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Integrable Optics Test Accelerator – Centerpiece of Transformational Accelerator R&D Program

Alexander Valishev

Fermilab

12 February 2015



Motivation and Strategy

- **Fundamental beam physics phenomena in circular proton machines** hinder the progress of accelerator based HEP with high intensity beams.
- Overcoming these limitations by using **innovative design approaches** will allow multi-MW beam power **at lower cost**.
- **There is a lack of dedicated ring-based accelerator test facilities** in the US for high intensity research.
- IOTA will become a unique machine for revolutionary proof-of-principle R&D towards future high intensity machines
 - push performance limits of rings by 3-5 times to enable multi-MW beam power – $\Delta Q_{SC} > 1$, lower losses, stable beams
 - become the focal point for collaboration and training

IOTA Roadmap

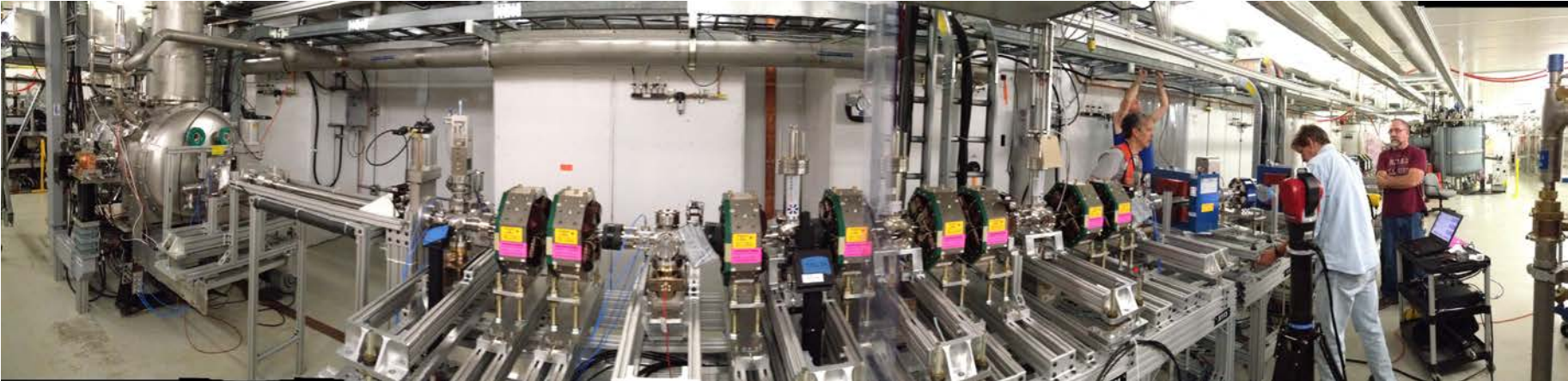
- I. **Construct and commission the Integrable Optics Test Accelerator (IOTA) storage ring and its proton and electron injectors, and establish reliable and time-effective operation of the facility for accelerator research program.**
- II. **Carry out transformative beam dynamics experiments**
 - a. Integrable optics with non-linear magnets and with electron lenses.
 - b. Space charge compensation with electron lenses and electron columns.
 - c. Optical stochastic cooling demonstration
- III. **Open new opportunities for training** young researchers

All **in collaboration with national and international partners** on corresponding modeling, design, and analysis efforts

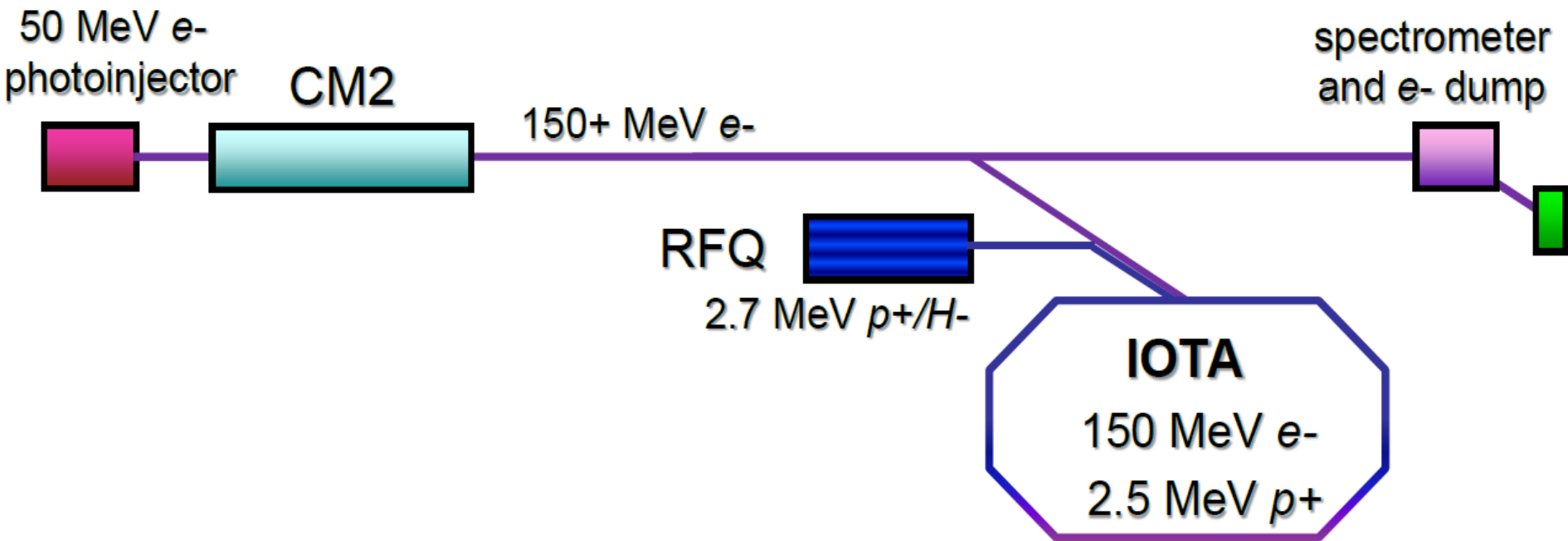
Existing Infrastructure

- **IOTA capitalizes on the investments** made by OHEP for highly successful ILC/SRF R&D Program.
- Construction of ASTA (formerly NML) began in 2006 as part of the ILC/SRF R&D Program and later American Recovery and Reinvestment Act (ARRA). The facility was motivated by the goal of building, testing and operating a complete ILC RF unit.
- **Multi-million (>\$90M) investment** resulted in the successful commissioning of 1.3 GHz SRF cryomodule (CM2).
 - Beam through low-energy photo injector
 - Facility nears completion
- The **addition of IOTA expands scope** to host high-intensity accelerator research.

