MC4BSM 2015

Monte Carlo Tools for Physics Beyond Standard Model

Calculational Algorithms for M_2 Variables

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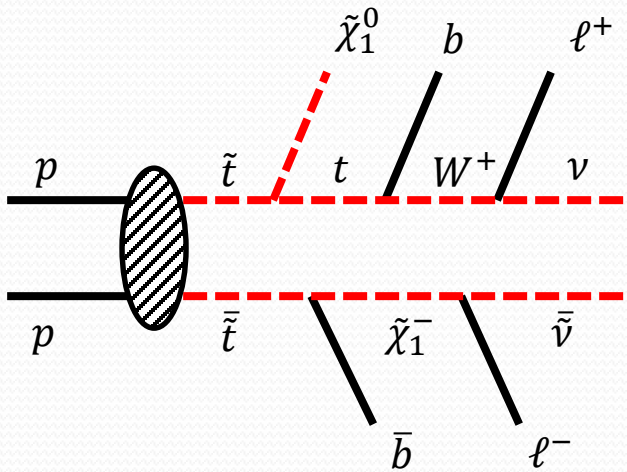
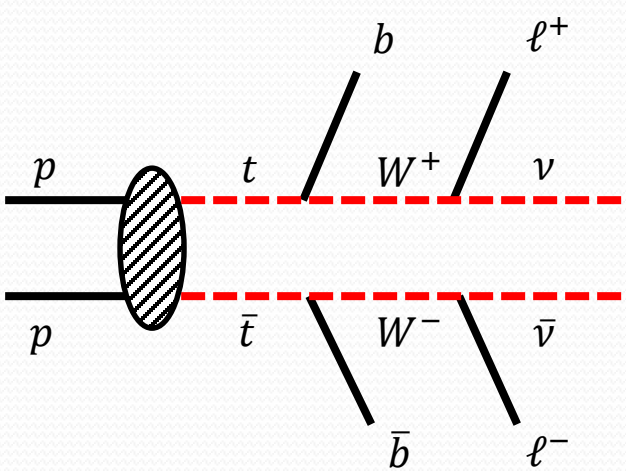
Fermilab LPC

Motivation

Stop search

- Pair-produced supersymmetric top partners in a compressed regime



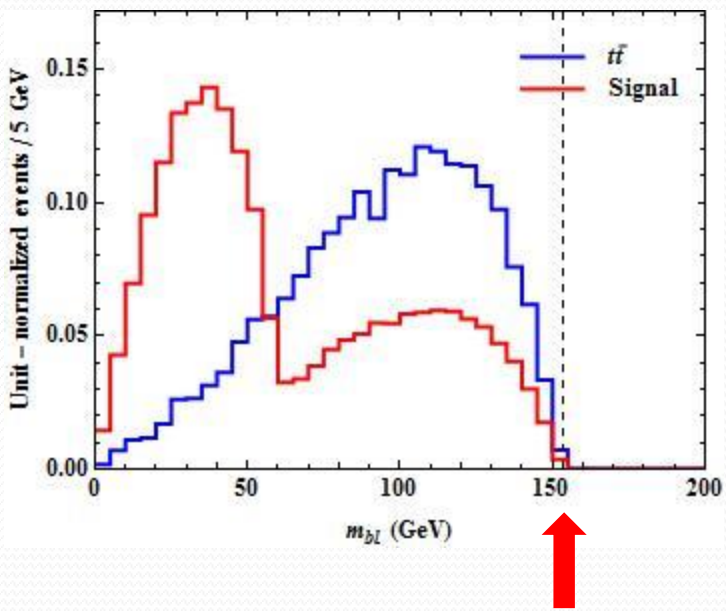


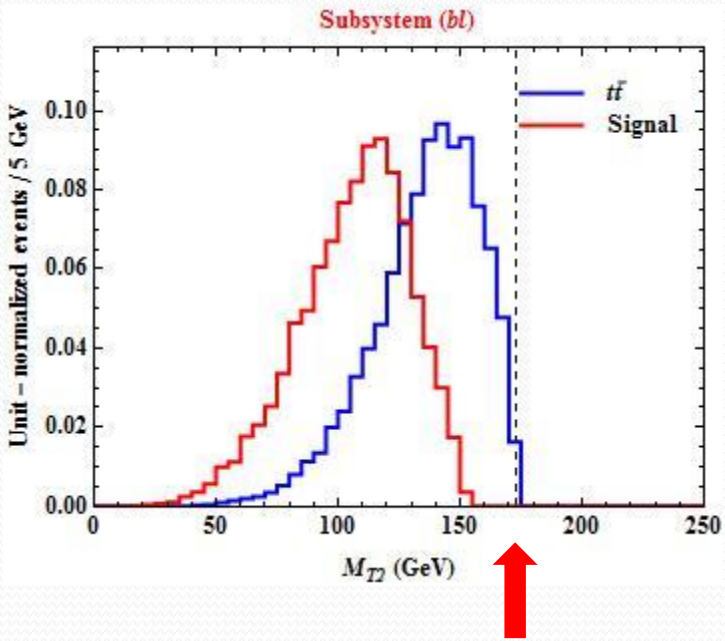
- Standard observables such as p_T^j, E_T^{miss} etc. → hard to separate/distinguish signal events from background ones

