Luminosity and Crab-Waist Collision Studies

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Abstract

In high energy physics, the luminosity is one useful value to characterize the performance of a particle collider. To gain more available data, we need to maximize the luminosity in most collider experiments. With the discussions of tune shift involved the beam dynamics and a recently proposed "crabbed waist" scheme of beam-beam collisions, I present some qualitative analysis to increase the luminosity. In addition, beam-beam tune shifts and luminosities of e+e-, proton-proton/proton-antiproton, and $\mu+\mu-$ colliders are discussed.

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- ✧ Electron-Positron Collider
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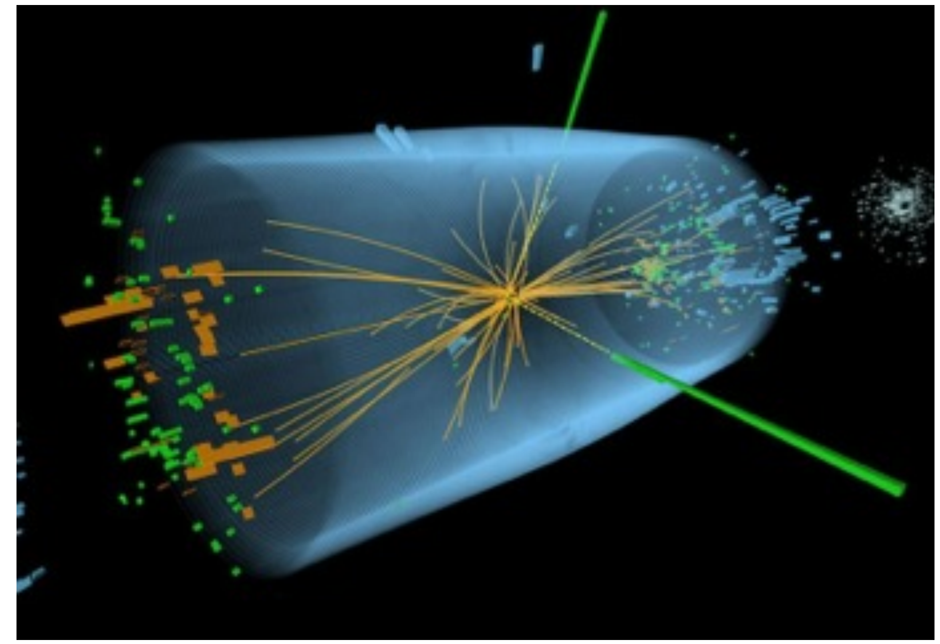
- ✧ Muon-Muon Collider
- ✧ Conclusion

