A Global Jet Finding Algorithm

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MC4BSM Workshop@LPC

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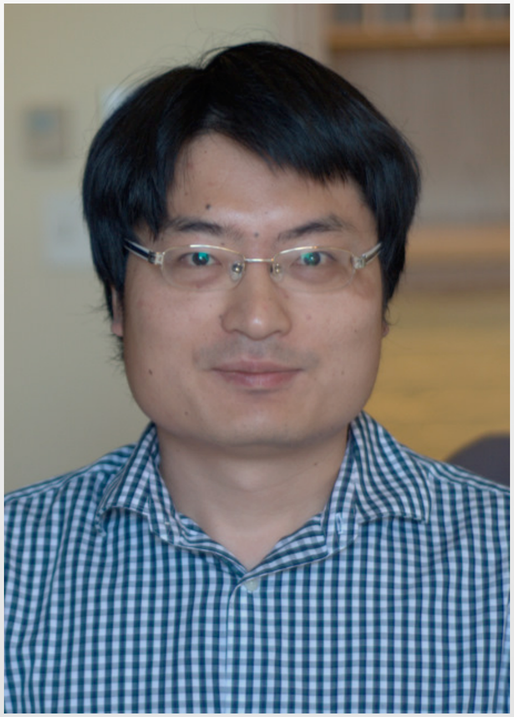
In collaboration with:



Zhenyu Han

University of Oregon

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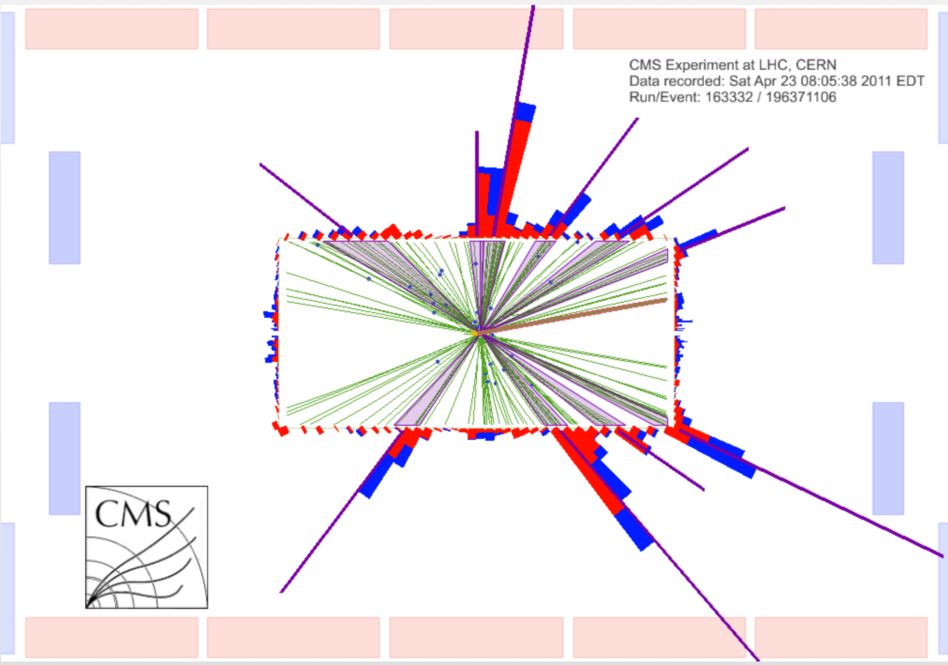
arxiv:1411.3705 and work in progress

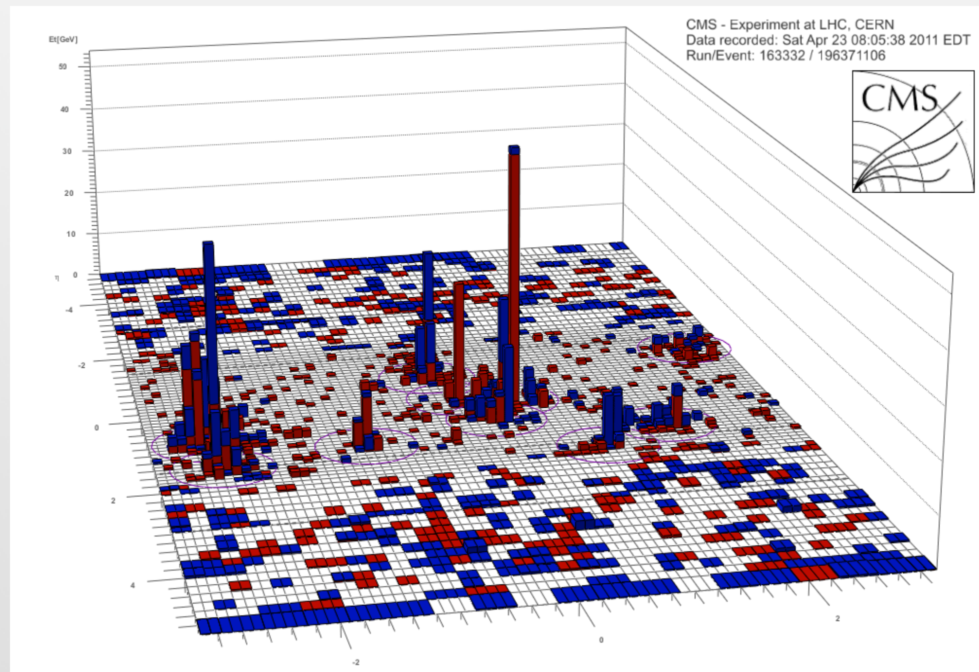
Outline

- Motivation: physics beyond the Standard Model
- Review of jet finding algorithms
- Introduction of a global definition
- Application to QCD-like jets
- Extension to boosted two-prong jets
- Conclusion

What is a jet?

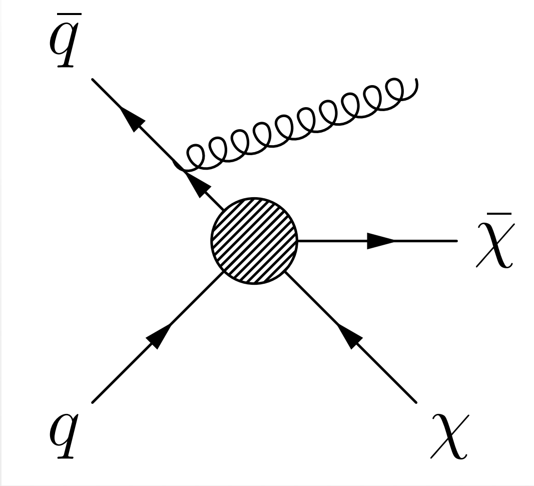
An ensemble of particles in detectors can be called a jet





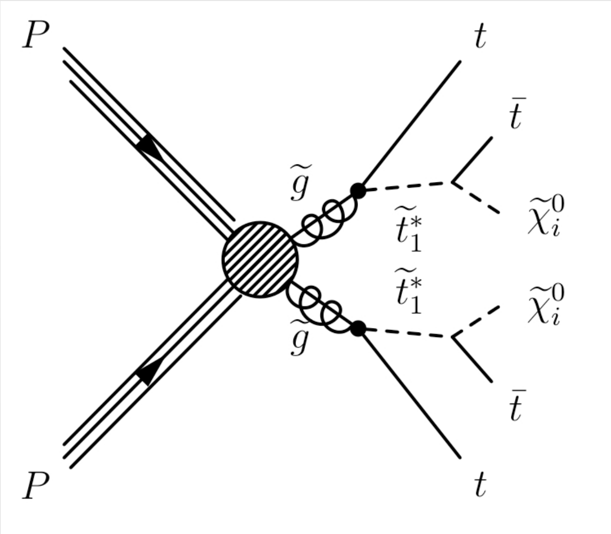
Jet-finding algorithm: how to group particles together?

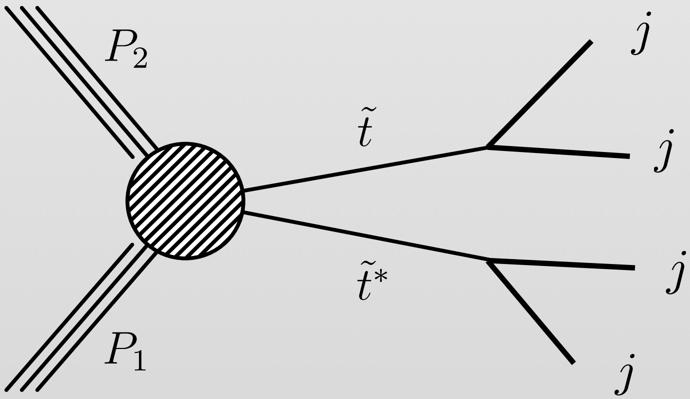
Jets in BSM



- Mono-jet plus MET events as the dark matter signature

- Multi-jets plus MET for RPC SUSY or without MET for RPV SUSY

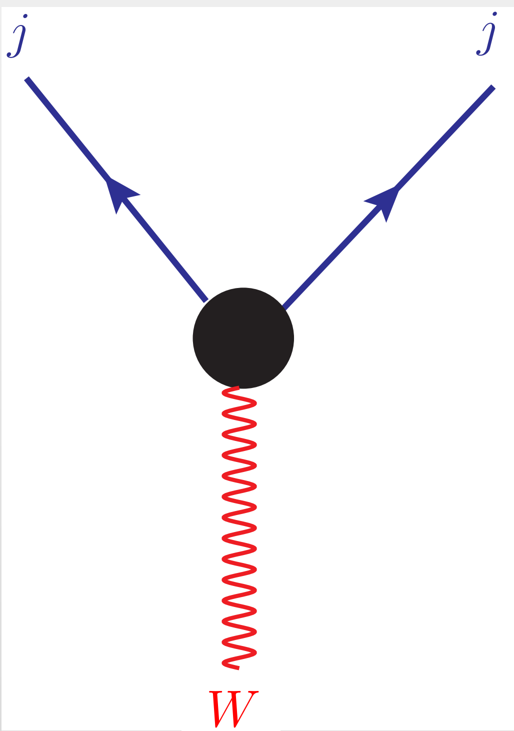




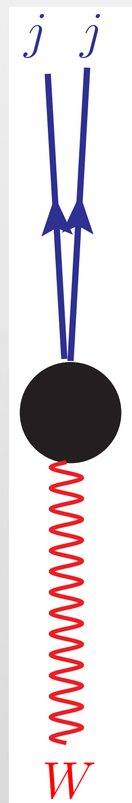
Fat-jet object

Search for a few TeV resonance decaying into t, W, Z, h …

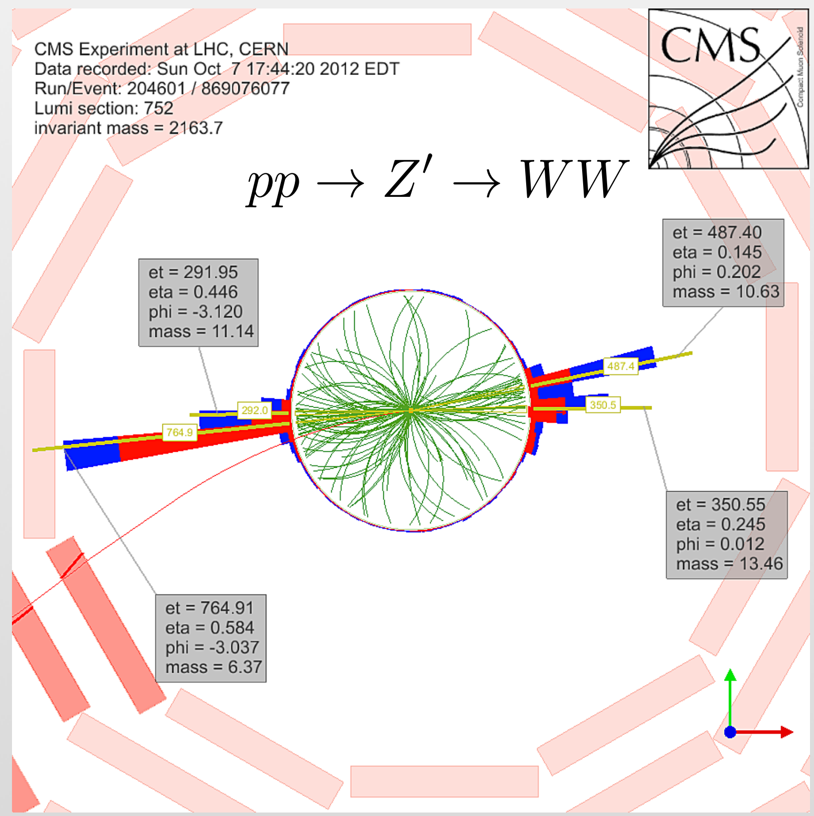
- Boosted top quark, W/Z, Higgs ……



$p_T(W) \sim M_W$



$p_T(W) \gg M_W$



Jet substructure

A jet may not be just a parton and it could have an internal structure

Many new objects: (incomplete list)