Motivation

Stop search

- Clever observables such as m_{bl} and M_{T2}



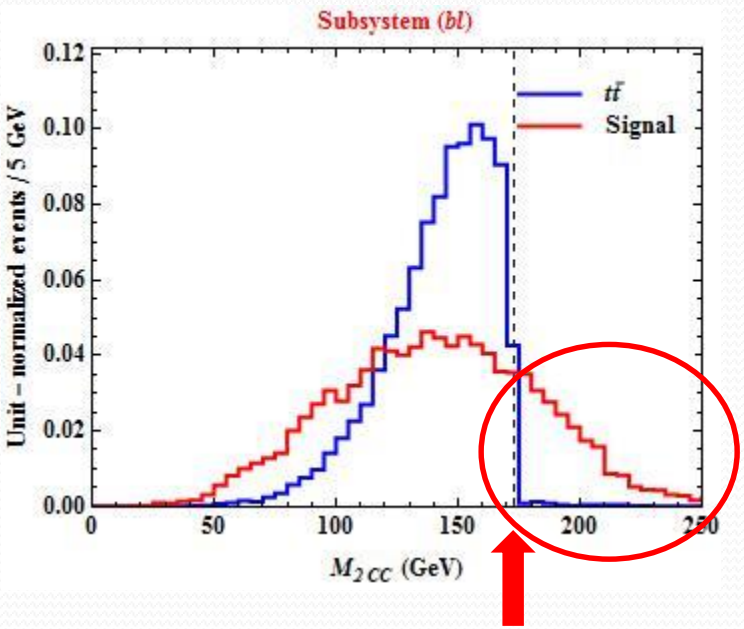


- Still hopeless to suppress background events while keeping a large number of signal ones

Motivation

Stop search using M_2 variables

- More clever (signal/background-targeted) observables (implementing characteristic features of signal/background): E.g., M_2 variables



Great for probing challenging region

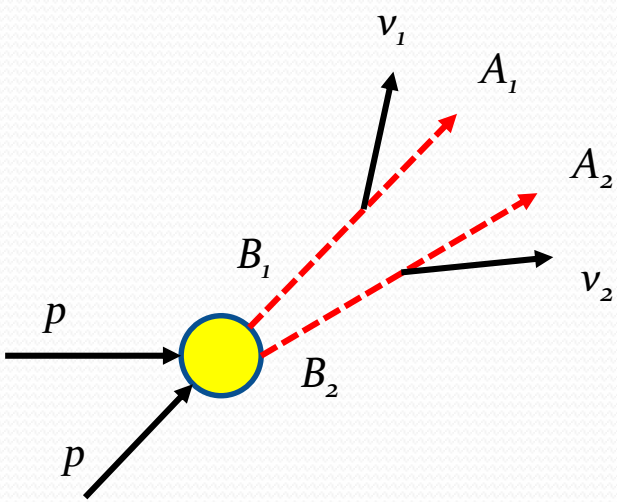
M_2 Variables

Definition

- M_2 variables as (3+1) dimensional analogue of the (2+1) dimensional M_{T2} variable: **minimization** of the two invariant mass quantities constructed with visible particles over the **unknown invisible momenta** subject to relevant **constraints** [**Cho, Gainer, DK, Matchev, Moortgat, Pape, and Park, '14**]

$$M_2(\tilde{m}) = \min_{\text{constraints}} [\max\{M^{(1)},\ M^{(2)}\}]$$

$$(M^{(i)})^2 = (p_{Ai} + p_{vi})^2$$



M_2 Variables

Why minimize?

- We want to "reconstruct" the given event in spite of existence of invisible particles
- In general, # of unknowns > # of constraints
- Scanning over solution space defined by constraints to obtain "best" guess/ansatz

Constraints

- Targeting at $t\bar{t}$-like event topology
  - ✓ MET condition relating the two decay sides: linear constraint, easily absorbed/implemented
  - ✓ **On-shell** intermediate states with **same** mass (via **full** momentum ansatz for invisible particles): non-linear constraints, (in general) highly non-trivial to implement/perform constrained minimization

Constrained minimization

Problem definition

$$\text{Min } f(x) \text{ subject to } c_i(x) = 0 \quad x \in R^n \quad i=1,2,\ldots,m \quad n>m$$

- $f(x)$: objective function
- x: variables to be minimized over
- $c_i(x)$: in general, inequality constraints possible

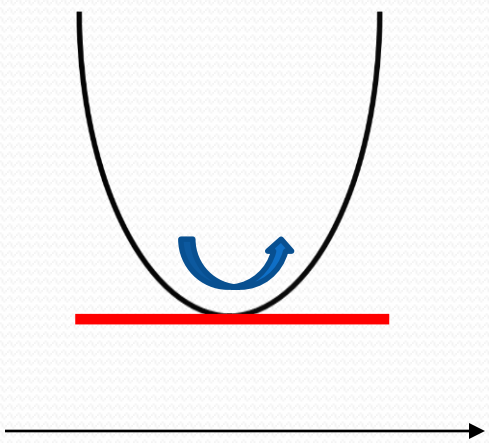
- Some constraints can be easily solved/reduced.
  - Linear constraints: Ex) MET in M_{T2}
  - Some non-linear constraints: Ex) $x^2+y^2=1$ through polar coordinate
- In general, this is not the case!