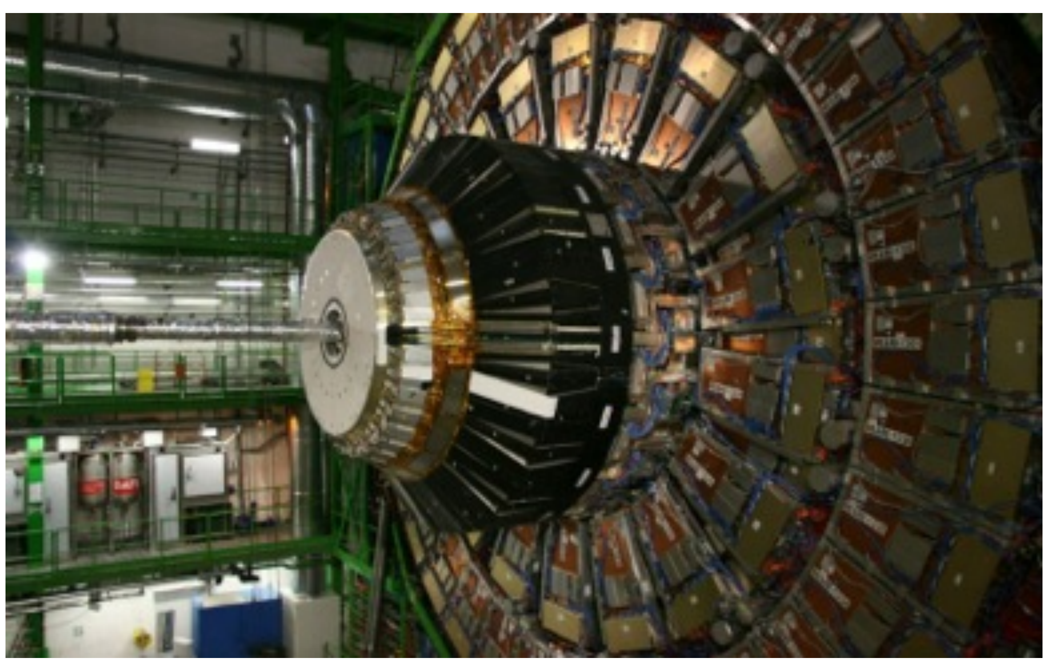
Introduction—Luminosity

☞ Two most important parameters that quantify the performance of particle colliders:

- ☆ beam energy
- ☆ luminosity



[Image: CMS/Cern]



[Image: news.sciencemag.org]

☞ The LHC smashes previous energy record: 13 TeV (*updated on June 5, 2015*)

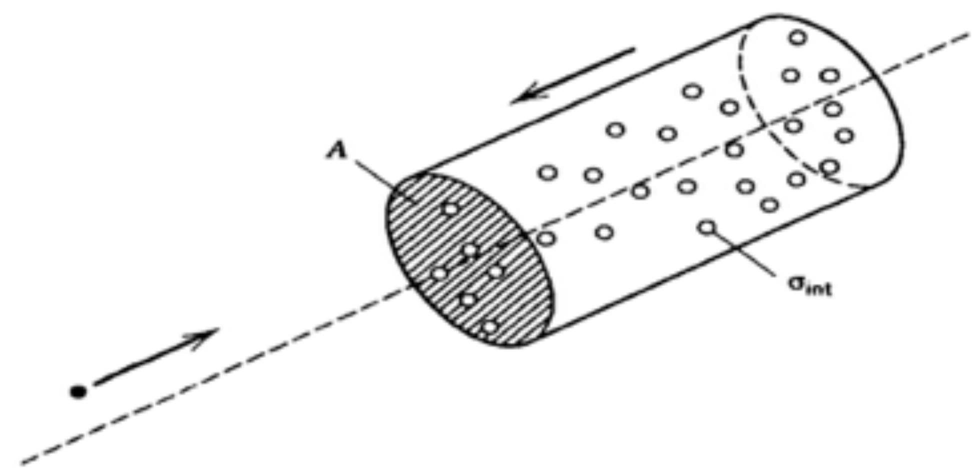
☞ High Luminosity LHC Project: increase the luminosity of the LHC by a factor of 10 beyond its design value by 2020

Introduction—Luminosity

The luminosity, $\mathscr{L}$, is defined as the interaction rate per unite cross section:

$$\mathscr{L} = \frac{R}{\sigma_{int}} = f\frac{N_1 N_2}{A}.$$

#: the number of particles in the accelerator per unit area per time, multiplied by a measure of the target's impenetrability to electromagnetic radiation



For a "Round Gaussian" particle distribution, the luminosity can be written as

$$\mathscr{L} = f\frac{N_1 N_2}{4\pi\sigma_x\sigma_y},$$

☞ *A glance at the luminosity formula reveals that to raise luminosity one must increase the collision frequency, bunch intensity and lower the beam cross sectional area.*

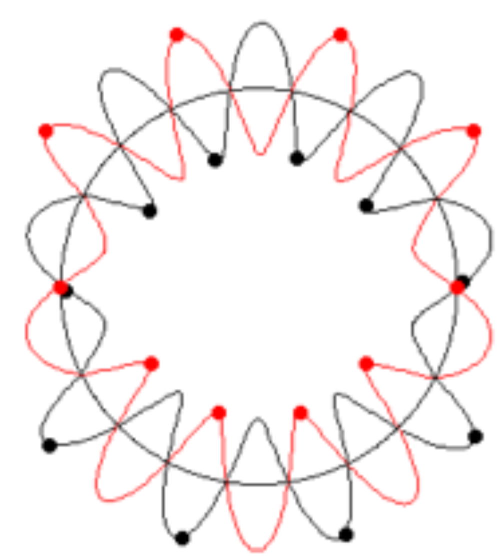
Introduction—Beam-Beam Tune Shift

☞ Tune shift: an important concept in a circular accelerator or synchrotron.