- …; Butterworth, Cox, Forshaw, WW scattering, hep-ph/0201098
- Butterworth, Davison, Rubin, Salam, boosted Higgs, 0802.2470
- Thaler and Wang, boosted top, 0806.0023
- Kaplan, Rehermann, Schwartz, Tweedie, boosted top, 0806.0848
- Almeida, Lee, Perez, Sterman, Sung, boosted top, 0807.0234;…

Many new variables or procedures:

- mass drop, N-subjettiness, pull, dipolarity, without trees, …
- Jet grooming: filtering, trimming, pruning …

Jet substructure: an example

Boosted Higgs for measuring the $h \to b\bar{b}$ decay

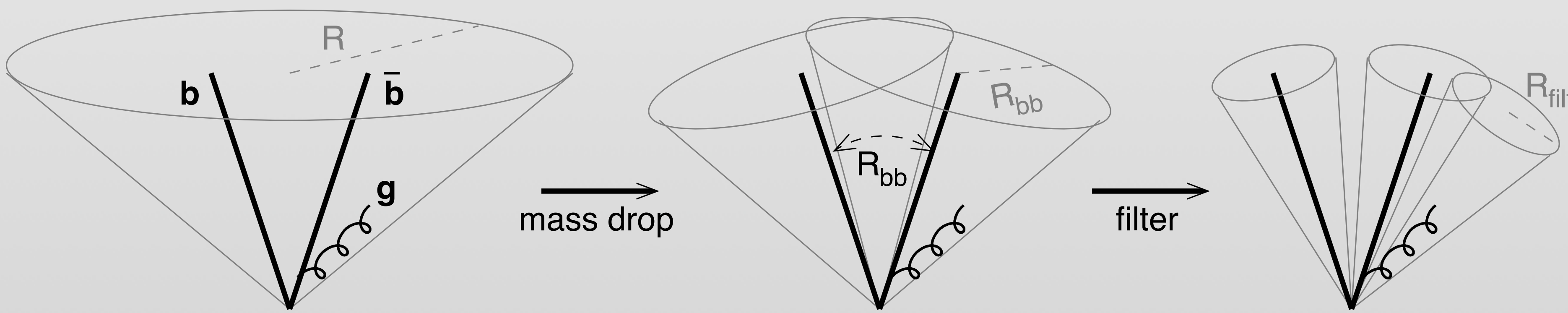
Two steps:

Butterworth et.al., 0802.2470

(1) start from a jet-finding algorithm (C/A) to cover a wider area

(2) mass-drop: (the QCD quark is massless) some subset of particles inside a Higgs-jet can have a much smaller mass.

Filter: (reduce underlying events) introduce a finer angular scale



Our motivation

Can we combine this two-step procedure into a single one?

- Hope: keep more hard process information and less underlying event contamination
- Method: define a new jet-finding algorithm suitable for a boosted heavy object

To proceed, let's start with traditional jet-finding algorithms for QCD jets

A brief review of jet-finding algorithms

★ Cone algorithm

- Started by Sterman and Weinberg in 70's
- CDF SearchCone, Mid point, SISCone …
- Used at UA1, Tevatron

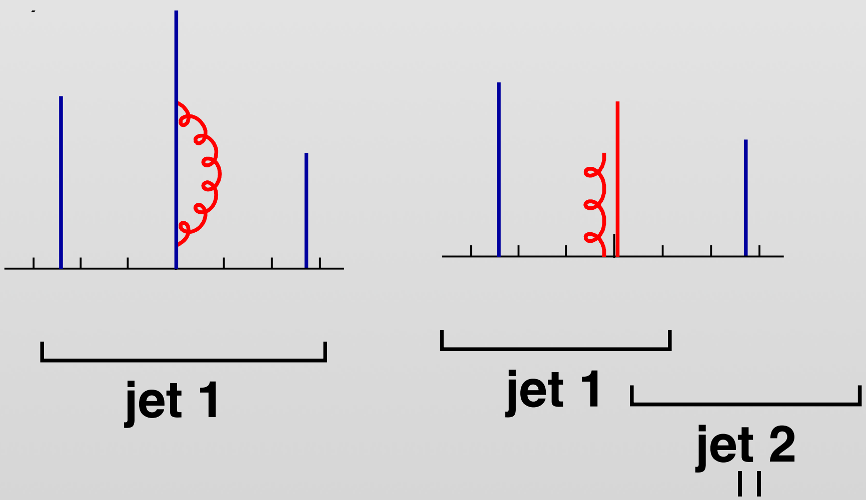
★ Sequential recombination algorithm

- Started by the JADE collaboration in 80's
- k_t, Cambridge/Aachen, $\mathrm{anti}\text{-}k_t$
- Extensively used at the LHC

Cone algorithm

Iterative process:

- choose particle with highest transverse momentum as the seed particle
- draw a cone of radius R around the seed particle
- sum the momenta of all particles in the cone as the jet axis
- if the jet axis does not agree with the original one, continue; otherwise find a stable cone and stop



Colinear safety?

SISCone (split-merge)

Anti-k_t algorithm

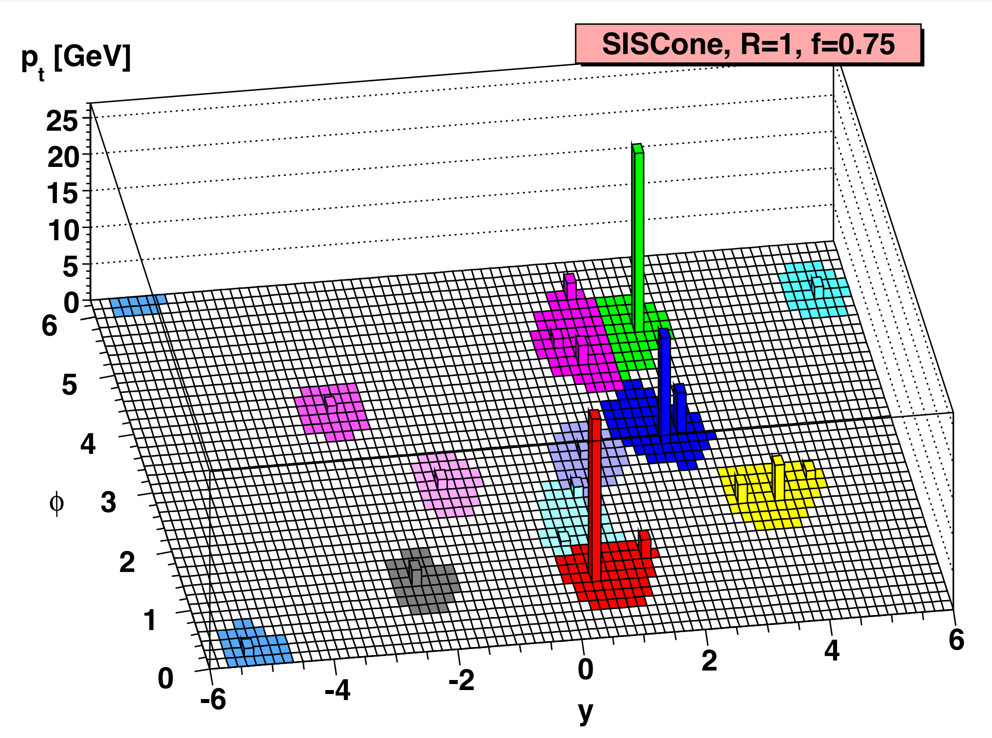
$$d_{ij} = \min(p_{ti}^{-2}, p_{tj}^{-2}) \frac{\Delta R_{ij}^2}{R^2} \qquad d_{iB} = p_{ti}^{-2} \qquad \Delta R_{ij}^2 = (y_i - y_j)^2 + (\phi_i - \phi_j)^2$$

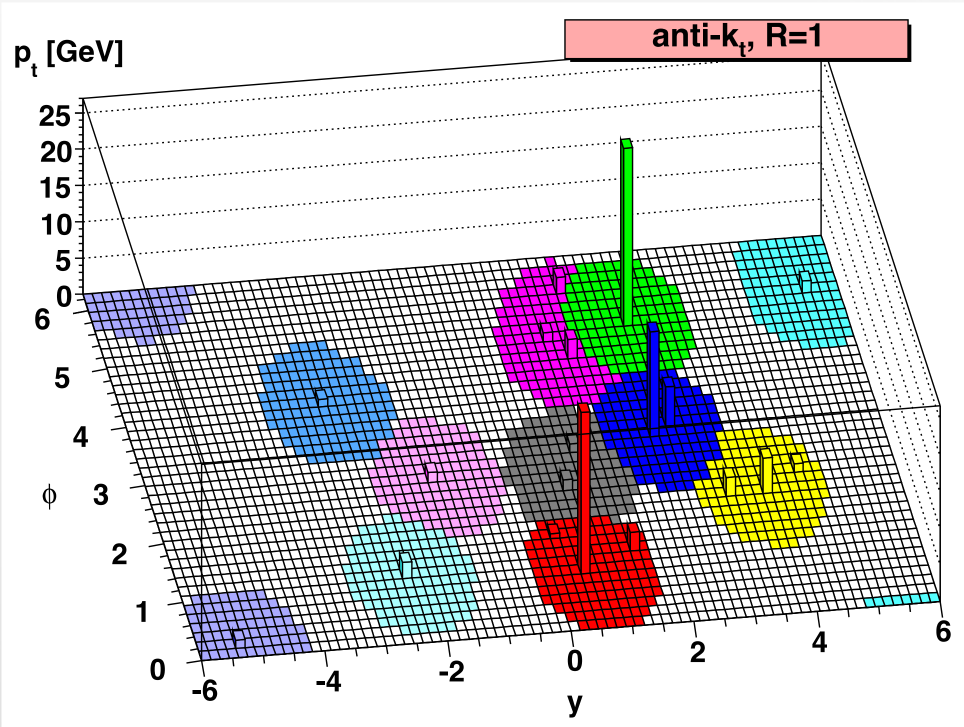
Iterative process:

- Find the minimum of the d_{ij} and d_{iB}
- If it is a d_{ij} , recombine i and j into a single new particle, and repeat
- otherwise, if it is a d_{iB}, declare i to be a jet, and remove it from the list of particles
- stop when no particles remain

Infrared and collinear safe !