ASTA Facility



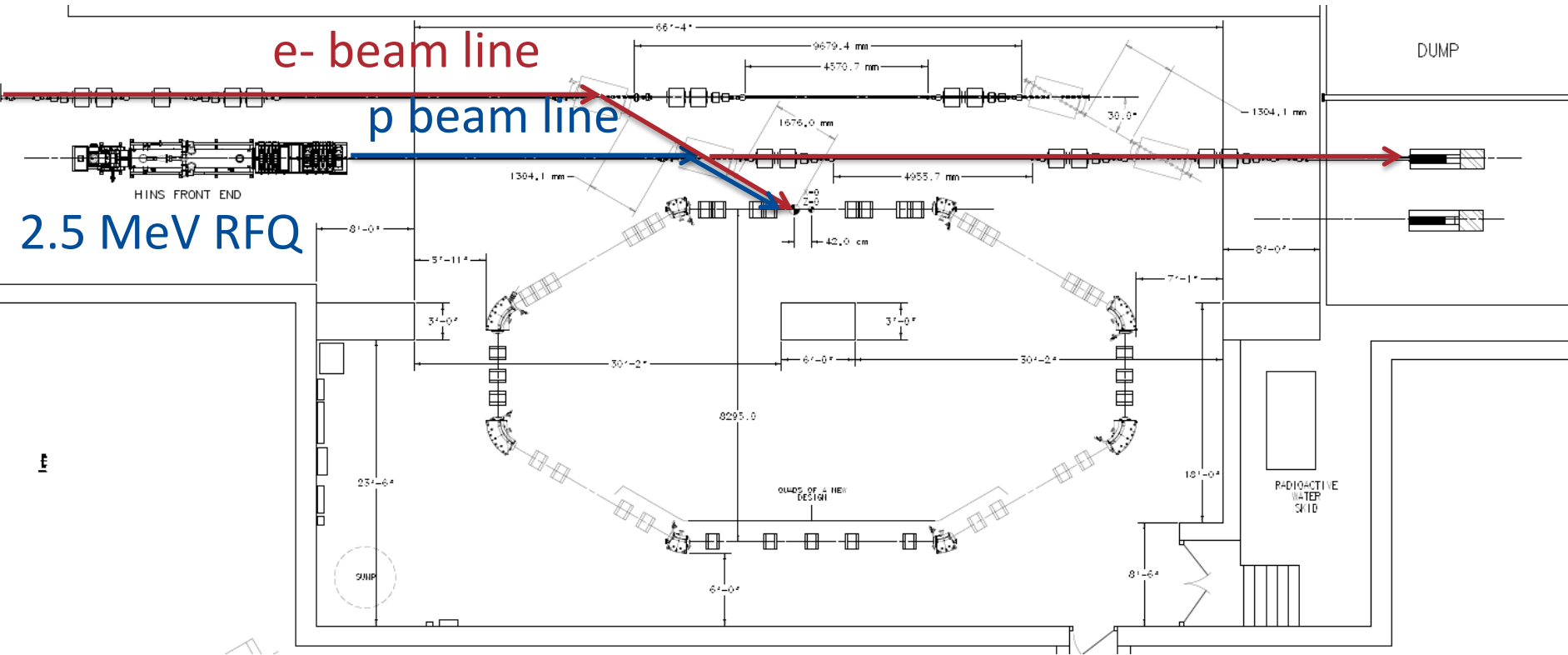
IOTA / ASTA Schematic



Integrable Optics Test Accelerator

- **Unique features:**
 - Can operate with either electrons or protons (up to 150 MeV/c momentum)
 - Large aperture
 - Significant flexibility of the lattice
 - Precise control of the optics quality and stability
 - Set up for very high intensity operation (with protons)
- **Based on conventional technology** (magnets, RF)
- **Cost-effective solution**
 - Balance between low energy (low cost) and discovery potential

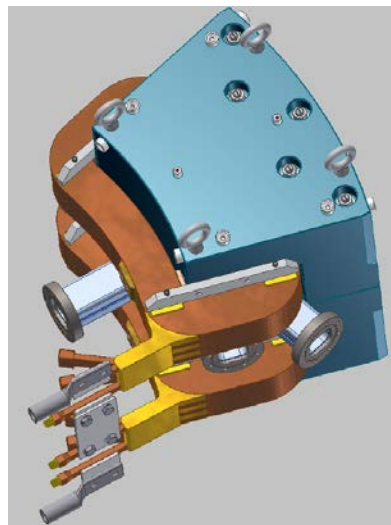
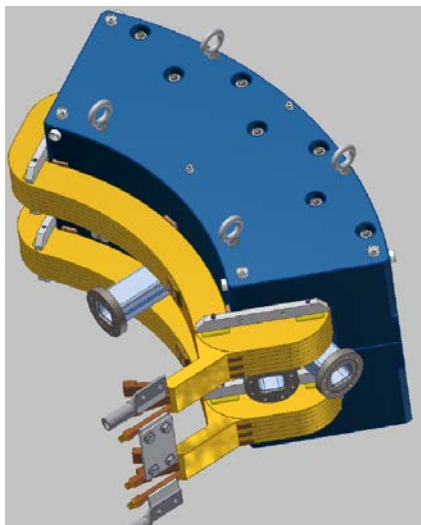
IOTA Ring



IOTA Parameters

| | |
|-------------------------------|--|
| Nominal kinetic energy | e ⁻ : 150 MeV, p ⁺ : 2.5 MeV |
| Nominal intensity | e ⁻ : 1×10^9 , p ⁺ : 1×10^{11} |
| Circumference | 40 m |
| Bending dipole field | 0.7 T |
| Beam pipe aperture | 50 mm dia. |
| Maximum b-function (x,y) | 12, 5 m |
| Momentum compaction | $0.02 \div 0.1$ |
| Betatron tune (integer) | $3 \div 5$ |
| Natural chromaticity | $-5 \div -10$ |
| Transverse emittance r.m.s. | e ⁻ : $0.1 \mu\text{m}$, p ⁺ : $2 \mu\text{m}$ |
| SR damping time | 0.6s (5×10^6 turns) |
| RF V,f,q | e ⁻ : 1 kV, 30 MHz, 4 |
| Synchrotron tune | e ⁻ : $0.002 \div 0.005$ |
| Bunch length, momentum spread | e ⁻ : 2 cm, 1.4×10^{-4} |

Ring Elements in Hand



Dipole magnets (ordered)



32 quads from **JINR (Dubna)** received



Vacuum chambers for dipoles (received)



Magnet support stands from **MIT** (received)

Also:

- BPM bodies and electronics
- Vacuum system
- Dipole power supply
- Corrector power supplies

IOTA Physics Drivers

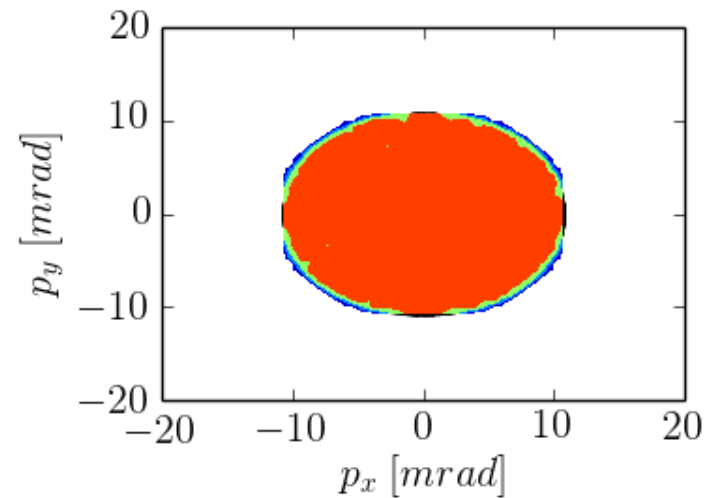
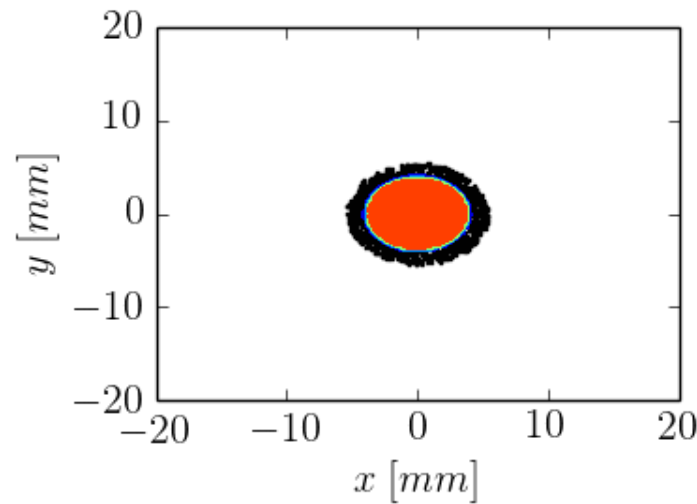
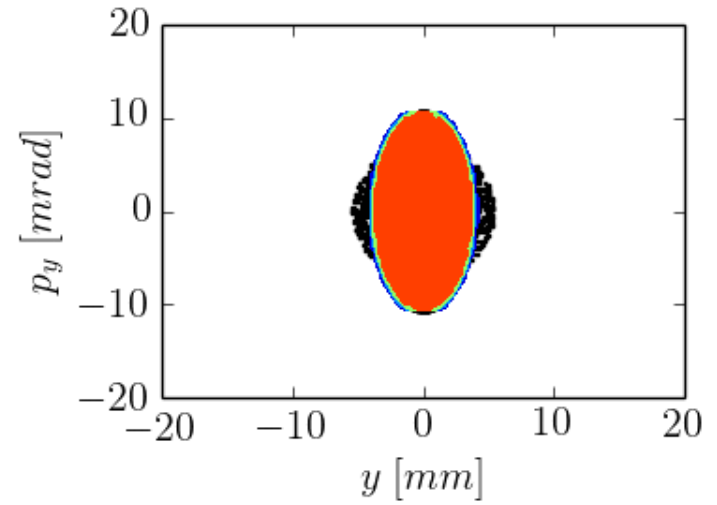
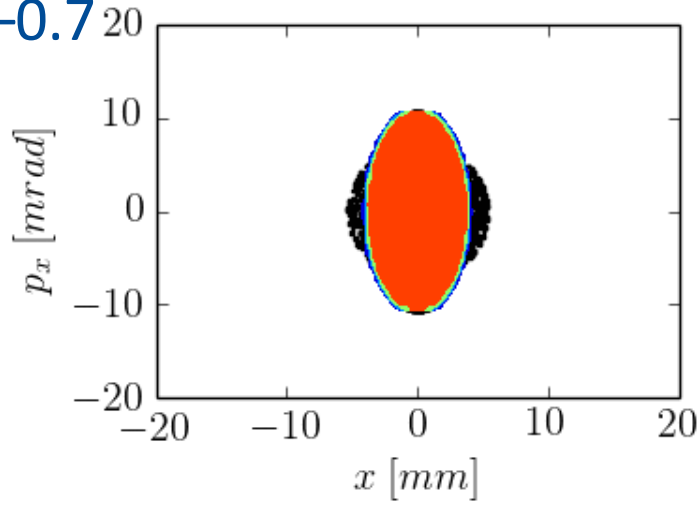
1. Experimental demonstration of **Nonlinear Integrable Optics lattice**
2. **Space Charge Compensation** in high intensity circular accelerators
 - Machines for neutrino research
 - Injectors for future high energy colliders (HL-LHC, FCC)