Tune: the number of betatron oscillations (bounded oscillatory motion about the design trajectory corresponding to the transverse stability) per turn in a synchrotron.

$$\nu = \frac{1}{2\pi} \oint \frac{ds}{\beta(s)},$$

☞ s is path length along the design trajectory and the quantity β(s) is usually referred to as the amplitude function.



[Cartoon: lns.cornell.edu/~dugan/

Introduction—Beam-Beam Tune Shift

☞ In a colliding beam accelerator, each time the beams cross each other, the particles in one beam feel the electric and magnetic forces due to the particles in the other beam. Since the particles in these two beams have opposite velocity directions, the electric and magnetic forces do not cancel but rather add, creating a net defocussing force.

For particles undergoing infinitesimal betatron oscillations in a highly relativistic Gaussian beam, the net force would be

$$F = \frac{e^2 N}{2\pi\epsilon_0\sigma^2} r,$$

The tune shift experienced by the particle would be:

$r_0 = e^2/(4\pi\epsilon_0 mc^2)$ is the classical radius of the particle.

$\epsilon_N = \pi\sigma^2(\gamma\beta)/\beta(s) = \pi(\gamma\beta)\sqrt{\epsilon_x\epsilon_y}$ is the normalized emittance.

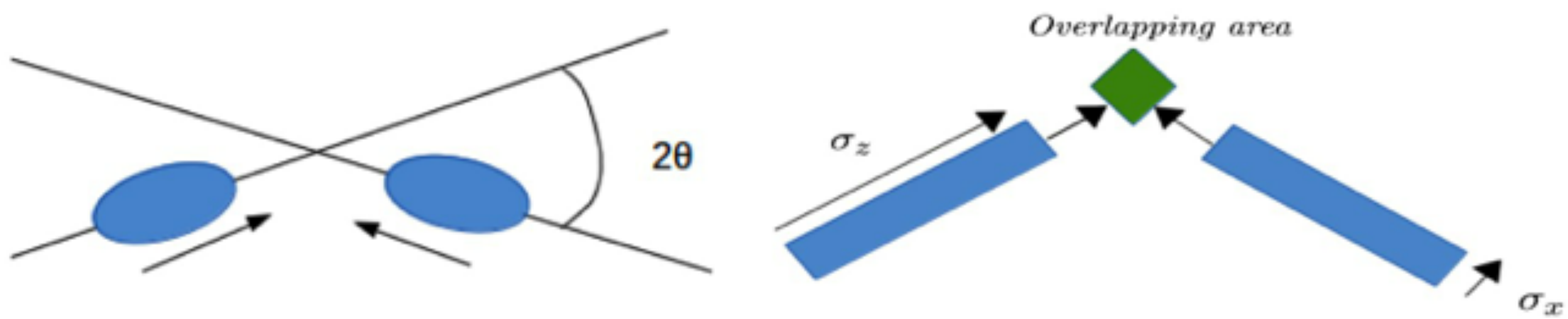
$$\Delta\nu = \frac{1}{4\pi}\frac{1}{pc}\frac{e^2}{2\pi\epsilon_0}\oint\frac{N\beta(s)}{\sigma^2(s)}ds = \frac{r_0}{2\epsilon_N}\times\frac{1}{2}\int N ds.$$

☞ the beam-beam tune shift per collision becomes

$$\boxed{\Delta\nu = \xi = \frac{N r_0}{4\epsilon_N}.}$$

Introduction—the “Crabbed Waist” Scheme

☞ Two kinds of Collisions: “head-on” and “crab-waist” collisions



The “crabbed waist” collision means that the crossing angle $2\theta \gg \sigma_x/\sigma_z$, in contrast, the “head-on” collision means that $2\theta \ll \sigma_x/\sigma_z$.