Behaviors of different algorithms

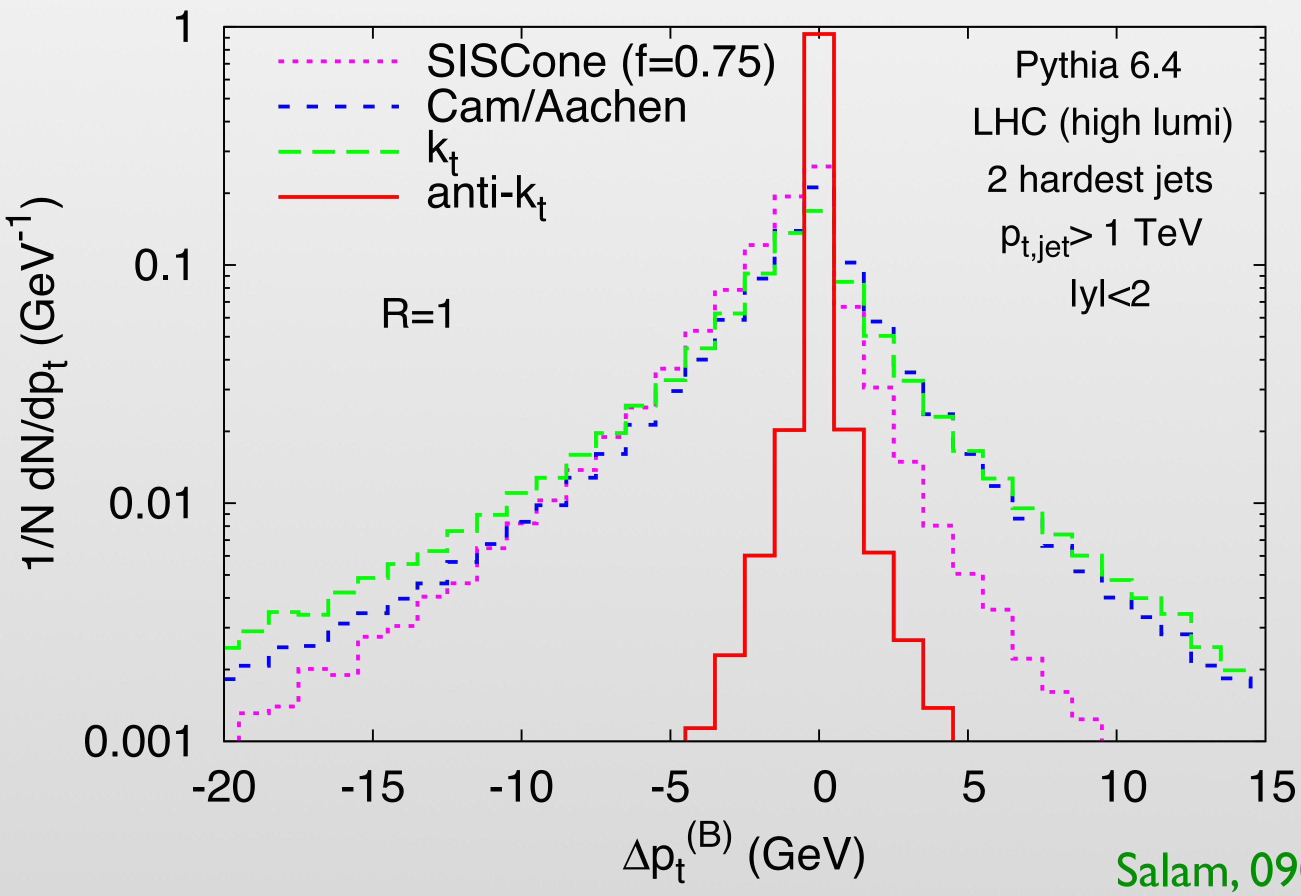




Salam, 0906.1833

Quantify the goodness of algorithms

Back-reaction: how much adding soft background particles changes the original particles in a jet



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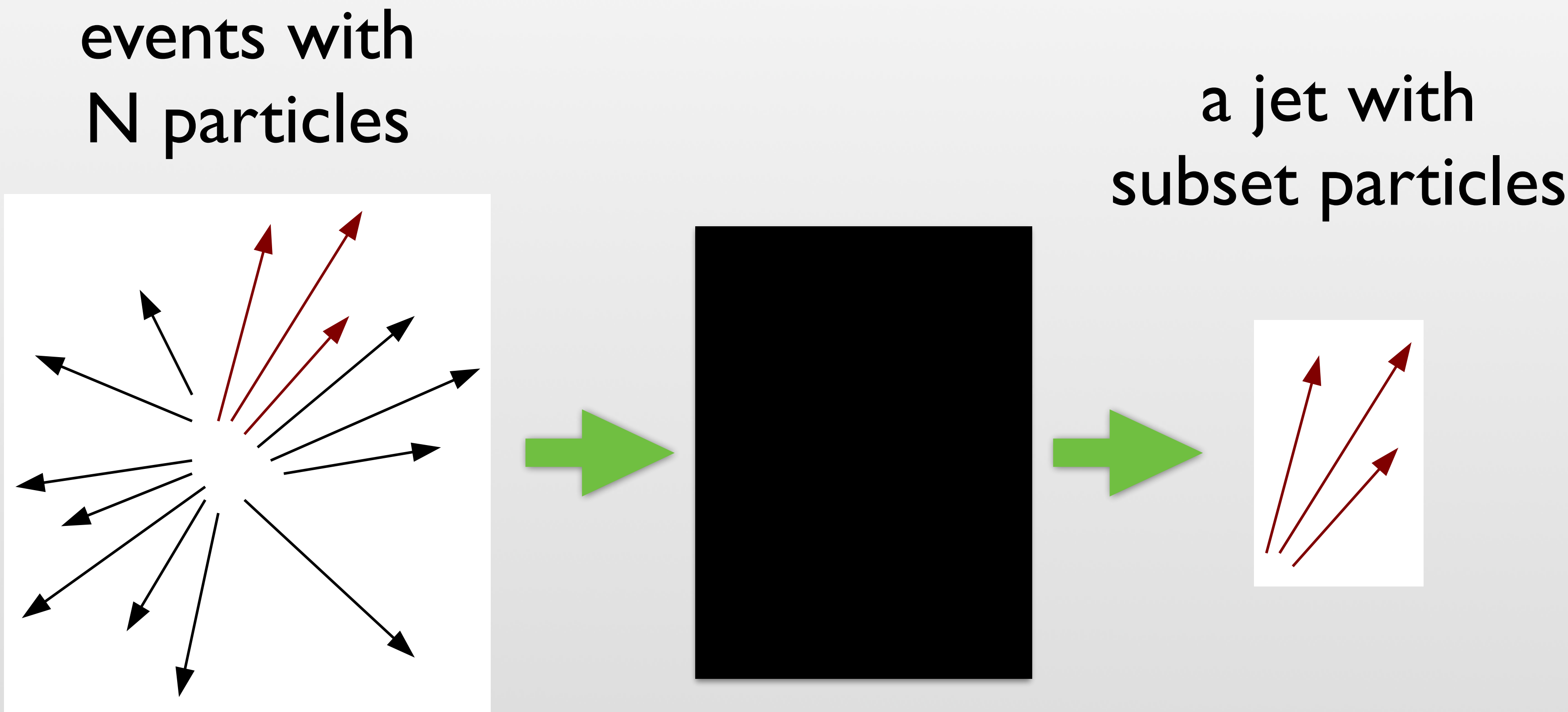
Can one has a more intuitive way to define a jet-finding algorithm?



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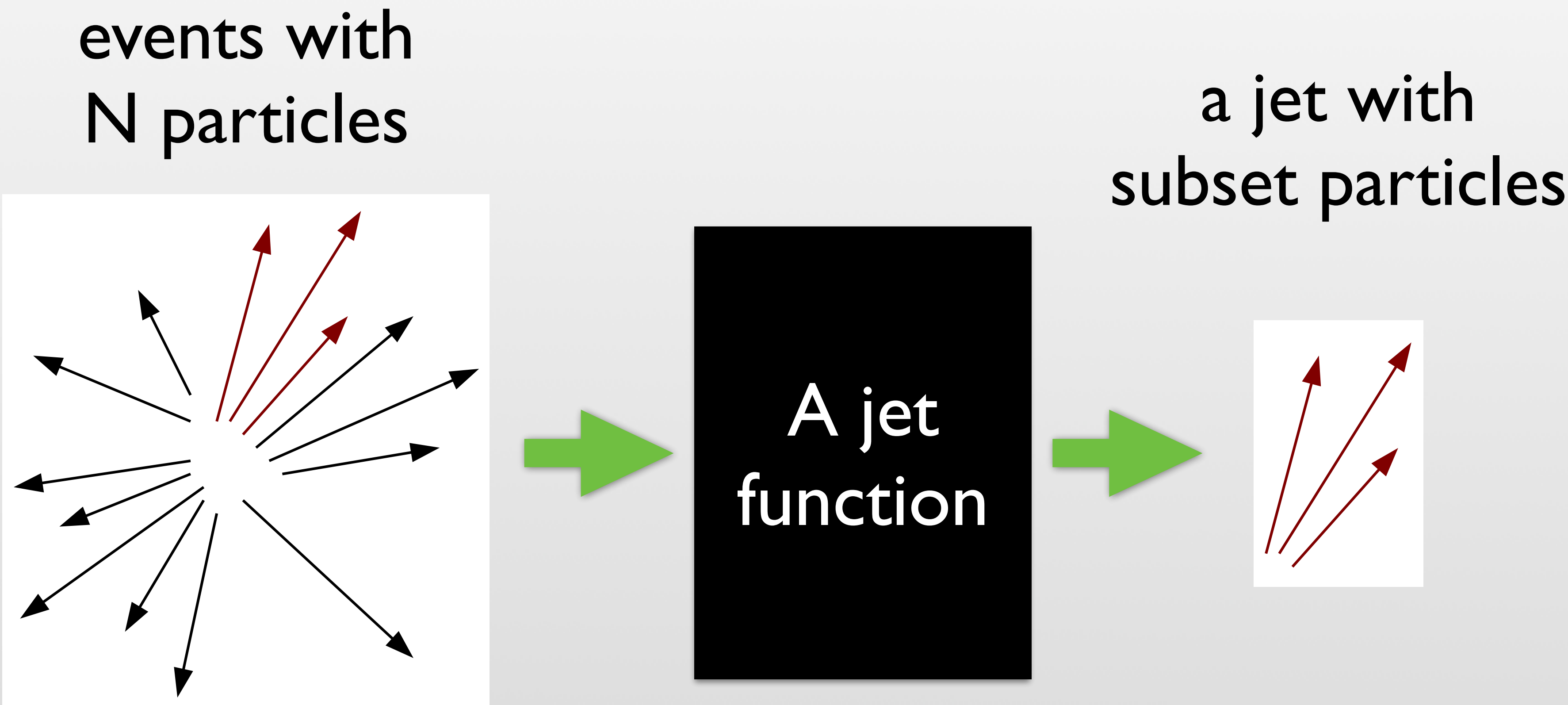

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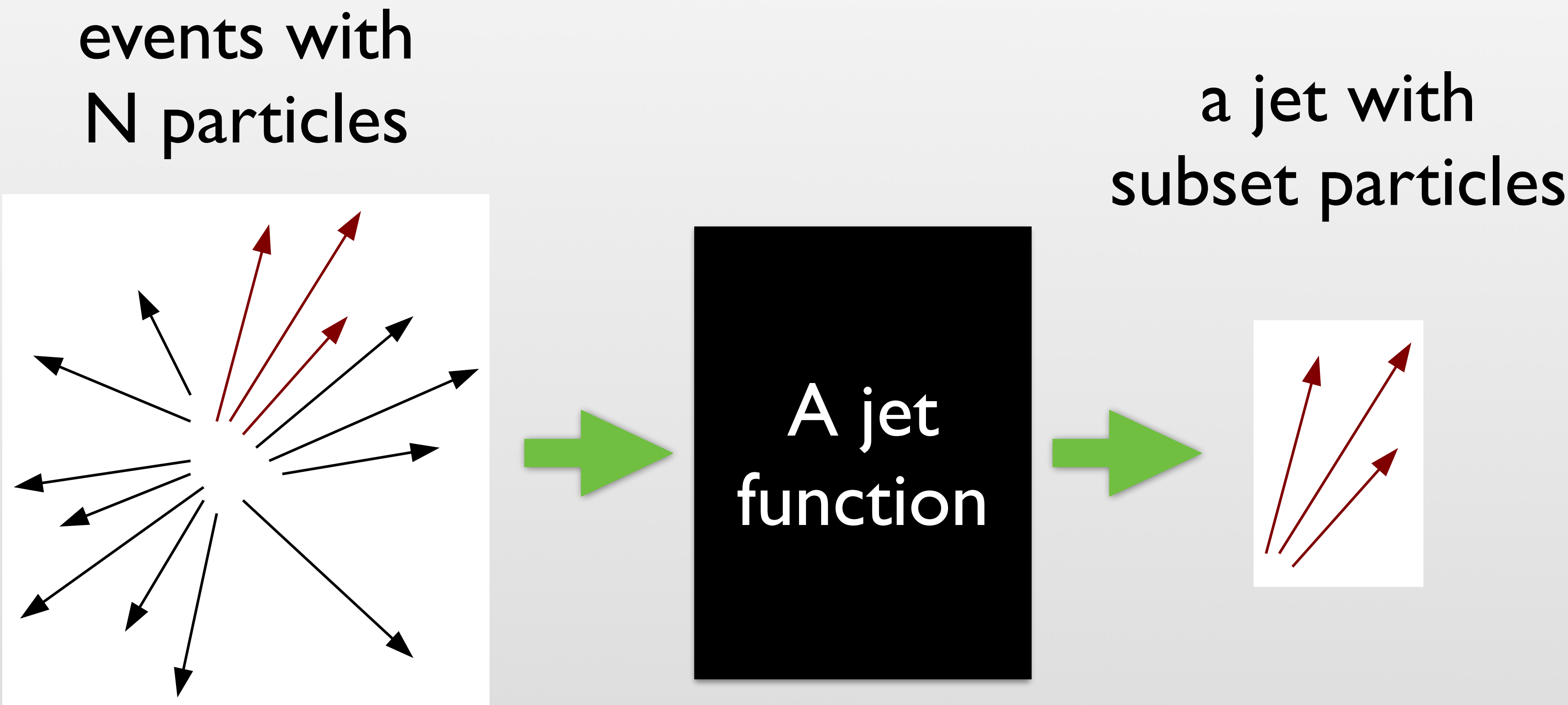

Look for a simple jet definition function

Start with a QCD jet

★ QCD partons are massless

★ The jet function should

- prefer increasing jet energy
- disfavor increasing jet mass

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★ The simple option at a lepton collider:

$$J(P^\mu) = E - \beta \frac{m^2}{E}$$

[H. Georgi, 1408.1161]

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[H. Georgi, 1408.1161]

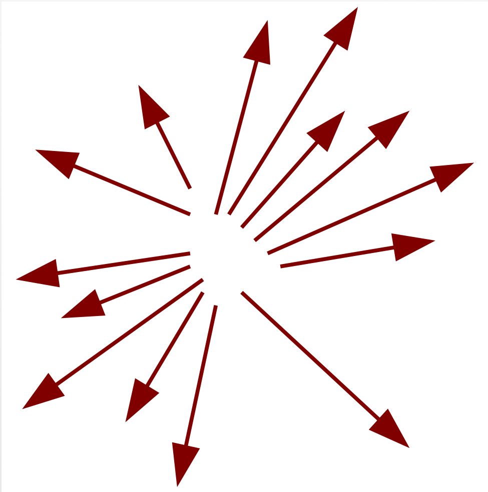
For N particles and 2^N possibilities, find the one maximizing this jet function. One does this iteratively to find all jets in one event.