1. Integrable Optics

Space Charge in Linear Optics

- System: linear FOFO 100 A linear KV w/mismatch
- Result: quickly drives test-particles into the halo

$$\Delta Q_{sc} \sim -0.7$$



Tech-X, RadiaSoft simulation

Introducing Landau Damping

COLLIDING BEAMS: PRESENT STATUS; AND THE SLAC PROJECT*

B. Richter

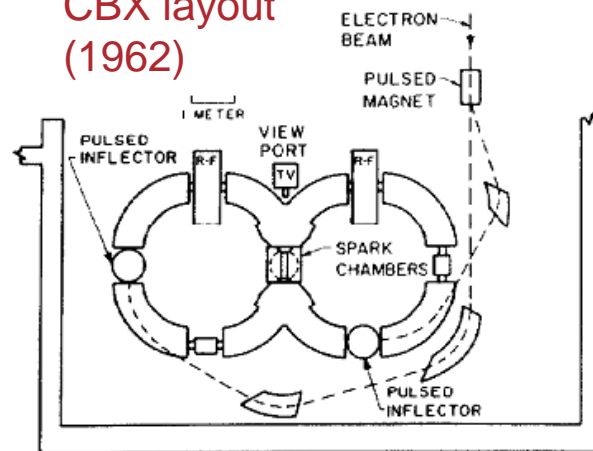
Stanford Linear Accelerator Center
Stanford University, Stanford, California 94305



The discovery in the early '60's at the Princeton-Stanford ring of what was thought to be the resistive wall instability brought the realization that circular accelerators are fundamentally unstable devices because of the interaction of the beam with its environment. Stability is achieved only through Landau damping and/or some external damping system.

- 1965 Princeton-Stanford CBX: First mention of an 8-pole magnet
 - Observed vertical resistive wall instability
 - With octupoles, increased beam current from ~5 to 500 mA
- CERN PS: In 1959 had 10 octupoles; not used until 1968
 - At 10^{12} protons/pulse observed (1st time) head-tail instability. Octupoles helped.
 - Once understood, chromaticity jump at transition was developed using sextupoles.
 - More instabilities were discovered; helped by octupoles, fb

CBX layout
(1962)



Stability of Linear Lattices

- The main feature of all present accelerators – **linear focusing lattice**: particles have nearly identical betatron frequencies (tunes) by design. Such machines are built using dipoles and quadrupoles.
 - **All nonlinearities** (both magnet imperfections and specially introduced) **are perturbations and make single particle motion unstable** due to resonant conditions

PHYSICAL REVIEW

VOLUME 88, NUMBER 5

DECEMBER 1, 1952

The Strong-Focusing Synchrotron—A New High Energy Accelerator*

ERNEST D. COURANT, M. STANLEY LIVINGSTON,† AND HARTLAND S. SNYDER
Brookhaven National Laboratory, Upton, New York

(Received August 21, 1952)

Strong focusing forces result from the alternation of large positive and negative n -values in successive sectors of the magnetic guide field in a synchrotron. This sequence of alternately converging and diverging magnetic lenses of equal strength is itself converging, and leads to significant reductions in oscillation amplitude, both for radial and axial displacements. The mechanism of phase-stable synchronous acceleration still applies, with a large reduction in the amplitude of the associated radial synchronous oscillations. To illustrate, a design is proposed for a 30-Bev proton accelerator with an orbit radius of 300 ft, and with a small magnet having an aperture of 1×2 inches. Tolerances on nearly all design parameters are less critical than for the equivalent uniform- n machine. A generalization of this focusing principle leads to small, efficient focusing magnets for ion and electron beams. Relations for the focal length of a double-focusing magnet are presented, from which the design parameters for such linear systems can be determined.

BETATRON OSCILLATIONS

RESTORING forces due to radially-decreasing magnetic fields lead to stable “betatron” and “syn-

chrotron” oscillations in synchrotrons. The amplitudes of these oscillations are due to deviations from the equilibrium orbit caused by angular and energy spread in the injected beam, scattering by the residual gas, magnetic inhomogeneities, and frequency errors. The strength of the restoring forces is limited by the

* Work done under the auspices of the AEC.

† Massachusetts Institute of Technology, Cambridge, Massachusetts.



Focusing: Linear vs. Nonlinear

- Accelerators are linear systems by design (frequency is independent of amplitude).
- In accelerators, nonlinearities are unavoidable (space charge, beam-beam) and some are useful (Landau damping).
- All nonlinearities (in present rings) lead to resonances and dynamic aperture limits.

- **Are there “magic” nonlinearities with zero resonance strength?**
- **The answer is – yes (we call them “*integrable*”)**