☞ A recently proposed “crabbed waist” scheme of beam-beam collisions (*[arXiv: physics/0702033; arXiv:0802.2667]*) can substantially increase the luminosity of a collider since it combines several potentially advantageous ideas.

$$\mathscr{L} \propto \frac{N\xi_y}{\beta_y^*} \propto \frac{1}{\sqrt{\beta_y^*}}; \;\; \xi_y \propto \frac{N\beta_y^*}{\sigma_x\sigma_y \cdot \sqrt{1+\phi^2}}; \;\; \xi_x \propto \frac{N}{\epsilon_x \cdot \sqrt{1+\phi^2}}.$$

Here, ϕ is the Piwinski angle, defined as $\phi = \dfrac{\sigma_z}{\sigma_x}\tan\theta \approx \dfrac{\sigma_z}{\sigma_x}\theta,$

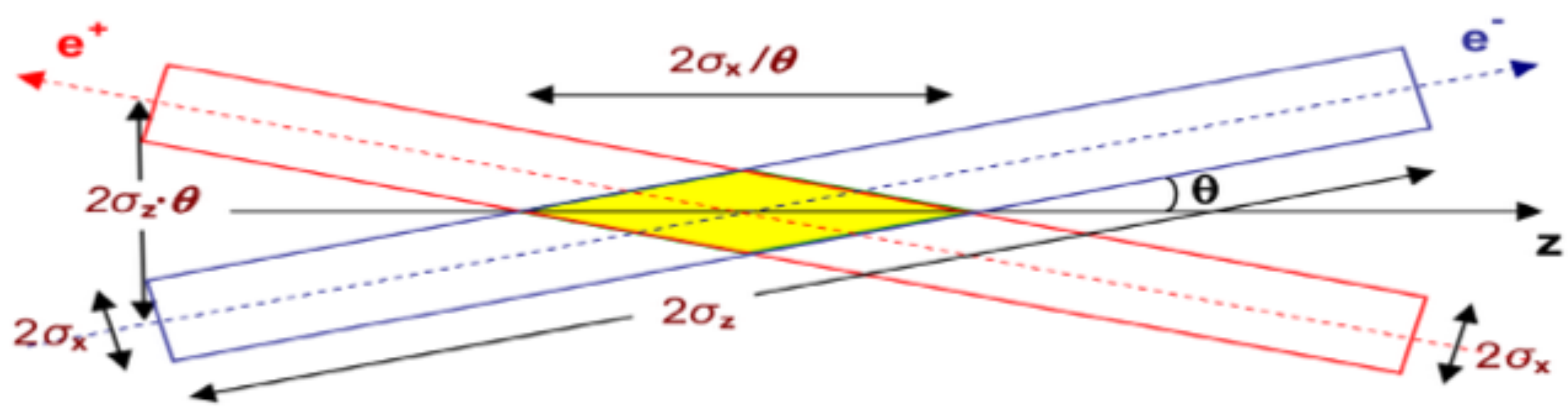
Introduction—the "Crabbed Waist" Scheme

- High luminosity requires short bunches with a very small vertical beta-function β_y^* at the IP and a high beam intensity I with small vertical emittance ϵ_y.

- Large horizontal beam size σ_x and large horizontal emittance ϵ_x can be tolerated.

- Though short bunches are usually hard to make, with large Piwinski angle ϕ the overlapping area becomes much smaller than σ_z, allowing significant β_y decrease and a very significant potential gain in the luminosity.

- The large Piwinski angle increases luminosity by allowing a small β_y^* , but decreases the luminosity by effectively decreasing N. However, the beam-beam tune shift is now small and a large gain in luminosity may be possible by raising the beam-beam tune shift to its limit.



[Figure: arXiv: physics/0702033]

Electron-Positron Colliders

Due to the synchrotron radiation, the energy loss by one electron in a circular orbit is

$$\delta E = \frac{8.85 \times 10^{-5} E^4 (GeV)}{R(meter)}.$$

Because an electron has a small mass, the transverse beam size is naturally damped by the synchrotron radiation. Electron-positron colliders are typically synchrotron wall power limited, so more particles cannot be added. One can take advantage of the crab waist crossing by lowering emittance.

