# Monte Carlo review 

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## MC4BSM 2015

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## Outline

1) Introduction
2) Parton showering, higher-order calculcations and their combination.
3) Soft physics: Multiple interactions and hadronisation.

## How will we find what is out there?

Know what we want to look for...

Know what we're facing...

Assess if there is a realistic chance with our current experiments ... and check before building a new experiment.

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Dark sectors?
New bound states?
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We need an accurate representation of "known" and
"unknown" physics that feels like data!
$\Longrightarrow$ Event generators


## Event generation: Start from hard process


...and emit gluons from incoming partons


## ...or outgoing partons



## ...or split gluons into quarks


... and how to do this arbitrarily often


## ... and emit photons from charged fermions


...and include multiple interactions between composite protons

...which again produce more radiation

...and add beam remnants to form a colourless state

...and form strings (colour flux tubes)

...and produce hadrons from strings and remnants

...and decay the excited hadrons

...which can again involve photons


And the detector records this...


## Standard event generator frameworks

The three commonly used General Purpose Event Generators are

## HERWIG

Basic ME generator

Angular ordered $\tilde{q} \quad p_{\perp}$-ordered dipoles shower and $p_{\perp}$-ordered CS dipole shower

YFS multipole QED MPI afterburner

Cluster hadronisation

PYTHIA

Basic ME generator with ME-corrections, VINCIA antenna shower

QED from shower Interleaved MPI

String hadronisation

## SHERPA

Mature ME generator
$p_{\perp}$-ordered CS dipole shower, ANTS antenna shower

YFS multipole QED
MPI afterburner

Cluster hadronisation
2) Parton showering, higher-order calculcations and their combination
a) Factorisation and parton showers
b) Why we need more...
c) NLO calculations and matching.
d) Many-jet calculations, combining many NLO calculations.

## Soft/collinear limits and splitting probabilities

Cross sections containing an additional collinear gluon factorise as

$$
d \sigma(\mathrm{pp} \rightarrow Y+\mathrm{g}+X) \approx d \sigma(\mathrm{pp} \rightarrow Y+X) \int \frac{d p_{\perp}^{2}}{p_{\perp}^{2}} \frac{d z}{z} \frac{\alpha_{s}}{2 \pi} \frac{f\left(\frac{X_{a}}{z}, t\right)}{f_{a}\left(x_{a}, t\right)} P(z)
$$

with the splitting kernels $P(z)$, independent of the process $\mathrm{pp} \rightarrow Y+X$.
Multi-parton cross sections can be approximated by "dressing up" low-multiplicity results with many collinear partons.

The splitting kernels have a probabilistic interpretation:

$$
\int_{p_{\perp \text { min }}^{2}}^{p_{\perp \text { max }}^{2}} \frac{d p_{\perp}^{2}}{p_{\perp}^{2}} \int_{z_{\text {min }}}^{z_{\text {max }}} d z \frac{\alpha_{s}}{2 \pi} P(z) \equiv \begin{aligned}
& \text { Probability of emitting a gluon with } \\
& \text { momentum fraction } 1-z \in\left[z_{\text {min }}, z_{\text {max }}\right] \text { and } \\
& \text { transverse momentum } p_{\perp} \in\left[p_{\perp \text { min }}, p_{\perp \text { max }}\right] .
\end{aligned}
$$

Note that $\frac{d p_{\perp}^{2}}{p_{\perp}^{2}}=\frac{d \theta^{2}}{\theta^{2}}=\frac{d Q^{2}}{Q^{2}} \Longrightarrow$ Evolution variable is up for debate.

## Parton showering

Use probabilities to dress partons with softer partons $\Longrightarrow$ Jet formation!
The "naive" probability of an emission in the interval $[t, t+\delta t]$ is

$$
\delta t \int_{0}^{1} d z P(z)
$$

...which gives the probability of no emission between two times

$$
1-\delta t \int d z P(z)
$$

....and the probability of no emissions in $n$ intervals of step size $\delta t / n$ :

$$
\left[1-\frac{\delta t}{n} \int d z P(z)\right]^{n} \underset{\substack{\frac{\delta t}{n} \rightarrow d t}}{=} \exp \left\{-\int_{t}^{t+\delta t} d t \int d z P(z)\right\}=\Pi(t+\delta t, t)
$$

Thus, we find

## Probability of no emission $\Pi\left(t_{0}, t_{1}\right)$ Probability of an emission at $t_{1}$ $\Pi\left(t_{0}, t_{1}\right) P(z)$

## Parton showers

Parton showers start from a (simple) input state:


## Parton showers

... and may produce no hard splitting, or a hardest splitting


## Parton showers

...and then no further splitting, or a second hardest splitting, etc.


The all-order factors $\Pi_{i}$ are called Sudakov form factors. They connect to resummation, and make jet cross sections well-behaved.

## Parton showers vs. fixed order

Parton showers give an approximate multi-parton (jet) cross section which...

+ is always finite.
+ is good for any number of emissions.
- but is only valid for very small relative $p_{\perp}$.

Is your signal affected by (many) jets ${ }^{1}$ ?
$\Longrightarrow$ Need good calculation for partonic jet seeds!
$\Longrightarrow$ Need something better than plain parton shower.
$\Longrightarrow$ Combine the strengths of showers and fixed-order calculations!

Parton showers start from lowest-multiplicity tree-level inputs. The next step is next-to-leading order.

[^0]
## Do you need ME+PS for BSM signals?



Figure: Jet $p_{\perp} s$ for squarks + jets. PS bands are obtained by varying between "wimpy" and "power shower", merged bands by varying the merging scale from $50-200 \mathrm{GeV}$ (taken from Phys.Rev. D87 (2013) 3, 035006 (Dreiner, Krämer, Tattersall)).
$\Longrightarrow$ Improved QCD pins down the transverse momenta.

## ...and how good is your exclusion?



Figure: Exclusion limits for squarks+jets. PS bands are obtained by varying between "wimpy" and "power shower", merged bands by varying the merging scale from $50-200 \mathrm{GeV}$ (taken from Phys.Rev.D87(2013)3,035006 (Dreiner, Krämer, Tattersall)). $\Rightarrow$ Improved QCD pins down jet momenta, reducing MC uncertainties.

Precision backgrounds: Do you worry about multi-jet states?


(Figure taken from EPJC 75 (2015) 2 82)

Precision backgrounds: Do you worry about deviations in cross sections?

Mar 2015


CMS summary (taken from https://twiki.cern.ch/twiki/bin/view/CMSPublic/PhysicsResultsCombined)

## Mission statement

Task: Combine multiple fixed-order calculations with each other and with PS into a single one-does-it-all prediction.

Keep highest accuracy for inclusive $n$-jet cross sections. Keep PS resummation for exclusive quantities.

The current state-of-the-art is NLO merging.