Constrained minimization

Basic/conceptual algorithm

- Schematically, from an **initial guess** the solution evolves by some **preferred algorithms** until it satisfies some **convergence criteria**

$$x_0 \to x_1 \to x_2 \to \cdots \to x_k$$



- Optimality (first order derivative)

$$\partial_x f(x_k) = 0$$

- Convexification (second order derivative)

$$\partial_x \partial_x f(x_k) > 0$$

- Feasibility

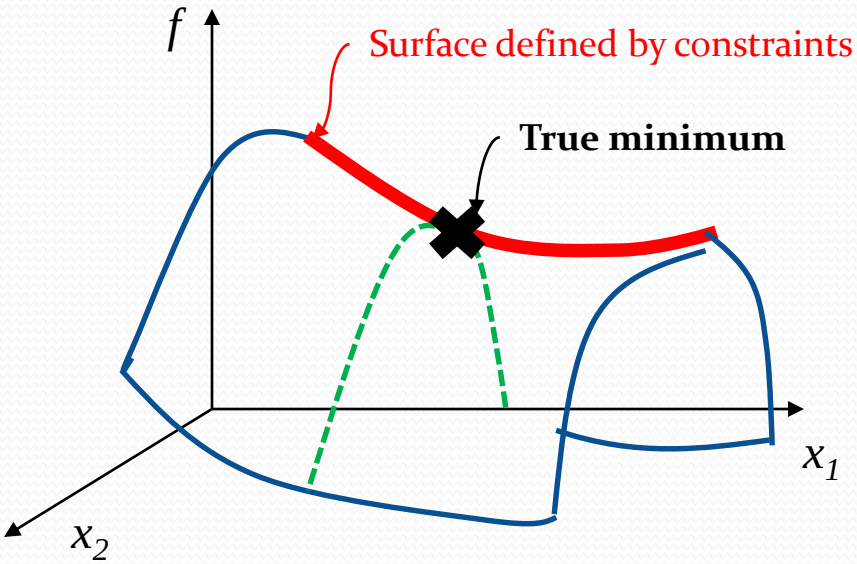
$$c_i(x_k) = 0$$

Constrained minimization

Basic algorithm

- Conceptually trivial, but not machine-friendly
  - Convexification sufficient for the hyper-surface defined by the constraints!
  - Hard to perform for the computer

- Find transformation/modification of constrained min. → unconstrained min. to guarantee the convexification in all directions.



Unconstrained minimization

Method of Lagrange multipliers

- Formally, the stationary points → KKT conditions

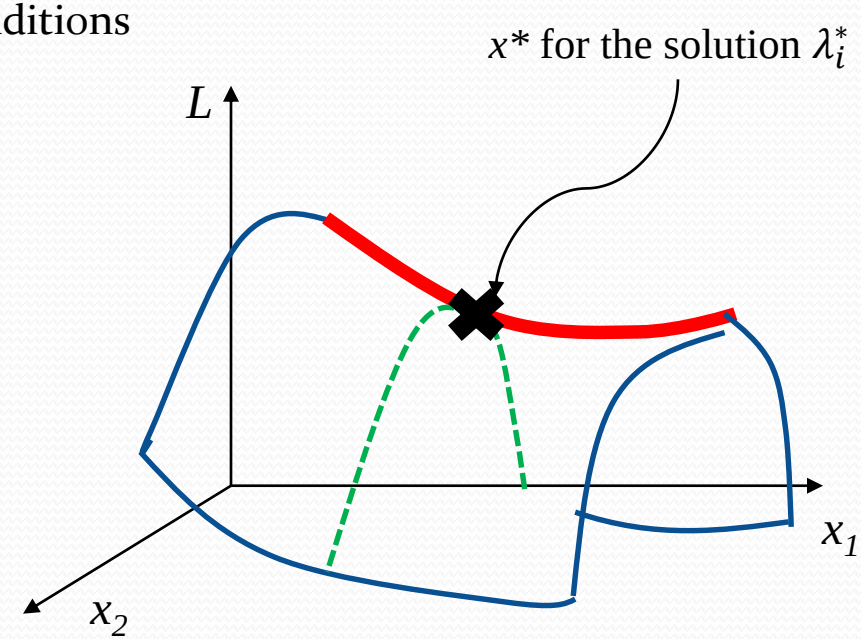
$$c_i(x_k) = 0$$

$$\partial_x f(x_k) - \lambda \cdot c(x_k) = 0$$

- Lagrange multiplier methods reproduce the KKT conditions successfully

$$L(x, \lambda) = f(x) - \lambda \cdot c(x)$$

- Convexification not guaranteed in the direction normal to the surface defined by the constraints