☞ "Head-on" Collision: beam-beam tune shift

$$\xi_x = \frac{N r_e \beta_x^*}{2\pi\gamma\sigma_x(\sigma_x + \sigma_y)}. \qquad \xi_y = \frac{N r_e \beta_y^*}{2\pi\gamma\sigma_y(\sigma_x + \sigma_y)}.$$

For "round beam cross sections", i.e., $\sigma_x \approx \sigma_y$, using the definition of normalized emittance $\epsilon_N = \pi\sigma^2(\gamma v/c)/\beta^*$, at a relativistic high energy ($v \to c$) we will have

$$\xi_y = \frac{N r_e}{4\epsilon_N}.$$

Electron-Positron Colliders

For "elliptical beam cross sections", i.e., $\sigma_x \neq \sigma_y$, with the assumption that the elliptical beams have the same cross sectional area and charge density as the round beams, we have $\sigma^2 = \sigma_x\sigma_y$ and the normalized emittance $\epsilon_N = \pi\gamma\sqrt{\epsilon_x\epsilon_y}$, where $\epsilon_x = \frac{\sigma_x^2}{\beta_x^*}$ and $\epsilon_y = \frac{\sigma_y^2}{\beta_y^*}$. The beam-beam tune shifts then become

$$\begin{aligned} \xi_x &= \frac{Nr_e\beta_x^*}{2\pi\gamma\sqrt{\beta_x^*\epsilon_x}(\sqrt{\beta_x^*\epsilon_x}+\sqrt{\beta_y^*\epsilon_y})} \\ &= \frac{Nr_e}{2\epsilon_N}\cdot\frac{1}{\sqrt{\frac{\epsilon_x}{\epsilon_y}}+\sqrt{\frac{\beta_y^*}{\beta_x^*}}} \end{aligned} \qquad \begin{aligned} \xi_y &= \frac{Nr_e\beta_y^*}{2\pi\gamma\sqrt{\beta_y^*\epsilon_y}(\sqrt{\beta_x^*\epsilon_x}+\sqrt{\beta_y^*\epsilon_y})} \\ &= \frac{Nr_e}{2\epsilon_N}\cdot\frac{1}{\sqrt{\frac{\beta_x^*}{\beta_y^*}}+\sqrt{\frac{\epsilon_y}{\epsilon_x}}} \end{aligned}$$

For the beam cross sections $\sigma_y \ll \sigma_x$, $\xi_y \approx \frac{Nr_e\beta_y^*}{2\pi\gamma\sigma_x\sigma_y}$, we will have

$$\xi_y = \frac{Nr_e}{2\epsilon_N}.$$

The luminosity then can be written as

$$\mathscr{L} = \frac{fN_1N_2}{4\pi\sigma_x\sigma_y} = \frac{fN_1N_2\gamma}{4\epsilon_N\beta_y^*} = \frac{fN_1\gamma\xi_y}{2r_e\beta_y^*}.$$

Electron-Positron Colliders

LEP Beam Parameters at Three Different Energies (*[Rept. Prog. Phys. 63 (2000) 939]*)

E (GeV)	N ($\times 10^{11}$)	k_b	$\mathscr{L}$ ($cm^{-1}s^{-2}$)	Q_s	Q	β^* (m)	ϵ (nm)	σ (μm)	ξ
45.6	1.18	8	1.51×10^{31}	0.065	90.31	2.0	19.3	197	0.030
					76.17	0.05	0.23	3.4	0.044
65	2.20	4	2.11×10^{31}	0.076	90.26	2.5	24.3	247	0.029
					76.17	0.05	0.16	2.8	0.051
97.8	4.01	4	9.73×10^{31}	0.116	98.34	1.5	21.1	178	0.043
					96.18	0.05	0.22	3.3	0.079

$$\epsilon_N = \pi \times \frac{97.8 \times 10^3}{0.511}\sqrt{21.1 \times 10^{-9} \times 0.22 \times 10^{-9}} = 1.295 \times 10^{-3}$$