## Next-to-leading order calculations

Pen-and-paper: Add Born + Virtual + Real.
$\langle\mathcal{O}\rangle^{\mathrm{NLO}}=\int \mathrm{B}_{n} \mathcal{O}\left(\Phi_{n}\right) d \Phi_{n}+\int \mathrm{V}_{n} \mathcal{O}_{n}\left(\Phi_{n}\right) d \Phi_{n}+\int \mathrm{B}_{n+1} \mathcal{O}\left(\Phi_{n}\right) d \Phi_{n+1}$

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Reality: Phase space integral separately divergent $\Rightarrow$ Add zero!
$\langle\mathcal{O}\rangle^{\text {NLO }}=\int\left[\mathrm{B}_{n}+\mathrm{V}_{n}+\int \mathrm{D}_{n+1}\right] \mathcal{O}\left(\Phi_{n}\right) d \Phi_{n}+\int\left[\mathrm{B}_{n+1} \mathcal{O}\left(\Phi_{n+1}\right)-\mathrm{D}_{n+1} \mathcal{O}\left(\Phi_{n}^{\prime}\right)\right] d \Phi_{n+1}$

## NLO is the new standard



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...then new unitarity techniques removed the "virtual matrix element" bottle-neck (BlackHat, HELAC-NLO)

## NLO is the new standard



Now, you can execute these calculations yourself with tools like (BlackHat, GOSAM, OpenLoops, NJet) + Sherpa, (GOSAM, MadLoop) + MG5_aMC!

## What did that mean for event simuation?



Many LO matrix elements available $\Rightarrow$ Multi-jet merging.

## What did that mean for event simuation?



Many NLO matrix elements available $\Rightarrow$ Automated NLO+PS matching, NLO merging.

## Next-to-leading order calculations (again)

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Real reality: States $\Phi_{n+1}$ and $\Phi_{n}^{\prime}$ are correlated. $\Rightarrow$ Problematic, since further manipulations (e.g. hadronisation) can spoil the cancellations

$$
\begin{aligned}
\langle\mathcal{O}\rangle^{\mathrm{NLO}} & =\int\left[\mathrm{B}_{n}+\mathrm{V}_{n}+\mathrm{I}_{n}+\int d \Phi_{\mathrm{rad}}\left(\quad-\mathrm{D}_{n+1}\right)\right] \mathcal{O}\left(\Phi_{n}\right) d \Phi_{n} \\
& +\int\left(\mathrm{B}_{n+1} \quad\right) \mathcal{O}\left(\Phi_{n+1}\right) \\
& +\int(\quad)
\end{aligned}
$$

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$\Rightarrow$ Add more zeros!
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$+\int\left(\mathrm{B}_{n+1}\right) \mathcal{O}\left(\Phi_{n+1}\right)$
$+\int\left(-\mathrm{B}_{n+1}^{\prime} \mathcal{O}\left(\Phi_{n}\right)\right)$

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$+\int\left(\mathrm{B}_{n+1}-\mathrm{B}_{n+1}^{\prime}\right) \mathcal{O}\left(\Phi_{n+1}\right)$
$+\quad \int\left(\mathrm{B}_{n+1}^{\prime} \mathcal{O}\left(\Phi_{n+1}\right)-\mathrm{B}_{n+1}^{\prime} \mathcal{O}\left(\Phi_{n}\right)\right) \longleftarrow$ That's the $\mathcal{O}\left(\alpha_{\mathrm{s}}\right)$ of a PS step!

## NLO matching

For NLO matching, we start out with a shower-dependent seed cross section and a shower Sudakov factor

$$
\begin{aligned}
\overline{\mathrm{B}}_{n} & =\mathrm{B}_{n}+\mathrm{V}_{n}+\mathrm{I}_{n}+\int d \Phi_{\mathrm{rad}}\left(\mathrm{~B}_{n+1}^{\prime}-\mathrm{D}_{n+1}\right) \\
\Delta^{B}\left(t_{0}, t_{\text {min }}\right) & =\exp \left(-\iint^{t_{0}} d \Phi_{\mathrm{rad}} \frac{\mathrm{~B}_{n+1}^{\prime}}{\mathrm{B}_{n}}\right)
\end{aligned}
$$

and perform a PS step on $\overline{\mathrm{B}}_{n}{ }^{1}$

$$
\begin{aligned}
& \overline{\mathrm{B}}_{n} \Delta^{B}\left(t_{0}, t_{\min }\right) \mathcal{O}_{0}\left(\Phi_{n}\right)+\int^{t_{0}} d \Phi_{\mathrm{rad}} \overline{\mathrm{~B}}_{n} \frac{\mathrm{~B}_{n+1}^{\prime}}{\mathrm{B}_{n}} \Delta^{B}\left(t_{0}, t\right) \mathcal{O}_{1}\left(\Phi_{n+1}\right) \\
& \quad+\left(\mathrm{B}_{n+1}-\mathrm{B}_{n+1}^{\prime}\right) \mathcal{O}_{1}\left(\Phi_{n+1}\right)
\end{aligned}
$$

At $\mathcal{O}\left(\alpha_{\mathrm{s}}^{n+1}\right)$, this gives back the NLO cross section. Common schemes are

$$
\begin{aligned}
& \text { POWHEG: } \mathrm{B}_{n+1}^{\prime}=\mathrm{B}_{n+1} \cdot \frac{h^{2}}{h^{2}+p_{\perp}^{2}}, t_{0}=s \\
& \text { MC@NLO: } \mathrm{B}_{n+1}^{\prime}=\mathrm{D}_{n+1} \cdot \Theta\left(\mu_{Q}-t\left(S_{+1}\right)\right), \mu_{Q}=k Q^{2}
\end{aligned}
$$



NLO+PS methods usually lead to a smaller differences. There are striking counter-examples where large differences are consistent with higher order effects.
Large differences usually appear in the "LO" part of the prediction
Good news: We can improve on this!

## Merging: Iterative improvements by slicing



1jet PS

Look at one-jet states: ...use PS for soft emissions ... use ME for hard emissions

## Merging: Iterative improvements by slicing



1jet PS

Now improve two-jet states: ...use PS for soft emissions ...use ME for hard emissions

## Merging: Iterative improvements by slicing



## Merging: Iterative improvements by slicing



Continue with three-jet states:
... use PS for soft emissions
... use ME for hard emissions

The dependence on the hard-soft separation (merging scale) is removed by resummation, i.e. by including Sudakov form factors and a running coupling. (Further tricks are often necessary)

## Differences merging/matching

NLO matching is NLO-correct.
$\Longrightarrow$ Good uncertainty estimate, limited applicability.
Merging can be used to combine "any number" of LO calculations.
$\Longrightarrow$ Questionable uncertainty, broad applicability.

We can be lucky if
...NLO matched calculation describes very exclusive data.
...merged calculations describe normalisations.
It would be unreasonable to expect
Luck in one process $=$ Luck in another process
$\Rightarrow$ Both strategies are incomplete and need to be combined for a satisfactory result.

## NLO matching $\otimes$ merging $=$ NLO merging

Any leading-order method $\mathbf{X}$ contains approximate $\mathcal{O}\left(\alpha_{\mathrm{s}}\right)$-corrections from the expansion of the necessary all-order factors (e.g. Sudakovs).

But we want to use more accurate NLO results whenever possible!

## NLO matching $\otimes$ merging $=$ NLO merging

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But we want to use more accurate NLO results whenever possible!

To do NLO multi-jet merging for your preferred LO scheme $\mathbf{X}$, do:
$\diamond$ Subtract approximate $\mathcal{O}\left(\alpha_{\mathrm{s}}\right)$-terms from merged calculation X, add multiple NLO calculations.
$\diamond$ Ensure that real-emission parts of fixed-order calculations do not overlap.
$\diamond$ Ensure that fixed-order and shower calculations do not overlap ...just as we did at leading order.
$\diamond$ Adjust higher orders to suit your other needs.
$\Rightarrow \mathbf{X @ N L O}$

## LHC Run II+ era theory predictions (H+jets)



Figure: $p_{\perp, H}$ and $\Delta \phi_{12}$ for $\mathrm{gg} \rightarrow \mathrm{H}$ after merging $(\mathrm{H}+0) @ \mathrm{NLO},(\mathrm{H}+1) @ N L O$, $(\mathrm{H}+2) @ \mathrm{NLO},(\mathrm{H}+3) @ L O$, compared to other generators.
$\Rightarrow$ The generators come closer together if enough fixed-order matrix elements are employed. Uncertainties in exclusive regions can still be large.