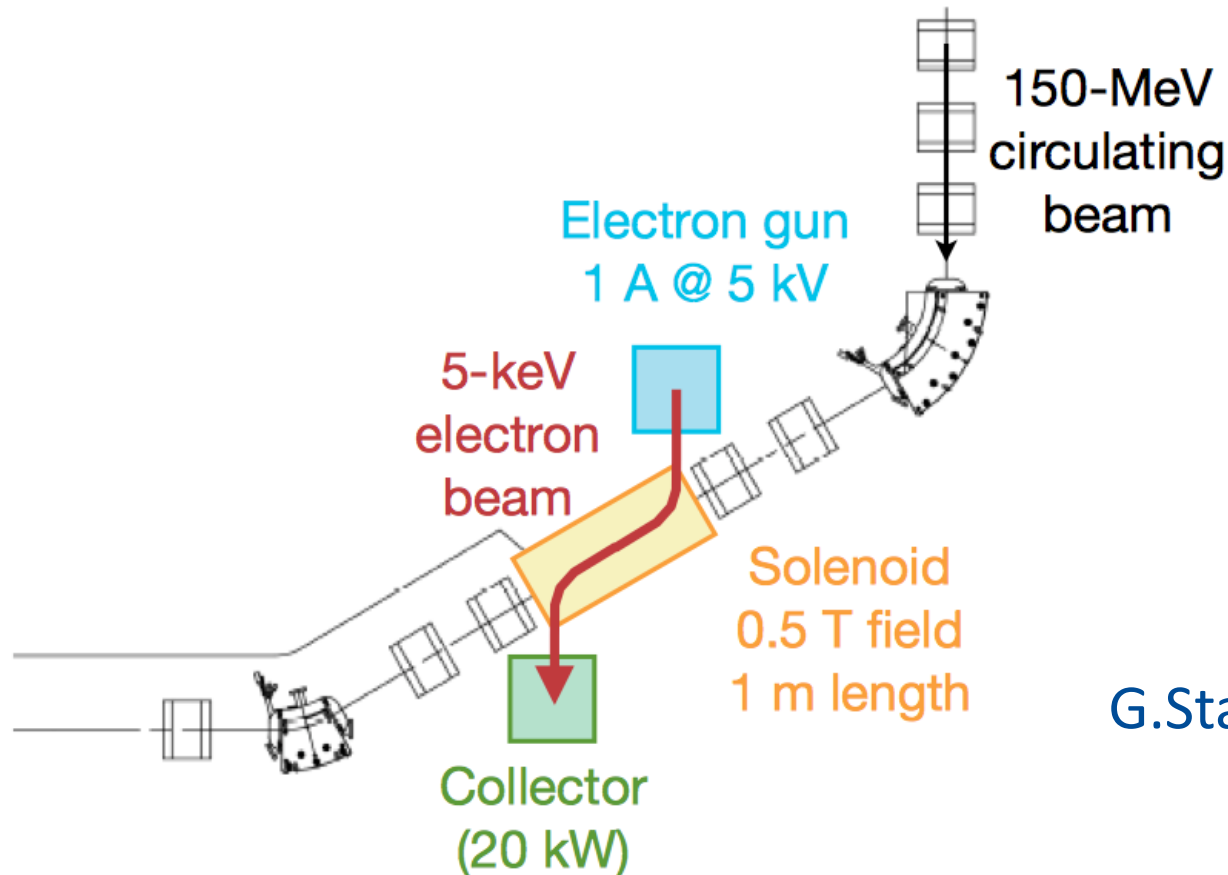
Do Accelerators Need to be Linear?

Search for solutions that are strongly nonlinear yet stable

- Orlov (1963)
- McMillan (1967) – 1D solution
- ✓ Perevedentsev, Danilov (1990) – generalization of McMillan case to 2D, round colliding beams. **Require non-Laplacian potentials to realize**
 - Round colliding beams possess 1 invariant – VEPP-2000 at BINP (Novosibirsk, Russia) commissioned in 2006. Record-high beam-beam tune shift ~ 0.25 attained in 2013
 - **Electron Lens**
- Chow, Cary (1994)
- ✓ Nonlinear Integrable Optics: Danilov and Nagaitsev solution for nonlinear lattice with 2 invariants of motion that **can be implemented with Laplacian potential**, i.e. with special magnets – *Phys. Rev. ST Accel. Beams* 13, 084002 (2010)

IOTA Electron Lens

- Capitalize on the Tevatron experience and recent LARP work
- Re-use Tevatron EL components



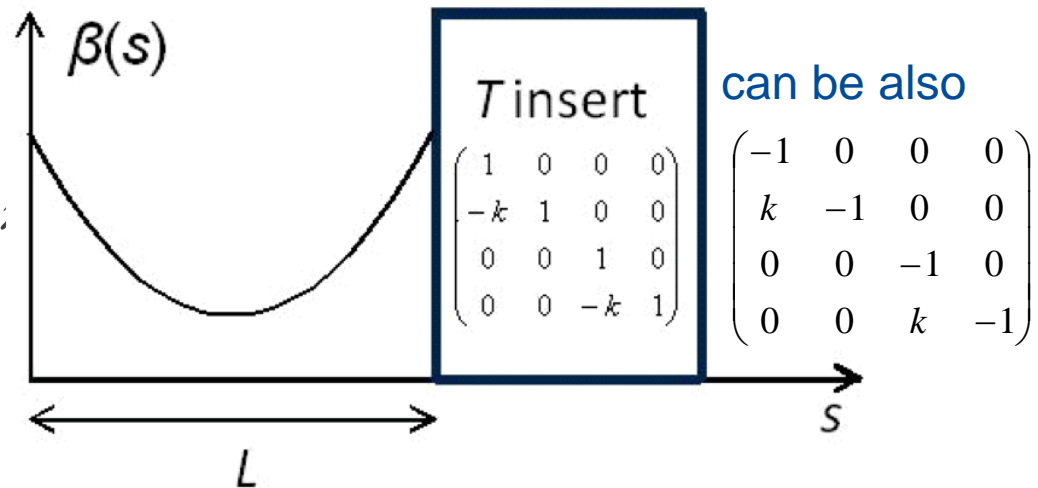
G.Stancari

Nonlinear Integrable Optics with Laplacian Potential

1 Start with a round axially-symmetric *linear* lattice (FOFO) with the element of periodicity consisting of

a. Drift L

b. Axially-symmetric focusing block “T-insert” with phase advance $n \times \pi$;



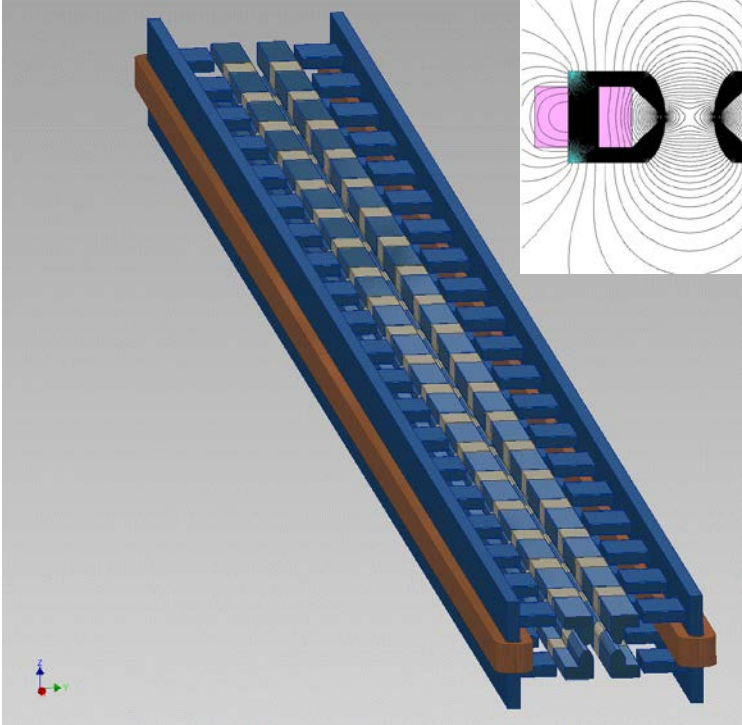
1 Add special nonlinear potential $V(x,y,s)$ in the drift such that

