \pm 2.5)\ \text{GeV}$$

Does our fate depend on m_t?

[see e.g. Coleman & Weinberg, Phys. Rev. D7, 1888 (1973)]



$$V(\phi) = \lambda\left(|\phi|^2 - v^2\right)^2 \simeq \frac{\lambda}{4}\,h^4$$

$$\phi = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v+h \end{pmatrix}$$

$$v \simeq 246\,\mathrm{GeV}$$

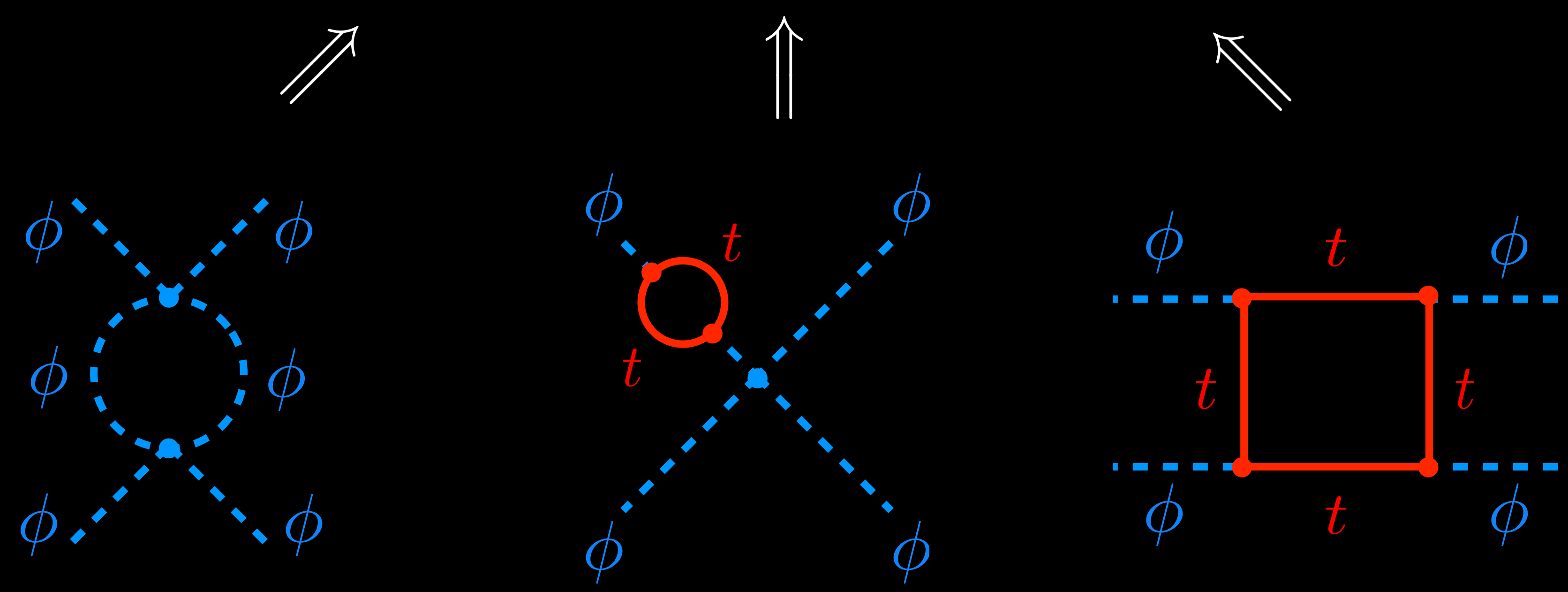
High-scale Higgs potential can be calculated via renormalization group (RG).
In quartic approximation, stability of $V(\phi)$ is equivalent to positivity of λ

RG evolution of Higgs quartic

[cf. Chetyrkin & Zoller, 1205.2892 for 3-loop results]

$$\frac{d\lambda(\mu)}{d\ln\mu} = \beta_\lambda(\mu)$$

$$\beta_\lambda(\mu) \simeq \frac{1}{(4\pi)^2}\left(24\lambda^2(\mu) + 12y_t^2(\mu)\lambda(\mu) - 6y_t^4(\mu)\right)$$



RG evolution of Higgs quartic

[cf. Chetyrkin & Zoller, 1205.2892 for 3-loop results]

$$\frac{d\lambda(\mu)}{d\ln\mu} = \beta_\lambda(\mu)$$

$$\beta_\lambda(m_t^{\rm pole}) \simeq \frac{1}{(4\pi)^2}\big(0.4 + 1.4 - 6.0\big) \simeq -0.03 < 0$$

$$\Uparrow$$

$$\lambda(m_t^{\rm pole}) = \frac{m_h^2}{2v^2} \simeq 0.13 \qquad y_t(m_t^{\rm pole}) = \frac{\sqrt{2}m_t^{\rm pole}}{v} \simeq 1.0$$