NLO merged results: The end of a 10-year journey


...but in general theory uncertainties decrease (from EPJC 75 (2015) 2 82) $49 / 77$

NLO merged results: The end of a 10-year journey



W(+jets) production at ATLAS (PRD 85 (2012) 092002) in PYTHIA8 UNLOPS.

## Back to the big picture

However
...showers are still only QCD/QED ${ }^{1}$
...and at "low" accuracy.
...there's more to a realistic state than $2 \rightarrow n$ scattering.
${ }^{1}$ No longer true! Electroweak effects are also included by now.
3) Soft physics: Multiple interactions and hadronisation.
a) Multiparton interactions
b) Hadronisation
c) Why should I care?

## Realistic final states (MPI)



Event


Scattering+MPI


Perturbative scattering

Assume we understand weak showers and matrix element merging. What if a state mixes "soft" MPI and hard perturbative physics?
At LHC, jets from MPI are relatively soft. $\Rightarrow$ Small (?) effects. But the effects are usually directly in the "resummation" region. $\Rightarrow$ Competition should be understood.
$\diamond$ Can we simply only look at jets with large $p_{\perp}$, i.e ignore competition?
$\diamond$ Can we improve the PS accuracy without worrying about MPI?

## Multiple interactions

## Multiple interactions between the composite protons are supported by 30 years of evidence:



FIG. 3. Charged-multiplicity distribution at 540 GeV , UA5 results (Ref. 32) vs simple models: dashed low $p_{T}$ only, full including hard scatterings, dash-dotted also including initial- and final-state radiation.


FIG. 5. Charged-multiplicity distribution at 540 GeV , UA5 results (Ref. 32) vs impact-parameter-independent multipleinteraction model: dashed line, $p_{T \min }=2.0 \mathrm{GeV}$; solid line, $p_{T_{\text {min }}}=1.6 \mathrm{GeV}$; dashed-dotted line, $p_{T_{\text {min }}}=1.2 \mathrm{GeV}$.

## Multiple interactions

Measurements indicate that the "underlying event" (UE) has a mini-jet structure. This leads to the following model.

1. Overlay QCD (QED) $2 \rightarrow 2$ transitions on top of the hard interaction.
2. Introduce (non-perturbative colour-screening) parameter $p_{\perp 0}$ into $2 \rightarrow 2$ cross section regularise divergence.
3. Order multiple scatterings in descending $p_{\perp}$ sequence, with cut-off $p_{\perp \text { min }}$ and scattering probability $\approx \frac{1}{\sigma_{\text {tooal }}} \frac{\mathrm{d} \sigma^{\text {reg }}(2 \rightarrow 2)}{d p_{\perp}}$
4. Ensure that energy, momentum, colour, flavour are conserved.
$\Rightarrow$ MPI model for the UE of soft jets in hadronic collisions with a handful of parameters. MPI models are tuned to UE measurements.

## Hadronisation

...our result still contains coloured partons $\Rightarrow$ Needs to be converted to hadrons! Two prescriptions have passed the test of time:

## Cluster

Form hadrons by decaying "preconfined" colourless clusters of partons.

Gluons split non-perturbatively to $q \bar{q}$

Many-parameter energy-momentum structure.

Few-parameter flavour chemistry.

## String

Colour flux tubes (strings, junctions) between partons break to form hadrons.

Gluons are kink on string.

Few-parameter energy-momentum structure.

Many-parameter flavour chemistry.

## Hadron decays

Fragmentation can produce excited hadrons, which will then decay, e.g.


Most particles are produced in this part.
$\Rightarrow$ Process has to be modelled for the correct jet structure by
... Hadronic matrix elements for some (important) decays.
...PDG decay tables for others. If tables are incomplete, be creative.

Does hadronisation matter for BSM physics?


Assume dark matter annihilates into bottom quarks.

Does hadronisation matter for BSM physics?


The quarks will radiate.

Does hadronisation matter for BSM physics?


If the quarks radiate a gluon, we find two colour lines

Does hadronisation matter for BSM physics?


The colour lines form strings, which form hadrons and photons in the region spanned by the string.

## Does hadronisation matter for BSM physics?



If a photon were radiated instead, the string would be spanned between the bottom quarks, and there would be no activity close to the "hard" photon.

## Does hadronisation matter for BSM physics?



This difference is called the string effect. It is model-dependent and may (partially) stem from tunes to data.

If your favorite DM primarily annihilates into quarks, and your primary concern is the photon spectrum, you might have to worry about hadronisation ${ }^{1}$. So be careful :)

[^1]
## Summary

- Event generation can be divided into subproblems.
- Massive progress in fixed-order calculations, and including accurate (multi-jet) results into parton showers.
- Background estimations rather reliable now ...but can (should?) equally well be applied to signal processes.
- Less dramatic progress on the all-order structure of showers although showers do continuously get better.
- Event simulation more than perturbative $2 \rightarrow n$ scatterings. Multiparton interactions are omnipresent at hadron colliders. Hadronisation is a must - at colliders and beyond.


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Most simple signals are excluded.
So we can finally have some fun!

## Back-up supplement

## ME+PS tools shopping list

LO merging
MLM available with Alpgen + (Herwig6, Pythia6/8), Madgraph + (Herwig++, Pythia6/8), Whizard + Pythia6
CKKW no longer available (?) in Sherpa, Herwig++
CKKW-L / METS available in Sherpa, (Alpgen, Madgraph,...) + Pythia8 UMEPS available in (Alpgen, Madgraph,...) + (Herwig++, Pythia8)
NLO matching
NLO merging
NNLO matching
Other improvements

## ME+PS tools shopping list

LO merging
NLO matching
POWHEG available in Sherpa, Herwig ++ , POWHEG-BOX + (Herwig6/++, Pythia6/8)
MC@NLO available in Sherpa, Herwig++, aMC@NLO + (Herwig6/++, Pythia6/8)
NLO merging
NNLO matching
Other improvements

## ME+PS tools shopping list

LO merging
NLO matching
NLO merging
MEPS@NLO available in Sherpa
UNLOPS available in Herwig++, (POWHEG-BOX, aMC@NLO) + Pythia8
FXFX available in aMC@NLO + (Herwig++, Pythia8)
NNLO matching
Other improvements

## ME + PS tools shopping list

LO merging
NLO matching
NLO merging
NNLO matching
UN²LOPS available as plugin to Sherpa
MiNLO-NNLOPS available through POWHEG-BOX
Other improvements

## ME+PS tools shopping list

LO merging
NLO matching
NLO merging
NNLO matching

## Other improvements

MiNLO available through POWHEG-BOX
Iterated ME corrections available through VINCIA
ME reweighting available in HEJ
KRKC proposed new NLO matching
GENEVA proposed higher-logs + fixed-order (NLO, NNLO) + showers