$$\Delta V(x, y, s) \approx \Delta V(x, y) = 0$$

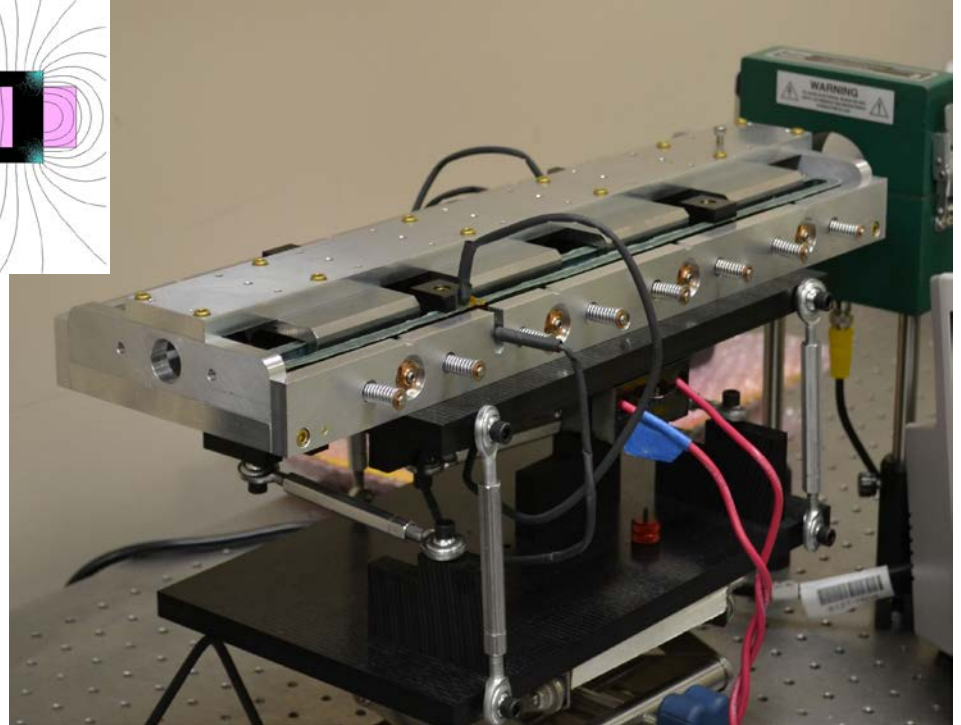
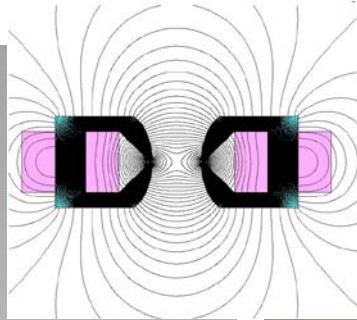
and the system possesses two integrals of motion

Nonlinear Magnet

- Joint effort with RadiaBeam Technologies (Phase I and II SBIR)



FNAL Concept: 2-m long nonlinear magnet



RadiaBeam short prototype. The full 2-m magnet will be designed, fabricated and delivered to IOTA in Phase II

What is Unique About These Solutions?

- One can **add the special potential** (Laplacian or E-Lens) **to a drift of a conventional accelerator** (albeit specially designed and carefully controlled) and make the lattice integrable.
 - **Does not require new technology for the significant portion of accelerator circumference – same cost!**

Stability of Nonlinear Lattices

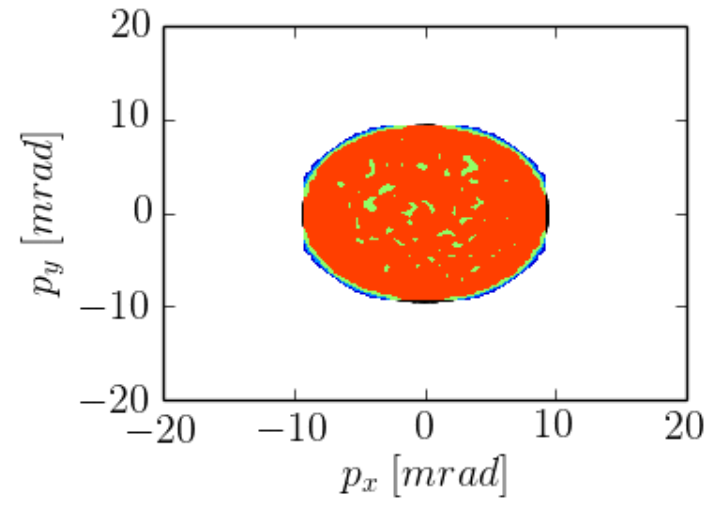
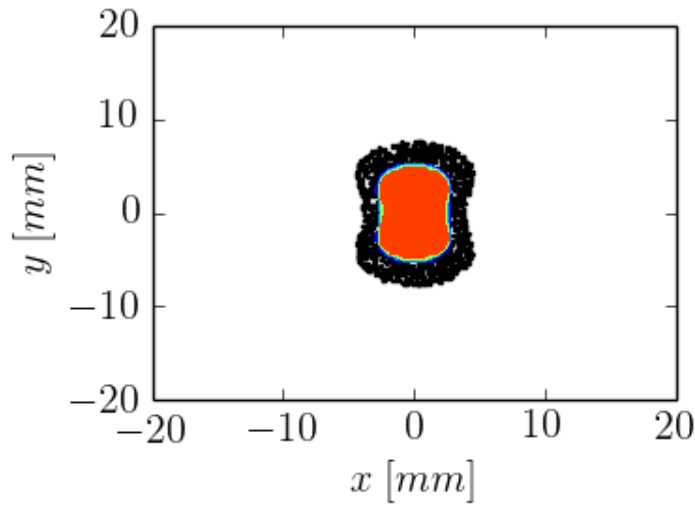
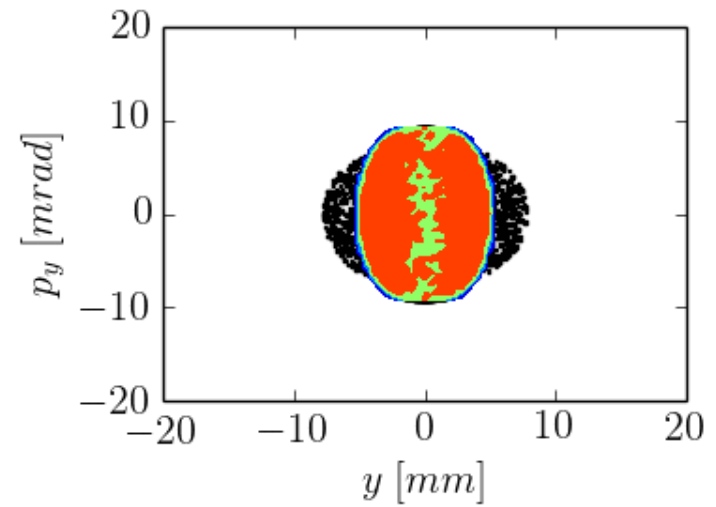
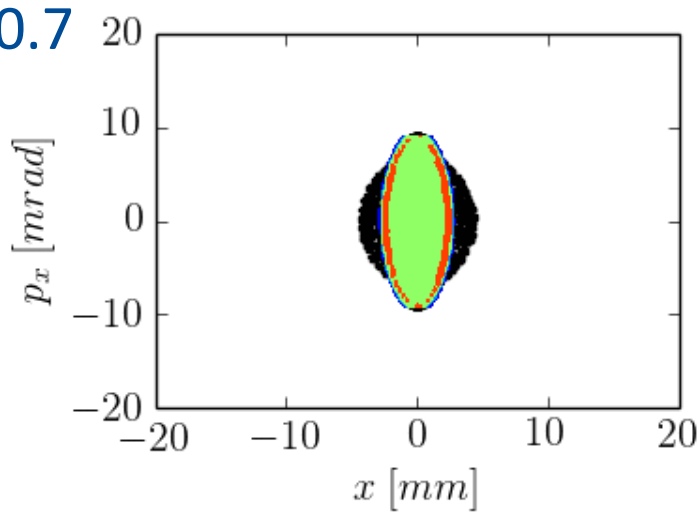
Nonlinear systems can be more stable!