Measured top & Higgs mass imply that beta function β_λ negative at low energies. Higgs quartic λ will thus approach zero at high energies

RG evolution of Higgs quartic

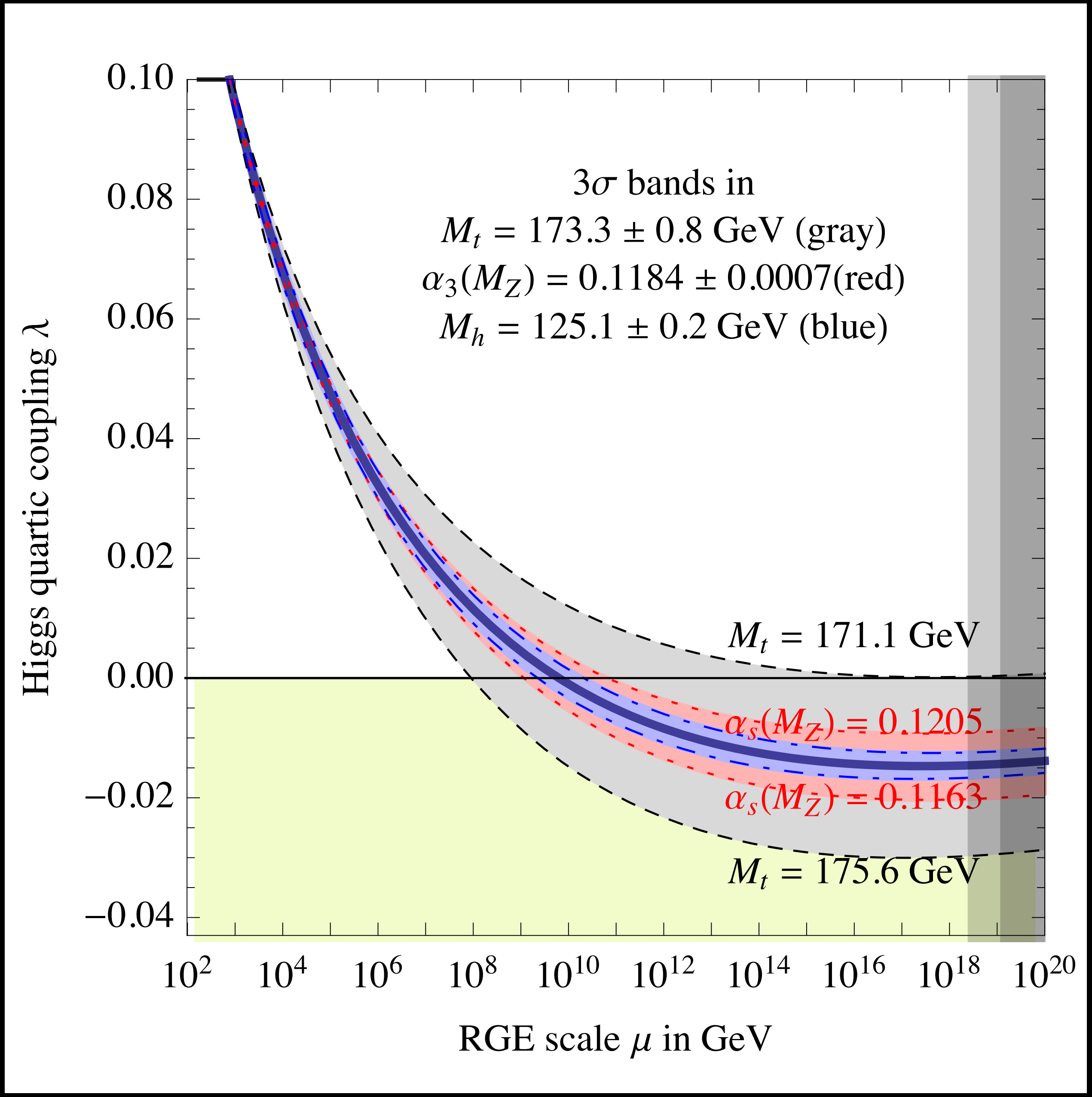
[Buttazzo et al., 1307.3536]

Fact 1: SM can be extrapolated up to Planck scale M_{Pl}

Fact 2: Higgs mass of 125 GeV leads to $\lambda(M_{Pl}) \approx 0$

Fact 3: β_λ vanishes close to M_{Pl}

Disclaimer: All this could be an accident & new physics can (is likely to) change these results by a bit or by a lot