Special cases

$$J(P^\mu) = E - \beta \frac{m^2}{E}$$

- $\beta = 0: \;\; J = E$

  include all particles in one jet



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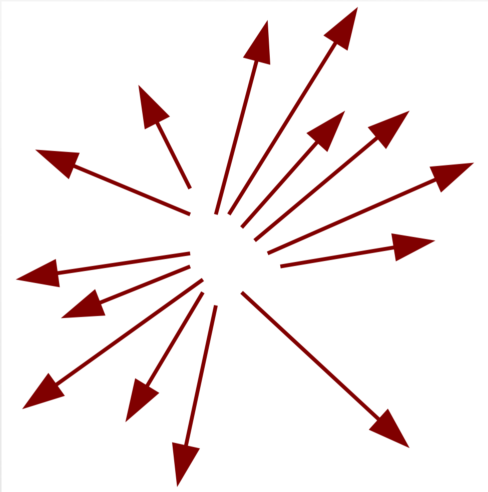
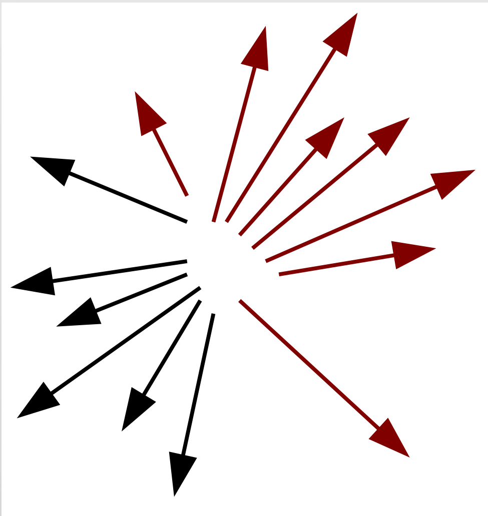


- $\beta = 1: J = |\vec{P}|$

  hemisphere way for two jets



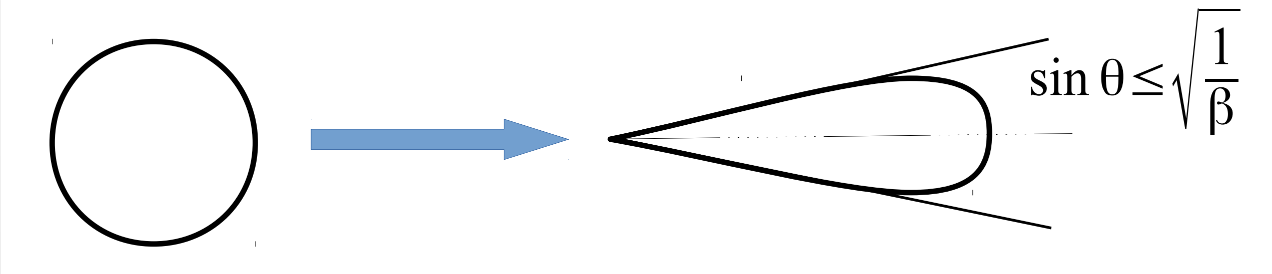
General cases

★ A group of particles will have a boost factor from its rest frame and has a jet function bigger than a soft particle

$$J = \frac{\left(E^2 - \beta m^2\right)}{E} = \left(\gamma^2 - \beta\right)\frac{m^2}{E} \geq 0$$

$$\gamma \geq \sqrt{\beta}$$

★ Relativistic beaming effect



★ The particles are inside a jet cone

A larger value of β means a smaller cone size

Extension to hadron colliders

★ The center-of-mass frame is likely to be highly boosted in the beam direction

★ The simplest way to extend the jet definition is

$$J_{E_T}(P_J^\mu) \equiv E_T - \beta \frac{m^2}{E_T}$$

★ One could also try other powers

$$J_{E_T}(P_J^\mu) \equiv E_T^\alpha (1 - \beta \frac{m^2}{E_T^2})$$

★ Does this new function has a similar cone geometry?

Try an "easier" function

★ For $\alpha = 2$,

$$J_{E_T^2} = E_T^2 - \beta m^2 = E^2 - P_z^2 - \beta m^2$$

★ Requiring $J_{E_T^2}(P_J^\mu) > J_{E_T^2}(P_J^\mu - p_j^\mu)$, the boundary satisfies

$$\frac{1}{|p||P|}\left(P_x p_x + P_y p_y + \left(1 - \frac{1}{\beta}\right) P_z p_z\right) = \frac{1}{v}\left(1 - \frac{1}{\beta}\right)$$

Try an "easier" function

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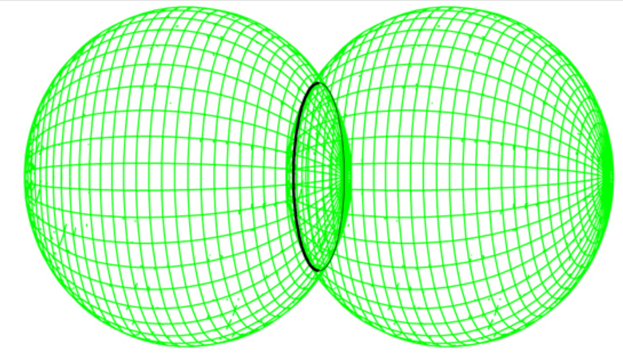
$$\begin{cases} p_x^2 + p_y^2 + p_z^2 = C_1(P) \\ (p_x - P_x)^2 + (p_y - P_y)^2 + \left(p_z - \left(1-\frac{1}{\beta}\right)P_z\right)^2 = C_2(P) \end{cases}$$



★ Can be interpreted as intersection of two spheres

Still a cone jet

★ For a general α, the boundary is

$$\frac{1}{|p||P|}\left(P_x p_x + P_y p_y + \kappa P_z p_z\right) = \frac{\kappa}{v} \qquad \kappa = 1 - \frac{\alpha}{2\beta} + \frac{\alpha-2}{2}\frac{m^2}{E_T^2}$$

the center is shifted from the jet momentum towards the central region

$$\vec{\hat{P}}_c = \frac{1}{\sqrt{1-(1-\kappa^2)\hat{P}_J^{z\,2}}}(\hat{P}_J^x, \hat{P}_J^y, \kappa\,\hat{P}_J^z) \qquad \kappa < 1$$

particles belong to the jet is within a cone from the center

$$z_c \geq \frac{\kappa}{v_J\sqrt{1-(1-\kappa^2)\cos^2\theta_J}}$$

Still a cone jet

★ For a general α, the boundary is

$$\frac{1}{|p||P|}\left(P_x p_x + P_y p_y + \kappa P_z p_z\right) = \frac{\kappa}{\nu}$$

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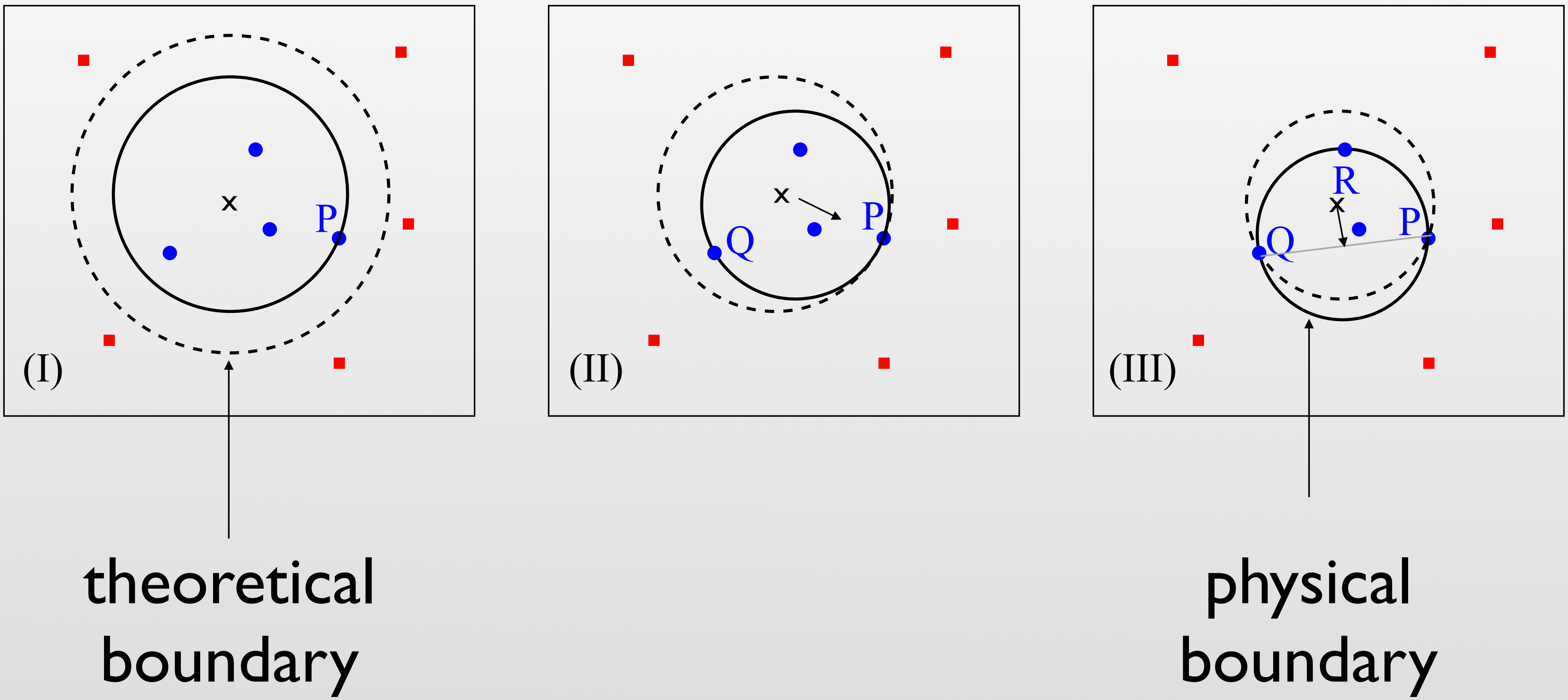
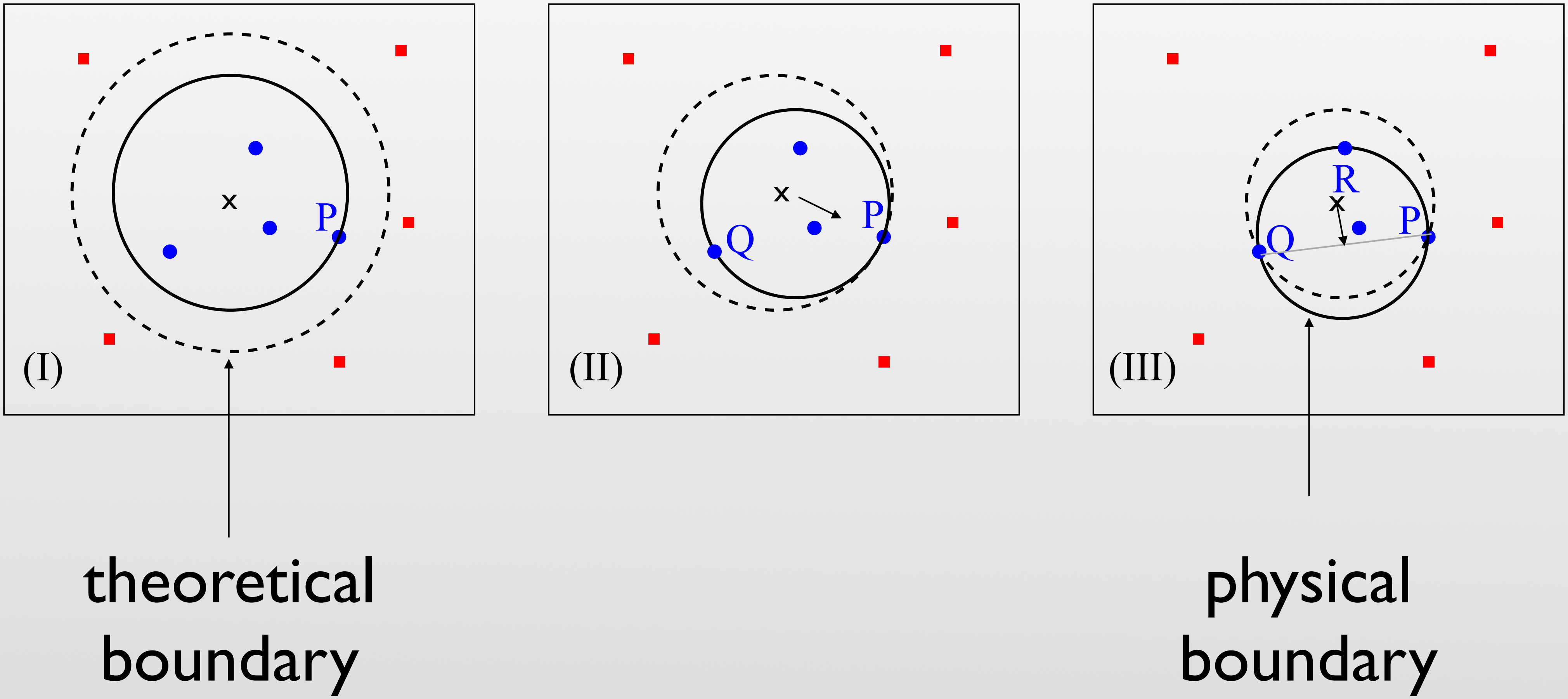
$$z_c \geq \frac{\kappa}{v_J\,\sqrt{1-(1-\kappa^2)\cos^2\theta_J}}$$