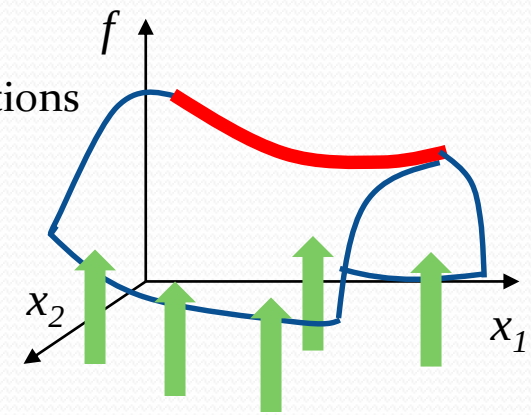
- Successful, but difficult to examine the convexification numerically
  - Modification on the objective function

Unconstrained minimization

Quadratic penalty method (QPM)

- Intuitively, enforced to be **convexified** in all directions

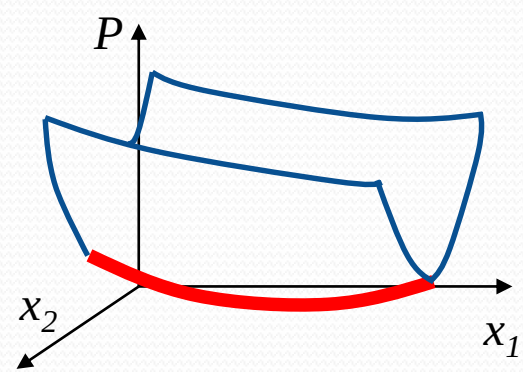
$$P(x,\mu) = f(x) + \frac{1}{2\mu}c(x)^2 \ (\mu > 0)$$



- KKT conditions and convexification **guaranteed**

$$\lambda_i \to -\frac{c_i(x)}{\mu} \ (\mu, c_i(x) \to 0 \text{ as } \lambda_i \to \lambda_i^*)$$

- Penalty parameter decreases as sub-minimizations are performed
- However, for a very small penalty parameter, too sensitive to $c(x)$: ***ill-conditioning***
  - Not make it too small but keep the good properties



Unconstrained minimization

Augmented Lagrangian method (ALM)

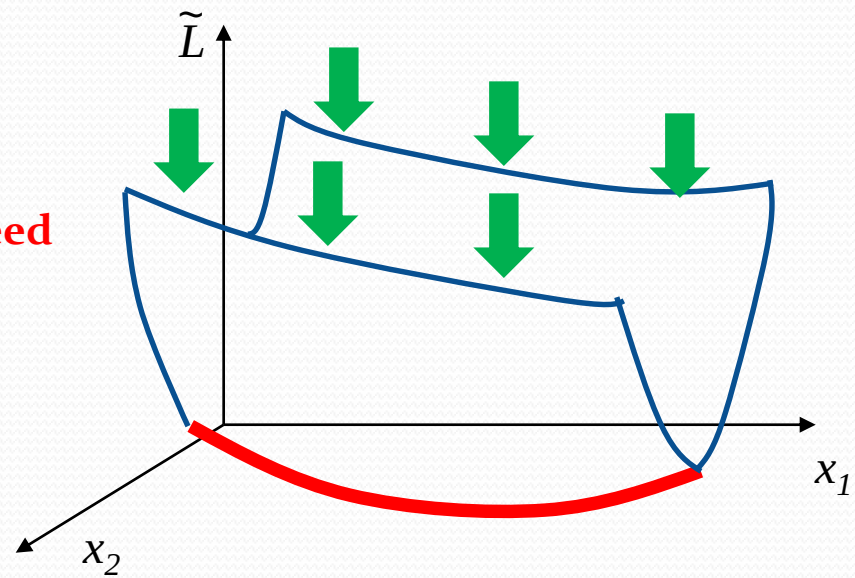
- Add Lagrange multipliers

$$\tilde{L}(x,\lambda) = f(x) - \lambda \cdot c(x) + \frac{1}{2\mu} c(x)^2$$

- KKT conditions and convexification **guaranteed**

$$\lambda_i \to \lambda_i - \frac{c_i(x)}{\mu} \left(c_i(x^*) \approx -\mu(\lambda_i^* - \lambda_i) \right)$$

- Penalty parameter decreases &
  Lagrange multipliers also evolves,
  as sub-minimizations are performed
- Without too small penalty parameter,
  can be approximated to λ_i^*: ***no ill-conditioning***