## References

Introduction
Good references for event generators in general are:
MCnet report (Phys. Rept. 504 (2011) 145-233)
Many older lectures of MCNet (montecarlonet.org) and CTEQ schools.
Peter Skands' TASI lectures (arXiv:1207.2389)
Stefan Höche's TASI lectures (http://slac.stanford.edu/ shoeche/tasi14/ws/tasi.pdf)
Factorisation: Divide and conquer
The book: Collins, Perturbative Quantum Chromodynamics
Collins, Soper, Sterman (Nucl.Phys.B250(1985)199)
Dipoles / antennae
Ariadne (CPC 71 (1992) 15)
Catani, Seymour (Nucl.Phys.B485(1997)291)
Kosower antennae (Phys.Rev. D57 (1998) 5410)
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## Parton shower basics

Parton showers are unitary all-order operators:

$$
\begin{aligned}
& \mathbf{P S}\left[\sigma_{+0}^{\mathrm{ME}}\right]=\sigma_{+0}^{\mathrm{PS}}+\sigma_{+1}^{\mathrm{PS}}+\sigma_{+\geq 2} \\
& =\sigma_{+0}^{\mathrm{ME}} \Pi_{S_{+0}}\left(\rho_{0}, \rho_{\text {min }}\right) \quad \longleftarrow 0 \text { emissions in }\left[\rho_{0}, \rho_{\text {min }}\right] \\
& +\sigma_{+0}^{\mathrm{ME}} \Pi_{S_{+0}}\left(\rho_{0}, \rho_{1}\right) \alpha_{s} w_{f}^{0} P_{0} \Pi_{S_{+1}}\left(\rho_{1}, \rho_{\text {min }}\right) \longleftarrow 1 \text { emission in }\left[\rho_{0}, \rho_{\text {min }}\right] \\
& +\sigma_{+0}^{\mathrm{ME}} \Pi_{S_{+0}}\left(\rho_{0}, \rho_{1}\right) \alpha_{\mathrm{s}} w_{f}^{0} P_{0} \Pi_{S_{+1}}\left(\rho_{1}, \rho_{2}\right) \alpha_{\mathrm{s}} w_{f}^{1} P_{1}\left[\Pi_{S_{+2}}\left(\rho_{2}, \rho_{\text {min }}\right)+\ldots\right] \\
& \uparrow \quad \uparrow \\
& 2 \text { or more emissions in [ } \rho_{0}, \rho_{\text {min }} \text { ] } \\
& \stackrel{!}{=} \sigma_{+0}^{M E}
\end{aligned}
$$

The no-emission probabilities

$$
\Pi_{S_{+i}}\left(\rho_{1}, \rho_{2}\right)=\exp \left\{-\int_{\rho_{2}}^{\rho_{1}} d \rho \alpha_{s} w_{f}^{i} P_{i}\right\}
$$

define exclusive cross sections and remove the overlap between samples!

## CKKW(-L)

Aim: Combine multiple tree-level calculations with each other and (PS) resummation. Fill in soft and collinear regions with parton shower.

$$
\begin{aligned}
\langle\mathcal{O}\rangle= & \mathrm{B}_{0} \mathcal{O}\left(S_{+0 \mathrm{j}}\right) \\
& \quad-\int d \rho \mathrm{~B}_{0} P_{0}(\rho) \Theta_{>}^{(1)} w_{f} w_{\alpha_{s}} \Pi_{S_{+0}}\left(\rho_{0}, \rho\right) \mathcal{O}\left(S_{+0 j}\right) \\
+ & \quad \int \mathrm{B}_{1} \Theta_{>}^{(1)} w_{f} w_{\alpha_{s}} \Pi_{S_{+0}}\left(\rho_{0}, \rho\right) \mathcal{O}\left(S_{+1 j}\right) \\
& \quad-\int d \rho \mathrm{~B}_{1} P_{1}(\rho) \Theta_{>}^{(2)} w_{f} w_{\alpha_{s}} \Pi_{S_{+0}}\left(\rho_{0}, \rho_{1}\right) \Pi_{S_{+1}}\left(\rho_{1}, \rho\right) \mathcal{O}\left(S_{+1 j}\right) \\
& +\int \mathrm{B}_{2} \Theta_{>}^{(2)} w_{f} w_{\alpha_{s}} \Pi_{S_{+0}}\left(\rho_{0}, \rho_{1}\right) \Pi_{S_{+1}}\left(\rho_{1}, \rho\right) \mathcal{O}\left(S_{+2 j}\right)
\end{aligned}
$$

Changes inclusive cross sections
$\Longrightarrow$ Can contain numerically large (sub-leading) logs.
$\Longrightarrow$ Needs fixing!

## Bug vs. Feature in CKKW(-L)

The ME includes terms that are not compensated by the PS approximate virtual corrections (i.e. Sudakov factors).
These are the improvements that we need to describe multiple hard jets!
If we simply add samples, the "improvements" will degrade the inclusive cross section: $\sigma_{\text {inc }}$ will contain $\ln \left(t_{\mathrm{MS}}\right)$ terms.

The inclusive cross section does not contain logs related TO CUTS ON HIGHER MULTIPLICITIES.

Traditional approach: Don't use a too small merging scale.
$\rightarrow$ Uncancelled terms numerically not important.
Unitary approach ${ }^{1}$ :
Use a (PS) unitarity inspired approach exactly cancel the dependence of the inclusive cross section on $t_{\mathrm{MS}}$.

## Unitarised ME + PS

Aim: If you add too much, then subtract what you add!
$\langle\mathcal{O}\rangle=B_{0} \mathcal{O}\left(S_{+0 j}\right)$
$-\int d \rho B_{1} \Theta_{>}^{(1)} w_{f} w_{\alpha_{s}} \Pi_{S_{+0}}\left(\rho_{0}, \rho\right) \mathcal{O}\left(S_{+0 j}\right)-\int d \rho \mathrm{~B}_{2} \Theta_{>}^{(2)} \Theta_{<}^{(1)} w_{f} w_{\alpha_{s}} \Pi_{S_{+0}}\left(\rho_{0}, \rho\right) \mathcal{O}\left(s_{+0 j}\right)$
$+\int \mathrm{B}_{1} \Theta_{>}^{(1)} w_{f} w_{\alpha_{s}} \Pi_{s_{+0}}\left(\rho_{0}, \rho\right) \mathcal{O}\left(S_{+1 j}\right)$
$-\int d \rho B_{2} \Theta_{>}^{(2)} w_{f} w_{\alpha_{s}} \Pi_{s_{+0}}\left(\rho_{0}, \rho_{1}\right) \Pi_{s_{+1}}\left(\rho_{1}, \rho\right) \mathcal{O}\left(s_{+1 j}\right)$
$+\int \mathrm{B}_{2} \Theta_{>}^{(2)} w_{f} W_{\alpha_{s}} \Pi_{S_{+0}}\left(\rho_{0}, \rho_{1}\right) \Pi_{S_{+1}}\left(\rho_{1}, \rho\right) \mathcal{O}\left(S_{+2 j}\right)+\int \mathrm{B}_{2} \Theta_{>}^{(2)} \Theta_{<}^{(1)} w_{f} w_{\alpha_{s}} \Pi_{S_{+0}}\left(\rho_{0}, \rho_{1}\right) \mathcal{O}\left(s_{+}\right.$
Inclusive cross sections preserved by construction.
Cancellation between different "jet bins".
$\Rightarrow$ Statistics needs fixing.