★ The beam direction always stays away from the jet and does not need any special treatment

Cone identification

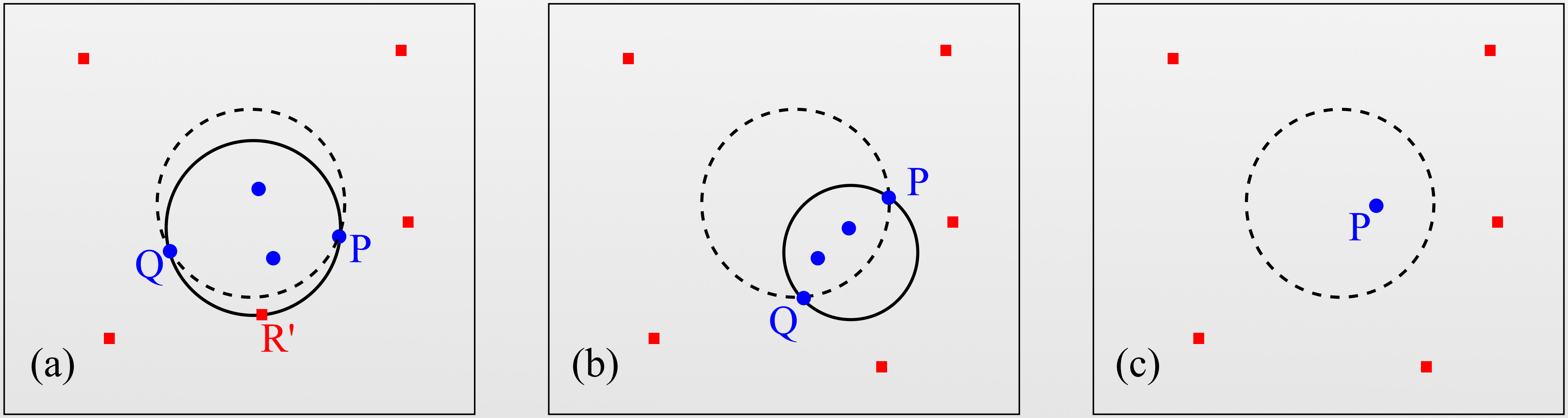


Cone identification



one can use three particles to identify a cone

Alternative boundaries



Numerical implementation

★ In general, we need to check all 2^N possible subsets of particles for a general function, which is not possible

★ Knowing the geometrical shape of jets, one only need to check all possible cones and choose the one maximizing the jet function — "global"

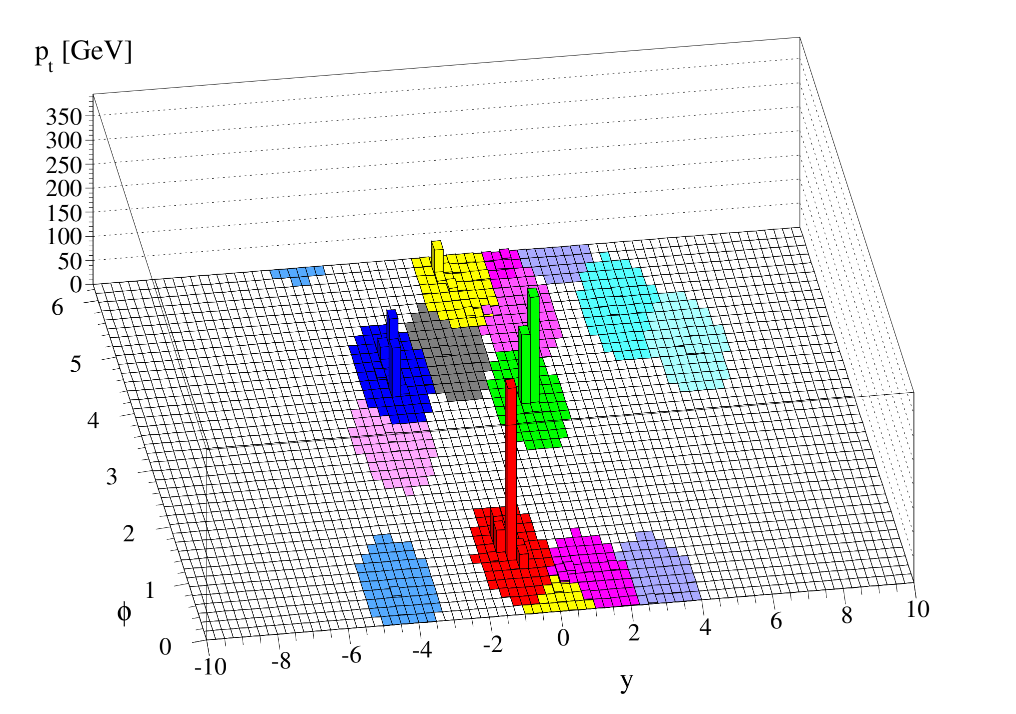
★ For each particle, one can also determine its fiducial region such that one only needs to check "n << N" nearby particles as a neighbor

★ For each particle, the physically distinct cones is $O(n^3)$, the total operation time is $O(N\, n^3)$