MINUIT

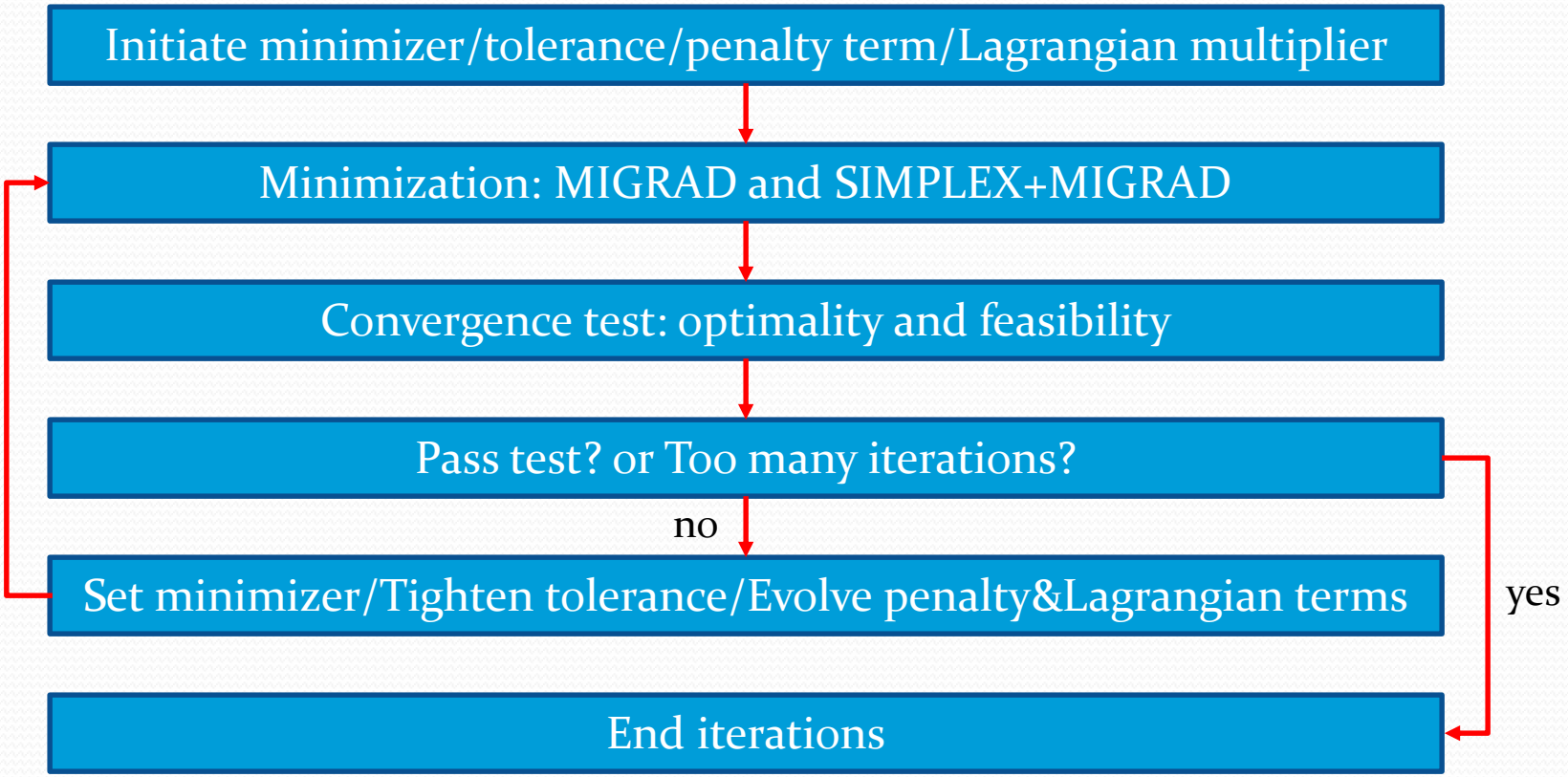
Tool for the unconstrained minimization

- Remaining job is to find a (at least) reasonable unconstrained minimization tool!
- We choose MINUIT framework
  - Could exist better options
  - Has been used in-and-outside HEP community

- Among minimization schemes, we use MIGRAD and SIMPLEX
  - MIGRAD: good at folded profile, M_2 develops folded regions
  - SIMPLEX: good at shallow profile, M_2 develops shallow regions

