# Fit of TBT data using MAD-X[a]

[a]Y.Alexahin et al., "Coupled Optics Reconstruction from TBT data using MAD-X", PAC07

# INTRODUCTION

**Through the Fourier transform of the measured TBT data**

$$
\begin{aligned}
x_n = {} & A_I\sqrt{\beta_{xI}}\cos(\phi_{xI}+\delta_I+2\pi nQ_I) + \\
& A_{II}\sqrt{\beta_{xII}}\cos(\phi_{xII}+\delta_{II}+2\pi nQ_{II}) \\
y_n = {} & A_I\sqrt{\beta_{yI}}\cos(\phi_{yI}+\delta_I+2\pi nQ_I) + \\
& A_{II}\sqrt{\beta_{yII}}\cos(\phi_{yII}+\delta_{II}+2\pi nQ_{II})
\end{aligned}
$$

**the coupled Mais-Ripken twiss functions ($\beta_{xI}, \beta_{xII}$ etc.) are known ( a part for $A_{I,II}$ and $\delta_{I,II}$).**

The **eigenvectors** of the coupled transport matrix are related to the Mais-Ripken twiss functions

$$
\begin{aligned}
V_{11} &\equiv \sqrt{\beta}_{xI} \cos \phi_{xI} & V_{12} &\equiv \sqrt{\beta}_{xI} \sin \phi_{xI} \\
V_{13} &\equiv \sqrt{\beta}_{xII} \cos \phi_{xII} & V_{14} &\equiv \sqrt{\beta}_{xII} \sin \phi_{xII} \\
V_{31} &\equiv \sqrt{\beta}_{yI} \cos \phi_{yI} & V_{32} &\equiv \sqrt{\beta}_{yI} \sin \phi_{yI} \\
V_{33} &\equiv \sqrt{\beta}_{yII} \cos \phi_{yII} & V_{34} &\equiv \sqrt{\beta}_{yII} \sin \phi_{yII}
\end{aligned}
$$

**Goal: adjust**

- quadrupole **gradient** and **tilt**
- BPMs **calibration** and **tilt**[a]
- $A_{I,II}$ and $\delta_{I,II}$

in order to fit the measured eigenvector values at the BPMs.

MAD-X is capable of matching coupled optics and allows user-defined expressions in matching constraints ("macros").

MAD-X **TWISS** uses Edwards-Teng formalism.
MAD-X **PTC_TWISS** uses Mais-Ripken formalism, but it is too slow for matching purposes.

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[a] The BPM reading is related to the actual beam position by

$$x^{meas} = \frac{x + y\tan\chi}{r_x} \qquad y^{meas} = \frac{y - x\tan\chi}{r_y}$$

with $\chi \equiv$ BPM tilt and $r_z \equiv z/z^{meas}$ $(z \equiv x, y)$.

The two formalism are of course related, the relationships between the two sets of twiss functions being

$$\beta_{xI} = \kappa\beta_1 \quad \beta_{yII} = \kappa\beta_2 \qquad \phi_{xI} = \varphi_1 \quad \phi_{yII} = \varphi_2$$

$$\beta_{xII} = \kappa[R_{22}(R_{22}\beta_2 + 2R_{12}\alpha_2) + R_{12}^2\gamma_2]$$

$$\beta_{yI} = \kappa[R_{11}(R_{11}\beta_1 - 2R_{12}\alpha_1) + R_{12}^2\gamma_1]$$

$$\phi_{xII} = \varphi_2 - \arctan[R_{12}/(R_{22}\beta_2 + R_{12}\alpha_2)]$$

$$\phi_{yI} = \varphi_1 + \arctan[R_{12}/(R_{11}\beta_1 - R_{12}\alpha_1)]$$

with $\kappa \equiv 1/(1 + |R|)$, $R$ being a $2 \times 2$ matrix, also computed by MAD-X.

Use MAD-X macros to define

- Mais-Ripken functions in terms of Edwards-Teng ones
- constraints & variables

## Application to Tevatron

- The model: Tevatron luminosity optics (converted by Norman to MAD-8 format from Sasha OPTIM file (September 06) and further converted to MAD-X format)
- Number of variable magnets: 216 normal and 216 skew 2$^{th}$ order multipoles (as in Sasha LOCO fits)
- Number of observation points: $2 \times 118$
- TBT data to fit: July 07 (horizontal and vertical kick)

The fit is very time consuming, therefore

- simplify input
- split fit

(see Sergey talk).