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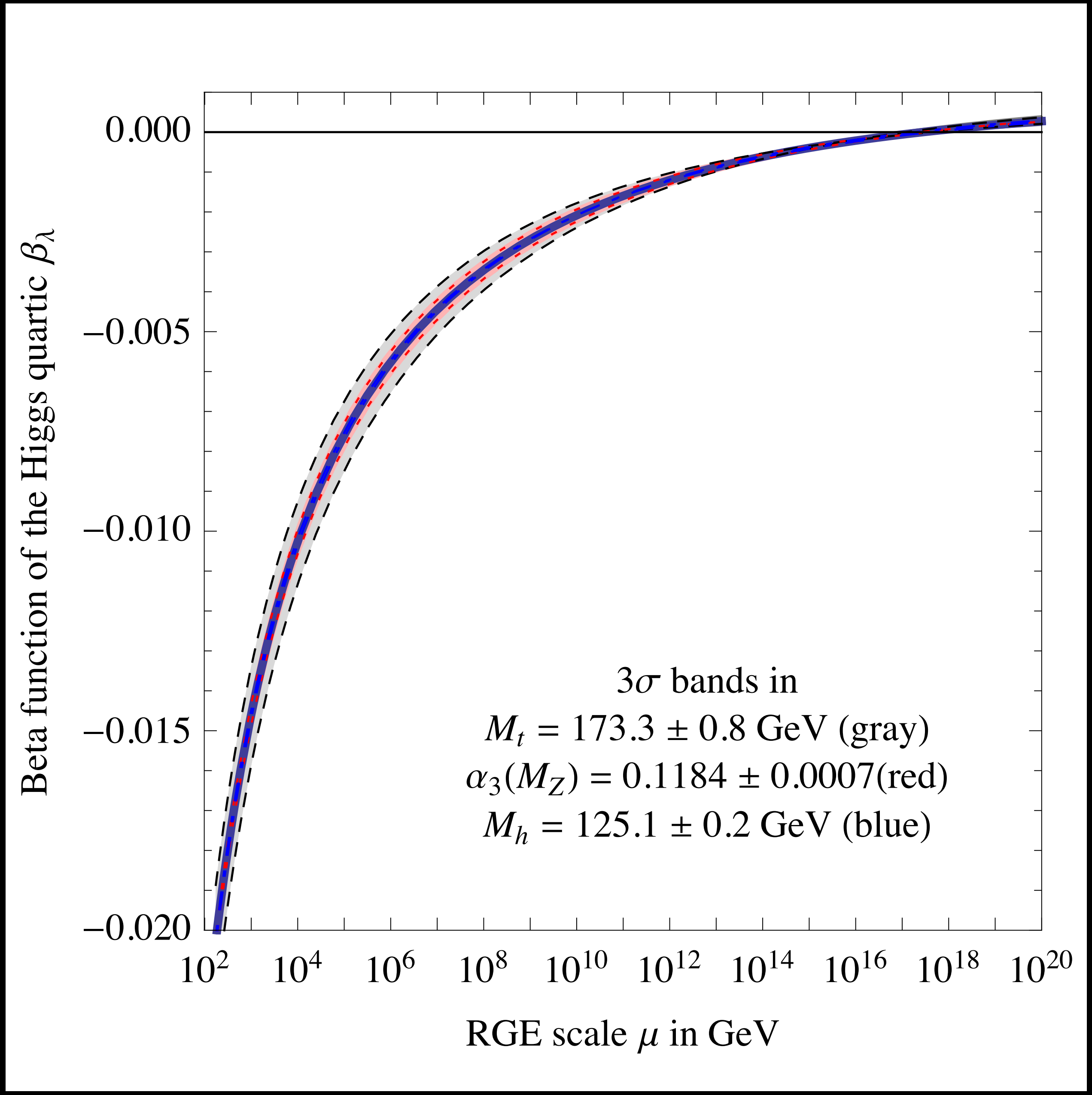
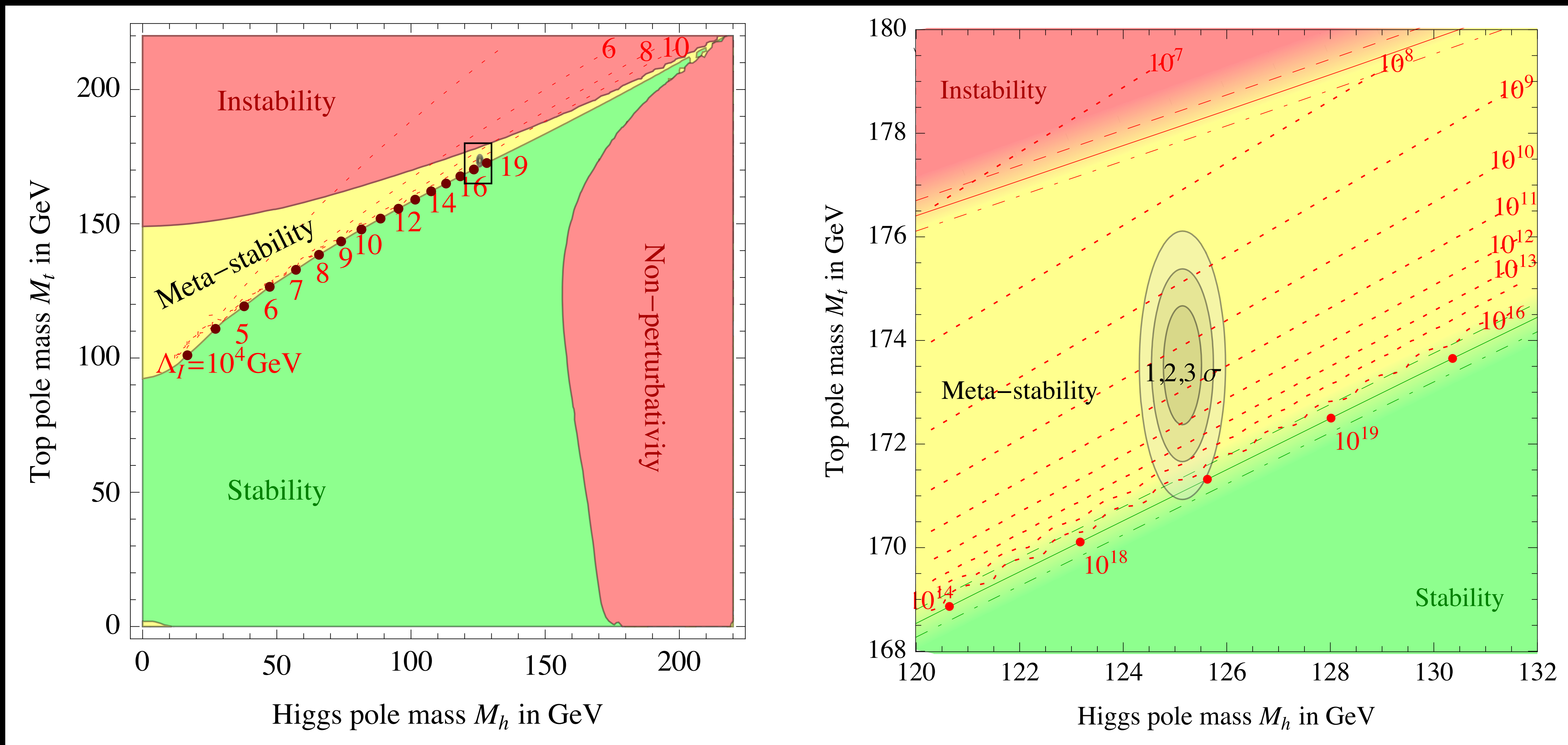