## NLO matching with MC@NLO

Aim: Achieve NLO for inclusive +0 -jet, and LO for inclusive +1 -jet observables and attach PS resummation.

To get there, remember that the (regularised) NLO cross section is

$$
\begin{aligned}
\mathrm{B}_{\mathrm{NLO}}= & {\left[\mathrm{B}_{n}+\mathrm{V}_{n}+\mathrm{I}_{n}\right] \mathcal{O}_{0}+\int d \Phi_{\mathrm{rad}}\left(\mathrm{~B}_{n+1} \mathcal{O}_{1}-\mathrm{D}_{n+1} \mathcal{O}_{0}\right) } \\
= & {\left[\mathrm{B}_{n}+\mathrm{V}_{n}+\mathrm{I}_{n}\right] \mathcal{O}_{0}+\int d \Phi_{\mathrm{rad}}\left(\mathrm{~S}_{n+1} \mathcal{O}_{0}-\mathrm{D}_{n+1} \mathcal{O}_{0}\right) } \\
& +\int d \Phi_{\mathrm{rad}}\left(\mathrm{~S}_{n+1} \mathcal{O}_{1}-\mathrm{S}_{n+1} \mathcal{O}_{0}\right)+\int d \Phi_{\mathrm{rad}}\left(\mathrm{~B}_{n+1} \mathcal{O}_{1}-\mathrm{S}_{n+1} \mathcal{O}_{1}\right)
\end{aligned}
$$

where $\mathrm{S}_{n+1}$ are some additional "transfer functions", e.g. the PS kernels.
Red term is the $\mathcal{O}\left(\alpha_{\mathrm{s}}\right)$ part of a shower from $\mathrm{B}_{n} . \Rightarrow$ Discard from $\mathrm{B}_{\mathrm{NLO}}$.
Thus, we have the seed cross section
$\widehat{\mathrm{B}}_{\mathrm{NLO}}=\left[\mathrm{B}_{n}+\mathrm{V}_{n}+\mathrm{I}_{n}+\int d \Phi_{\mathrm{rad}}\left(\mathrm{S}_{n+1}-\mathrm{D}_{n+1}\right)\right] \mathcal{O}_{0}+\int d \Phi_{\mathrm{rad}}\left(\mathrm{B}_{n+1}-\mathrm{S}_{n+1}\right) \mathcal{O}_{1}$
This is not the NLO result...but showering the $\mathcal{O}_{0}$-part will restore this!

## UMEPS, MC@NLO-style (Plätzer)

Aim: Combine multiple tree-level calculations with each other and (PS) resummation. Fill in soft and collinear regions with parton shower.
$\langle\mathcal{O}\rangle=B_{0} \Pi_{S_{+0}}\left(\rho_{0}, \rho_{\mathrm{MS}}\right) \mathcal{O}\left(S_{+\mathrm{S}_{\mathrm{j}}}\right)$

$$
\begin{aligned}
& -\int d \rho\left[\mathrm{~B}_{1}-\mathrm{B}_{0} P_{0}(\rho)\right] \Theta_{>}^{(1)} w_{f} w_{\alpha_{s}} \Pi_{S_{+0}}\left(\rho_{0}, \rho\right) \mathcal{O}\left(S_{+0 j}\right) \\
+ & \int \mathrm{B}_{1} \Theta_{>}^{(1)} w_{f} w_{\alpha_{s}} \Pi_{S_{+0}}\left(\rho_{0}, \rho\right) \Pi_{S_{+1}}\left(\rho, \rho_{\mathrm{Ms}}\right) \mathcal{O}\left(S_{+1 j}\right) \\
& -\int \mathrm{d} \rho\left[\mathrm{~B}_{2}-\mathrm{B}_{1} P_{1}(\rho)\right] \Theta_{>}^{(2)} w_{f} w_{\alpha_{s}} \Pi_{S_{+0}}\left(\rho_{0}, \rho_{1}\right) \Pi_{S_{+1}}\left(\rho_{1}, \rho\right) \mathcal{O}\left(s_{+1 j}\right) \\
+ & \int \mathrm{B}_{2} \Theta_{>}^{(2)} w_{f} w_{\alpha_{s}} \Pi_{S_{+0}}\left(\rho_{0}, \rho_{1}\right) \Pi_{S_{+1}}\left(\rho_{1}, \rho\right) \mathcal{O}\left(S_{+2 j}\right)+\int \mathrm{B}_{2} \Theta_{>}^{(2)} \Theta_{<}^{(1)} w_{f} w_{\alpha_{S}} \Pi_{S_{+0}}\left(\rho_{0}, \rho_{1}\right) \mathcal{O}\left(s_{+}\right.
\end{aligned}
$$

Inclusive cross sections preserved by construction.
Less cancellation between different "jet bins" fixed.
$\Longrightarrow$ Statistics okay.

## The UnLOPS method

## Start with UMEPS:

$$
\begin{array}{ccc}
\langle\mathcal{O}\rangle=\int d \phi_{0}\left\{\mathcal { O } ( s _ { + 0 j } ) \left(\mathrm{~B}_{0}+\right.\right. & -\int \widehat{\mathrm{B}}_{1 \rightarrow 0} & \left.-\int \widehat{\mathrm{B}}_{2 \rightarrow 0}\right) \\
& \left.+\int \mathcal{O}\left(s_{+1 j}\right)\left(\begin{array}{cc} 
\\
& \widehat{\mathrm{B}}_{1} \\
& -\int \widehat{\mathrm{B}}_{2 \rightarrow 1} \\
&
\end{array}\right)+\iint \mathcal{O}\left(s_{+2 j}\right) \widehat{\mathrm{B}}_{2}\right\}
\end{array}
$$

## The UnLOPS method

Remove all unwanted $\mathcal{O}\left(\alpha_{\mathrm{s}}^{n}\right)$ - and $\mathcal{O}\left(\alpha_{\mathrm{s}}^{n+1}\right)$-terms:

$$
\begin{aligned}
\langle\mathcal{O}\rangle=\int d \phi_{0}\left\{\mathcal{O}\left(S_{+0 j}\right)( \right. & -\left[\int \widehat{\mathrm{B}}_{1 \rightarrow 0}\right]_{-1,2} \\
& \left.\left.+\int \mathcal{O}\left(S_{+1 j}\right)\left(\widehat{\mathrm{B}}_{1}\right]_{-1,2}-\left[\int \widehat{\mathrm{B}}_{2 \rightarrow 1}\right]_{-2}\right)+\int \mathcal{O}\left(S_{+2 j}\right) \widehat{\mathrm{B}}_{2}\right\}
\end{aligned}
$$

## The UnLOPS method

## Add full NLO results:

$$
\begin{gathered}
\langle\mathcal{O}\rangle=\int d \phi_{0}\left\{\mathcal { O } ( s _ { + 0 j } ) \left(\begin{array}{cc}
\widetilde{\mathrm{B}}_{0} & -\left[\int \widehat{\mathrm{B}}_{1 \rightarrow 0}\right]_{-1,2} \\
\left.+\int \mathcal{O}\left(s_{+1 j}\right)\left(\widetilde{\mathrm{B}}_{1}+\left[\widehat{\mathrm{B}}_{1}\right]_{-1,2}-\left[\int \widehat{\mathrm{B}}_{2 \rightarrow 1}\right]_{-2}\right)+\iint \mathcal{O}\left(s_{+2 j}\right) \widehat{\mathrm{B}}_{2}\right\}
\end{array}\right.\right.
\end{gathered}
$$