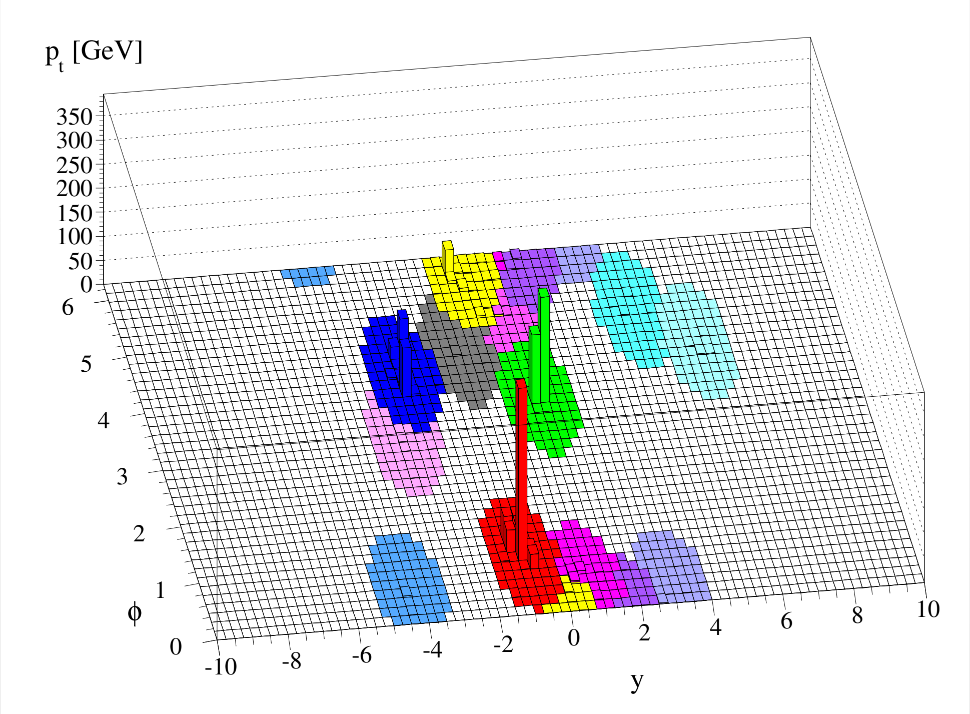
https://github.com/LHCJet/JET

Comparison: shape

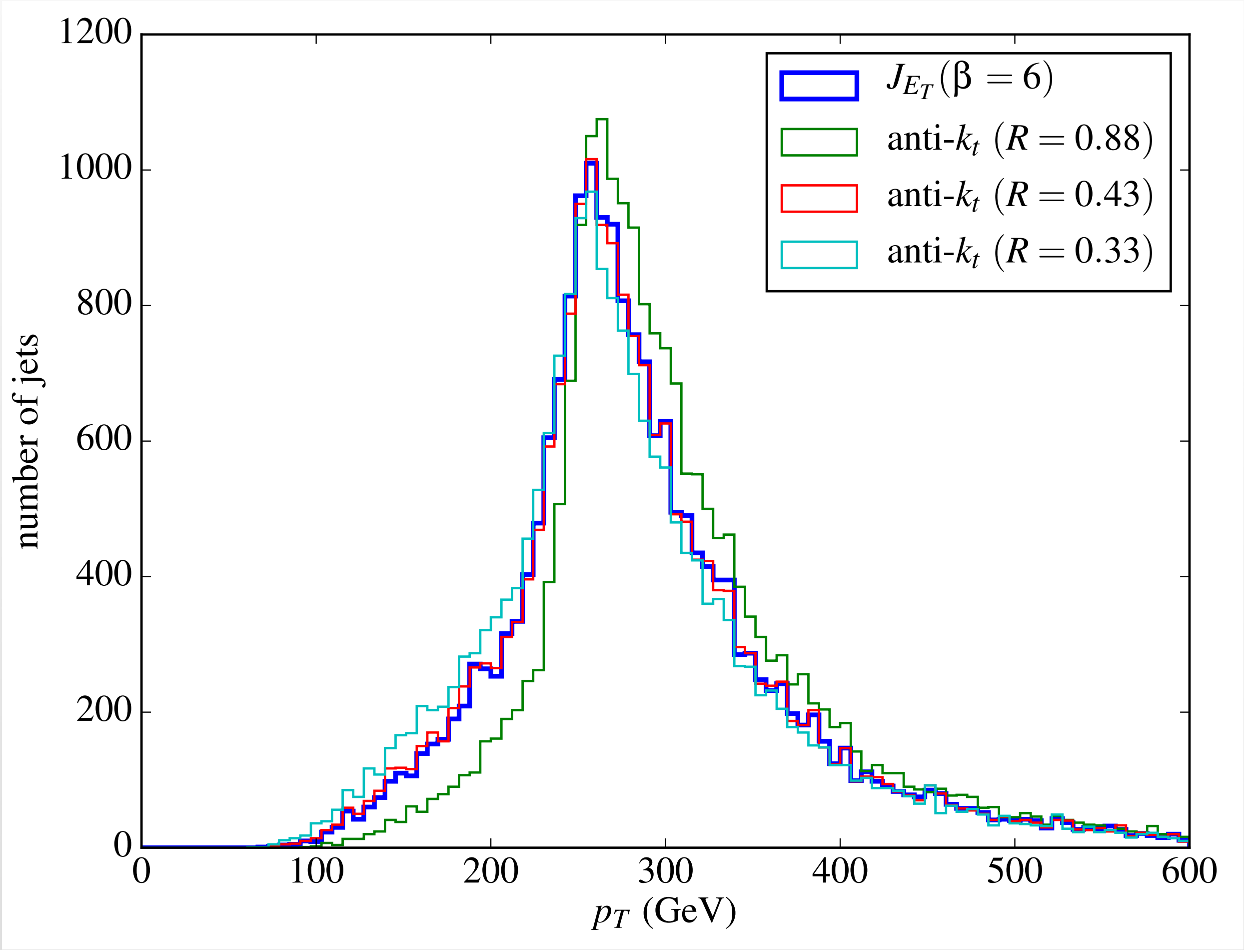
J_{E_T} with $\beta = 1.4$



anti-k_T with $R = 1.0$

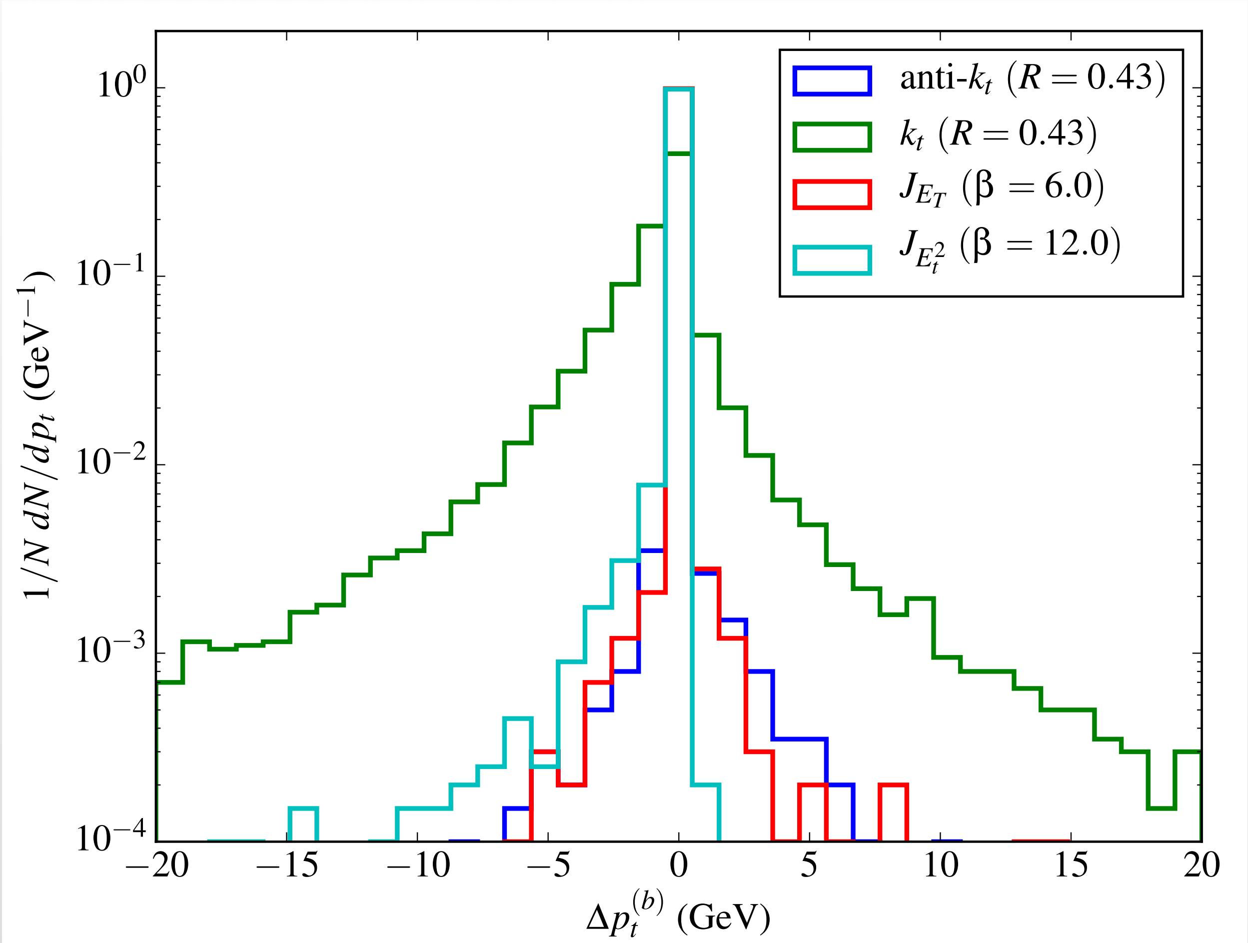


Comparison: size



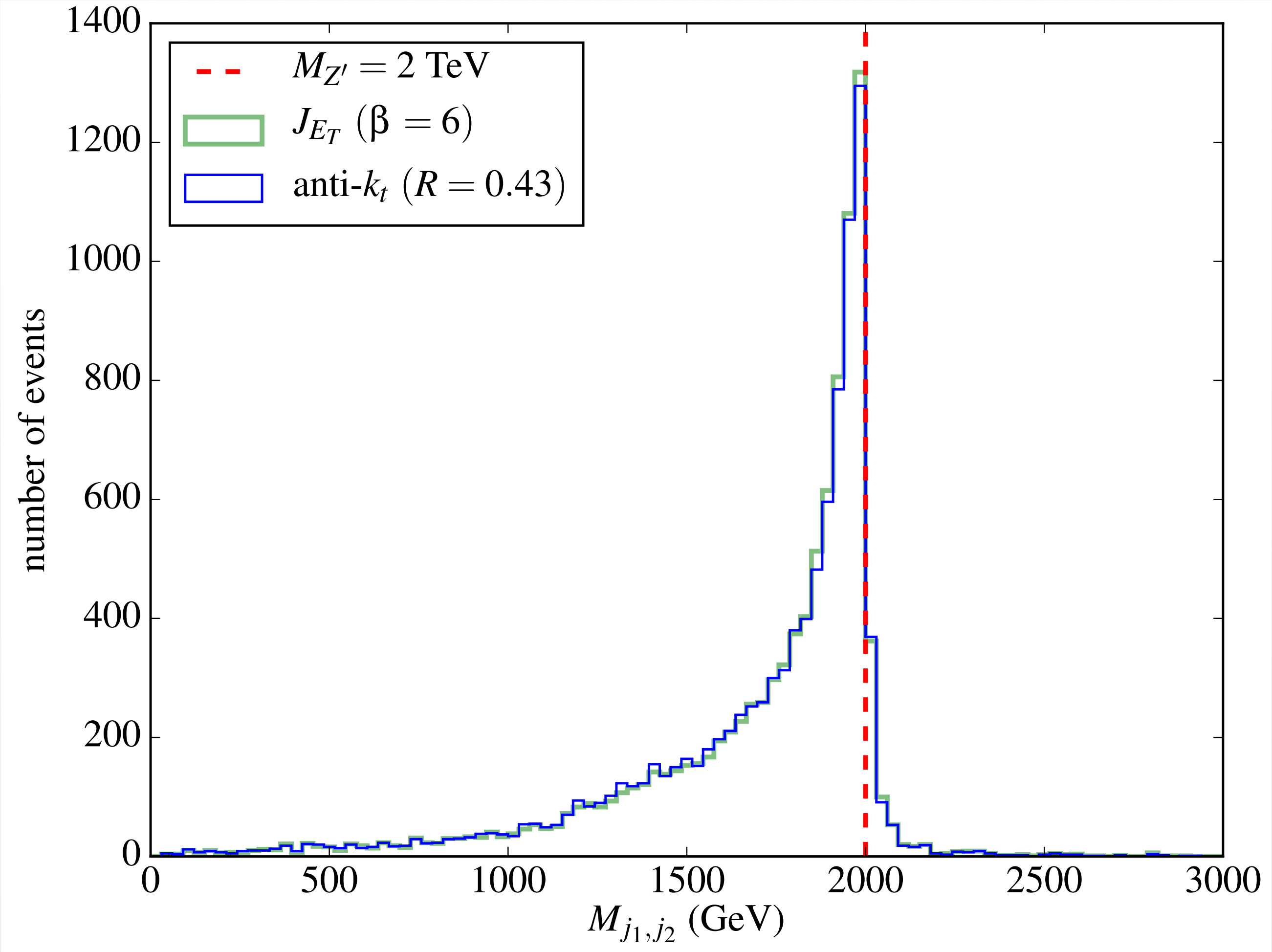
match anti-kt results very well for a QCD jet