Vacuum (meta)stability

[Buttazzo et al., 1307.3536]



SM parameters rather special, in sense that vacuum in a near-critical condition, at border between stability & metastability. Is this significant?

Conclusions

- Discovery of top in 1995 at Fermilab & Higgs in 2012 at CERN completes SM. Without doubt, one of the greatest scientific achievements of mankind

- Quantum effects involving top play a crucial role in SM, since they drive flavor breaking in quark sector & represent leading source of violation of custodial symmetry. While well tested, meaning (if any) not understood

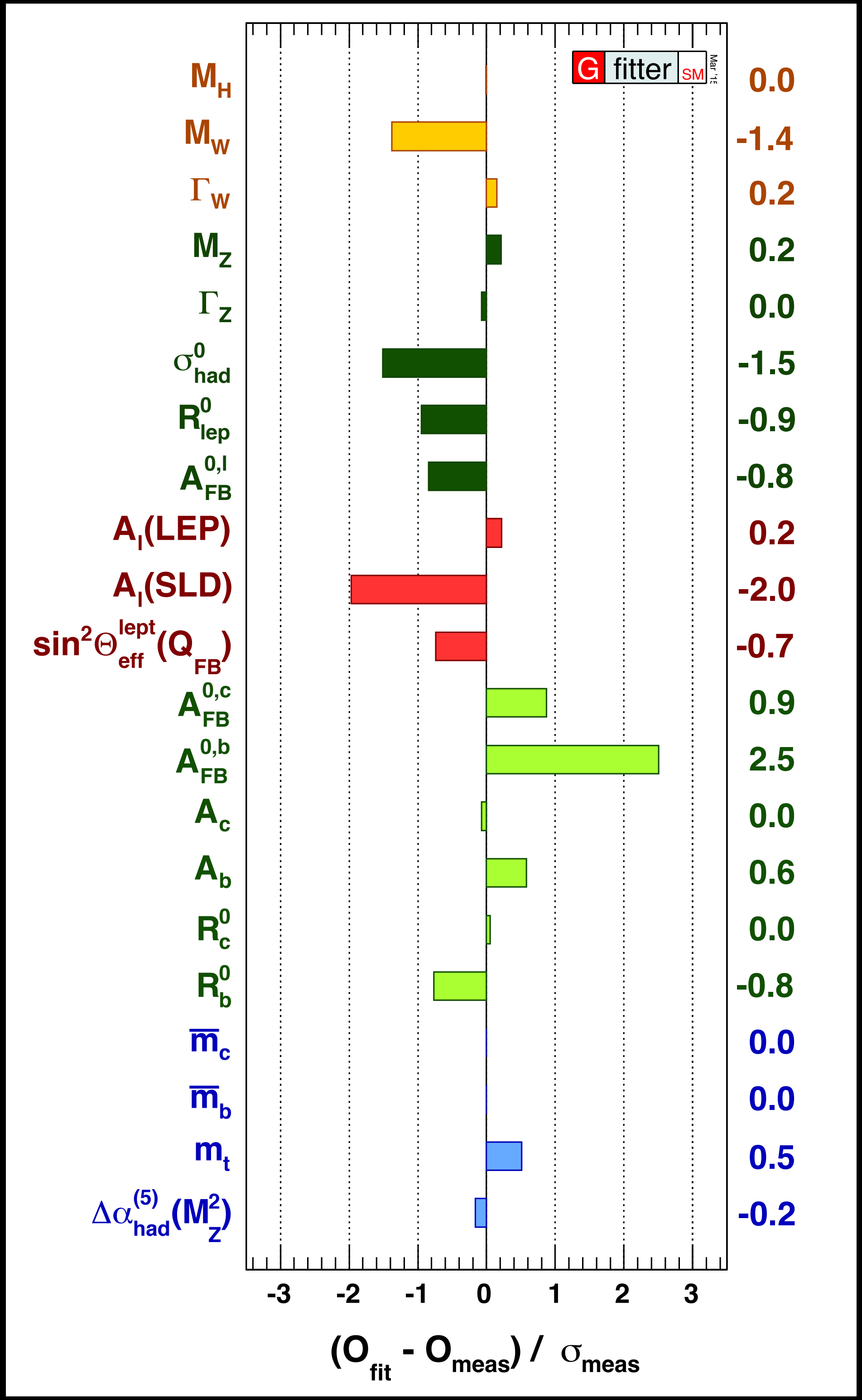
- Intriguing observation that top & Higgs mass conspire to make EW vacuum close to critical. Whether this special condition, allowing for a prolonged vacuum lifetime, is just a numerical coincidence or an important feature of SM is an open question

Results of current global EW fit

[Gfitter, March 2015]