Implementing the algorithm

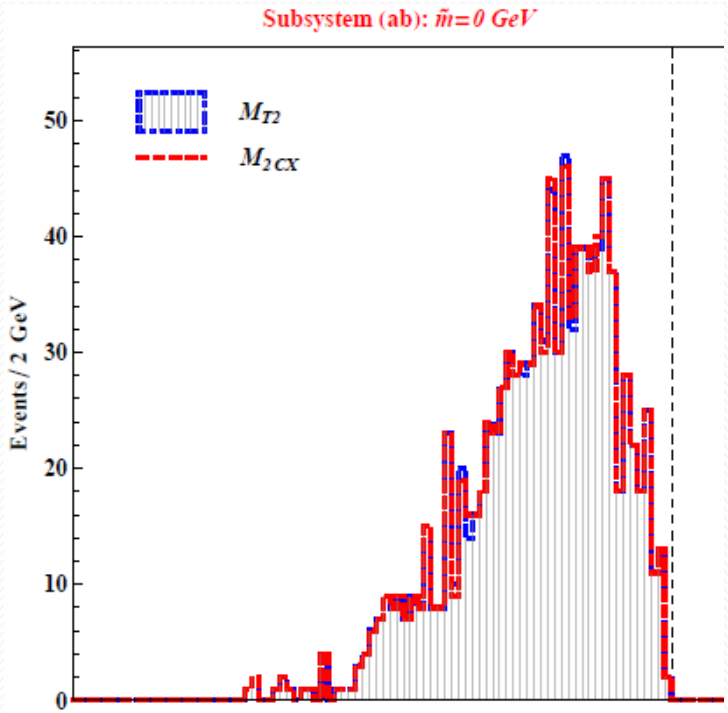
- **Flow chart**

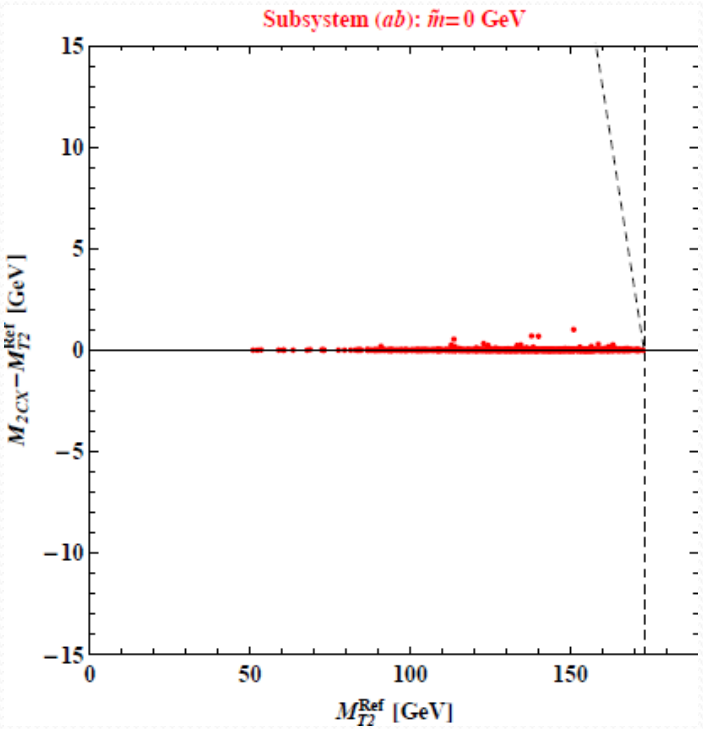


Validity of the algorithm

Results (preliminary)

- M_{T2} vs. M_{2CX}: test of the reliability of the constrained minimization with $t\bar{t}$ sample

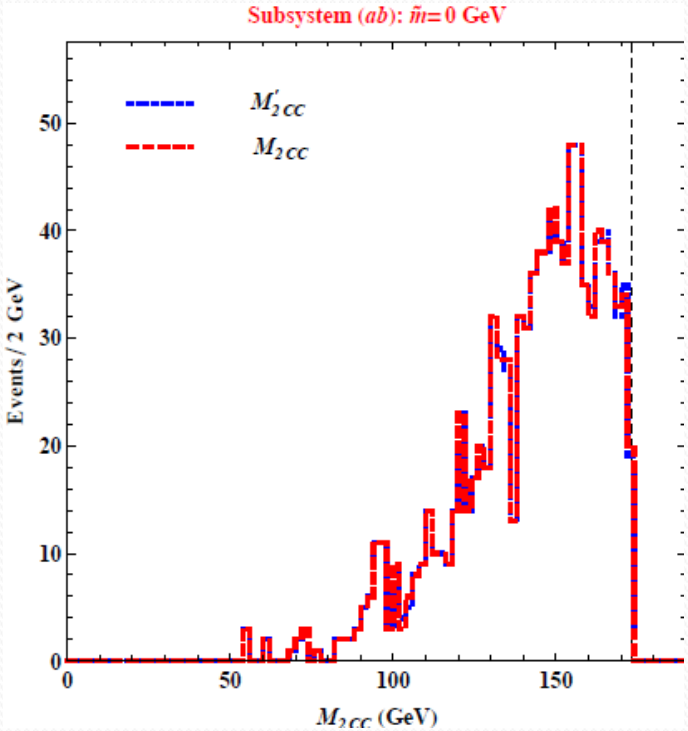


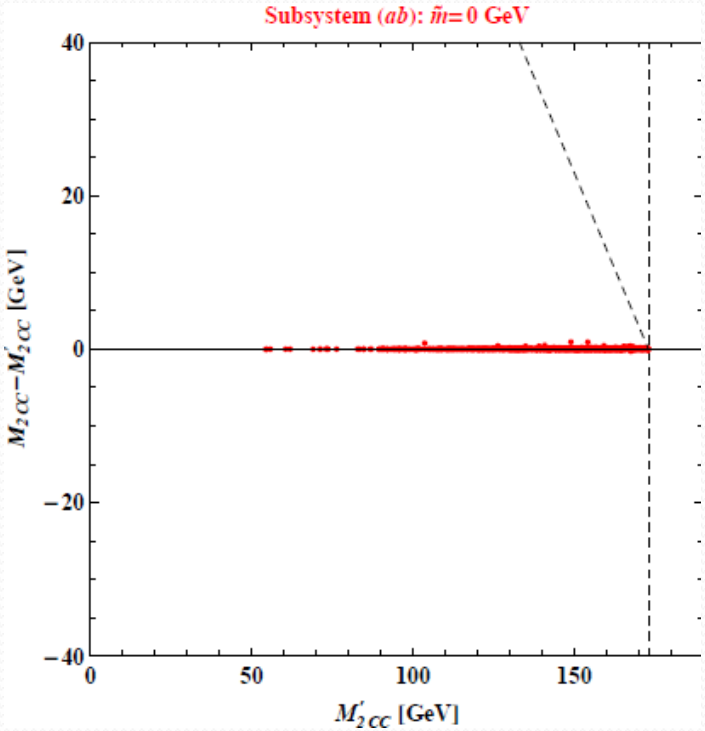


Validity of the algorithm

Results (preliminary)