The beam-beam tune shift is

$$\begin{aligned}
\xi_y &= \frac{N r_e}{2\epsilon_N} \cdot \frac{1}{\sqrt{\frac{\beta_x^*}{\beta_y^*}} + \sqrt{\frac{\epsilon_y}{\epsilon_x}}} \\
&= \frac{4.01 \times 10^{11} \times 2.82 \times 10^{-15}}{2 \times 1.295 * 10^{-3}} \times \frac{1}{\sqrt{\frac{1.5}{0.05}} + \sqrt{\frac{0.22}{21.2}}} \\
&\approx 0.078
\end{aligned}$$

For a specific electron-positron bunch collision, the luminosity $\mathscr{L}$ can be given by

$$\mathscr{L} = \frac{1}{4er_e}\frac{\xi_y}{\beta_y^*}\gamma I,$$

where $r_e = e^2/(4\pi\epsilon_0 mc^2) = 2.82 \times 10^{-15}$ m is the classical radius of the electron, γ is the relativistic scaling factor and $I = 2feN$ is the total beam current of both beams.

Electron-Positron Colliders

☞ "Crab-waist" Collision: beam-beam tune shift

$$\xi_y = \frac{N r_e \beta_y^{*2}}{2\pi\gamma\sigma_x\sigma_y\sigma_z}.$$

e^+e^- Collider Parameters

Parameter	LEP	VLLC	$CrabWaist_{200}$	$CrabWaist_{250}$
Circumference (m)	26 658.9	233 000.0	233 000.0	233 000.0
β_x^*, β_y^* (cm)	150, 5	100, 1	2, .06	2, .06
Luminosity ($cm^{-2}\ sec^{-1}$)	9.73×10^{31}	8.8×10^{33}	1.5×10^{35}	9.7×10^{34}
Energy (GeV)	97.8	200.0	200.0	250.0
γ	191 000	391 000	391 000	489 000
Emittances ϵ_x, ϵ_y (nm)	21.1, 0.220	3.09, 0.031	.9, .0017	.9, .00067
rms beam size IP σ_x^*, σ_y^* (μm)	178.0, 3.30	55.63, 0.56	4.25, 0.0321	4.25, 0.0201
Bunch intensity/I (/mA)	$\frac{4.01\times10^{11}}{0.720}$	$\frac{4.85\times10^{11}}{0.1}$	$\frac{4.85\times10^{11}}{0.1}$	$\frac{4.85\times10^{11}}{0.04}$
Number of bunches per beam	4	114	114	46
Total beam current (mA)	5.76	22.8	22.8	9.34
Beam-beam tune shift ξ_x, ξ_y	0.043, 0.079	0.18, 0.18	0.034, 0.18	0.027, 0.23
Dipole field (T)	0.110	0.0208	0.0208	0.0260
E loss / $e^\pm$ / turn (GeV)	2.67	4.42	4.42	10.8
Bunch length (mm)	11.0	6.67	6.67	6.67
Revolution frequency (kHz)	11.245	1.287	1.287	1.287
Synch rad pwr (b.b.) (MW)	14.5	100.7	100.7	100.7

[Table: arXiv: 1112.1105]

Electron-Positron Colliders

- By comparing the result from the crabbed waist collision and that from head-on collision, we already see these advantages of the crabbed waist scheme.
- However, beamstrahlung puts an additional condition on the value of $N/(\sigma_x \sigma_z)$, and thus on the luminosity of high energy e^+e^- colliders.
- It turns out that the crabbed waist scheme is of marginal benefit for a $240 GeV$ Higgs factory circular collider where $e^+e \to Z^0 h^0$. The scheme is useful at the Z^0, where one might search for rare $Z^0 \to \tau^\pm e^\mp$or $Z^0 \to \tau^\pm \mu^\mp$decays.

☞ SuperKEKB in Japan

☞ Belle II will have 40x the luminosity of Belle or BaBar with only a factor of 2.2 increase in beam currents, as compared to Belle. *([PTEP 2013 (2013) 03A011])*

☞ Introducing a large crossing angle and small βy∗ lowers the beam-beam tune shift and hence the luminosity. But the luminosity can be restored by reducing the vertical size, σy, of e+e− bunches, i.e. by using nanobeams to increase the vertical tune shift to as high a value as the storage rings will tolerate.