## The UnLOPS method

## Unitarise:

$$
\begin{gathered}
\langle\mathcal{O}\rangle=\int d \phi_{0}\left\{\mathcal{O}\left(S_{+0 j}\right)\left(\quad \widetilde{\mathrm{B}}_{0}-\int_{S} \widetilde{\mathrm{~B}}_{1 \rightarrow 0}+\int_{S} \mathrm{~B}_{1 \rightarrow 0}-\left[\int \widehat{\mathrm{B}}_{1 \rightarrow 0}\right]_{-1,2}-\int_{S} \mathrm{~B}_{2 \rightarrow 0}^{\uparrow}-\int \widehat{\mathrm{B}}_{2 \rightarrow 0}\right)\right. \\
\left.+\int \mathcal{O}\left(S_{+1 j}\right)\left(\widetilde{\mathrm{B}}_{1}+\left[\widehat{\mathrm{B}}_{1}\right]_{-1,2}-\left[\int \widehat{\mathrm{B}}_{2 \rightarrow 1}\right]_{-2}\right)+\iint \mathcal{O}\left(S_{+2 j}\right) \widehat{\mathrm{B}}_{2}\right\}
\end{gathered}
$$

## Comparison of NLO merging schemes

FxFx: Restricts the range of merging scales. Cross section changes thus numerically small.
Probably fewest counter events.

MEPS@NLO: Improved, colour-correct Sudakov of MC@NLO for the first emission. Larger $t_{\mathrm{MS}}$ range. Smaller cross section changes. Improved resummation in process-independent way.

UNLOPS: Inclusive observables strictly NLO correct. Further shower improvements also directly improve the results. Many counter events if done naively.

MinLO: applies analytical (N)NLL Sudakov factors, which cancel problematic logs, only merging two multiplicities. Was moulded into an NNLO matching.

## The next step(s): Matching @ NNLO

Aim: For important processes - lumi monitors like Drell-Yan, precision studies (ggH, ZH, WBF,...) - reduce uncertainties and remove personal bias. But make sure all other improvements stay intact!

Observation: If an NLO merged calculation leads to a well-defined zero-jet inclusive cross section, it is easy to upgrade this cross section to NNLO.
$\Longrightarrow$ Fulfilled by MiNLO and UNLOPS
$\Longrightarrow$ NNLO+PS schemes have been implemented (MiNLO-NNLOPS and UN ${ }^{2}$ LOPS)

## Deriving an UN ${ }^{2}$ LOPS matching

We basically follow a "merging strategy":

- Pick calculations to combine (two MC@NLOs) with each other and with the PS resummation.
- Remove kinematic overlaps between the two MC@NLOs by dividing the one-jet phase space.
- Reweight one-jet MC@NLO (to make it exclusive $\leftrightarrow$ want to describe hardest jet with this), remove all undesired terms at $\mathcal{O}\left(\alpha_{\mathrm{s}}^{1+1}\right)$ and make sure that the whole thing is numerically stable. Reweight subtractions with $\Pi_{S_{+0}}$ to be able to group them with virtuals.
- Add and subtract reweighted one-jet MC@NLO, ( $\rightarrow$ unitarise) to ensure inclusive zero-jet cross section is unchanged w.r.t. NLO.
- Remove all terms up to $\mathcal{O}\left(\alpha_{\mathrm{s}}^{2}\right)$ in the zero-jet contribution, replace by NNLO jet-vetoed cross section.


## UN ${ }^{2}$ LOPS matching

Aim: Combine just two NLO calculations, then upgrade to NNLO directly.
Start over again, now combining MC@NLO's because those are resonably stable. Thus:
$\diamond$ Use 0-jet matched (MC@NLO 0 ) and 1-jet matched calculation (MC@NLO ${ }_{1}$ ).
$\diamond$ Remove hard ( $q_{T}>\rho_{\text {MS }}$ ) reals in MC@NLO 0 .
$\diamond$ Reweight $\mathrm{B}_{1}$ of MC@NLO $1_{1}$ with "zero-jet Sudakov" factor $\Pi_{S_{+0}} / \alpha_{s}$ running.
$\diamond$ Reweight NLO part $\widetilde{\mathrm{B}}_{1}^{\mathrm{R}}$ of MC@NLO ${ }_{1}$ with "zero-jet Sudakov" factor.
$\diamond$ Subtract erroneous $\mathcal{O}\left(\alpha_{\mathrm{s}}^{+1}\right)$ terms multiplying $\mathrm{B}_{1}$.
$\diamond$ Reweight subtractions with $\Pi_{S_{+0}}$ to be able to group them with $\widetilde{\mathrm{B}}_{1}^{\mathrm{R}}$.
$\diamond$ Put $\rho_{\text {MS }} \rightarrow \rho_{c}<1 \mathrm{GeV}$. ( $\rightarrow$ MC@NLO ${ }_{0}$ becomes exclusive NLO)
$\diamond$ Unitarise by subtracting the processed MC@NLO ${ }_{1}^{\prime}$ from the "zero- $q_{T}$ bin".
$\diamond$ Remove all terms up to $\alpha_{s}^{2}$ from the "zero- $q_{T}$ bin" and add the $q_{T}$-vetoed NNLO cross section.
$\Rightarrow \sigma_{\text {inclusive }}$ @ NNLO, resummation as accurate as Sudakov, stats fine.
NNLO logarithmic parts from $q_{T}$-vetoed TMDs (EFT calculation), hard coefficients from $q_{T}$-subtraction (i.e. DYNNLO, HNNLO), power corrections from MC@NLO $1_{1}$.

## UN ${ }^{2}$ LOPS matching

$$
\begin{aligned}
& \mathcal{O}^{(\mathrm{UN} 2 \mathrm{LOPS})}=\int d \Phi_{0} \overline{\mathrm{~B}}_{0}^{q_{T, \text { cut }}}\left(\Phi_{0}\right) O\left(\Phi_{0}\right) \\
& +\int_{q_{T}, \text { cut }} d \Phi_{1}\left[1-\Pi_{0}\left(t_{1}, \mu_{Q}^{2}\right)\left(w_{1}\left(\Phi_{1}\right)+w_{1}^{(1)}\left(\Phi_{1}\right)+\Pi_{0}^{(1)}\left(t_{1}, \mu_{Q}^{2}\right)\right)\right] \mathrm{B}_{1}\left(\Phi_{1}\right) O\left(\Phi_{0}\right) \\
& +\int_{q T, \text { cut }} d \Phi_{1} \Pi_{0}\left(t_{1}, \mu_{Q}^{2}\right)\left(w_{1}\left(\Phi_{1}\right)+w_{1}^{(1)}\left(\Phi_{1}\right)+\Pi_{0}^{(1)}\left(t_{1}, \mu_{Q}^{2}\right)\right) \mathrm{B}_{1}\left(\Phi_{1}\right) \overline{\mathcal{F}}_{1}\left(t_{1}, O\right) \\
& +\int_{q T, \text { cut }} d \Phi_{1}\left[1-\Pi_{0}\left(t_{1}, \mu_{Q}^{2}\right)\right] \tilde{\mathrm{B}}_{1}^{\mathrm{R}}\left(\Phi_{1}\right) O\left(\Phi_{0}\right)+\int_{q_{T}, \text { cut }} d \Phi_{1} \Pi_{0}\left(t_{1}, \mu_{Q}^{2}\right) \tilde{\mathrm{B}}_{1}^{\mathrm{R}}\left(\Phi_{1}\right) \overline{\mathcal{F}}_{1}\left(t_{1}, 0\right) \\
& +\int_{q T, \text { cut }} d \Phi_{2}\left[1-\Pi_{0}\left(t_{1}, \mu_{Q}^{2}\right)\right] \mathrm{H}_{1}^{\mathrm{R}}\left(\Phi_{2}\right) O\left(\Phi_{0}\right)+\int_{q_{T, \text { cut }}} d \Phi_{2} \Pi_{0}\left(t_{1}, \mu_{Q}^{2}\right) \mathrm{H}_{1}^{\mathrm{R}}\left(\Phi_{2}\right) \mathcal{F}_{2}\left(t_{2}, 0\right) \\
& +\int_{q_{T}, \text { cut }} d \Phi_{2} \mathrm{H}_{1}^{\mathrm{E}}\left(\Phi_{2}\right) \mathcal{F}_{2}\left(t_{2}, O\right)
\end{aligned}
$$