- M_{2CC}: internal test of the accuracy between two independently implemented codes

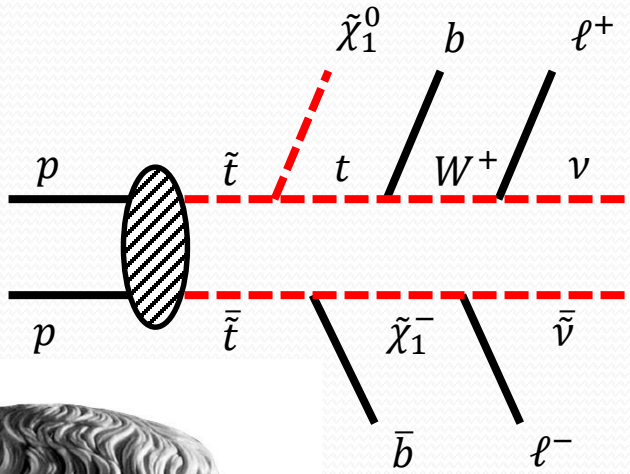




Application

Stop search

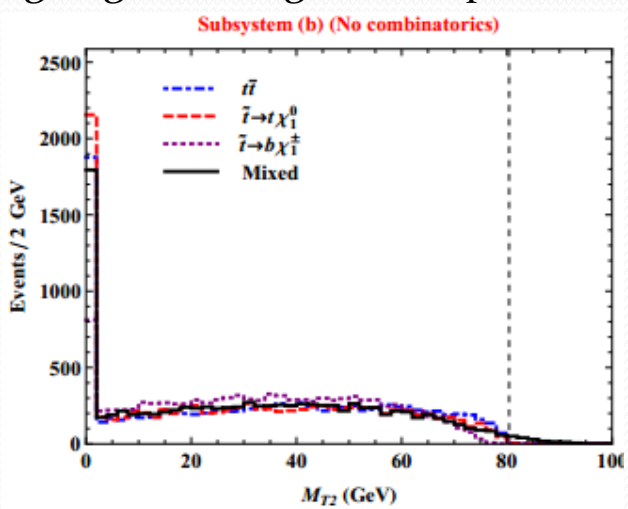
- **Asymmetric event topology**: pair-produced stops going through different decay processes
- Main idea [**Cho, Gainer, DK, Matchev, Moortgat, Pape and Park, '14**]
  - Model assumptions targeted for ttbar decay topology, i.e., symmetric decay process
  - Signal process having an asymmetric decay topology, i.e., contradiction to the model assumption
  - Expecting a huge **endpoint violation**

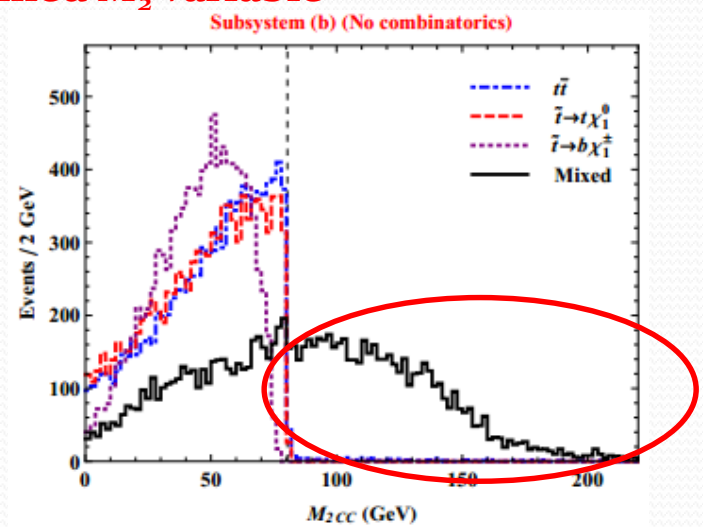


Application

Sample Result

- $m_{\tilde{t}} = 174$ GeV, $m_{\tilde{\chi}} = 1$ GeV, $m_{\tilde{\chi}^{\pm}} = 150$ GeV, $m_{\tilde{\nu}} = 110$ GeV
- Standard approach hopeless/poor signal sensitivity, e.g., M_{T2}
  - Imposing a cut (dashed line) to suppress backgrounds
- Huge signal-background separation with a **constrained M_2 variable**



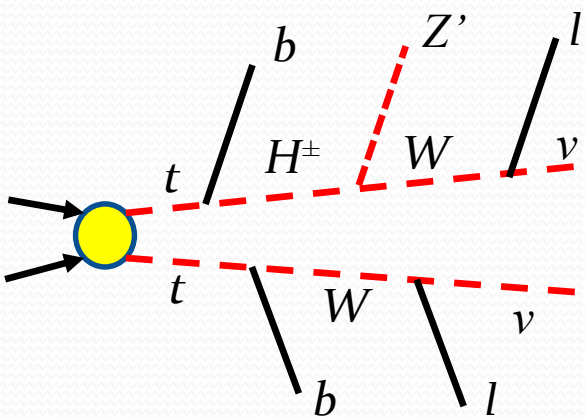


[Cho, Gainer, DK, Matchev, Moortgat, Pape and Park, '14]