- 1D systems: non-linear (unharmonic) oscillations can remain stable under the influence of periodic external force perturbation. Example: $\ddot{z} + \omega_0^2 \sin(z) = a \sin(\omega_0 t)$
- 2D: The resonant conditions $k\omega_1(J_1, J_2) + l\omega_2(J_1, J_2) = m$ are valid only for certain amplitudes.
- Nekhoroshev's condition guarantees detuning from resonance and, thus, stability.
 - *An Exponential Estimate of the Time of Stability of Nearly-Integrable Hamiltonian Systems*. Russian Math. Surveys 32:6 (1977) from Uspekhi Mat. Nauk 32:6 (1977)

Space Charge in NL Integrable Optics

- System: linear FOFO 100 A linear KV w/mismatch
- Result: nonlinear decoherence suppresses halo

$$\Delta Q_{sc} \sim -0.7$$



Tech-X, RadiaSoft simulation

IOTA Goals for Integrable Optics

The IOTA experiment has the **goal to demonstrate the possibility to implement nonlinear integrable optics** with a large betatron frequency spread $\Delta Q > 1$ and stable particle motion **in a realistic accelerator design**

Benefits of nonlinear integrable optics include

- Increased Landau damping
- Improved stability to perturbations
- Resonance detuning



IOTA Staging – Phase I

Phase I will concentrate on the academic aspect of single-particle motion stability using e-beams

- **Achieve large nonlinear tune shift/spread** without degradation of dynamic aperture **by “painting”** the accelerator aperture **with a “pencil” beam**
- Suppress strong lattice resonances = cross the integer resonance by part of the beam without intensity loss
- Investigate stability of nonlinear systems to perturbations, develop practical designs of nonlinear magnets
- The measure of success will be the achievement of high nonlinear tune shift = 0.25

IOTA Staging – Phase I

- The magnet quality, optics stability, instrumentation system and optics measurement techniques must be of highest standards in order to meet the requirements for integrable optics
 - 1% or better measurement and control of β -function, and 0.001 or better control of betatron phase
- This is why **Phase I needs pencil e⁻ beams** as such optics parameters are not immediately reachable in a small ring operating with protons

IOTA Staging – Phase II

After the IOTA commissioning, we will move the existing 2.5 MeV proton/H⁻ RFQ into the ASTA hall to inject protons into the IOTA ring.

$\Delta Q_{SC} = 0.6$ for one-turn injection

*multi-turn injection possible



- Allows tests of Integrable Optics with protons and realistic space charge beam dynamics studies
- **Allows space charge compensation experiments**
- Unique capability

2. Space Charge Compensation

Space Charge Compensation

$$\xi_{SC} = \frac{B_f r_p N_{tot}}{4\pi\epsilon_n \beta \gamma^2}$$

$$\mathbf{B} = \beta \mathbf{E}$$

Net force

$$\mathbf{E} - \beta \mathbf{B} = \mathbf{E} / \gamma^2$$

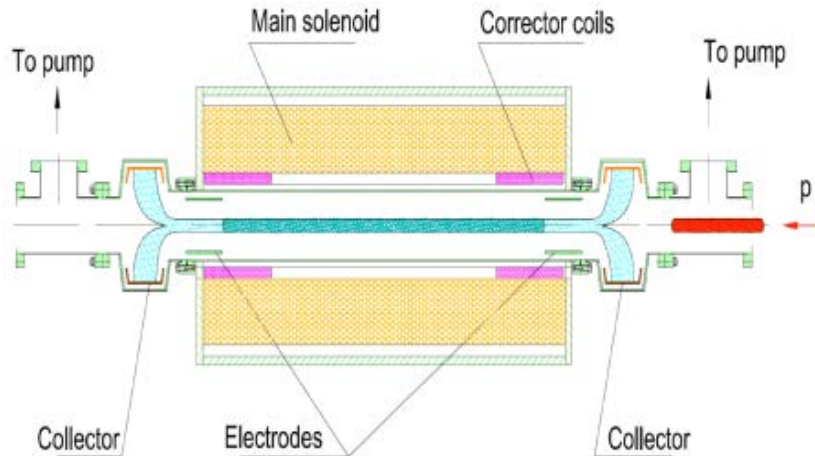
protons

r , across the beam

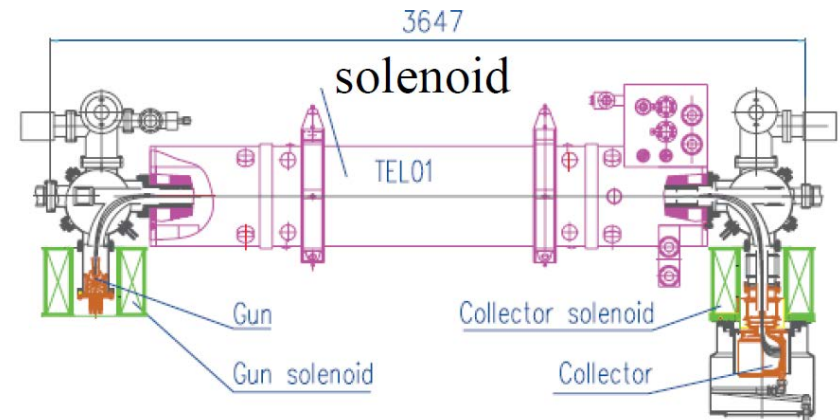
A. Burov, G. Foster, V. Shiltsev, FNAL-TM-2125
(2000)

Possible Implementations

E-column concept



E-lens concept



1. The impact of electrons is equal to the total impact of space-charge over the ring

$$|\Delta v_{sc}| = \frac{N_{b,tot} r_{cb}}{2\pi\beta_b^2 \gamma_b^3 \epsilon} \frac{\hat{I}}{\bar{I}} = \Delta v_e = \frac{N_e r_{cb}}{2\pi\beta_b^2 \gamma \epsilon} \quad \frac{N_e}{N_{b,tot} (\hat{I}/\bar{I})} = \frac{1}{\gamma_b^2} = \eta_0 \frac{N_{ec} L_{ec}}{C}$$

2. The transverse profile of the electron is made the same as that of the proton beam
→ use of solenoid
3. The system of magnetized electrons and protons is now dynamically stable

Roadmap to High-Intensity Rings

- Theory and modeling to develop basis for high intensity circular machines
- Proof-of-principle experiments at IOTA
- Ultimately, **develop a recipe for a new generation rapid cycling synchrotron for super-high beam intensity (\times 3-5 present)**
 - Self-consistent or compensated space-charge
 - Strong non-linearity (for Landau damping) to suppress instabilities
 - Stable particle motion at large amplitudes

This strategic initiative received support through LDRD

Plan of Activities

Phase 1: FY15-17

1. Construction of main elements of the IOTA facility: a) IOTA ring; b) electron injector based on existing ASTA electron linac; c) proton injector based on existing HINS proton source; d) special equipment for AARD experiments.
2. Commissioning of the IOTA ring with electron beam.
3. Study of single-particle dynamics in integrable optics with electron beams.

Plan of Activities

Phase 2: FY18-20

1. Commission IOTA operation with proton beams.
2. Carry out space-charge compensation experiments with nonlinear optics and electron lenses.

Phase 3: FY21 and beyond

1. Study the application of space-charge compensation techniques to next generation high intensity machines.
2. Expand the program beyond these high priority goals to allow Fermilab scientists and a broader accelerator HEP community to utilize unique proton and electron beam capabilities of the IOTA facility

Collaboration



participants of the 2nd ASTA Collaboration Meeting, June 2014

Collaboration

- A lot of interest to participate in IOTA from the accelerator community
 - 2 annual Collaboration Meetings, ~60 participants
- Significant intellectual and in-kind contributions, expressions of interest
 - **NIU, UMD, RadiaSoft, CERN, ORNL, BINP, Colorado State, Univ. Mexico** – integrable optics, space charge effects, phase space manipulation
 - **LBNL, ANL** – optical stochastic cooling demonstration
 - **UMD** – multi-pickup beam profile monitor for IOTA
 - **JINR** – integrable optics and space charge, contributed quadrupole magnets for IOTA
 - **Univ. Frankfurt** – electron lens

International *Space Charge Collaboration* at IOTA

- Collaborating institutions (at present): **Fermilab, ORNL, CERN, RadiaSoft, UMD**
- Work on the scientific case, hardware development, simulations, planning and execution of space charge compensation experiments with protons in IOTA
- Major topics
 - Operation of IOTA with protons, injection, and space charge measurements
 - Space charge compensation in nonlinear integrable lattice
 - Special magnets
 - Electron lens
 - Space charge compensation with electron columns
 - Space charge suppression with circular modes – **for FCC**