## UN ${ }^{2}$ LOPS matching

$$
\begin{aligned}
& \mathcal{O}^{\left(\mathrm{UN} N^{2} \mathrm{LOPS}\right)}=\int d \Phi_{0} \overline{\mathrm{~B}}_{0}^{q_{T, \text { cut }}}\left(\Phi_{0}\right) O\left(\Phi_{0}\right) \\
& +\int_{q T, \text { cut }} d \Phi_{1}\left[1-\Pi_{0}\left(t_{1}, \mu_{Q}^{2}\right)\left(w_{1}\left(\Phi_{1}\right)+w_{1}^{(1)}\left(\Phi_{1}\right)+\Pi_{0}^{(1)}\left(t_{1}, \mu_{Q}^{2}\right)\right)\right] \mathrm{B}_{1}\left(\Phi_{1}\right) O\left(\Phi_{0}\right) \\
& +\int_{q T, \text { cut }} d \Phi_{1} \Pi_{0}\left(t_{1}, \mu_{Q}^{2}\right)\left(w_{1}\left(\Phi_{1}\right)+w_{1}^{(1)}\left(\Phi_{1}\right)+\Pi_{0}^{(1)}\left(t_{1}, \mu_{Q}^{2}\right)\right) \mathrm{B}_{1}\left(\Phi_{1}\right) \overline{\mathcal{F}}_{1}\left(t_{1}, 0\right) \\
& +\int_{q T, \text { cut }} d \Phi_{1}\left[1-\Pi_{0}\left(t_{1}, \mu_{Q}^{2}\right)\right] \tilde{\mathrm{B}}_{1}^{\mathrm{R}}\left(\Phi_{1}\right) O\left(\Phi_{0}\right)+\int_{q_{T}, \text { cut }} d \Phi_{1} \Pi_{0}\left(t_{1}, \mu_{Q}^{2}\right) \tilde{\mathrm{B}}_{1}^{\mathrm{R}}\left(\Phi_{1}\right) \overline{\mathcal{F}}_{1}\left(t_{1}, 0\right) \\
& +\int_{q_{T}, \text { cut }} d \Phi_{2}\left[1-\Pi_{0}\left(t_{1}, \mu_{Q}^{2}\right)\right] \mathrm{H}_{1}^{\mathrm{R}}\left(\Phi_{2}\right) O\left(\Phi_{0}\right)+\int_{q_{T}, \text { cut }} d \Phi_{2} \Pi_{0}\left(t_{1}, \mu_{Q}^{2}\right) \mathrm{H}_{1}^{\mathrm{R}}\left(\Phi_{2}\right) \mathcal{F}_{2}\left(t_{2}, 0\right) \\
& +\int_{q_{T}, \text { cut }} d \Phi_{2} \mathrm{H}_{1}^{\mathrm{E}}\left(\Phi_{2}\right) \mathcal{F}_{2}\left(t_{2}, 0\right)
\end{aligned}
$$

Note that this is just an extention of the old Sudakov veto algorithm: Run trial shower on the reconstructed zero-jet state, If trial shower produces an emission, keep zero-jet kinematics and stop; else start PS off one-jet state.

## UN ${ }^{2}$ LOPS matching

$$
\begin{aligned}
& \mathcal{O}^{\left(\mathrm{UN} \mathrm{~N}^{2} \mathrm{LOPS}\right)}=\int d \Phi_{0} \overline{\overline{\mathrm{~B}}}_{0}^{q_{T, \text { cut }}}\left(\Phi_{0}\right) O\left(\Phi_{0}\right) \\
& +\int_{q_{T}, \text { cut }} d \Phi_{1}\left[1-\Pi_{0}\left(t_{1}, \mu_{Q}^{2}\right)\left(w_{1}\left(\Phi_{1}\right)+w_{1}^{(1)}\left(\Phi_{1}\right)+\Pi_{0}^{(1)}\left(t_{1}, \mu_{Q}^{2}\right)\right)\right] \mathrm{B}_{1}\left(\Phi_{1}\right) O\left(\Phi_{0}\right) \\
& +\int_{q_{T}, \text { cut }} d \Phi_{1} \Pi_{0}\left(t_{1}, \mu_{Q}^{2}\right)\left(w_{1}\left(\Phi_{1}\right)+w_{1}^{(1)}\left(\Phi_{1}\right)+\Pi_{0}^{(1)}\left(t_{1}, \mu_{Q}^{2}\right)\right) \mathrm{B}_{1}\left(\Phi_{1}\right) \overline{\mathcal{F}}_{1}\left(t_{1}, 0\right) \\
& +\int_{q_{T}, \text { cut }} d \Phi_{1}\left[1-\Pi_{0}\left(t_{1}, \mu_{Q}^{2}\right)\right] \tilde{\mathrm{B}}_{1}^{\mathrm{R}}\left(\Phi_{1}\right) O\left(\Phi_{0}\right)+\int_{q_{T}, \text { cut }} d \Phi_{1} \Pi_{0}\left(t_{1}, \mu_{Q}^{2}\right) \tilde{\mathrm{B}}_{1}^{\mathrm{R}}\left(\Phi_{1}\right) \overline{\mathcal{F}}_{1}\left(t_{1}, 0\right) \\
& +\int_{q_{T}, \text { cut }} d \Phi_{2}\left[1-\Pi_{0}\left(t_{1}, \mu_{Q}^{2}\right)\right] \mathrm{H}_{1}^{\mathrm{R}}\left(\Phi_{2}\right) O\left(\Phi_{0}\right)+\int_{q_{T} \text { cut }} d \Phi_{2} \Pi_{0}\left(t_{1}, \mu_{Q}^{2}\right) \mathrm{H}_{1}^{\mathrm{R}}\left(\Phi_{2}\right) \mathcal{F}_{2}\left(t_{2}, 0\right) \\
& +\int_{q_{T}, \text { cut }} d \Phi_{2} \mathrm{H}_{1}^{\mathrm{E}}\left(\Phi_{2}\right) \mathcal{F}_{2}\left(t_{2}, O\right)
\end{aligned}
$$

Note: $\left[1-\Pi_{0}\left(t_{1}, \mu_{Q}^{2}\right)\right] \tilde{\mathrm{B}}_{1}^{\mathrm{R}}$ etc. comes from using $q_{T}$-vetoed cross sections.