Proton-Proton Colliders

- For e^+e colliders, the transverse beam size is naturally damped by synchrotron radiation. However, hadrons colliders cannot enjoy fast damping due to the synchrotron radiation, at least for energies less than 10 TeV.

- Hence for the quest for high luminosity (smaller beam emittances) of pp or $p\bar{p}$ colliders, we may either generate low emittance beams in the sources or arrange beam cooling (phase space reduction, usually at low or medium energy accelerators in the injector chain), using either stochastic cooling or electron cooling methods.

For "round beam cross sections", consider the shift for the Tevatron collider, the typical numbers are $N_p = 2.5 \times 10^{11}$, $\epsilon_{N,p} = 2.8\pi\, mm\ mrad = 3\pi \times 10^{-6}\ m$ and for the proton, $r_p = 1.53 \times 10^{-18}$ meters.

$$\xi_{\bar{p}} = \frac{N_p r_p}{4\epsilon_{N,p}} = \frac{2.5 \times 10^{11} \times 1.53 \times 10^{-18}\, m}{4 \times 2.8\,\pi \times 10^{-6}\, m} = 0.011 \,\text{per IP}$$

☞ For hadron colliders it is difficult to lower the emittance to raise the tune shift, but the number of hadrons per bunch might be raised even if this requires lowering the number of bunches.

Muon-Muon Colliders

✧ The μ+μ− collider is attractive because the muon is a point-like particle, just like the electron but 200 times heavier.
✧ The beamstrahlung and synchrotron radiation are not important unless energy is extremely high.
✧ The short lifetime of muons (around 2.2μs in the rest frame) makes a muon collider challenging technologically.
✧ There are still many challenges to increase the luminosity, e.g., emittance reduction, targeting and neutrino radiation.

☞ It is still possible to combine design ideas of e+e− and proton-proton/proton-antiproton colliders and take their advantages.

☞ If the "crabbed waist" scheme allows 3x more muons per bunch, the luminosity increase 9x. This assumes that the muons are available.

☞ **a "round beam" muon-muon collider**

E (TeV)	1.5	3
luminosity $\mathscr{L}$ $(10^{34}\ cm^{-2}s^{-1})$	0.92	3.4
beam-beam Tune Shift ξ_y	≈ 0.087	≈ 0.087
number of particles per bunch N (10^{12})	2	2
muon transverse emittance $(\pi mm\ mrad)$	25	25

[Table: MAP-DOC - 4318 (2011)]

$$\xi_y = \frac{N r_\mu}{4\epsilon_N} = \frac{2\times 10^{12}\times 2.82\times 10^{-15}\times \frac{0.511}{105.658}}{4\times 25\pi\times 10^{-6}} = 0.087.$$

Conclusion

- For e^+e^- colliders, the "crabbed waist" scheme is possible to gain luminosity due to the significant decrease of β_y^* (with constant ξ_y achieved by lower emittance) at low energy level.
- For $pp/p\bar{p}$ and $\mu^+\mu^-$ colliders, the way to exploit the "crabbed waist" to increase the luminosity requires higher beam intensity, i.e., increasing the number of particles of each bunch.
- At high energy level (above the Z^0 mass), the "crabbed waist" scheme would be of marginal benefit because of the beamstrahlung for e^+e^- colliders.
- Though small beam sizes would bring possible high luminosity, operation of the colliders with smaller and smaller beams would also bring up many issues relevant to alignment of magnets, vibrations and long-term tunnel stability, which should be dealt with seriously.
- The expression of the beam-beam tune shift of e^+e^- colliders is different from that of $pp/p\bar{p}$ and $\mu^+\mu^-$ colliders by a factor 1/2 because of the beam is flat.

☞ Relative article : *"Luminosity and Crab Waist Collision Studies" , Wanwei Wu & Dan Summers, arXiv:1505.06482 [physics.acc-ph]*

☞ Acknowledge: *Thanks for Dr. Don Summers for his useful discussion and kind help.*