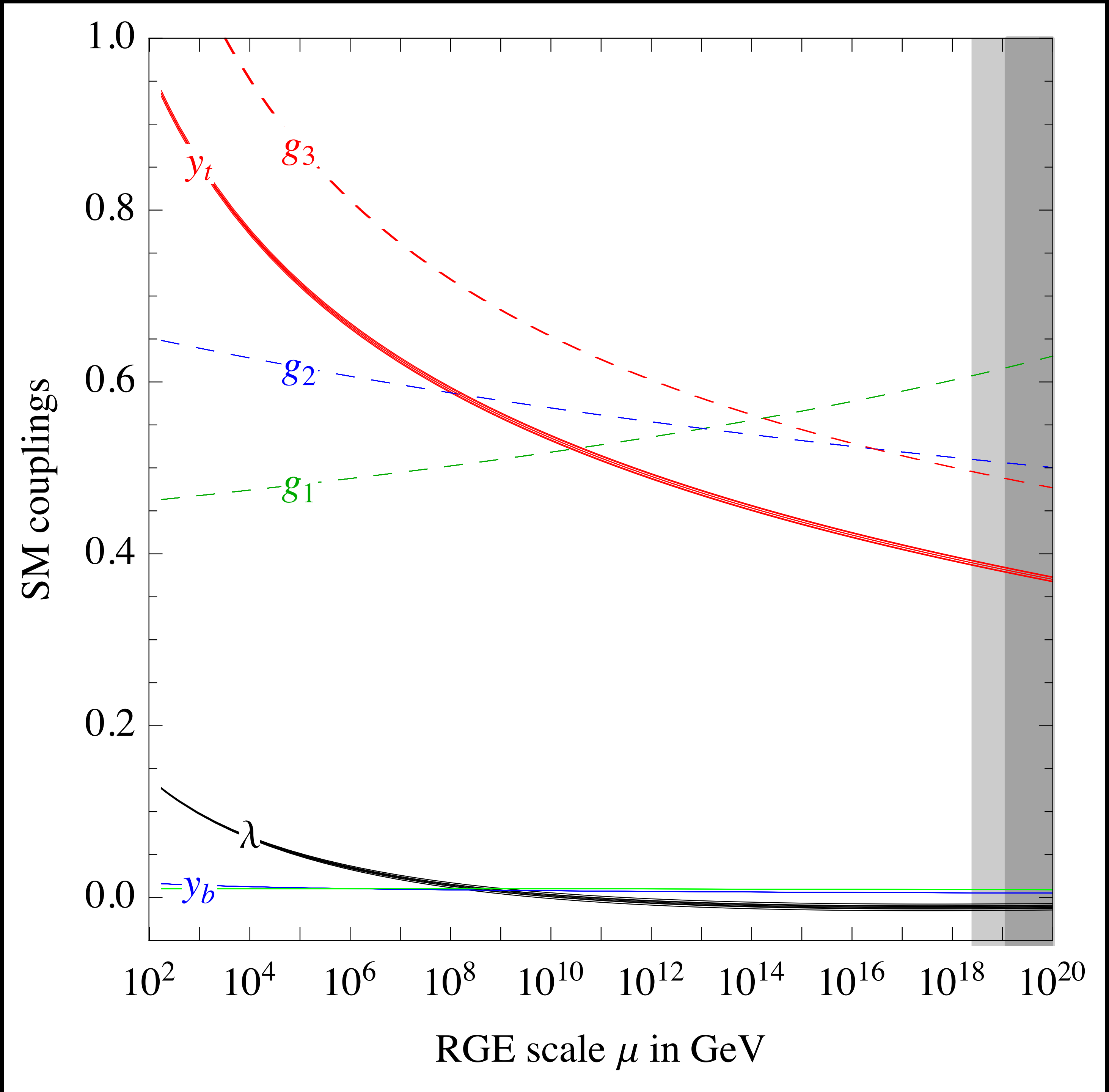
$$\frac{\chi^2_{\min}}{\mathrm{dof}} = \frac{17.8}{14}$$