## UN ${ }^{2}$ LOPS matching

$$
\begin{aligned}
& \mathcal{O}^{(\mathrm{UN} 2 \mathrm{LOPS})}=\int d \Phi_{0} \overline{\overline{\mathrm{~B}}}_{0}^{q_{T, \mathrm{cut}}}\left(\Phi_{0}\right) O\left(\Phi_{0}\right) \\
& +\int_{q_{T, \text { cut }}} d \Phi_{1}\left[1-\Pi_{0}\left(t_{1}, \mu_{Q}^{2}\right)\left(w_{1}\left(\Phi_{1}\right)+w_{1}^{(1)}\left(\Phi_{1}\right)+\Pi_{0}^{(1)}\left(t_{1}, \mu_{Q}^{2}\right)\right)\right] \mathrm{B}_{1}\left(\Phi_{1}\right) O\left(\Phi_{0}\right) \\
& +\int_{q_{T, \text { cut }}} d \Phi_{1} \Pi_{0}\left(t_{1}, \mu_{Q}^{2}\right)\left(w_{1}\left(\Phi_{1}\right)+w_{1}^{(1)}\left(\Phi_{1}\right)+\Pi_{0}^{(1)}\left(t_{1}, \mu_{Q}^{2}\right)\right) \mathrm{B}_{1}\left(\Phi_{1}\right) \overline{\mathcal{F}}_{1}\left(t_{1}, 0\right) \\
& +\int_{q_{T, c u t}} d \Phi_{1}\left[1-\Pi_{0}\left(t_{1}, \mu_{Q}^{2}\right)\right] \tilde{\mathrm{B}}_{1}^{\mathrm{R}}\left(\Phi_{1}\right) O\left(\Phi_{0}\right)+\int_{q_{T}, \mathrm{cut}} d \Phi_{1} \Pi_{0}\left(t_{1}, \mu_{Q}^{2}\right) \tilde{\mathrm{B}}_{1}^{\mathrm{R}}\left(\Phi_{1}\right) \overline{\mathcal{F}}_{1}\left(t_{1}, 0\right) \\
& +\int_{q_{T, \text { cut }}} d \Phi_{2}\left[1-\Pi_{0}\left(t_{1}, \mu_{Q}^{2}\right)\right] H_{1}^{\mathrm{R}}\left(\Phi_{2}\right) O\left(\Phi_{0}\right)+\int_{q_{T, \text { cut }}} d \Phi_{2} \Pi_{0}\left(t_{1}, \mu_{Q}^{2}\right) H_{1}^{\mathrm{R}}\left(\Phi_{2}\right) \mathcal{F}_{2}\left(t_{2}, O\right) \\
& +\int_{q_{T, \text { cut }}} d \Phi_{2} \mathrm{H}_{1}^{\mathrm{E}}\left(\Phi_{2}\right) \mathcal{F}_{2}\left(t_{2}, 0\right) \\
& \overline{\overline{\mathrm{B}}}_{0}^{q_{T, \text { cut }}}+\tilde{\mathrm{B}}_{1}^{\mathrm{R}}+\mathrm{H}_{1}^{\mathrm{R}}+\mathrm{H}_{1}^{\mathrm{E}}=\mathrm{B}_{\mathrm{NNLO}}
\end{aligned}
$$

Other terms drop out in inclusive observables.

## UN ${ }^{2}$ LOPS matching

$$
\begin{aligned}
& \mathcal{O}^{\left(\mathrm{UN}^{2} \mathrm{LOPS}\right)}=\int d \Phi_{0} \overline{\overline{\mathrm{~B}}}_{0}^{q_{T, \text { cut }}}\left(\Phi_{0}\right) O\left(\Phi_{0}\right) \\
& +\int_{q_{T}, \text { cut }} d \Phi_{1}\left[1-\Pi_{0}\left(t_{1}, \mu_{Q}^{2}\right)\left(w_{1}\left(\Phi_{1}\right)+w_{1}^{(1)}\left(\Phi_{1}\right)+\Pi_{0}^{(1)}\left(t_{1}, \mu_{Q}^{2}\right)\right)\right] \mathrm{B}_{1}\left(\Phi_{1}\right) O\left(\Phi_{0}\right) \\
& +\int_{q_{\mathrm{T}, \mathrm{cut}}} d \Phi_{1} \Pi_{0}\left(t_{1}, \mu_{Q}^{2}\right)\left(w_{1}\left(\Phi_{1}\right)+w_{1}^{(1)}\left(\Phi_{1}\right)+\Pi_{0}^{(1)}\left(t_{1}, \mu_{Q}^{2}\right)\right) \mathrm{B}_{1}\left(\Phi_{1}\right) \overline{\mathcal{F}}_{1}\left(t_{1}, 0\right) \\
& +\int_{q_{T}, \text { cut }} d \Phi_{1}\left[1-\Pi_{0}\left(t_{1}, \mu_{Q}^{2}\right)\right] \tilde{\mathrm{B}}_{1}^{\mathrm{R}}\left(\Phi_{1}\right) O\left(\Phi_{0}\right)+\int_{q_{T}, \text { cut }} d \Phi_{1} \Pi_{0}\left(t_{1}, \mu_{Q}^{2}\right) \tilde{\mathrm{B}}_{1}^{\mathrm{R}}\left(\Phi_{1}\right) \overline{\mathcal{F}}_{1}\left(t_{1}, 0\right) \\
& +\int_{q_{T}, \text { cut }} d \Phi_{2}\left[1-\Pi_{0}\left(t_{1}, \mu_{Q}^{2}\right)\right] \mathrm{H}_{1}^{\mathrm{R}}\left(\Phi_{2}\right) O\left(\Phi_{0}\right)+\int_{q_{T}, \text { cut }} d \Phi_{2} \Pi_{0}\left(t_{1}, \mu_{Q}^{2}\right) \mathrm{H}_{1}^{\mathrm{R}}\left(\Phi_{2}\right) \mathcal{F}_{2}\left(t_{2}, 0\right) \\
& +\int_{q_{T}, \text { cut }} d \Phi_{2} \mathrm{H}_{1}^{\mathrm{E}}\left(\Phi_{2}\right) \mathcal{F}_{2}\left(t_{2}, 0\right)
\end{aligned}
$$

Orange terms do not contain any universal $\alpha_{S}$ corrections present in the PS. $\mathrm{H}_{1}$ do not contribute in the soft/collinear limit.
$\Longrightarrow \mathrm{PS}$ accuracy is preserved.

## Weak reals in PYTHIA 8 arXiv:1401.6364




- Small effect at LHC, larger at FCC.
- Effect mostly from the first (few) weak bosons.

Weak virtuals in HERWIG++ arXiv:1401.3964


- Multiply (full!) electro-weak virtual corrections as phase-space dependent K-factor $K(\hat{s}, \hat{t})$. No real emissions included.
- Effect on ("QCD-cleaned") vetoed observables large.


## Weak reals in SHERPA arXiv:1403.4788







- Splitting kernels: Massive CS dipoles (CDST). Ordered in $p_{\perp}$.
- Boosted techniques at LHC can discriminate between pure QCD and jets containing hadronically decaying W's.


[^0]:    ${ }^{1}$ Translation: You need to apply $n_{j e t s}, p_{\perp j e t}, H_{T}$ cuts or use "kinematic endpoint variables" like $M_{T 2}$.

[^1]:    ${ }^{1}$ The same effect can also be obtained from perturbative physics - it's not obvious if the photons are imprinted by peturbative or non-perturbative effects.