Comparison: back-reaction



again, similar to the anti-kt results

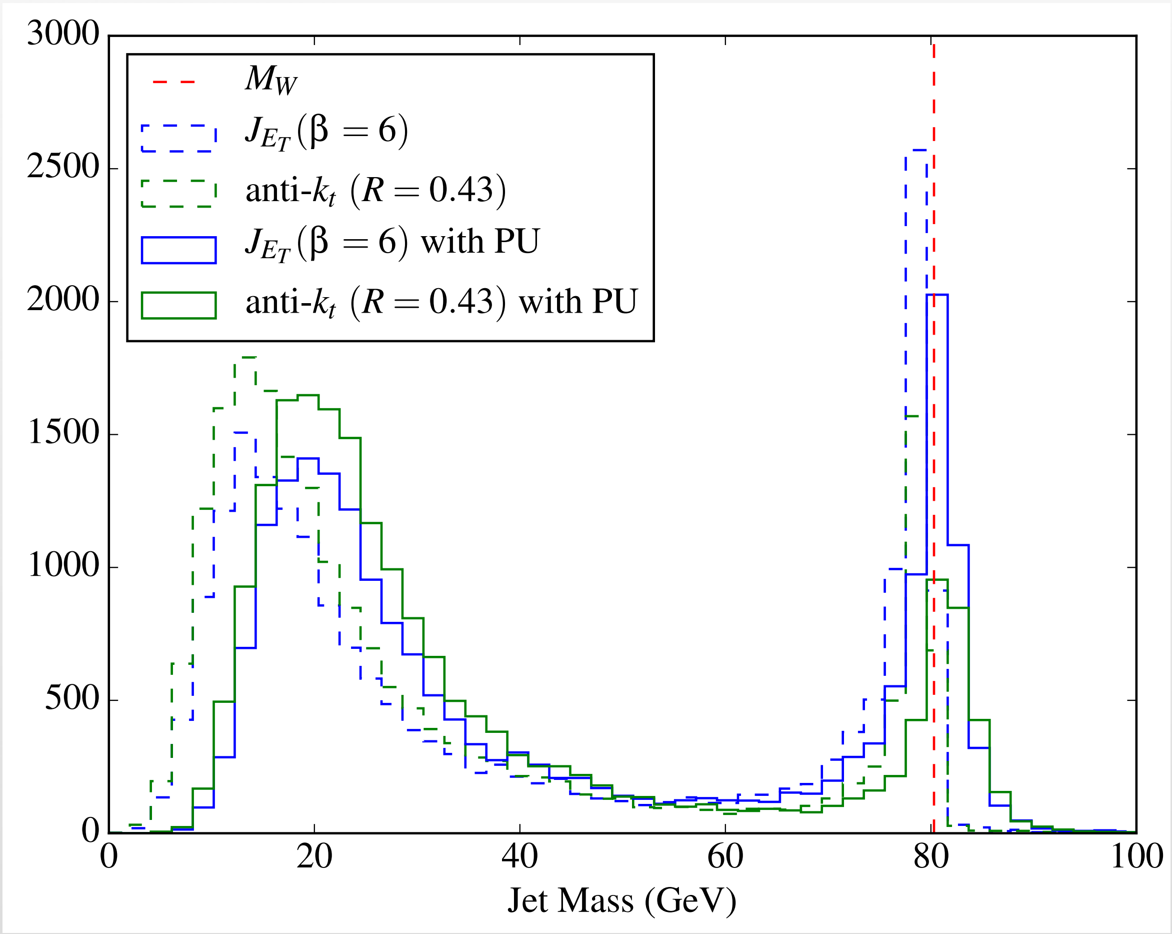
Comparison: dijet Z' mass



again, similar to the anti-kt results

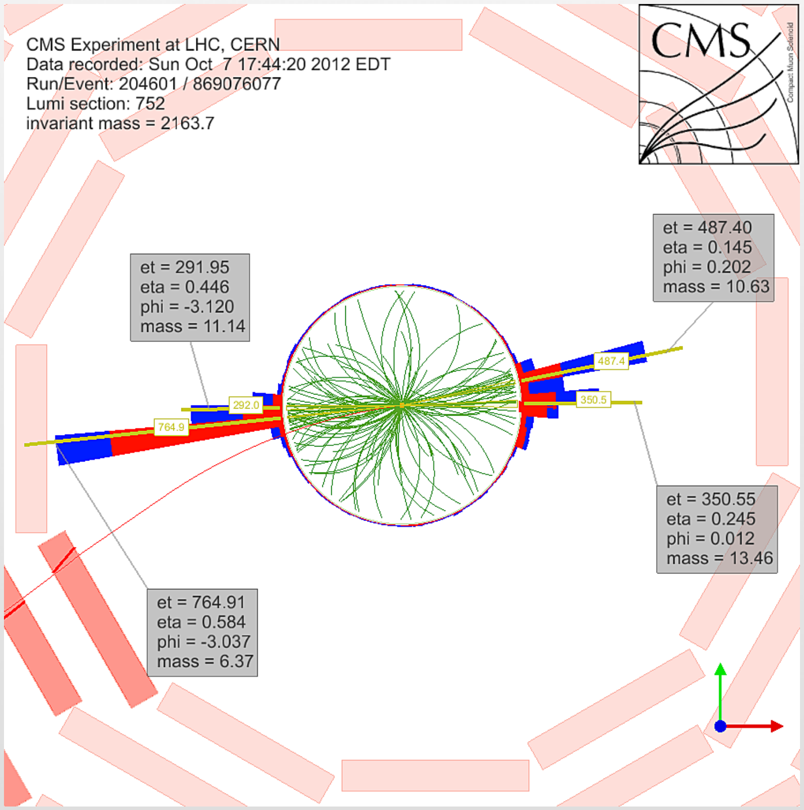
A naive comparison for W-jet

$p_T(W) > 250$ GeV



our jet-finding algorithm is designed for QCD jets so far

Design a W-jet-finding function



- A boosted W-jet contains a two-prong structure
- Need to incorporate a jet shape in the function
- The existing part of J_{E_T} may be kept

Design a W-jet-finding function

$$J_{E_T}^{W}(P_J^{\mu}) = E_T^{\alpha}\left[1-\beta\frac{m^2}{E_T^2}+\gamma\overline{H}_{2,J}\right]$$

- The new function need to prefer two-prong

Design a W-jet-finding function

$$J^{W}_{E_T}(P^{\mu}_J) = E^{\alpha}_T \left[1 - \beta \frac{m^2}{E^2_T} + \gamma \overline{H}_{2,J} \right]$$

- The new function need to prefer two-prong
- try the jet energy correlation functions:

$$\sum_{i \neq k} \frac{|\vec{p}_i||\vec{p}_k|}{E^2_J} |\sin\varphi_{ik}|^a (1 - |\cos\varphi_{ik}|)^{1-a}$$

Banfi, Salam, Zanderighi, hep-ph/0407286

$$\mathrm{ECF}(N, \beta) = \sum_{i_1 < i_2 < \ldots < i_N \in J} \left(\prod_{a=1}^{N} p_{T i_a} \right) \left(\prod_{b=1}^{N-1} \prod_{c=b+1}^{N} R_{i_b i_c} \right)^{\beta}$$

Larkoski, Salam, Thaler, 1305.0007

A working function

$$\overline{H}_{2,J} \equiv \frac{H_{2,J}}{E_T^2} = \frac{1}{E_T^2} \sum_{i,k} \frac{\left[m_J^2\, p_i \cdot p_k - (P_J \cdot p_i)(P_J \cdot p_k)\right]^2}{m_J^2 (P_J \cdot p_i)(P_J \cdot p_k)}$$

- It is Lorentz invariant except the overall factor
- It becomes transparent in the jet rest frame

$$\overline{H}_{2,J} = \left(\sum_{i,k} \frac{|\vec{p}_i||\vec{p}_k|}{E_T^2} \cos^2 \varphi_{ik} \right)_{\mathrm{rest}} = \left(\sum_{i,k} \frac{(\vec{p}_i \cdot \vec{p}_k)^2}{E_T^2\, |\vec{p}_i||\vec{p}_k|} \right)_{\mathrm{rest}}$$

- One can easily show that this function reaches its maximum for a two-prong structure

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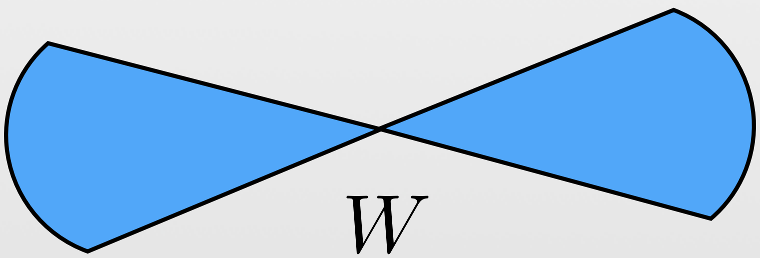
- One can easily show that this function reaches its maximum for a two-prong structure
- The function in rest frame is the Fox-Wolfram moment, introduced as an event shape at lepton colliders

Double-cone shape

$$J^W_{E_T}(P^\mu_J) = E^\alpha_T \left[1 - \beta \frac{m^2}{E_T^2} + \gamma \overline{H}_{2,J} \right]$$

in the rest frame



in the lab frame

- a double-cone structure with the subjet size determined dynamically

Double-cone shape

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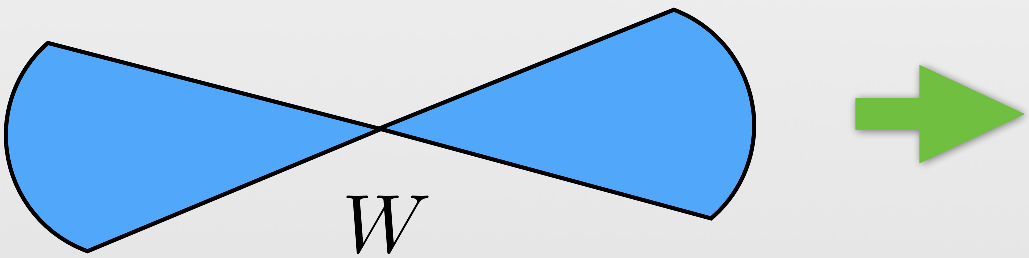
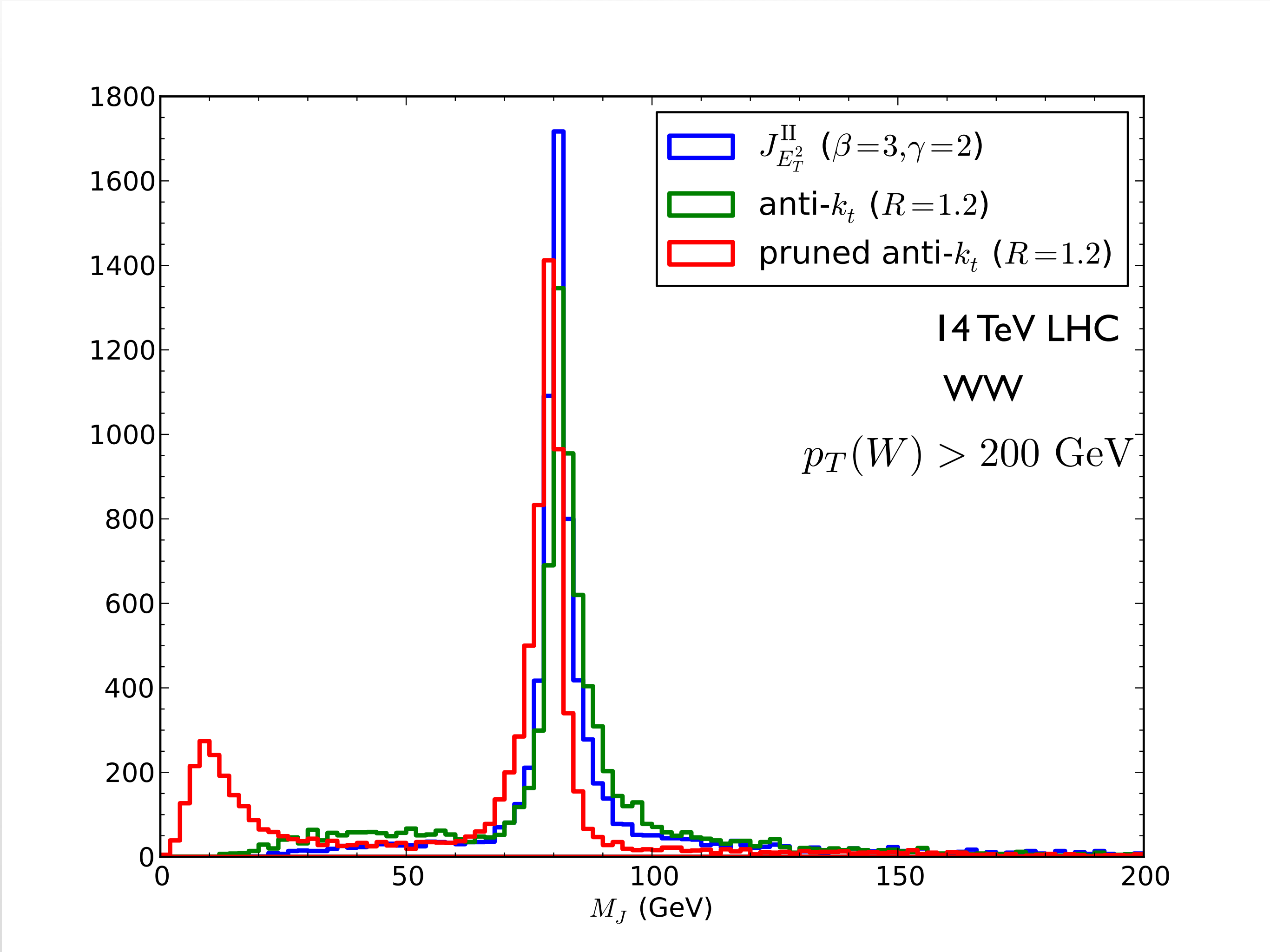


in the lab frame

- a double-cone structure with the subjet size determined dynamically
- $1/\sqrt{\beta}$ controls the subjet size and $1/(\beta - \gamma)$ controls the fat jet size