p-value 21%



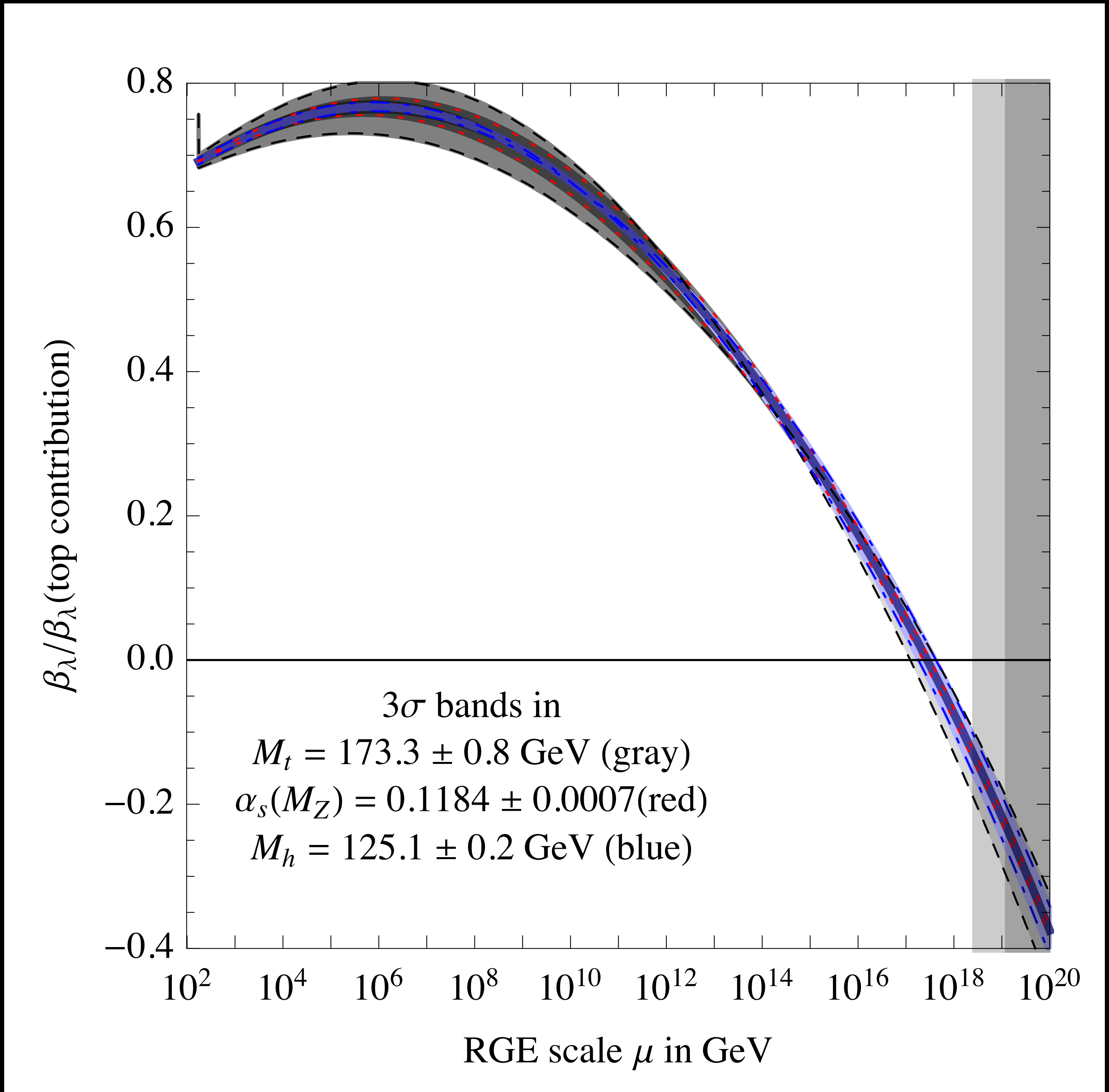
Running SM couplings

[Degrassi et al., 1205.6497]