Application

Dark Z search

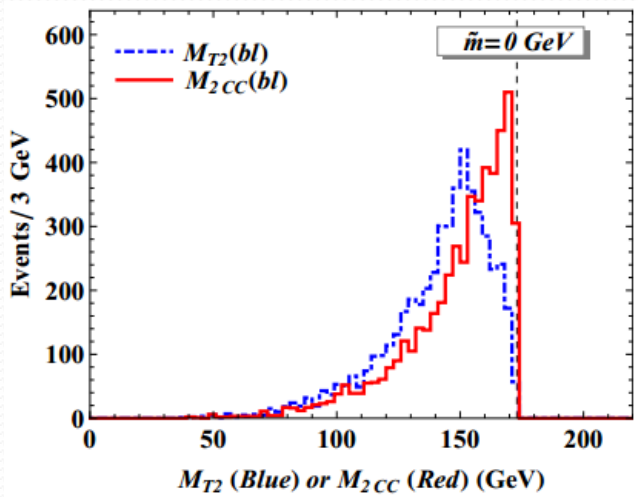
- New physics searches via **rare decays of top quark**, e.g., light invisible Z' [Davoudiasl, Lee and Marciano, '12]
- Signal: **asymmetric** process naturally emerging due to a tiny branching fraction
- (Typically) soft bottom from the rare decay of top quark
  - Hard to discriminate the signal and background by a simple cut and count in standard observables, e.g., p_T^b, E_T^{miss}

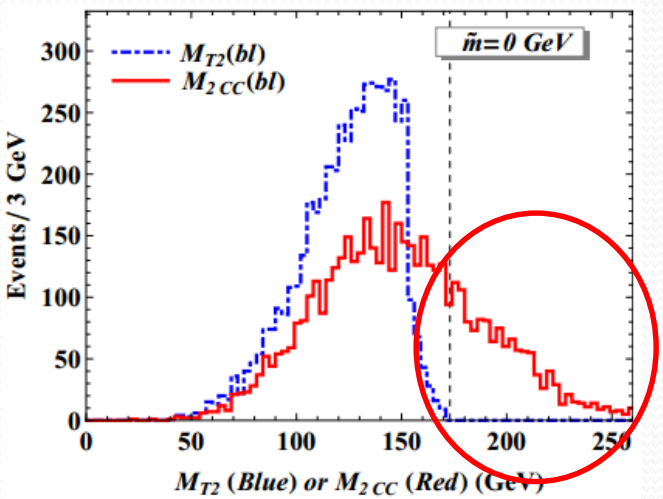


Application

Result

- $m_{H^\pm} = 130$ GeV, $m_{Z'} = 1$ GeV
- No endpoint violation for background vs. significant overflow for signal
  - Imposing a cut (dashed line) to suppress backgrounds
- Substantial signal-background discrimination with a **constrained M_2 variable**





[DK, Lee and Park, '14]

Code(β ver.) available

Library of Optimized Masses $\bar{M}$ for Event Topologies

Code of the Optimized Mass, $\bar{M}$

http://www.phys.ufl.edu/~cho/Optimized_Mass/OptM_introduction.html

be found in "Description of the $\bar{M}$". More detailed class reference and member function/variables

Contact

Links

for each optimized mass can be found in **"Dictionary of $\bar{M}$"**.

The constrained minimization process for the optimization of the $\bar{M}$ is based on the algorithm of the augmented Lagrangian method, realized with the power of the unconstrained minimization algorithms of **MINUIT** for the case of the $\bar{M}_2$ variable. In particular, the constrained-$\bar{M}_2$ variable, which is a good example of the optimized mass, had been surveyed in the series of papers:

Guide to transverse projections and mass-constraining variables A. J. Barr, T. J. Khoo, P. Konar, K. Kong, C. G. Lester, K. T. Matchev, M. Park

On-shell constrained $\bar{M}_2$ variables with applications to mass measurements and topology disambiguation, W. S. Cho, J. S. Gainer, D. Kim, K. T. Matchev, F. Moortgat, L. Pape and M. Park, arXiv:1401.1449 [hep-ph].

As optimized in a topology-by-topology basis, our code provides the routine for various optimized masses, especially for the well-known background/signal events of the Standard Model/Beyond the Standard Model. The list of the embodied $\bar{M}$ can be found in the **"Description of**

Conclusions

Summary and outlook

- Better kinematic variables can be more sensitive to signal processes. Sometimes, they require a constrained optimization procedure which is, in general, non-trivial to perform. One example is the recently proposed M_2 variable.
- To compute its value, the non-trivial constrained minimization should be transformed to the (machine-friendly) unconstrained minimization. We adapt the augmented Lagrange methods with the aid of MINUIT framework.
- Performance of the code is quite remarkable for the M_2 variable. Using the code, the M_2 variable can be studied along with various applications. We also expect that the algorithms can be applicable to other observables/variables/functions involving constrained optimizations.

Thank you!