IOTA Optical Stochastic Cooling Demonstration

Example of

- Broad research potential of the facility
- Collaboration-driven research (with **NIU, LBNL and ANL**)
- Goal – Experimental demonstration of the optical stochastic cooling technique
- Why IOTA
 - Low energy (~ 100 MeV – minimal synchrotron radiation damping) is ideal for demonstration purposes
 - Flexible lattice e^- storage ring
- Motivation – **Beam cooling for future high energy accelerators/colliders**

Training and University Collaboration

- Excellent connection to the university community through the Joint Fermilab/University PhD program
 - Already 9 graduate students doing thesis research at ASTA/IOTA
 - 7 NIU, 1 U.Chicago, 1 IIT, 2 more to join soon
- Partnership with university groups
 - NIU – DOE GARD grant on OSC
 - Univ. of Maryland – NSF grant for IOTA-related work
 - Univ. Frankfurt – IOTA electron lens
 - Univ. Mexico – ASTA linac commissioning
 - Colorado State – ASTA gun stability
 - Interest from: UC Berkeley, MIT, Oxford



Summary

- Experimental accelerator R&D at **IOTA** is one of the **cornerstones of the proposed national R&D thrust** “Multi-MW Beams and Targets”, and is well aligned with P5 priorities
- IOTA offers a **unique scientific program** aiming at breakthrough research to allow for **x3-5 increase of beam intensity in future proton rings**
 - IOTA augments the US program lacking ring facilities for accelerator research and training
- IOTA experiments are a great opportunity to explore something **truly novel with circular accelerators**
- IOTA will be a strong **driver of national and international collaboration and training**

Acknowledgments

- A.Burov, K.Carlson, A.Didenko, N.Eddy, V.Kashikhin, V.Lebedev, J.Leibfritz, M.McGee, S.Nagaitsev, L.Nobrega, G.Romanov, V.Shiltsev, S.Wesseln, D.Wolff (FNAL)
- D.Shatilov, A.Romanov (BINP)
- G. Kafka (IIT)
- S. Danilov (ORNL),
- S. Antipov (U of Chicago)
- J. Cary (Tech-X)
- D. Bruhwiler, S. Webb (RadiaSoft)
- F.O'Shea, A.Murokh (RadiaBeam)
- R.Kishek, K.Ruisard (UMD)
- JINR, Dubna

Backup slides

Halo Formation from Beam Mismatch

Phase Space from Tomography

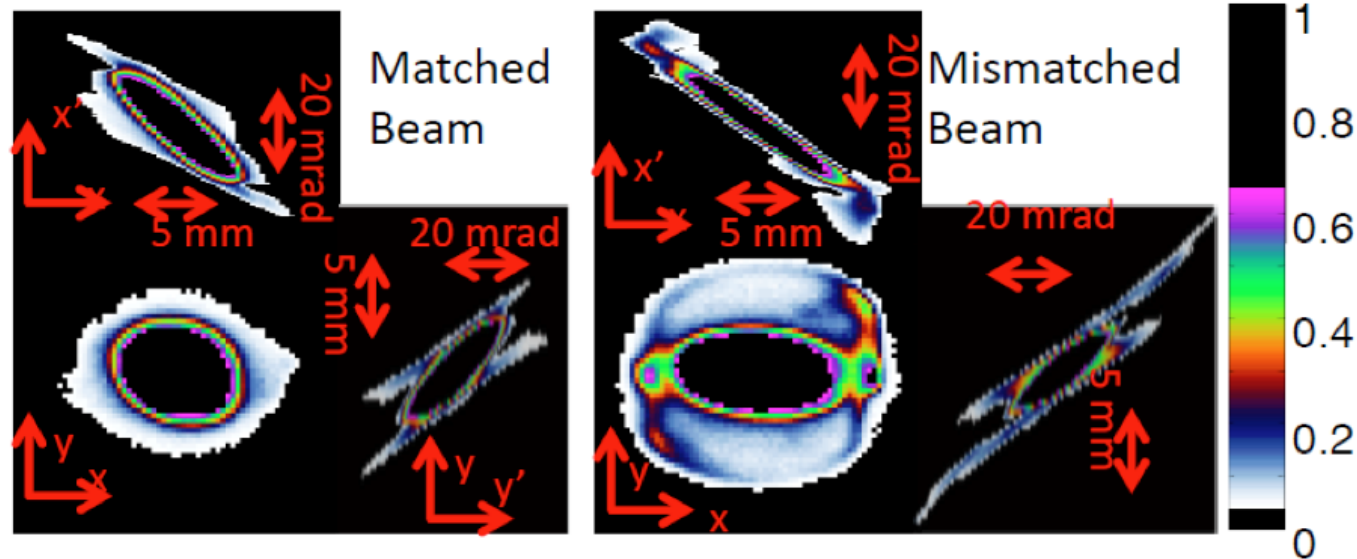


image courtesy
UMD

- LEDA and UMER (NA PAC 13, H.D. Zhang et al.) have confirmed the theoretical basis for halo formation in space-charge dominated beams
- **Linear optics + breathing mode – parametric resonance is the “engine” driving particles into halo**

2D Generalization of McMillan Mapping

SOME THOUGHTS ON STABILITY
IN NONLINEAR PERIODIC FOCUSING SYSTEMS

Edwin M. McMillan

September 5, 1967



- 1D – thin lens kick

$$x_i = p_{i-1}$$

$$f(x) = -\frac{Bx^2 + Dx}{Ax^2 + Bx + C}$$

$$p_i = -x_{i-1} + f(x_i) \quad Ax^2 p^2 + B(x^2 p + xp^2) + C(x^2 + p^2) + Dxp = \text{const}$$

- 2D – a thin lens solution can be carried over to 2D case in axially symmetric system

1. The ring with transfer matrix

$$\begin{pmatrix} cI & sI \\ -sI & cI \end{pmatrix} \begin{pmatrix} 0 & \beta & 0 & 0 \\ -\frac{1}{\beta} & 0 & 0 & 0 \\ 0 & 0 & 0 & \beta \\ 0 & 0 & -\frac{1}{\beta} & 0 \end{pmatrix} \quad \begin{aligned} c &= \cos(\phi) \\ s &= \sin(\phi) \\ I &= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \end{aligned}$$

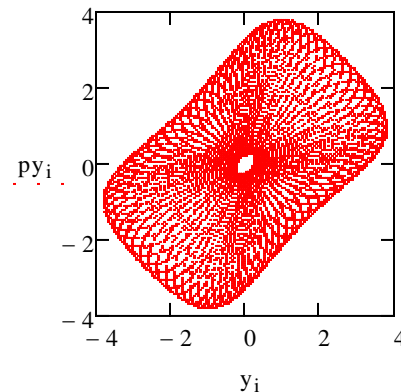
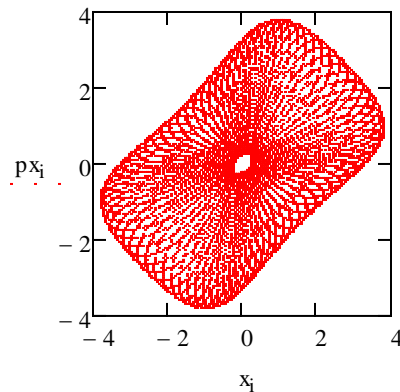
2. Axially-symmetric thin kick

$$\theta(r) = \frac{kr}{ar^2 + 1}$$

can be created with electron lens

2D Generalization of McMillan Mapping

- The system is integrable. Two integrals of motion (transverse):
 - Angular momentum: $xp_y - yp_x = \text{const}$
 - McMillan-type integral, quadratic in momentum