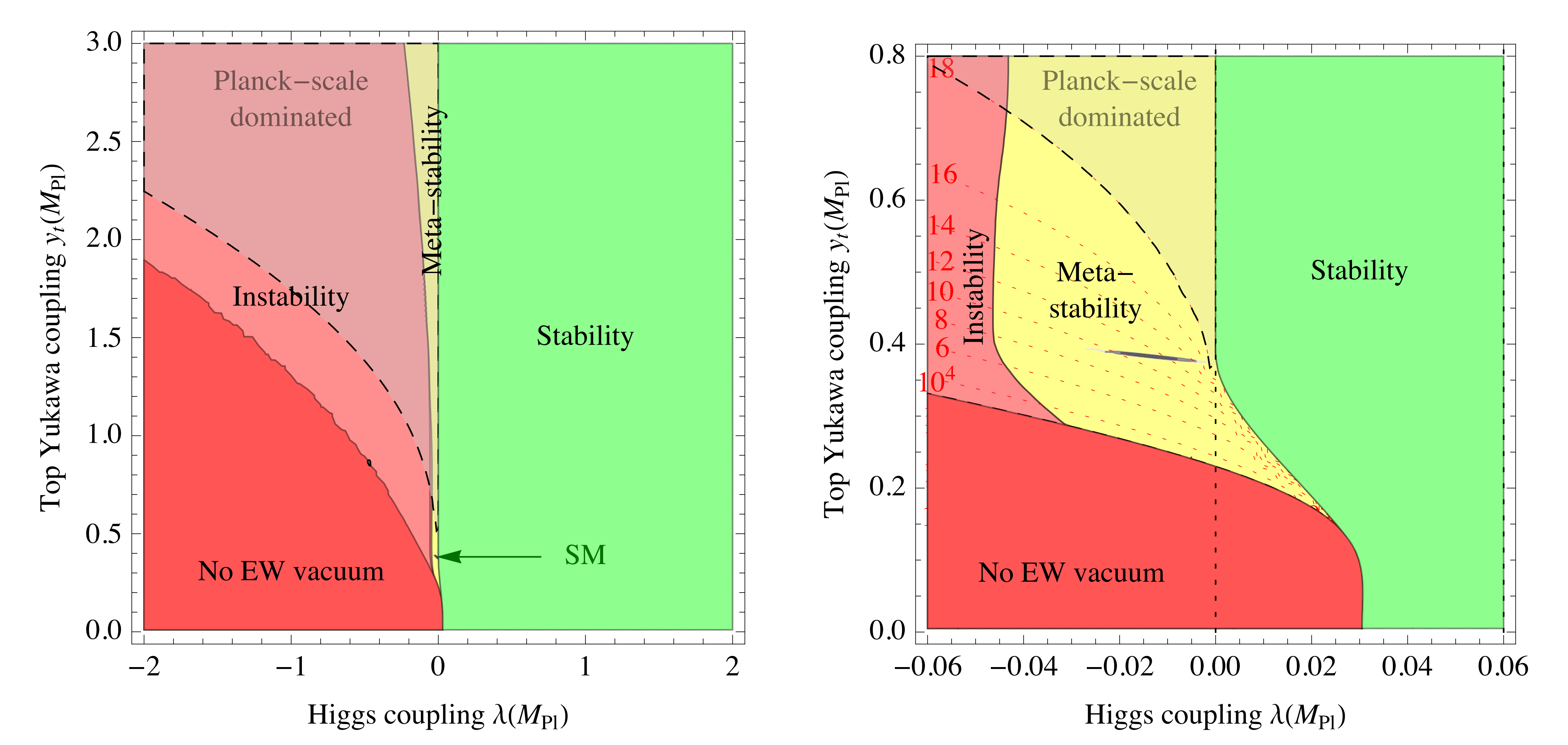
Top contribution to β_λ

[Buttazzo et al., 1307.3536]