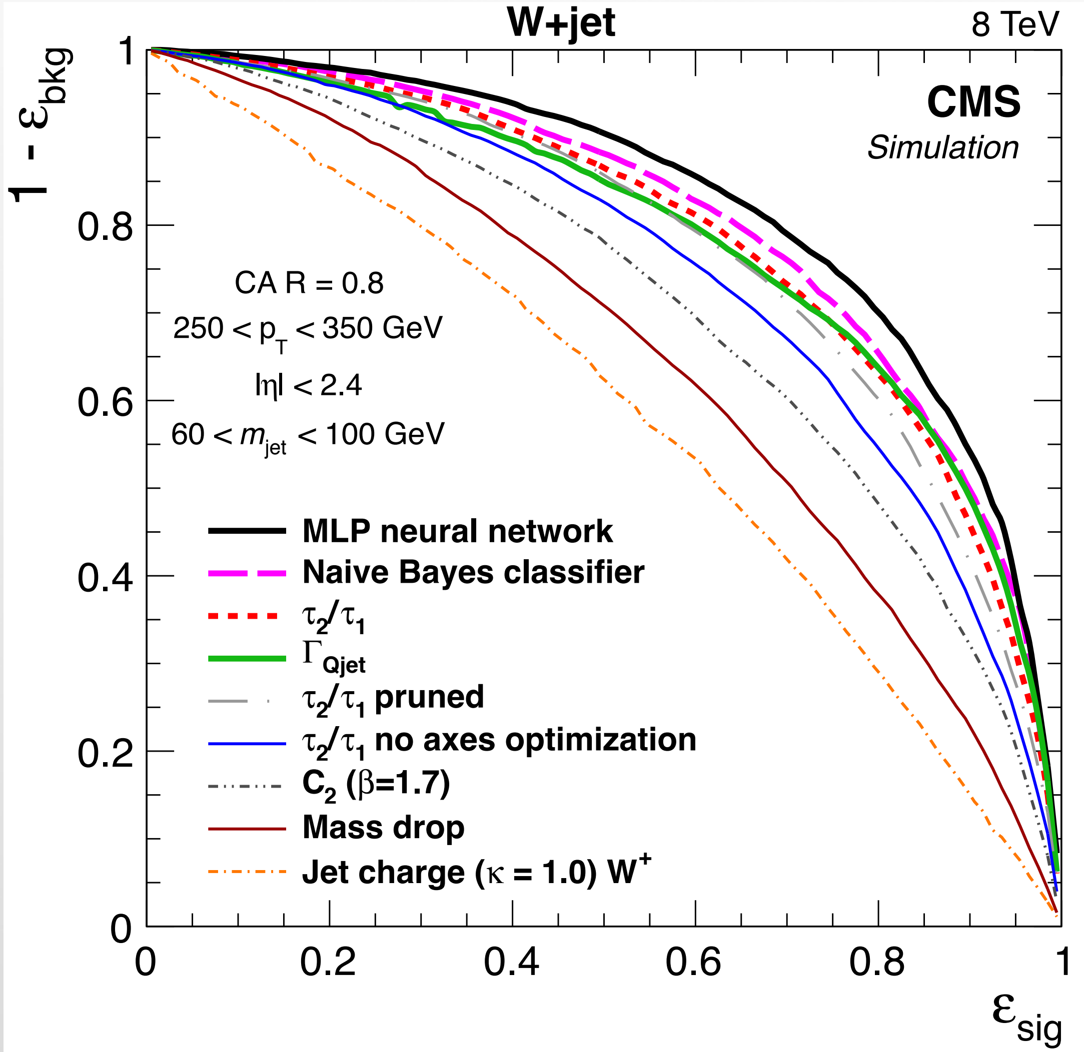
$J^{W}_{E_T^2}$ results



pruning jet: S. Ellis, Vermilion, Walsh; 0912.0033

no pile-up included yet

Variables used in CMS

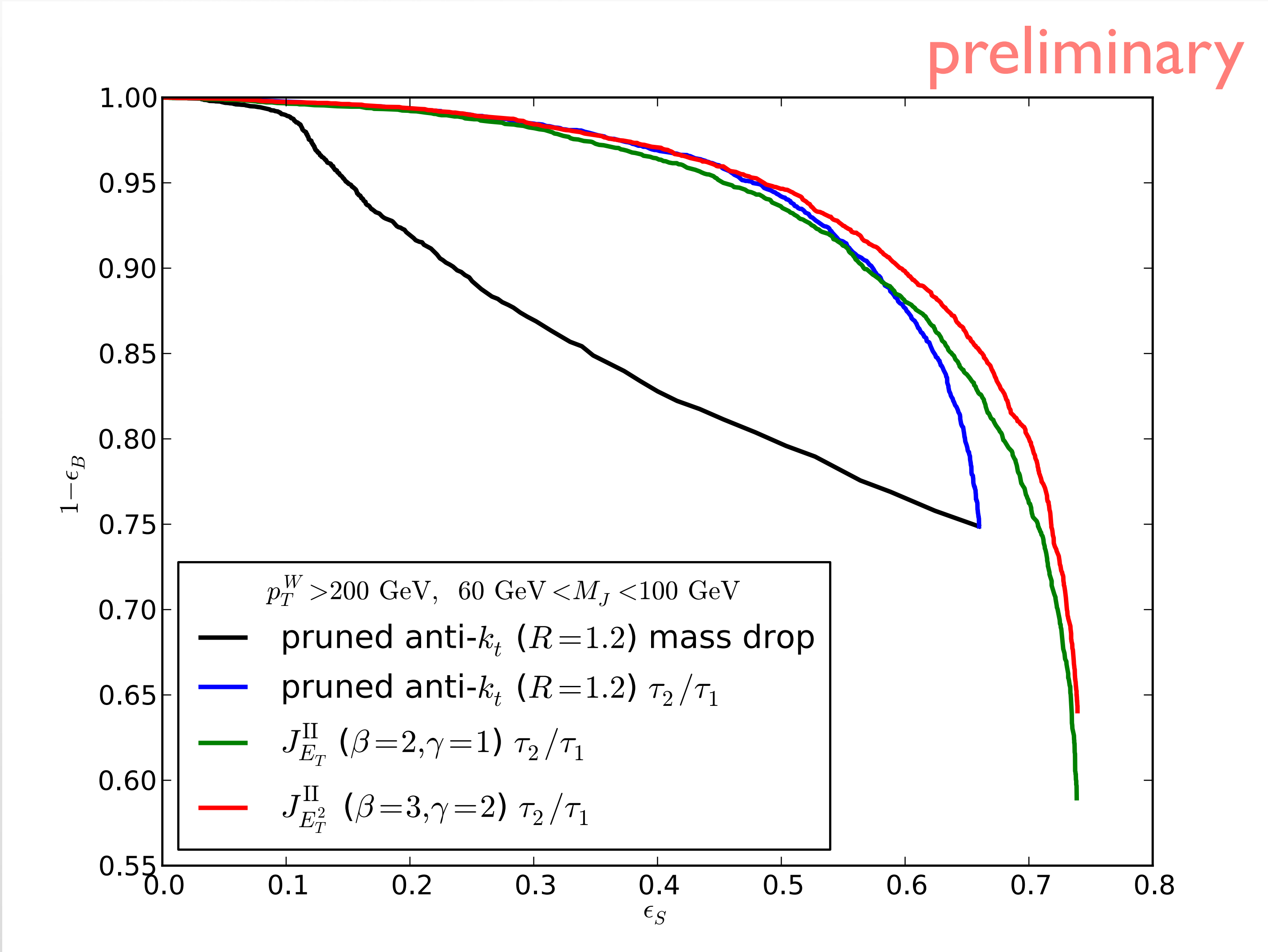


CMS; 1410.4227

N-subjettiness: Thaler and Tilburg; 1011.2268

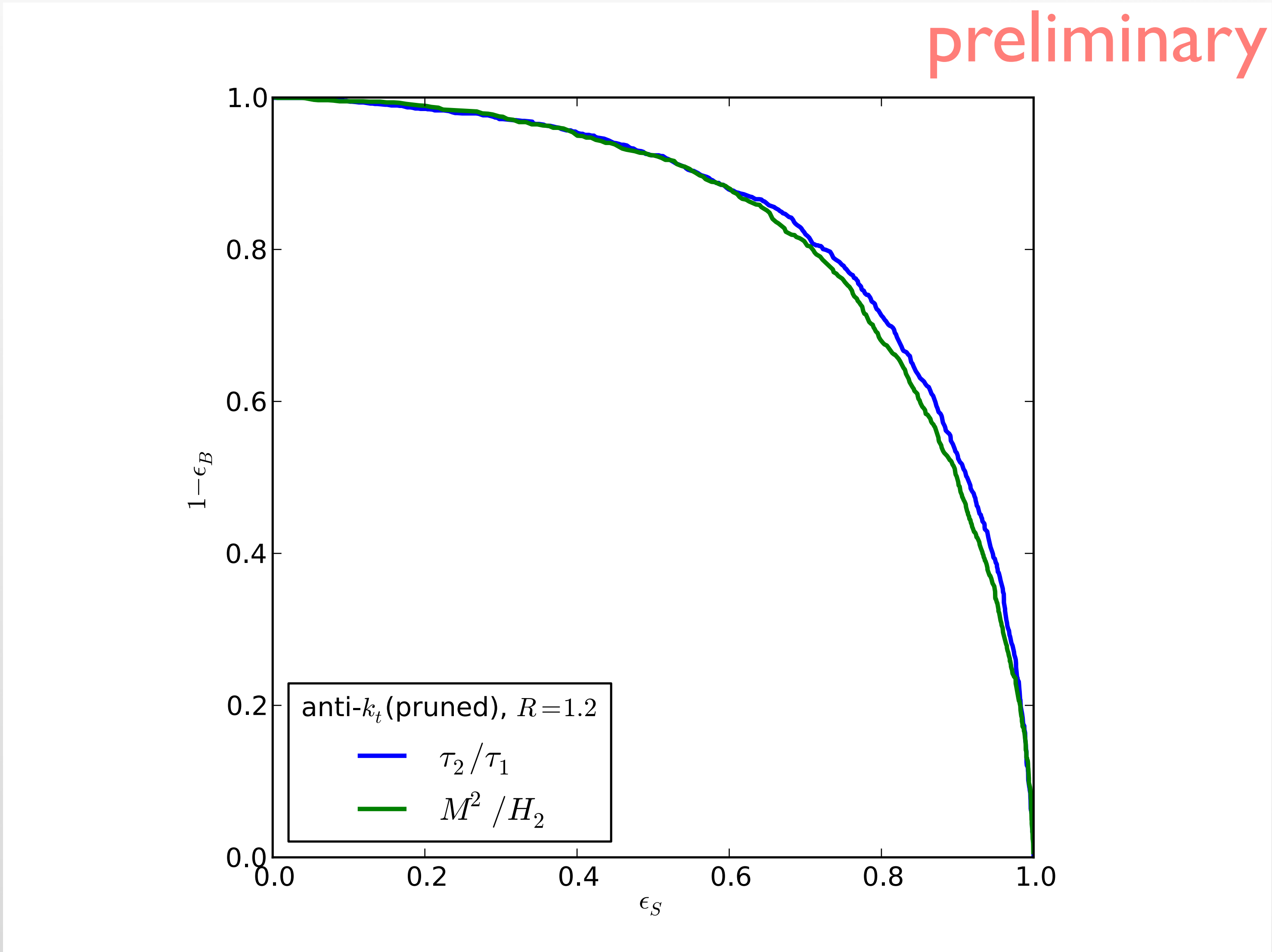
Q-jets: Ellis, Hornig, Roy, Krohn, Schwartz; 1201.1914

Performance w. Jet-sub. Variables



A better jet-finding algorithm makes some improvement

Byproduct: A New Event Shape Variable



Conclusions

- ★ A global jet-finding algorithm for maximizing a jet function works for a QCD jet
- ★ Our preliminary results show that our W-jet function can tag a W-jet very well
- ★ We are finalizing the numerical code with a trade-off between finding a global maximum and running speed
- ★ Other jet functions to tag top quark, black-hole multi-jets and new conformal gauge sector signatures are also interesting to explore

Thanks

Real proof for a cone jet

★ Check the angular distance of a soft particle from the jet momentum

$$z = \cos\theta = \frac{p_x P_x + p_y P_y + p_z P_z}{|p||P|}$$

★ For a soft particle j belongs to the jet:

$$J(P) > J(P - p_j)$$

$$1 - \beta(1 - v_\alpha^2) > 1 - r_j - \beta \frac{1 - v_\alpha^2 - 2 r_j (1 - z v_\alpha)}{1 - r_j}$$

$$z > \frac{\beta(1 + v_\alpha^2) - (1 - r_j)}{2\beta v_\alpha} > \frac{\beta(1 + v_\alpha^2) - 1}{2\beta v_\alpha} = \frac{1}{v_\alpha}\left(1 - \frac{1}{2\beta}\left(1 + \beta\frac{m^2}{E^2}\right)\right)$$

Real proof for a cone jet

★ For a soft particle j belongs to the jet:

$$z > \frac{\beta\left(1+v_\alpha^2\right)-\left(1-r_j\right)}{2\beta v_\alpha} > \frac{\beta\left(1+v_\alpha^2\right)-1}{2\beta v_\alpha} = \frac{1}{v_\alpha}\left(1-\frac{1}{2\beta}\left(1+\beta\frac{m^2}{E^2}\right)\right)$$

★ For a soft particle k not belongs to the jet:

$$z < \frac{\beta\left(1+v_\alpha^2\right)-\left(1+r_k\right)}{2\beta v_\alpha} < \frac{\beta\left(1+v_\alpha^2\right)-1}{2\beta v_\alpha} = \frac{1}{v_\alpha}\left(1-\frac{1}{2\beta}\left(1+\beta\frac{m^2}{E^2}\right)\right)$$

★ So, a cone-like boundary for individual jets

★ Soft particles are on the boundary; very IR safe