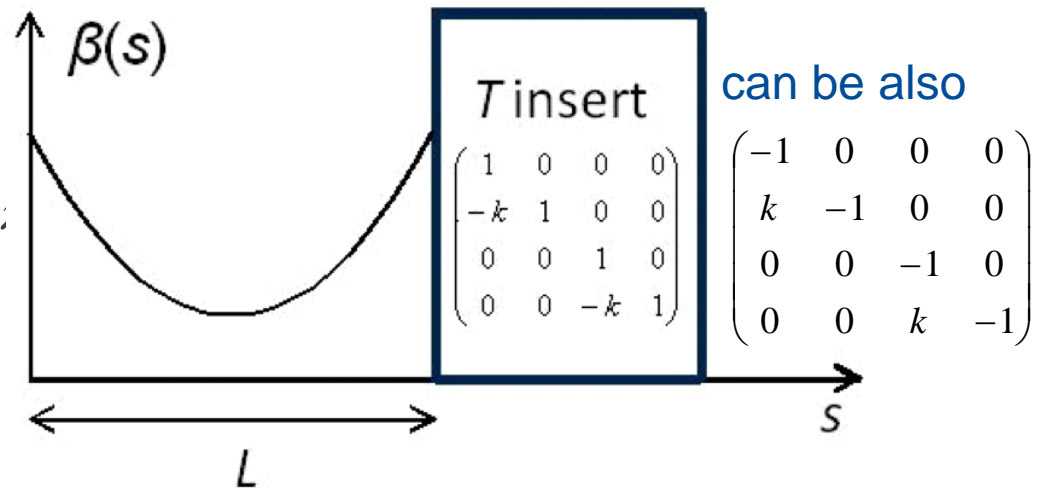
- For large amplitudes, the fractional tune is 0.25
- For small amplitude, the electron (defocusing) lens can give a **tune shift of ~ -0.3 per cell !**
- Potentially, can cross an integer resonance

Nonlinear Integrable Optics with Laplacian Potential

1 Start with a round axially-symmetric *linear* lattice (FOFO) with the element of periodicity consisting of

a. Drift L

b. Axially-symmetric focusing block “T-insert” with phase advance $n \times \pi$;



1 Add special nonlinear potential $V(x,y,s)$ in the drift such that

$$\Delta V(x, y, s) \approx \Delta V(x, y) = 0$$

Time-Independent Hamiltonian

- Start with a Hamiltonian

$$H = \frac{p_x^2}{2} + \frac{p_y^2}{2} + K(s) \left(\frac{x^2}{2} + \frac{y^2}{2} \right) + V(x, y, s)$$

- Choose s -dependence of the nonlinear potential such that H is time-independent in normalized variables

$$z_N = \frac{z}{\sqrt{\beta(s)}},$$

$$p_N = p\sqrt{\beta(s)} - \frac{\beta'(s)z}{2\sqrt{\beta(s)}},$$

$$H_N = \frac{p_{xN}^2 + p_{yN}^2}{2} + \frac{x_N^2 + y_N^2}{2} + \beta(\psi)V(x_N\sqrt{\beta(\psi)}, y_N\sqrt{\beta(\psi)}, s(\psi))$$

$$H_N = \frac{p_{xN}^2 + p_{yN}^2}{2} + \frac{x_N^2 + y_N^2}{2} + U(x_N, y_N, \psi)$$

- This results in H being the integral of motion
- Note there was no requirement on V – can be made with any conventional magnets, i.e. octupoles

Special Potential – Second Integral of Motion

- Find potentials that result in the Hamiltonian having a second integral of motion quadratic in momentum
 - All such potentials are separable in some variables (cartesian, polar, elliptic, parabolic)
 - First comprehensive study by Gaston Darboux (1901)

$$I = Ap_x^2 + Bp_x p_y + Cp_y^2 + D(x, y) \quad A = ay^2 + c^2, B = -2axy, C = ax^2$$

- Darboux equation

$$xy(U_{xx} - U_{yy}) + (y^2 - x^2 + c^2)U_{xy} + 3yU_x - 3xU_y = 0$$

- General solution in elliptic variables ξ, η , with f and g arbitrary

- Solution that satisfies the Laplace equation

$$f_2(\xi) = \xi \sqrt{\xi^2 - 1} (d + t \operatorname{acosh}(\xi)) \quad g_2(\eta) = \eta \sqrt{1 - \eta^2} (q + t \operatorname{acos}(\eta))$$

Maximum Tune Shift

- Multipole expansion of U :

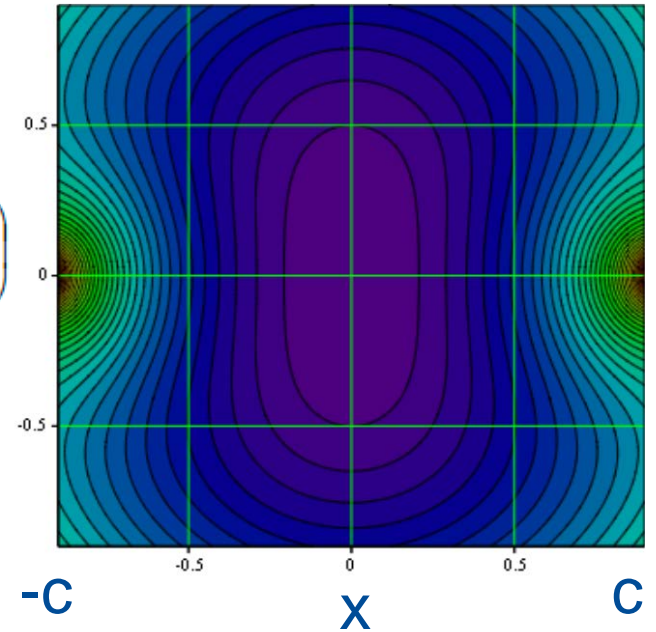
$$U(x, y) \approx \frac{x^2}{2} + \frac{y^2}{2}$$

$$+ t \operatorname{Re} \left((x + iy)^2 + \frac{2}{3} (x + iy)^4 + \frac{8}{15} (x + iy)^6 + \frac{16}{35} (x + iy)^8 + \dots \right)$$

- For small-amplitude motion to be stable*, $t < 0.5$

$$v_1 = v_0 \sqrt{1 + 2t} \quad v_2 = v_0 \sqrt{1 - 2t}$$

- Theoretical maximum nonlinear tune shift per cell is
 - ▣ 0.5 for mode 1, **or 50% per cell**
 - ▣ 0.25 for mode 2, **or 25% per cell**



Principles of OSC

- Optical stochastic cooling can deliver damping rates 4 orders of magnitude larger than usual (microwave) stochastic cooling

microwave “slicing”



sample length
~10 cm

$$N_s = N \frac{\Delta \ell}{\ell_b}$$

optical “slicing”



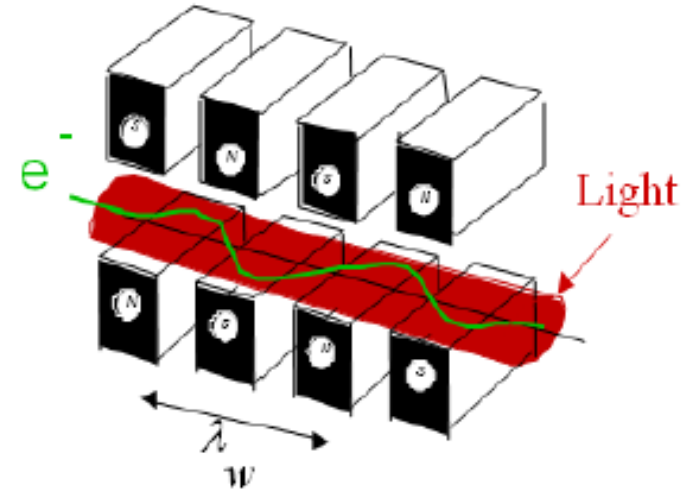
sample length
~10 μm

- Undulator can be used both as a pickup and as a kicker
 - Effective for longitudinal kicks
 - Transverse kick is suppressed for ultra-relativistic beam

$$(\mathbf{F} = e(\mathbf{E} - [\boldsymbol{\beta}\mathbf{B}]) \xrightarrow{\beta \rightarrow c} 0)$$

Mikhailichenko, Zolotorev, PRL 71, 4146 (1993)

Zolotorev, Zholents, PRE 50, 4, 3087 (1994)



V. Lebedev