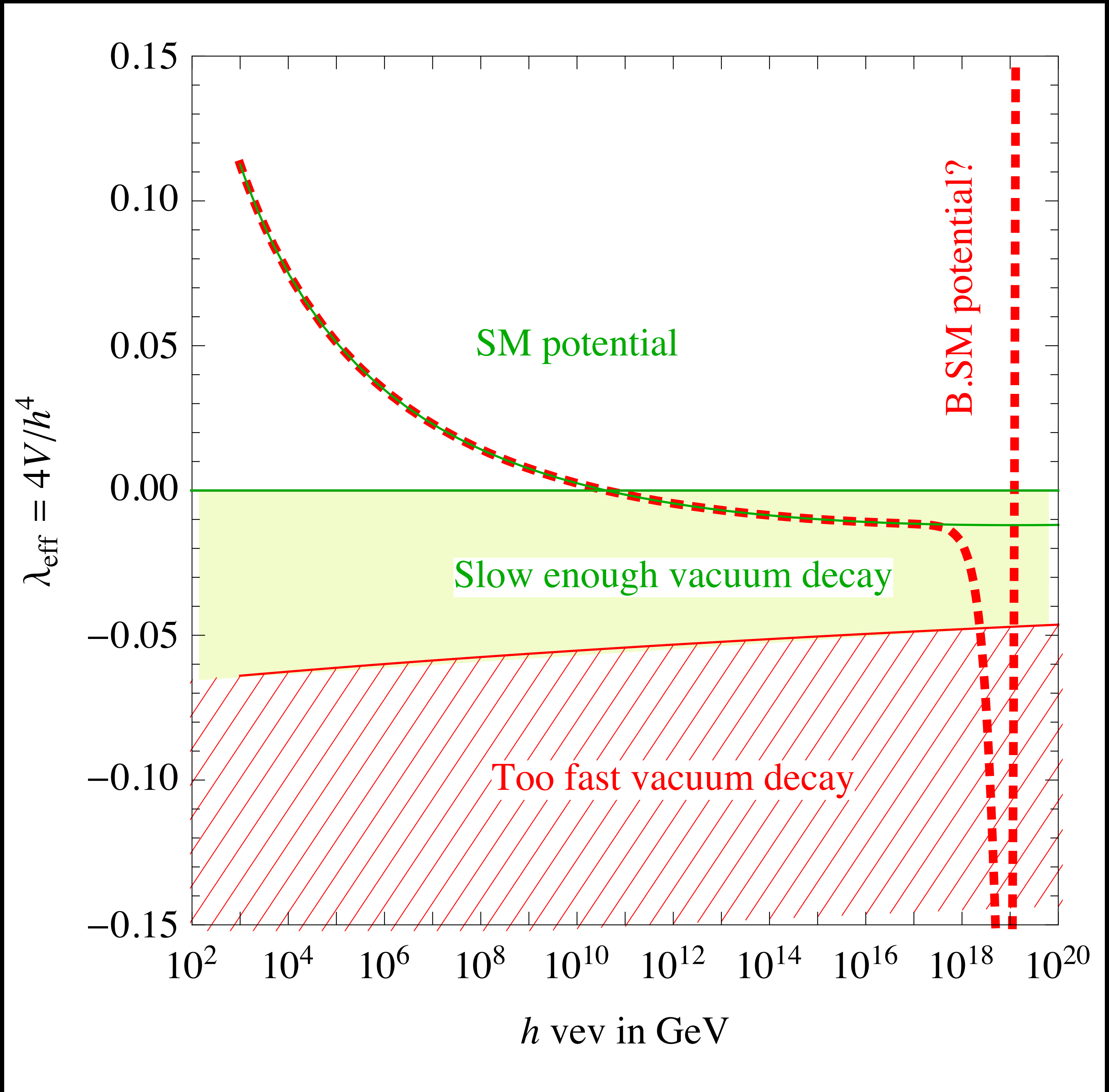
SM vacuum phase diagram

[Buttazzo et al., 1307.3536]



Vacuum decay

[Strumia, Moriond EW 2015]



Lifetime of EW vacuum

[Buttazzo et al., 1307.3536]